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Multinational Expansion in Time and Space

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Keywords: Multinational firms, foreign direct investment, firm dynamics, sunk costs

JEL Classification: F1

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1 Introduction

Many questions in international economics involve the complex activities of multinational enterprises (MNEs) over time and space. One recent example is the United Kingdom abandoning the European Union ("Brexit"). Undoubtedly, this shock had global consequences: It affected not only the behavior of MNEs operating in the United Kingdom, but also the behavior and expansion strategies of MNEs operating in countries nearby — or connected to – the United Kingdom. The Brexit shock may also have affected MNEs differently in the short and long run, as these firms make long-lasting investments in productive capacity. Answering these and other similar questions requires an understanding of the MNE expansion over time and across space, as well as of the nature of the costs these firms face.

Despite their importance for the global economy and in the policy arena, the behavior of MNEs and their affiliates over time *and* space has received relatively scarce attention in the literature.[1] On the empirical side, this is primarily due to data limitations. On the theoretical side, the nature of the costs of MNE activities—whether variable, fixed, or sunk, and whether host- or destination-country specific—poses challenges to tractability, particularly in multi-country dynamic settings where MNEs can separate the locations of production and sales. In this paper, we introduce a multi-country general equilibrium dynamic model, which is informed by a new set of facts on the behavior of foreign affiliates of US MNEs over time and space. The rich spatial and dynamic structure of the model is aimed at answering quantitatively questions about the effects of globalization shocks on MNE expansion.

Our empirical analysis uses a long panel of US MNEs and their foreign affiliates from the Bureau of Economic Analysis (BEA).[2] First, we document that virtually all US MNE affiliates are born with sales to the host market, which is the main destination of sales over their entire life; exports start later in life. Second, affiliate sales, relative to parent sales, are flat over the affiliate's life, except for a one-time jump in the year when the affiliate starts exporting. Third, the probability of MNE entry into a market is indistinguishable from that probability conditional on having entered similar markets (e.g. markets in the same continent or with a similar income level). Fourth, sales of existing affiliates to their host market do not change when sibling affiliates (i.e. affiliates of the same MNE) are created in a different market, when sibling affiliates start exporting to the affiliate's host market, or when sibling affiliates start exporting to the same affiliate's export market. Finally,

[1] See Antrás and Yeaple (2014) for a detailed survey on the main facts and theories about MNEs.

[2] Not only is the United States the main source of MNEs in the world, but MNE affiliates are also the main channel through which US firms reach foreign consumers. In 2009, for instance, majority-owned affiliates of US MNEs abroad accounted for 75 percent of US sales to foreign customers; 40 percent of those affiliates' sales were exports, i.e., sales to customers outside the affiliate's host market (Yeaple, 2013).

aggregate shocks and firm-level time-invariant characteristics account for more than eighty percent of the variation in affiliate sales.

Guided by these facts, we build a multi-country general equilibrium dynamic model of firm expansion. Firms decide whether, when, and where to open foreign affiliates, which, in turn, can sell both to their host market and to any other market, subject to sunk, fixed, and variable costs. The MNE decisions of whether to set up an affiliate in a market, and whether to export from it, are shaped by the interaction of firm-specific characteristics, persistent aggregate shocks, and the array of costs.

While the static components of the model are standard and follow Melitz (2003), the formulation of the dynamic problem of the MNE is new to the international trade literature. We build on insights from the literature on real options on how to solve models of investment under uncertainty (Dixit and Pindyck, 1994). Concretely, our model is based on a *compound option* structure: opening an affiliate in a country is an option, which, if exercised, gives access to a set of additional options, such as exporting from the affiliate to any other location. We couple this dynamic structure with a standard Armington-type assumption by which goods are firm- and location-specific.

Our modeling assumptions are key to obtaining tractability while preserving the rich heterogeneity necessary for quantitative analysis. First, the compound option structure allows us to introduce dynamic interdependence in the location choices of the MNE: the decision to open an affiliate in a country depends on the set of countries that the affiliate can export to. Hence, shocks to the cost of operating, or establishing, an export network from a given market affect the MNE entry decisions into that market. Second, due to the continuous-time formulation of the real-option problem, the value functions can be solved in closed form as simple additive functions of the present discounted value of the firm's profit flow, the option value of expansion, and the option value of exit. Third, we prove that, in this class of real-option problems, firms behave as-if they were myopic, a feature that dramatically reduces the dimensionality of the dynamic problem. Finally, the Armington-type assumption implies that the MNE entry decision into a market is separable across locations, avoiding a high-dimensional combinatorial problem. Together, these features make aggregation possible and allow us to solve for the model's general equilibrium.

We calibrate the model to static and dynamic moments related to the behavior of US MNE affiliates located in the top-ten host countries for US Foreign Direct Investment (FDI), over thirty years. Our calibration implies that opening and operating foreign affiliates is more costly than exporting from them, for most host countries. Affiliate exports to the United States (the Home country) are generally associated with lower barriers than exports to other destinations. Heterogeneity, however, is large across countries, sales type, and type of frictions.

Armed with the calibrated model, we perform various quantitative exercises that demonstrate the importance of including both dynamics and spatial heterogeneity for evaluating the reallocation of MNE activities after a globalization shock. First, including the possibility of affiliate exports—captured by the compound option structure—dramatically increases the MNE incentives to operate in a country, and delivers different long-run responses compared to what a standard dynamic model of only horizontal FDI predicts, after a change in the costs of entering (and operating) in a country. Second, our model predicts that an increase in export costs between the United Kingdom and EU countries —mimicking "Brexit" — would have a static effect due to the higher costs of accessing the export network, a dynamic effect on entry and exit due to the presence of sunk costs, and a general equilibrium effect. The strength of each effect on the reallocation of MNE activities depends on the nature of the export cost. For instance, an increase in per-period fixed costs would decrease the share of UK-based affiliates selling to the EU by twice as much as a similar increase in sunk export costs. These different responses could not be captured with a static model of the MNE, since one-time sunk and fixed per-period costs would be indistinguishable. Additionally, after a globalization shock, sunk costs create slower dynamics than a (static) model without sunk costs. Finally, thanks to the general equilibrium structure, the model creates substitution effects between exports and MNE affiliate sales, as predicted by the "proximity-concentration tradeoff." Put together, our quantitative exercises highlight the importance of considering both the time *and* space dimensions of MNE expansion decisions, without neglecting general equilibrium responses.

Our paper is related to the existing literature in several ways. First, most contributions in the literature have analyzed MNE behavior in space, but not in time. Papers such as Ramondo and Rodríguez-Clare (2013), Tintelnot (2017), Arkolakis et al. (2018), Alviarez (2019), Head and Mayer (2019), Fan (2023), Arkolakis et al. (2023), Oberfield et al. (2024), and Castro-Vincenzi (2024), have made substantial progress in building static general equilibrium models with a rich geography, in particular, by allowing firms to set up affiliates in locations —countries or regions— that differ from the destinations of their sales.[3] Making progress in dynamic setups, while keeping the spatial complexity (and tractability) of the static models, requires restricting the problem of the MNE. The sharp patterns that we document from observing US MNE affiliates over time guide us on how to simplify this problem: thanks to the compound-option structure, we are able to reduce the choice set of firms in a way that is consistent with the data. In this way, we are able to make substantial progress towards modeling the dynamics of MNE expansion, without sacrificing the spatial richness of static models.

[3] In static setups, the model of trade and FDI in Helpman et al. (2004) and its quantitative version in Irarrazabal et al. (2013) assume that the locations of production and sales of the MNE coincide (i.e. MNE activities are restricted to horizontal sales).

Second, there is a growing literature that analyzes different aspects of the dynamic behavior of the MNE. Papers in this literature, however, limit the spatial dimension of the problem. Gumpert et al. (2020) focus on the life-cycle dynamics of exporters and MNEs as alternative ways of serving a foreign market. Given the nature of their question, the analysis does not consider export platforms, and focuses on life-cycle, rather than aggregate, firm dynamics.[4] Fillat and Garetto (2015) build a dynamic two-country model of exporters and MNEs, where they introduce the idea that MNE activities can be treated as a real option that gets exercised once an affiliate is opened abroad.[5] Fillat et al. (2015) extend this idea to a multi-country setup. Both papers focus on the link between the MNE expansion decisions and asset prices, and both assume that the activities of affiliates are restricted to their market of operation. Our model treats MNE activities as a compound, rather than a simple, option. In this way, we preserve the tractability of the multi-country dynamic problem, and expand on the spatial dimension by separating the locations of MNEs' production and sales.[6] Finally, McGrattan and Waddle (2020) use a neoclassical growth model augmented by technology capital (McGrattan and Prescott, 2010) to analyze the effects of Brexit. While their paper analyzes similar quantitative exercises, the technology-capital growth model is quite different from ours, notably, because we include the extensive margin of both export and MNE decisions, a rich spatial export network, and uncertainty — all features dictated by the data, and relevant when analyzing counterfactual scenarios.

Third, our paper is naturally related to the large literature on export dynamics, which has been primarily concerned with quantifying the various costs of export activities and their welfare implications (see Alessandria et al., 2021, for a review).[7] The nature of the MNE problem, however, is more complex than the nature of the exporter problem: MNEs choose not only which markets to serve, as an exporter does, but also the location from where to serve each of those markets. Our compound-option structure allows us to implement quantitatively the complex spatial problem

[4] Our facts on US MNEs complement the facts in Gumpert et al. (2020), who compare the life-cycle dynamics of (non-MNE) exporters and MNEs, for France and Norway. While the BEA data are very detailed on the activity of affiliates abroad of US MNEs, they do not provide any information about (non-MNE) US exporters. In contrast, the French and Norwegian data used in Gumpert et al. (2020) contain detailed information on all firms operating in those economies, while they lack detailed information on the activity of MNEs outside their home country.

[5] Impullitti et al. (2013) use a real option model to study the entry and exit patterns of exporters.

[6] Other papers in the MNE literature limit both the spatial and dynamic dimension of the analysis by considering only horizontal FDI sales and only two periods (see, for instance, Ramondo et al., 2013; Egger et al., 2014; Conconi et al., 2016).

[7] Earlier contributions by Baldwin and Krugman (1989), Roberts and Tybout (1997), Das et al. (2007), and Alessandria and Choi (2007) find evidence of large sunk costs of exporting by focusing on observed patterns of export entry and exit. Subsequent analyses, such as Eaton et al. (2008) and Ruhl and Willis (2017), incorporate facts related to the life-cycle dynamics of new exporters and find that those costs are much lower. Alessandria et al. (2021) calculate the welfare gains from trade in a dynamic setting that matches well the life-cycle export facts. Arkolakis (2016) presents rich evidence on firm selection and export growth that supports dynamic theories of endogenous entry costs. Finally, Fitzgerald et al. (2023) show that the life-cycle growth patterns of export prices and quantities are quite different.

of the MNE in a dynamic setup. Admittedly, to be able to solve the multi-country model in general equilibrium, we must restrict the model to only include aggregate shocks, and exclude firm-level shocks, which are instead used in most of the literature on exporters dynamics. But overall, our analysis complements the literature on export dynamics by quantifying the frictions to MNE expansion and their implications in terms of aggregate firm dynamics.

Finally, our paper relates to the large literature that analyzes the dynamics of domestic firms, which goes back to Davis et al. (1996), and more recently Decker et al. (2014, 2016). While our framework can be applied more generally to dynamic problems of multi-establishment firms, our facts suggest that the dynamics of MNE affiliates are different from the dynamics of domestic firms. These differences may be indicative of US firms facing frictions of different magnitude in the domestic and foreign markets.

2 Evidence on US MNE Expansion

Our empirical evidence focuses on the expansion of affiliate activities over time, the expansion of the MNE over space, and the nature of the shocks that MNEs are exposed to. We start with a description of the data.

2.1 Data

Our empirical analysis uses firm-level data on the operations of US MNEs from the Bureau of Economic Analysis (BEA). The data include detailed information on the operations of MNEs in the United States and their affiliates abroad, for the period 1987-2011. We restrict the sample to majority-owned affiliates that do not operate in tax haven countries, have manufacturing as their primary activity, and belong to a US parent operating in any sector. Additionally, we consolidate affiliates belonging to the same parent and operating in the same country and 3-digit industry. Online Appendix O.1 provides more details on the data coverage and sample construction.

Crucially, the BEA data break down affiliate sales by destination: the host market of operation (horizontal sales) and other markets (exports). The data distinguish between affiliate exports to the United States and to other markets. Every five years, the data further distinguish affiliate exports to Canada, the United Kingdom, and Japan.

Table 1 shows the number of observations with horizontal and export sales in our sample. 96 percent of our affiliate-year observations have some horizontal sales, while about two-thirds of them have

Table 1: Summary Statistics.

	Horizontal sales	Export sales
No. of observations	132,493	132,493
with positive sales	127,220	88,060
	(96%)	(67%)
of pure type	44,433	5,273
	(34%)	(4.0%)
Sales accounted by pure type	16%	7.7%
Average share of total affiliate sales	72%	28%
Average affiliate sales over parent sales	5.6%	4.3%

Notes: Observations are at the affiliate-year level, for new majority-owned affiliates in manufacturing. A pure-type affiliate is an affiliate for which at least 99 percent of sales are either only horizontal or only export sales.

some exports. More than one-third of the observations correspond to affiliates with horizontal sales only, while the share of affiliates with only exports is four percent.

On average, about 72 percent of affiliate sales are directed to the country where the affiliate is located, while the remaining 28 percent are exports. Furthermore, on average, both horizontal and export affiliate sales are small relative to parent sales.

2.2 Affiliate activities over time

In this section we present evidence on the composition of affiliate sales over time. To establish the evidence from time series data, we need to observe affiliates for a number of years from birth. Thus, in this section only, we focus on affiliates that open during our sample period and that survive for at least ten consecutive years. This restriction implies that we exclude affiliates that open in 2003 or later, as well as observations belonging to the affiliate's eleventh year of life, or greater. Online Appendix O.1 provides a detailed description of this sample.

I. MNE affiliates' activities start in their host market and expand into export markets.

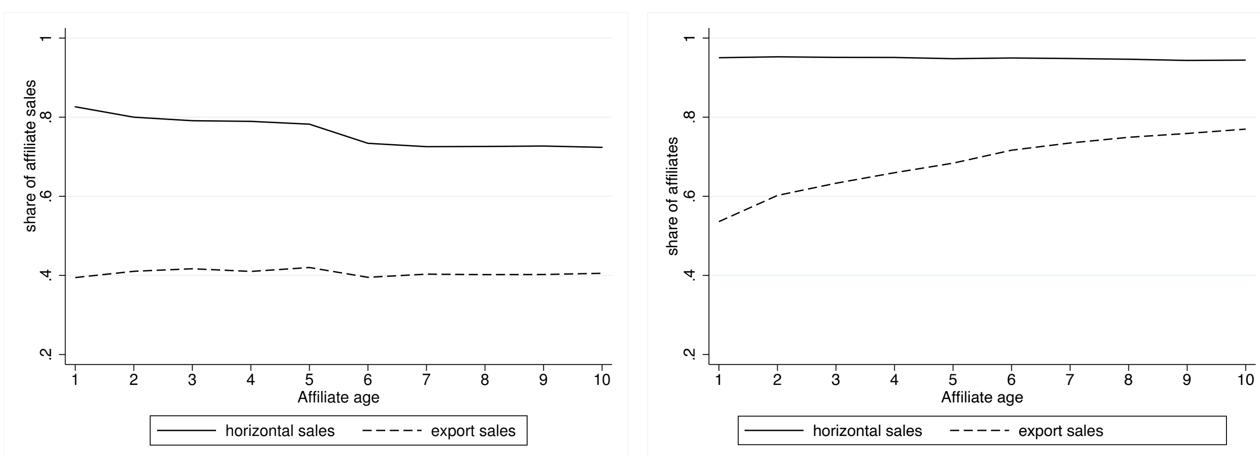
Figure 1 shows the evolution of the intensive and extensive margins of horizontal and export sales of US MNE affiliates. The intensive margin refers to the average share of horizontal (export) sales in total affiliate sales for firms with positive horizontal (export) sales. The extensive margin refers to the share of affiliates with positive horizontal (export) sales. Figure 1a shows that, on average, horizontal sales account for about 80 percent of affiliate sales at birth and decrease by ten percentage points over the first ten years of life of the affiliate, while the export share is flat at 40 percent.[8] Figure 1b shows that while the share of affiliates with horizontal sales is stable at more than 95 percent, the share of exporting affiliates increases from 50 to 70 percent. The patterns in Figure 1 are confirmed by OLS regressions that include a battery of fixed effects (see Online Appendix Table O.4).

Together, these findings suggest that, for horizontal activities, changes in sales shares are coming from the intensive margin, while export shares increase only because of affiliates that start exporting. Over time, many affiliates incorporate export sales into their activities, but they never stop selling to their host market.

These findings motivate the following assumption of our model: all affiliates sell their output in the host market, starting at birth and throughout their life, while they may start exporting at birth or later in life.

Figure 1: Affiliate sales and number of affiliates: horizontal vs export sales.

(a) Affiliate sales shares | (b) Share of affiliates



Notes: Sample of new majority-owned affiliates that survive for at least ten consecutive years, in manufacturing. Horizontal and export sales refer, respectively, to sales to the market where the affiliate is located, and to sales to markets outside the local market. (1a): average sales, as a share of total affiliate sales, including only affiliates with positive horizontal and export sales, respectively. (1b): share of affiliates with positive horizontal and export sales, respectively.

II. Affiliate sales, as a share of parent's, grow only at the time of export entry.

In Figure 2a, we show that affiliate horizontal sales, relative to parent sales, are close to their long-run value at birth and remain stable over the affiliate life span, growing only ten percent in

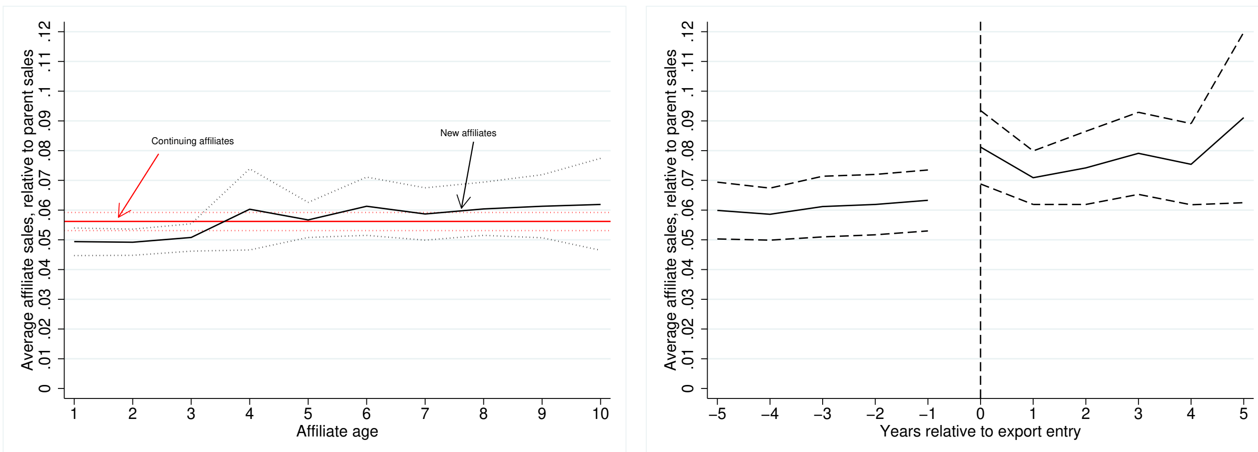
[8] Note that these shares add up to more than 100 percent. This is because we exclude zeros separately for each type of sales, and the number of affiliates with positive horizontal sales is larger than the ones with positive export sales. Including zeros, the share of export sales increases from 21 percent at age one to almost 32 percent at age 10.

their first four years. At the same time, Figure 2b shows that there is a jump in affiliate-to-parent sales in the year when the affiliate starts exporting.

Figure 2: Affiliate sales relative to parent sales.

(a) Horizontal sales, new affiliates

(b) Total sales, new exporters



Notes: Figure 2a reports the average value of affiliate horizontal sales relative to the domestic sales of the US parent, for new affiliates surviving at least ten years (black) and for all affiliates of all ages (red). Figure 2b reports the average value of affiliate total sales, relative to the domestic sales of the US parent, for a subsample of affiliates that are born with only horizontal sales and start exporting at a later age. Dashed lines denote 95% confidence intervals.

While Figure 2 shows the raw data, these patterns are robust to including country-year and affiliate fixed effects in an OLS regression (see Online Appendix Table O.5).[9]

Taken together, the results in Figure 2 show that affiliates are born with sales, relative to the parent's, which are as large as incumbent affiliates sales, and that grow significantly only when the affiliate expands to export markets. This fact suggests the presence of entry costs, both in the host and export markets, and the importance of the extensive margin for MNE expansion: (relative) growth occurs discretely when entering new markets, including the host market.[10] Our modeling strategy will focus on these margins, while our quantitative exercises will evaluate the magnitude of their responses to trade shocks.

[9]Results are robust to using different subsamples of affiliates (see Online Appendix Table O.6): first, initial affiliates do not grow faster than subsequent affiliates of the same parent, suggesting that the age of the firm at the time an affiliate opens is not driving the results; second, affiliates with intra-firm exports do not grow differently than affiliates without intra-firm exports, suggesting that being part of a global value chain or not does not drive our baseline result; and finally, affiliates established through greenfield FDI or mergers and acquisitions (M&A) both have flat sales ratios, suggesting that the lack of growth is not driven by pre-existing affiliates that became part of a new firm through M&A.

[10]Online Appendix Table O.14 presents additional evidence supporting the importance of sunk costs. Analogously to the literature on trade dynamics (see Bernard and Jensen 1999, Roberts and Tybout 1997, and Alessandria and Choi 2007, among others), we document that MNEs' horizontal and export activities are persistent, and that US parents are larger at the time of affiliate entry than at the time of affiliate exit.

2.3 Geography, affiliate entry, and affiliate sales

In this section, we document the expansion of US MNEs over space. Using the full sample of affiliates of US MNEs (not only affiliates surviving for at least ten years), we show that the location of a new affiliate barely depends on the location of preexisting affiliates of the same MNE, and that the sales of affiliates in a host country do not change with local and export activities from affiliates of the same MNE located in other countries.

III. Affiliate entry follows a weak "extended gravity" pattern.

Figure 3 considers the sample of affiliates located in the ten most popular host countries for US MNEs and belonging to US parents with at least two foreign affiliates. Figure 3a shows that, for a given US parent, there is an extremely small difference between the unconditional probability of opening an affiliate in a country and the probability of opening an affiliate conditional on already having an affiliate in a country located in the same continent, or in a country with similar income per capita. This is particularly true for richer host economies. Differences are slightly larger, but still small in magnitude, for less developed host economies, which are typically more engaged in global value chains (GVCs).[11]

To lend further credibility to our results, we implement an instrumental variable approach similar to the one used by Kovak et al. (2021). In the first stage, we predict the existence of a "sibling" affiliate in other countries using the existence of a bilateral tax treaty (BTT) between the United States and the host country of the affiliate at time t.[12] We define a sibling affiliate as an affiliate of an MNE operating in the same three-digit industry as another affiliate of the same firm. Figure 3b shows that results are virtually unchanged relative to OLS.[13]

Our findings are in stark contrast with the analogous findings for exporter entry in Morales et al. (2019). For instance, they find that the unconditional probability of exporting to a given country is 0.7 percent and increases to 2.8 percent if the firm is already exporting to a country in the same continent. We find that the unconditional probability of opening an affiliate in the United

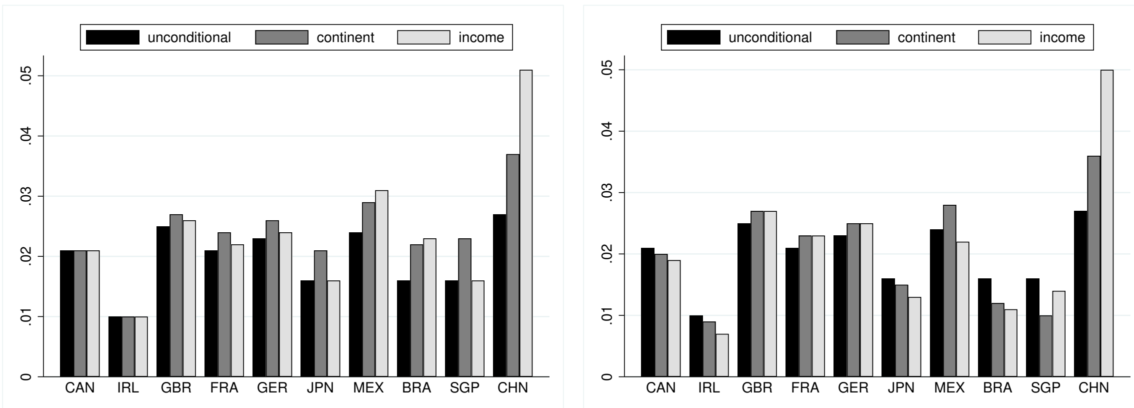
[11] Online Appendix Table O.7 reports the estimated coefficients, also including as "similarity" variables sharing a border and sharing a language; these conditional probabilities are also extremely similar — and in most cases statistically equal — to unconditional probabilities. Furthermore, Online Appendix Table O.9 shows that the weak pattern of extended gravity is more pronounced among non-GVC affiliates (i.e., affiliates with zero intra-firm exports) than among GVC affiliates (i.e., affiliates with positive intra-firm exports). However, differences between conditional and unconditional entry probabilities are still small and often insignificant for both GVC and non-GVC affiliates. Finally, Online Appendix Table O.10 shows that differences between unconditional and conditional probabilities become smaller when we restrict the sample to MNEs with more than five, and more than ten, affiliates.

[12] Kovak et al. (2021) show that BTTs are exogenous to MNE activities and can predict employment at foreign affiliates of US MNEs. Online Appendix O.3 presents a description of the first-stage results.

[13] Online Appendix Table O.8 includes other "similarity" variables using the IV strategy.

Figure 3: Unconditional and conditional probability of affiliate entry.

(a) OLS estimation (b) 2SLS estimation



Notes: Probabilities of affiliates' entry into the top-ten most popular destinations of US MNEs. Conditional probabilities refer to the probability of observing an MNE opening an affiliate in a country given that the parent already has an affiliate in another country in the same continent or in a country with similar income per capita. Similarity in terms of income per capita follows the group classification from the World Bank. The sample is restricted to parents with at least two affiliates worldwide.

Kingdom, for example, is 2.5 percent and increases to only 2.7 percent if the MNE already has an affiliate in the same continent. In general, while differences between conditional and unconditional probabilities for exporter entry range between 2 and 4 *times*, differences for MNE entry range between 2 and 20 *percent*.[14]

The pattern of affiliate entry documented in this section motivates a key assumption in our model: affiliate entry decisions in each market are independent across host markets.

IV. Horizontal sales of existing affiliates do not change with sibling activities.

Our last fact shows that horizontal and export sales from sibling affiliates may coexist in a destination market, and that entry of new sibling affiliates into a market does not affect the sales of existing affiliates operating in a different market. For this analysis, we restrict affiliate horizontal sales to *unaffiliated* parties to make sure that our findings are not contaminated by GVCs within the MNE.[15]

First, an affiliate may shrink, or expand, in its country of operation when a sibling affiliate opens in a different host country. Figure 4a shows the average horizontal sales across affiliates located in

[14] This finding does not contradict the fact that US MNEs open affiliates in closer and larger markets first, as documented in Egger et al. (2014) for German MNEs, and as we show in Online Appendix Table O.13 for US MNE affiliates.

[15] Online Appendix Figure O.1 shows results for total horizontal sales, both to unaffiliated and affiliated parties. Results barely change. As documented by Ramondo et al. (2016), sales to affiliated parties anywhere are only six percent of affiliate total sales, for benchmark year 2004.

host market j, five years before and after a new affiliate of the same parent starts operating in the same industry but in a different host country. Our estimates show that horizontal sales of existing affiliates do not change when their parent firm opens a sibling affiliate in the same industry but in a different country.

Second, an MNE may serve a destination from sibling affiliates operating in the same 3-digit industry and located in different host countries. The BEA collects affiliate export sales data by destination for various geographic areas covering all countries in the benchmark surveys conducted during our sample period. However, because Canada, United Kingdom, and Japan are the only individual countries among the geographic areas for which the data are collected, for this exercise we restrict our sample to affiliates that are located in, or export to, those three countries. Figure 4b reports the average horizontal sales across affiliates located in country j, five years before and after an affiliate of the same parent and in the same industry, but located in a different country, starts exporting to country j.[16] As the figure shows, the horizontal sales of an affiliate do not significantly change when a sibling affiliate in the same industry starts exporting to that market from a different location.

Lastly, Figure 4c reports the average affiliate export sales to country j before and after an affiliate of the same parent operating in the same industry but located in a different country starts exporting to j. These data are available for benchmark survey years so that we show exports 5, 10, 15, and 20 years before and after a sibling starts exporting to j. Also in this case, export sales of the affiliates do not significantly change when a sibling affiliate in the same industry but located in a different country starts exporting into the preexisting affiliate's export market.

The lower panels of Figure 4 shows results using the same instrumental variable approach as in Figure 3b. Once again, results are virtually unchanged relative to OLS.

The patterns of MNE entry and sales in a market documented in this section motivate a key assumption of our model: both entry and sales decisions in each market (host and export), are independent across markets.

2.4 MNE shocks

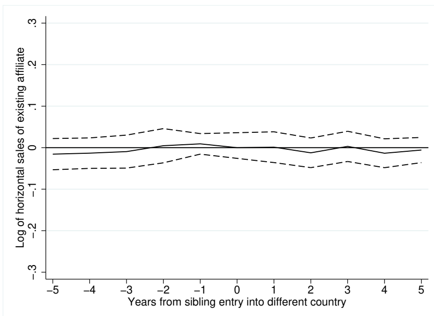
An important choice in our modeling strategy is the inclusion of different types of shocks that the MNE is subject to. In order to help assess the relative importance of aggregate versus firm-level

[16] Of the 20,359 affiliates that exported to Canada in 1999, 64 percent have a sibling affiliate located in Canada; 70 percent of the 5,017 affiliates that exported to the United Kingdom have siblings located there; for Japan, this share was 47 percent, out of the 5,224 affiliates that exported to this market in 1999.

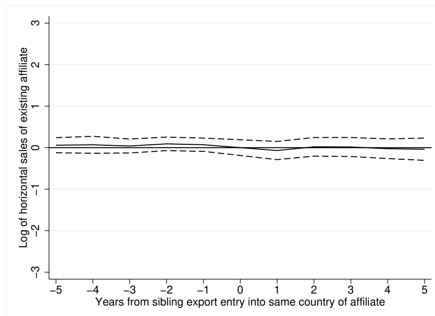
Figure 4: Horizontal and export sales of existing affiliates and siblings activities.

OLS estimation

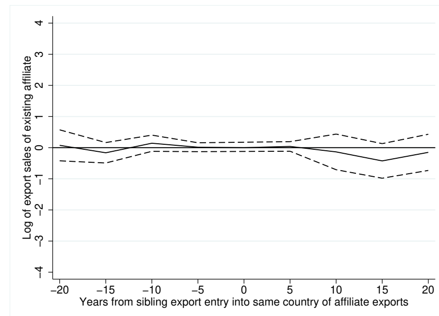
(a)



(b)

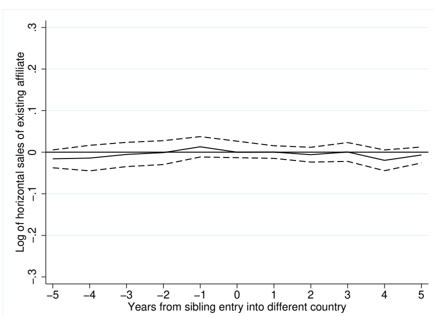


(c)

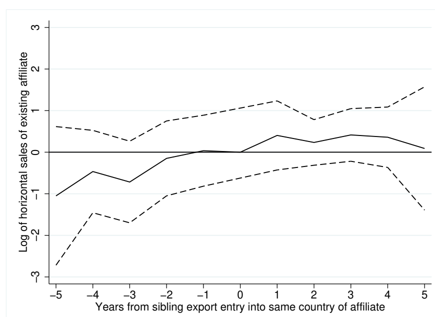


2SLS estimation

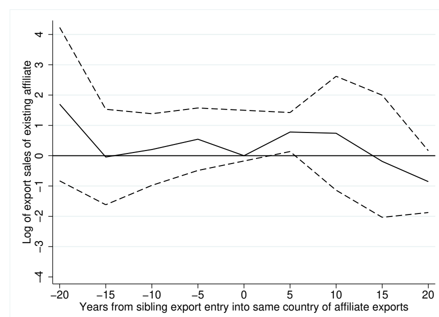
(d)



(e)



(f)



Notes: Horizontal sales refer to sales to unaffiliated parties in the host market of the affiliate. Export sales refer to exports to unaffiliated parties. Figure 4a (4d) includes all affiliates of US MNEs in manufacturing. Figure 4b (4e) includes affiliates located in Canada, United Kingdom, and Japan. Figure 4c (4f) includes affiliates exporting to Canada, United Kingdom, and Japan. Figures 4a (4d) and 4b (4e) show coefficients (solid line) from regressing the log of horizontal sales for an affiliate in country j and 3-digit industry h belonging to parent p at time $t \in \{-5, ...5\}$ on a set of dummies indicating: in Figure 4a (4d), time from opening an affiliate in country $j' \neq j$ belonging to the same parent p and industry h; and in Figure 4b (4e), time from export entry into j of an affiliate located in country $j' \neq j$ belonging to the same parent p and industry h, with $j = Canada, UK, Japan$. Figure 4c (4f) shows coefficients (solid line) from regressing the log of export sales to country j by affiliates located in country $k \neq j$ and 3-digit industry h belonging to parent p at time $t \in \{-20, -15, ...15, 20\}$ on a set of dummies indicating time from export entry into j of an affiliate located in country $k' \neq j$ and $k' \neq k$ belonging to the same parent p and industry h as the affiliate located in k and exporting to j, with $j = Canada, UK, Japan$. We include the log of the MNE global sales, the log of the US parent sales, affiliate fixed effects and year fixed effects. Standard errors are clustered at the affiliate level. Dashed lines denote 95% confidence intervals.

uncertainty, and time-varying versus time-invariant shocks, we examine the types of shocks that drive variation in affiliate sales. Concretely, in Table 2, we show OLS estimates from regressing the log of horizontal sales of an affiliate belonging to parent p, located in country j and industry h, at time t, on different combinations of fixed effects. We analyze the increase in the R-squared as we introduce these fixed effects. Aggregate uncertainty, captured by host country-year and industry-year fixed effects, accounts for almost 20 percent of the variation in horizontal affiliate sales, while adding a parent fixed effect brings the R-squared to 0.44. Adding parent sales to capture time-varying parent-level shocks barely changes the R-squared. In contrast, adding affiliate fixed effects, which capture invariant characteristics of the affiliate, doubles the R-squared, explaining almost half of the variation in affiliate sales.

Guided by this evidence, our model includes aggregate time-varying uncertainty and a firm-level characteristic, captured by a firm-specific productivity term. We leave out time-varying firm-level shocks, which according to the R-squared of our estimates account for around 12-15 percent of the variation in affiliate sales. We discuss in Section 3.2 how these assumptions on the sources of MNE uncertainty are key for the model's tractability.

Table 2: MNE shock structure. OLS.

Dependent variable	log of horizontal affiliate sales			
Host-year fixed effect	yes	yes	yes	yes
Industry-year fixed effect	yes	yes	yes	yes
Parent fixed effect	no	yes	yes	no
Parent sales	no	no	yes	yes
Affiliate fixed effect	no	no	no	yes
R-squared	**0.19**	**0.44**	**0.44**	**0.88**
Adjusted R-squared	0.17	0.41	0.41	0.85

Notes: Full sample. Number of observations: 132,493.

3 A Dynamic Model of MNE Expansion

We build a dynamic model where firms open affiliates, locally and abroad, over time. Affiliates sell in their host markets of operations, and they choose whether to export to other markets from there. We impose assumptions that are guided by the facts documented in Section 2, and introduce a compound option formulation, which allows us to characterize the richness of the decisions of MNEs in time and space. This formulation is novel to the international trade literature, and is key for the model's tractability.

3.1 Preferences and technology

The economy consists of N countries. In each country k, a fixed mass of L_k consumers have preferences over a differentiated good,

$$U_k = \int_0^\infty e^{-\rho t} C_k(t) dt, \tag{1}$$

where ρ denotes the subjective discount rate, and $C_k(t)$ is defined as

$$C_k(t) \equiv Y_k(t) Q_k(t). \tag{2}$$

Here, $Y_k(t)$ is an aggregate demand shock, and $Q_k(t)$ aggregates a continuum of tradable varieties, indexed by v,

$$Q_k(t) = \left[\sum_i \sum_j \int_{\Omega_{ijk}(t)} q_{ijk}(v,t)^{\frac{\eta-1}{\eta}} dv \right]^{\frac{\eta}{\eta-1}}, \tag{3}$$

with $\eta > 1$. The variable $q_{ijk}(v,t)$ denotes consumption of variety $v \in \Omega_{ijk}(t)$, where $\Omega_{ijk}(t)$ is the set of varieties sold to country k and produced by affiliates located in j belonging to firms from i, at time t.

Assumption 1 (Armington). *Varieties are firm and location specific.*

As in Armington (1969), Assumption 1 states that consumers perceive differently varieties produced in different locations by the same firm, a standard assumption in the literature.[17]

Each country is populated by a continuum of firms. Firms decide where to establish affiliates, including their own local market, and whether to export or not from each of them.

Labor is the only factor of production. Each firm produces with a linear technology, characterized by a productivity parameter φ. When a firm operates an affiliate in a foreign country, the affiliate inherits the home-country firm productivity φ. Under monopolistic competition, the profit-maximizing price of a variety produced in country j destined to k is given by a mark-up over the unit cost of production,

$$p_{ijk}(\varphi, t) = \frac{\eta}{\eta - 1} \frac{\tau_{ijk} W_j(t)}{\varphi}. \tag{4}$$

The variable $W_j(t)$ denotes the wage in country j, at time t, while τ_{ijk} is the iceberg-type cost of

[17] The implications of this assumption for firm entry are isomorphic to assuming that firm productivity is a random variable drawn from an independent Pareto distribution over varieties and production locations, as in Arkolakis et al. (2018). In the static model in Arkolakis et al. (2018), this case implies that the probability of opening an affiliate in market j is independent from the probability of opening an affiliate in market j'.

producing goods in j with a technology from i and shipping them to k, with $\tau_{ijk} \geq 1$, $\forall j \neq k$, and $\tau_{jjj} = 1$. This cost includes the efficiency loss of using a technology for production in a different location from its origin (Ramondo and Rodríguez-Clare, 2013), the standard trade cost of shipping goods internationally, and losses of selling in a different market from the headquarter country of the MNE (Head and Mayer, 2019; Wang, 2021).

When a firm from home country i starts serving market k from production location j, it must pay a sunk cost, F_{ijk}, in units of market-j labor. Additionally, firms from i face a per-period (marketing) fixed cost, f_{ijk}, to serve market k, also paid in units of labor in j.

Profits from sales to destination k for a firm from i with productivity φ producing in location j at time t are

$$\pi_{ijk}(\varphi, t) = \frac{1}{\eta} x_{ijk}(\varphi, t) - W_j(t) f_{ijk}, \tag{5}$$

where

$$x_{ijk}(\varphi, t) = \left(\frac{p_{ijk}(\varphi, t)}{P_k(t)} \right)^{1-\eta} X_k(t) \tag{6}$$

are revenues, $P_k(t)$ is the CES price index in k, and $X_k(t) \equiv P_k(t) Q_k(t) Y_k(t)$ is total expenditure in k. For $j = k$, $\pi_{ijk}(\varphi, t)$ denotes profits from domestic sales by local firms ($i = j$) or horizontal sales by foreign affiliates ($i \neq j$). For $j \neq k$, $\pi_{ijk}(\varphi, t)$ denotes profits from export sales, either by local firms ($i = j$) or foreign affiliates ($i \neq j$).

Our setup has two important implications. First, Assumption 1, coupled with CES preferences and monopolistic competition, implies that there is no cannibalization of sales within the MNE, since goods produced in different locations are different goods. This implication is consistent with our fourth fact in Section 2, as well as with evidence from the automobile industry in Head and Mayer (2019) where each vehicle model (a "variety") is produced in—and sourced from—a single location. Second, consistent with our evidence on weak extended gravity in affiliate entry, Assumption 1 also implies that the MNE location decision is separable across locations. However, as it will be evident in the next section, entry decisions into a location depend on other locations through the compound-option formulation of the dynamic location problem and general equilibrium effects.

3.2 The MNE dynamic problem: the compound option

At each point in time, a firm decides whether and where to open an affiliate, and whether and where to export from the existing affiliates. Affiliate exports may include exporting back to the home market. A firm may also decide to shut down affiliates, or to exit any of its export markets.

We use the notion of a *compound option* to model the dynamic problem of the MNE. Opening an affiliate is an option that, when exercised, gives access to another set of options, namely the possibility of expanding to each export destination. Hence, the decision to open an affiliate in country j depends on the set of countries where the affiliate can export to, introducing dynamic interdependence in the MNE location choices.

The compound option structure allows us to easily solve the firm's problem backwards, as suggested by Dixit and Pindyck (1994, chap. 10). Conditional on the firm having production operations in country j, one can solve for the value of exports to each destination and for the policy functions that induce the affiliate to start, or stop, exporting to each country $k \neq j$. Together with the value of sales to the host market, the value of exports determines the value of an affiliate in country j. One can then solve for the policy functions that induce the firm to open, or shut down, the affiliate.

Guided by the empirical observations in Section 2.2, we make the following assumption.

Assumption 2 (Sequential affiliate activities). *A new affiliate must sell to the market where production occurs in order to eventually export from there.*

Notice that because the model is specified in continuous time, opening an affiliate and exporting from it can happen simultaneously –consistent with the data.

Shock processes. Following Ghironi and Melitz (2005), we assume that firm's productivity φ is the product of a time-invariant firm-specific component, z, and a time-varying home country-specific component, Z_i: $\varphi \equiv z \;\, Z_i$. The term z is drawn from a time-invariant distribution, $G(z)$, as in Melitz (2003). The term Z_i follows a geometric Brownian motion,

$$\frac{dZ_i}{Z_i} = \mu_i^Z dt + \sigma_i^Z dW_i^Z, \tag{7}$$

for $\mu_i^Z \in \Re$, $\sigma_i^Z > 0$, and dW_i^Z denoting a standard Wiener process. When a firm operates an affiliate in a foreign country, it transfers both the aggregate and the idiosyncratic components of productivity to the host market. In this way, MNE operations contribute to the transmission of productivity shocks across countries, in the spirit of Cravino and Levchenko (2017).

We further assume that the aggregate demand shifter for country k, Y_k, evolves according to a geometric Brownian motion,

$$\frac{dY_k}{Y_k} = \mu_k^Y dt + \sigma_k^Y dW_k^Y, \tag{8}$$

where $\mu_k^Y \in \Re$, $\sigma_k^Y > 0$, and dW_k^Y denotes a standard Wiener process, possibly correlated with the home-country aggregate productivity shock.

Our shock structure is based on analytical and computational convenience, as well as on empirical observations. Analytically, the specifications in (7) and (8) are equivalent to assuming that both origin-country aggregate productivity growth and destination-country demand growth behave as a random walk and that productivity and demand growth are each independently and identically distributed. We allow, however, for non-zero correlation between demand and productivity shocks across countries.

This shock structure guarantees the tractability of the model solution. Computationally, relying only on aggregate shocks makes feasible the aggregation of individual firms' decisions and the computation of equilibrium variables for many countries; we do not need to keep track of changes in the firms' productivity distribution over time, which significantly reduces the dimensionality of the state space. Empirically, this shock specification is consistent with the main sources of variation of US MNE affiliate sales documented in Section 2.4, whereby only a small share of that variation is driven by parent- and affiliate-level time-varying shocks. Moreover, the persistence of the aggregate shocks, together with aggregate productivity growing over time ($\mu_i^Z \geq 0$), gives rise to the dynamic patterns documented in Figure 1: affiliates start serving their host market, and later on, they start expanding to export markets. Lastly, the introduction of country-specific demand shocks helps the model match the evolution of affiliate sales shares in different host countries.

With this shock structure, profits of a firm from i from sales to country k depend, both directly (linearly) and indirectly, through the effect on aggregate variables, on a composite aggregate shock,

$$Y_{ik} \equiv Z_i^{\eta-1} Y_k, \tag{9}$$

which captures the effect of both home- and destination-country aggregate shocks on the firm's profits. This composite shock is also a geometric Brownian motion, with drift and variance

$$\mu_{ik} = (\eta-1)\mu_i^Z + \mu_k^Y + (\eta-1)(\eta-2)\frac{{\sigma_i^Z}^2}{2} + (\eta-1)\gamma_{ik}\sigma_i^Z\sigma_k^Y\,, \tag{10}$$

$$\sigma_{ik}^2 = (\eta-1)^2{\sigma_i^Z}^2 + {\sigma_k^Y}^2 + 2(\eta-1)\gamma_{ik}\sigma_i^Z\sigma_k^Y\,, \tag{11}$$

where γ_{ik} denotes the correlation coefficient between Z_i and Y_k.

The state of the economy is then described by the matrix of composite shocks $\mathbf{Y} \equiv \{Y_{ik}\}_{i,k=1,\cdots,N}$, a property that we use next to write the Bellman equations.

Bellman equations. Let $\mathcal{V}_i(z, \mathbf{Y})$ denote the expected net present value of a firm from country i

with productivity z that follows an optimal policy when the state of the economy is $\mathbf{Y}$,

$$\mathcal{V}_i(z, \mathbf{Y}) = \sum_{j=1}^{N} \max \left\{ V_{ij}^o(z, \mathbf{Y}), V_{ij}^a(z, \mathbf{Y}) \right\}. \tag{12}$$

The function $V_{ij}^o(z, \mathbf{Y})$ is the option value of an affiliate in country j, and $V_{ij}^a(z, \mathbf{Y})$ is the value of an existing affiliate in country j, regardless of the destination of its sales. In turn, the value of an affiliate in country j is given by:

$$V_{ij}^a(z, \mathbf{Y}) = V_{ij}^h(z, \mathbf{Y}) + \sum_{k \neq j} \max \left\{ V_{ijk}^o(z, \mathbf{Y}), V_{ijk}^e(z, \mathbf{Y}) \right\}. \tag{13}$$

The function $V_{ij}^h(z, \mathbf{Y})$ is the value of horizontal sales —i.e. sales to country j by a firm located in j— while $V_{ijk}^o(z, \mathbf{Y})$ is the option value of exporting to country k for a firm located in j, and $V_{ijk}^e(z, \mathbf{Y})$ is the value of exports to country k from j. Equations (12) and (13) reflect the sequentiality of Assumption 2. The problem is formulated as a compound option because opening an affiliate is equivalent to exercising an option that gives access to another set of options — the options to export to any other country.

Next, we make explicit the Bellman equations that each of the value functions satisfy. We adopt a recursive formulation and omit the dependence on time t from the notation.

If a firm has not yet opened an affiliate in country j, all the value of its operations in j is option value—i.e., the value of the possibility of entering j in the future,

$$V_{ij}^o(z, \mathbf{Y}) = \max \left\{ \frac{1}{1 + \rho \Delta t} E[V_{ij}^o(z, \mathbf{Y}') | \mathbf{Y}]; V_{ij}^a(z, \mathbf{Y}) - W_j F_{ijj} \right\}, \tag{14}$$

where $\mathbf{Y}'$ denotes the matrix of realizations of the composite shock next period. A firm may keep the option of entering market j, in which case it gets the continuation value of that option, or may enter country j by opening an affiliate there, in which case it pays the entry cost, $W_j F_{ijj}$, and gets the value of having an affiliate in country j, $V_{ij}^a(z, \mathbf{Y})$. Thanks to Assumption 1, a firm evaluates entry into each location separately.

Since all affiliates sell in the market where they are located, the value of horizontal sales for an affiliate in country j of a firm from i is given by

$$V_{ij}^h(z, \mathbf{Y}) = \max \left\{ \frac{1}{1 + \rho \Delta t} \left[\pi_{ijj}(z, \mathbf{Y}) \Delta t + E[V_{ij}^h(z, \mathbf{Y}') | \mathbf{Y}] \right]; V_{ij}^o(z, \mathbf{Y}) \right\}. \tag{15}$$

An affiliate located in j may survive and make profits from selling to j, or may shut down and get the value of the option of opening an affiliate in j, $V^o_{ij}(z, \mathbf{Y})$. For $i = j$, (15) describes the value of sales to the firm's home market.

As indicated by (13), the value of an affiliate is given by the value of its horizontal plus its export sales. The Bellman equation describing the value of the option to export to country k for a firm from i with an affiliate in country j is given by

$$V^o_{ijk}(z, \mathbf{Y}) = \max\left\{\frac{1}{1+\rho\Delta t}E[V^o_{ijk}(z, \mathbf{Y}')|\mathbf{Y}]; V^e_{ijk}(z, \mathbf{Y}) - W_j F_{ijk}\right\}. \tag{16}$$

An affiliate may keep the option of exporting to country k—and get the continuation value of that option—or may start exporting to country k, in which case it pays the entry cost $W_j F_{ijk}$ and gets the value of exporting to k from j, $V^e_{ijk}(z, \mathbf{Y})$. In turn, this value is given by

$$V^e_{ijk}(z, \mathbf{Y}) = \max\left\{\frac{1}{1+\rho\Delta t}\left[\pi_{ijk}(z, \mathbf{Y})\Delta t + E[V^e_{ijk}(z, \mathbf{Y}')|\mathbf{Y}]\right]; V^o_{ijk}(z, \mathbf{Y})\right\}. \tag{17}$$

An affiliate may keep exporting to country k—and get the continuation value of that option—or may stop exporting to country k, in which case it gets the value of the option of exporting to k from j, $V^o_{ijk}(z, \mathbf{Y})$. For $i = j$, (16) and (17) describe the value of exports from the firm's home market.

3.3 Solution to the firm's problem: value functions and equivalence result

The problem of the firm can be solved first for $V^o_{ijk}(z, \mathbf{Y})$ and $V^e_{ijk}(z, \mathbf{Y})$, conditional on the firm having an affiliate in country j, and then for $V^a_{ij}(z, \mathbf{Y})$ and $V^o_{ij}(z, \mathbf{Y})$. The solution depends on the firm's productivity z, the matrix of shocks $\mathbf{Y}$, and the vectors of aggregate variables $\mathbf{P}$, $\mathbf{Q}$, and $\mathbf{W}$. These variables are endogenous and, in turn, depend on the realization of the composite shocks. In principle, one should write the solution to this problem taking into account the equilibrium magnitudes of the aggregate variables evaluated at the specific shock realization, e.g. $P = P(\mathbf{Y})$. However, we appeal to a result in Leahy (1993) and Dixit and Pindyck (1994), ch. 8-9, which we extend to accommodate our setup in Theorem 1 below. The result shows that a firm can ignore the effects of its own and of other firms' actions when solving its entry and exit problems, and hence, ignore the dependance of equilibrium aggregate variables on the matrix of shocks. To lighten the notation, we omit altogether the dependance of the solution of the firm's problem on equilibrium variables. Details are in Appendix A.

The solution to the export problem is a simple case of interlinked options (see Dixit and Pindyck, 1994, ch. 7). The option value of exporting and the value of exports, conditional on the firm's location decision, are given by, respectively:

$$V^o_{ijk}(z, Y_{ik}) \quad = \quad B^o_{ijk}(z) Y_{ik}^{\beta_{ik}}, \tag{18}$$

$$V^e_{ijk}(z, Y_{ik}) \quad = \quad \frac{\tilde{\pi}_{ijk}(z)}{\rho - \mu_{ik}} Y_{ik} - \frac{W_j f_{ijk}}{\rho} + A^e_{ijk}(z) Y_{ik}^{\alpha_{ik}}, \tag{19}$$

where

$$\tilde{\pi}_{ijk}(z) \equiv \frac{1}{\eta} \left(\frac{\eta}{\eta - 1} \right)^{1-\eta} \left(\frac{\tau_{ijk} W_j}{z} \right)^{1-\eta} P_k^\eta Q_k. \tag{20}$$

The terms $\tilde{\pi}_{ijk}(z)$, $B^o_{ijk}(z) > 0$ and $A^e_{ijk}(z) > 0$ depend on the firm's productivity and on aggregate variables, while the parameters $\alpha_{ik} < 0$ and $\beta_{ik} > 1$ are the roots of the fundamental quadratic equation, a standard object in problems of investment under uncertainty formulated as real options. While the option value of exporting to country k for a firm z from country i located in j, $B^o_{ijk}(z) Y_{ik}^{\beta_{ik}}$ in (18), is increasing in the realization of the composite shock, the option value of quitting that export market, given by $A^e_{ijk}(z) Y_{ik}^{\alpha_{ik}}$ in (19), is decreasing in the realization of the composite shock—i.e., the option of entering (exiting) an export market has a larger value in "good times" ("bad times").

Equations (18) and (19) are the value functions in their continuation regions. It remains to solve for the policy functions, which are cutoff values in the realization of the composite shock that induce a firm z to start and stop exporting. We denote these cutoff values by $\bar{Y}^e_{ijk}(z)$ and $\underline{\mathrm{Y}}^e_{ijk}(z)$, respectively. For each triplet (i, j, k) and for each firm z, the option value terms $B^o_{ijk}(z)$ and $A^e_{ijk}(z)$, and the policy functions $\bar{Y}^e_{ijk}(z)$, $\underline{\mathrm{Y}}^e_{ijk}(z)$, can be recovered from the corresponding system of value-matching and smooth-pasting conditions (see Equations (A.5)-(A.6) in the Appendix).

Following a similar procedure, the value of horizontal sales for an affiliate located in country j is given by the present discounted value of profits associated with horizontal sales plus the option value of shutting down the affiliate,

$$V^h_{ij}(z, Y_{ij}) = \frac{\tilde{\pi}_{ijj}(z)}{\rho - \mu_{ij}} Y_{ij} - \frac{W_j f_{ijj}}{\rho} + A^h_{ij}(z) Y_{ij}^{\alpha_{ij}}, \tag{21}$$

where $A^h_{ij}(z) > 0$ depends on the firm's productivity and aggregate variables. As a result, the value of an affiliate in country j can be written as

$$V^a_{ij}(z, \mathbf{Y}) \qquad = V^h_{ij}(z, Y_{ij}) + \sum_{k \in \mathcal{E}_{ij}(z)} V^e_{ijk}(z, Y_{ik}) + \sum_{k \notin \mathcal{E}_{ij}(z)} V^o_{ijk}(z, Y_{ik}), \tag{22}$$

where $\mathcal{E}_{ij}(z)$ is the set of countries where a firm z from country i located in country j exports to. Inspecting (22) makes clear that the compound option structure introduces interdependence in the firm's location choices: the value of an affiliate depends on the set of export destinations available to the affiliate from the host country.

Finally, the option value of locating production in country j is

$$V_{ij}^{o}(z, Y_{ij}) = B_{ij}^{o}(z) Y_{ij}^{\beta_{ij}}, \tag{23}$$

where $B_{ij}^{o}(z) > 0$ depends on firm-level productivity and aggregate variables.

Analogously to export decisions, we denote the policy functions, or the cutoffs in the realization of the composite shock that induce a firm z to open and shut down an affiliate, by $\bar{Y}_{ij}^{h}(z)$ and $\underline{Y}_{ij}^{h}(z)$, respectively. Hence, for each country-pair (i, j) and productivity z, $B_{ij}^{o}(z)$, $A_{ij}^{h}(z)$, $\bar{Y}_{ij}^{h}(z)$, and $\underline{Y}_{ij}^{h}(z)$, can be recovered from the corresponding system of value-matching and smooth-pasting conditions (see Equations (A.12)-(A.13) in the Appendix).

We now generalize the equivalence result in Leahy (1993) and Dixit and Pindyck (1994), which we have used to solve the firm's problem. This result states that the optimal solution to the firm's problem is the same as the solution to the problem of a myopic firm — i.e. a firm that ignores the dynamics of aggregate prices and quantities when calculating expected profits. While the previous literature has shown the equivalence for a perfectly competitive economy, our proof extends the result to the case of monopolistic competition.

Theorem 1. *Consider the entry and exit problem described in (18)-(19) for a firm with productivity z. The optimal solution of this problem, given by $B_{ijk}^{o}(z)$, $A_{ijk}^{e}(z)$, $\bar{Y}_{ijk}^{e}(z)$, and $\underline{Y}_{ijk}^{e}(z)$, is equivalent to the solution of the problem of a myopic firm.*

Proof. See Appendix A.

The proof proceeds in two steps. First, we characterize the problem of a social planner who chooses the optimal mass of firms in a market (efficient entry). Second, we show that the solution of the planner's problem is equivalent to the solution of the entry problem for the least productive firm in the market, which is equivalent to the problem of a myopic firm. To complete the proof, we apply a result in Dhingra and Morrow (2019), who show for a static setting that the competitive equilibrium under monopolistic competition and CES preferences is socially efficient.

Summing up, thanks to Theorem 1, we can treat equilibrium prices and quantities as parameters when solving the firm's problem. This property, together with the compound option structure, will be extremely useful for calibrating the model and performing counterfactual exercises.

3.4 Sorting and firm size.

At each point in time, our model has empirically relevant sorting properties. First, more productive firms have lower entry thresholds, both into a host market and into export markets. Under the assumptions that $\bar{Y}^h_{ij}(z) < \bar{Y}^e_{ijk}(z)\ \forall k \neq j$, and $\mu_{ik} > 0$, this result implies that affiliates that are exporters from birth have larger horizontal sales than non-exporting affiliates, and that affiliates that start exporting later in life have lower horizontal sales than affiliates that start earlier. Second, the same property implies that MNEs with larger parent sales enter more foreign markets through FDI; and the number of MNEs with affiliates in n host markets is decreasing in n, so that there is a negative correlation between the number of firms with affiliates in n markets and their parent sales. Third, for a firm with productivity z, the affiliate entry threshold, both to host and export markets, is increasing in the iceberg-type cost and sunk entry cost. Under the assumption that $\mu_{ik} > 0$, the model predicts that an MNE opens first its largest affiliates, conditional on the size of the host market, and affiliates in markets that are less costly to enter.

We formally derive these results in Propositions 1 and 2 in Appendix A.3 under the additional assumption that fixed costs of affiliate operations are "small," so that there is no endogenous exit of affiliates, either from export markets or from their production locations. Even though we cannot derive formally the results for the general case with arbitrary fixed costs, our quantitative analysis reveals that their implications hold in the general case where exit thresholds are active. Moreover, we contrast these additional model predictions with the data, and exploit these sorting properties for implementing our numerical algorithm, and for the model aggregation, to which we turn next.

3.5 Aggregation and equilibrium

We now specify the evolution of the mass of firms in each location and destination. Define $\bar{z}^h_{ij}(t) \equiv \bar{z}^h_{ij}(Y_{ij})$ ($\underline{z}^h_{ij}(t) \equiv \underline{z}^h_{ij}(Y_{ij})$) as the minimum (maximum) productivity of firms that open (shut down) production in j, while $\bar{z}^e_{ijk}(t) \equiv \bar{z}^e_{ijk}(Y_{ik})$ ($\underline{z}^e_{ijk}(t) \equiv \underline{z}^e_{ijk}(Y_{ik})$) are the minimum (maximum) productivity of firms producing in j that start (stop) exporting to k at time t.[18] Additionally, define $G_{ijk}(z;t) \equiv \int_{\Omega_{ijk}(t)} dG(z)$, the productivity distribution of firms from i with affiliates in j selling in k at time t. The perfect sorting properties of the model imply that the sets $\Omega_{ijk}(t)$ are connected intervals (see Proposition 3 in Appendix A), hence $G_{ijk}(z;t)$ is a truncation of the exogenous distribution $G(z)$, and we can easily keep track of the firm distribution by status.

[18] These productivity thresholds are the inverse of the policy functions. Hence, they are conditional on the realization of the composite shock. Existence is guaranteed by the monotonicity of the composite shock thresholds (see Fillat and Garetto, 2015).

The mass of firms from i located in j evolve as

$$M_{ij}(t+\Delta t) = M_{ij}(t)\left[1 - G_{ijj}(\underline{z}^h_{ij}(t+\Delta t);t)\right] + M^E_{ij}(t+\Delta t), \tag{24}$$

where $M^E_{ij}(t+\Delta t) \equiv [M_i - M_{ij}(t)]\left[1 - G_{ijj}(\bar{z}^h_{ij}(t+\Delta t);t)\right]$ are new affiliates from i located in j, and we assume that the potential mass of firms in each country i, M_i, is fixed.

The mass of firms from i located in j that export to k evolve as

$$M_{ijk}(t+\Delta t) \quad = \quad M_{ijk}(t)\left[1 - G_{ijk}(\underline{z}^e_{ijk}(t+\Delta t);t)\right] + M^E_{ijk}(t+\Delta t), \tag{25}$$

where $M^E_{ijk}(t+\Delta t) \equiv [M_{ij}(t) - M_{ijk}(t)]\left[1 - G_{ijk}(\bar{z}^e_{ijk}(t+\Delta t);t)\right]$ is the mass of new exporters with home country i selling to k and located in j. For $i=j$, the expressions in (24) and (25) simply capture the evolution of the mass of domestic firms and direct exporters, respectively.

It is convenient to define the aggregate productivity index in an origin-location-destination country triplet as

$$z_{ijk}(t) \equiv \left[\int_{\Omega_{ijk}(t)} z^{\eta-1} dG(z)\right]^{\frac{1}{1-\eta}}.$$

Aggregate sales to destination k of firms from i producing in j are given by

$$X_{ijk}(t) = \frac{(\tau_{ijk}W_j(t))^{1-\eta}\,(Z_i(t)z_{ijk}(t))^{\eta-1}}{\sum_{i',j'}\left(\tau_{i'j'k}W_{j'}(t)\right)^{1-\eta}\left(Z_i(t)z_{i'j'k}(t)\right)^{\eta-1}} X_k(t), \tag{26}$$

where $X_k(t) = \sum_{i,j} X_{ijk}(t)$, and the denominator is proportional to $P_k(t)^{1-\eta}$.

Lastly, we present the market clearing conditions for labor and goods markets. Labor market equilibrium entails that total payments to labor in country j at each time t add up to payments to production workers, marketing workers, and workers devoted to the sunk entry cost,

$$W_j(t)L_j = \frac{\eta-1}{\eta}\sum_{i,k} X_{ijk}(t) + W_j(t)\sum_i\left[\sum_{k\neq j} f_{ijk}M_{ijk}(t) + f_{ijj}M_{ij}(t)\right] + W_j(t)\sum_i\left[\sum_{k\neq j} F_{ijk}M^E_{ijk}(t) + F_{ijj}M^E_{ij}(t)\right], \tag{27}$$

where $\sum_{i,k} X_{ijk}(t)$ denotes total output in j. Total absorption in country j has to be equal to national income, at each time t,

$$X_j(t) = W_j(t)L_j + \sum_{l,k}\Pi_{jlk}(t), \tag{28}$$

where $\Pi_{jlk}(t)$ are the aggregate profits associated with sales X_{jlk},

$$\Pi_{jlk}(t) = \frac{1}{\eta} X_{jlk}(t) - W_l(t) \left[f_{jlk} M_{jlk}(t) + F_{jlk} M^E_{jlk}(t) \right]. \tag{29}$$

At each time t, the equilibrium conditions of our model are analogous to the conditions of static models of trade and multinational production (e.g. Arkolakis et al., 2018). Dynamics enter through changes in the sets $\Omega_{ijk}(t)$ over time. But thanks to our assumptions on the shock structure and the continuous-time formulation, together with Theorem 1, aggregation is feasible, and hence, we can solve for the general equilibrium of the model.

Traditionally, general equilibrium models of trade dynamics include firm-level shocks but exclude sunk costs, as in Luttmer (2007) and Arkolakis (2016), among others. In contrast, existing dynamic models that include sunk costs characterize the discrete-choice problem and equilibrium dynamics for a single firm, as in Das et al. (2007) and Morales et al. (2019), focus on stationary equilibria where aggregate variables do not change over time, as in Alessandria and Choi (2007), or restrict the geography to a two-country world, as in Impullitti et al. (2013). Since these models are often formulated in discrete time, the firm's value function itself needs to be solved numerically. The continuous time formulation, coupled with unit root aggregate shocks, and the equivalence result in Theorem 1, allow us to solve for the value functions in closed form, up to the option-value terms $B^o_{ijk}(z)$ and $A^e_{ijk}(z)$. Relative to the literature on exporter dynamics, we lose firm-level time-varying shocks, but by including only aggregate shocks, we do not need to keep track of the evolution of the firms' productivity distribution, and we gain the ability to solve for general-equilibrium aggregate variables for many countries.

4 Calibration

We calibrate the model to match the expansion of US MNEs during the period 1987-2011. Since we only observe MNEs headquartered in the United States, we restrict the source of MNEs to the US. All the other countries host US MNEs and are populated by domestic firms that produce only locally, but can also export to foreign destinations.

We construct data moments using the sample of all manufacturing affiliates operating in the top-ten host countries for US FDI: Brazil, Canada, China, France, United Kingdom, Germany, Ireland, Japan, Mexico, and Singapore. This sample includes 83,214 affiliate-year observations, which account for almost 70 percent of all sales by foreign affiliates of US MNEs.[19]

[19] Online Appendix O.5 shows that using this smaller sample does not change the facts documented in Section 2.

4.1 Procedure

We set the parameters capturing preferences and technology using estimates from the literature and direct observations from the data. Next, we jointly calibrate the rich set of barriers to MNE expansion included in the model to match static and dynamic moments from the BEA data.

We set the elasticity of substitution to $\eta = 5$, in line with estimates in the literature (e.g. Broda and Weinstein, 2006). We set the time preference rate to $\rho = 0.085$ so that it does not violate the technical condition that ensures that the present discounted value of profits does not diverge ($\rho > \mu_{ik}$ for all i, k).[20] We assume that the distribution of firm productivities z is Pareto with location parameter one and shape parameter $\vartheta = 4.25$, calculated by Kondo et al. (2023) using Census data for US firms.[21]

We calibrate the composite shock process for each country in our sample using data on manufacturing value added (in current US dollars) and employment, from the World Bank, together with data on manufacturing trade flows, from Feenstra and Romalis (2014), for the period 1987-2011. We construct absorption in manufacturing, in each country and year, as manufacturing output minus the value of net manufacturing exports.[22]

We assume that the drift of the aggregate composite shock is common across countries, and equal to the average growth rate of manufacturing absorption among the countries in our sample, $\mu_{ik} = 0.064$, for all i, k, over 1987-2011.[23]

We compute the standard deviation of the composite shock, σ_{ik}, using (11). We first set the standard deviation of aggregate productivity, σ_i^Z, to match the standard deviation of the growth in manufacturing value added per worker in country i, and the standard deviation of the aggregate demand shock in country k, σ_k^Y, to match country k's standard deviation of growth in manufacturing absorption. Next, we set the correlation between aggregate productivity growth in country i and

[20] This value might appear high, but it may include variables other than just the time preference rate. For example, if the model included an exogenous death rate, this parameter would be added to the time preference rate and the technical condition would allow for a lower time preference rate.

[21] Since productivity is Pareto, the firms' sales distribution is also Pareto with shape $\tilde{\vartheta} \equiv \vartheta/(\eta - 1)$. Hence, using firm-level data, this parameter can be estimated by regressing the log-rank on the log-size of a firm. Kondo et al. (2023) estimate $\tilde{\vartheta}$ ranging from 0.75 to 1.25, depending on the cutoff used for employment on the right tail. We choose a value around the middle range of these estimates, which corresponds to larger firms as US parent firms are, and given $\eta = 5$, satisfies the condition $\vartheta > \eta - 1$.

[22] We use the share of value added in manufacturing, as reported by OECD STAN, to transform value added in manufacturing into gross output.

[23] Growth rates of manufacturing value added per worker across countries are highly heterogeneous, ranging from 0.029 for France to 0.163 for China. Growth rates of manufacturing absorption display an even wider range, from 0.026 for the United Kingdom to 0.222 for China. Using these country-specific values for the calibration of μ_{ik} would force us to use an extremely high value of ρ (to ensure that $\rho - \mu_{ik} > 0$), and would imply unreasonably high effective discount rates for profits for the countries with the lowest μ_{ik}.

demand growth in country k, γ_{ik}, to match the correlation between country k's manufacturing absorption growth and country i's manufacturing value-added per worker growth. To initialize the shock processes, we normalize the initial value of the productivity shock in the United States to $Z_{US}(0) = 1$. We then set $Z_i(0)$, for all $i \neq US$, to be equal to the average of country i's manufacturing value added per worker relative to the United States over the period 1987-2011.

We set: the potential mass of firms in each country to one, M_i =1 for all i; L_k to match the observed average manufacturing employment levels, for each country relative to the United States, over the period 1987-2011; and $Y_k(0) = 1$ for all k.

It remains to calibrate the parameters related to the costs of MNE expansion: the per-period fixed costs, f_{ijk}; the sunk entry costs, F_{ijk}; and the iceberg-type costs, τ_{ijk}. We impose the following restrictions.

First, since we do not observe entry and exit of US firms in the data, we impose that $F_{US,US,US} = f_{US,US,US} = 0$, which implies that the measure of US firms is fixed and all US firms operate in the US market. We allow for changes in the mass of domestic firms in the other countries in our sample, even if we do not observe those margins in the data. To such end, we assume that the sunk and fixed costs for firms entering and serving market j are the same for US MNE affiliates and domestic firms: $F_{US,jj} = F_{jjj} = F_j^h$ and $f_{US,jj} = f_{jjj} = f_j^h$. Furthermore, since we only observe MNEs headquartered in the United States, $F_{ijk}, f_{ijk} \to \infty$ for $i \neq US$ and $i \neq j$, for all k.

Second, we assume that the fixed and sunk costs of US MNE affiliate exports are symmetric across all destination countries, except for the United States: $f_{US,jk} = f_j^e$ and $F_{US,jk} = F_j^e$, for $k \neq j$, and $k \neq US$; and $f_{US,j,US} = f_{j,US}^e$ and $F_{US,j,US} = F_{j,US}^e$, for $j \neq US$. Since we do not observe export entry and number of exporters systematically for all the other countries, we assume that domestic firms in each country $j \neq US$ face the same sunk and fixed export costs to each destination, including the United States, as affiliates of US MNEs operating in j.

Lastly, we assume that the iceberg-type costs to each host market for US MNEs are $\tau_{ijj} = \tau_j > 1$ for $i = US$ and $i \neq j$, while for firms from all other countries are $\tau_{ijj} \to \infty$, for $i \neq US$, and $i \neq j$. There are no iceberg costs associated with domestic sales, $\tau_{jjj} = 1$ for all j. For $i = j \neq k$, iceberg-type costs are the standard trade costs of exporting from j to k, $\tau_{jjk} = \tau_{jk}$. Since we observe export sales of US MNE affiliates from market j to Canada, Japan, the United Kingdom, and the United States, we use those sales as targets to calibrate τ_{jk} for $k \neq j$ and $k = CAN, JPN, UK, US$. For the remaining destinations, we assume that these costs are proportional to the geographic distance between j and k, an exporter-specific dummy, and an error term,

$$\log \tau_{jk} = D_j + \beta^d \log Dist_{jk} + u_{jk}. \tag{30}$$

Under the assumption that productivity z is Pareto and given the properties of the sets $\Omega_{ijk}(t)$, export sales in (26), at each time t, are CES, and hence, have a gravity structure as defined by Arkolakis et al. (2012). Leveraging this structure, we calculate the distance elasticity β^d by estimating a standard gravity equation for the log of bilateral trade flows in manufacturing, using data from Feenstra and Romalis (2014), including importer and exporter fixed effects, and assuming a trade elasticity of $\eta - 1 = 4$.[24] The dummy D_j is chosen to match the export share of country j, an average across the years in our sample period. Notice that domestic exporters in each host country j, including exporters located in the United States, also face these iceberg trade costs.

We are left with 123 parameters to calibrate, for which we target 123 moments from the data. Even though the model does not have a one-to-one mapping from each parameter to each moment, and parameters are jointly calibrated, because of the model's closed-form solutions, it is relatively easy to isolate the moment that drives the identification of a given parameter. Specifically, affiliate entry rates and the share of US MNE affiliates in each country help identify the sunk and fixed MNE costs, F_j^h and f_j^h. Similarly, export entry rates and the share of exporting affiliates help identify the sunk and fixed export costs, F_j^e and f_j^e. Export sales shares drive the identification of the iceberg trade cost τ_{jk}, while horizontal sales of US MNE affiliates in country j, relative to US parent sales, help identify the efficiency loss τ_j.

We choose the values of the parameters to best fit the data moments, for each country. To this end, we simulate the model 100 times, each time for a different realization of the vector of aggregate shocks. Each simulation amounts to solving the model for 500 firms and 30 years. Computationally, this entails solving $2N^2 + 3N$ systems of four equations (given by value-matching and smooth-pasting conditions) in four unknowns (given by option value terms and thresholds in the realization of the shocks), for each firm, time period, and simulation, as well as solving for price indexes $P_j(t)$, aggregate quantities $Q_j(t)$, and wages $W_j(t)$, for $j = 1, ...N$ and all t. Thanks to Theorem 1, aggregate equilibrium variables can be solved period by period, so that we can use a similar algorithm as for static models of trade and multinational firms (e.g. Alvarez and Lucas, 2007). Online Appendix O.6 describes the algorithm in detail, while Online Appendix O.7 reports the full set of simulated and data moments, as well as the calibrated parameters, for each host country.

[24] For this estimation, we use a large cross-section of countries, much larger than the ten countries used in the other parts of the calibration.

4.2 Results

Table 3 reports simulated and data moments, averaged across the top-ten host countries for US FDI and across years. The model matches extremely well static moments such as the affiliate sales shares to the different destinations and the share of affiliates serving those destinations. The calibrated model underestimates the affiliate entry margin, while it overestimates the export entry margin.

The table also includes three sets of non-targeted moments: affiliate size advantage, MNE sorting patterns, and affiliate exit.

First, the moments capturing the affiliate size advantage in panel 4 of Table 3 are related to the sorting properties of the model.[25] While in the data the average horizontal sales of an affiliate that exports from birth are 6.3 times larger than those of an affiliate that never exports, our calibrated model generates a premium six times larger than the one observed in the data. Similarly, the observed average horizontal sales of an affiliate that exports from birth are 3.7 times larger than those of an affiliate that starts exporting later in life, but in the calibrated model this early-exporter premium reaches 9.5. In contrast, the model is able to reproduce quite accurately the fact that the horizontal sales of a first affiliate of an MNE are larger than those of subsequent affiliates. As discussed in various papers in the literature (e.g. Armenter and Koren, 2015), trade models with perfect sorting and Pareto sales fail to match the exporter premium observed in the data, conditional on matching the share of exporters. Our model suffers from a similar shortcoming: it creates large firms that are too large, a feature evident when comparing average and median moments.

Second, in panel 5 of Table 3, we show moments analogous to the ones in Eaton et al. (2011) for exporters. Our model predicts that the MNEs with the largest US parent sales should enter more markets, including less popular ones. The calibrated model captures fairly accurately the magnitude of the elasticity of US parent sales with respect to the number of countries where US MNE affiliates operate, but it delivers an elasticity of average parent size to market popularity of more than double the one observed in the data.

Finally, as shown in the last panel of Table 3, the model underestimates exit rates of affiliates from their host market, but less so from their export markets. This result in not surprising given the absence of an exogenous death rate and of time-varying firm-level shocks in the model; exit rates are driven only by firms' endogenous responses to aggregate shocks.

[25]See Online Appendix Figures O.2 and O.3 for more details.

Table 3: Moments: model versus data, averages.

	Data	Model
Targeted Moments		
1. Intensive margin		
1.1 Affiliate sales share to host country	0.026	0.026
1.2 Affiliate export share to the US	0.139	0.129
1.3 Affiliate export share to third countries	0.276	0.251
1.4 Affiliate export share to Canada	0.015	0.005
1.5 Affiliate export share to the U.K.	0.069	0.060
1.6 Affiliate export share to Japan	0.033	0.033
2. Extensive margin		
2.1 Share of MNEs with affiliates in j	0.287	0.287
2.2 Share of affiliates in j exporting to US	0.566	0.565
2.3 Share of affiliates in j exporting to third countries	0.671	0.669
3. Entry		
3.1 Share of MNEs opening affiliates in j	0.035	0.021
3.2 Share of affiliates in j that start exporting to the US	0.030	0.039
3.3 Share of affiliates in j that start exporting to third countries	0.030	0.045
Non-Targeted Moments		
4. Affiliate size advantage		
4.1 Exporter size advantage	6.27	37.8 ($25.4^{\dagger}$)
4.2 Early exporter size advantage	3.68	9.54 ($7.50^{\dagger}$)
4.3 First affiliate size advantage	2.57	2.33
5. MNE sorting		
5.1 Elasticity of average sales in US w.r.t. # of markets entered	0.736	1.042
5.2 Elasticity of average sales in US w.r.t # of firms entering multiple markets	-0.424	-1.046
6. Exit		
6.1 Share of MNEs shutting down affiliates in j	0.113	0.037
6.2 Share of affiliates in j that stop exporting to the US	0.025	0.011
6.3 Share of affiliates in j that stop exporting to third countries	0.027	0.017

Notes: Averages across host countries and years. Data moments for Japan, Canada, and the United Kingdom are averages over benchmark year surveys only. Denominators are: in 1.1, US parent's sales; in 1.2-1.6, total horizontal sales of affiliates in j; in 2.1, the total number of MNEs; in 2.2 and 2.3, the total number of affiliates in j; in 3.1, total number of MNEs in period before entry; in 3.2 (3.3), total number of affiliates in j in period before export entry into US (third countries); in 6.1, total number of affiliates in j in period before exit; and in 6.2 (6.3), the total number of affiliates in j that export to the US (third countries) in the period before export exit. In 4.1, exporter size advantage refers to the average size of exporting MNE affiliates relative to the average size of non-exporting MNE affiliates, an average across countries and years. In 4.2, early exporter affiliate size advantage refers to the average size of exporting MNE affiliates that start exports in their first year of life relative to the average size of exporting MNE affiliates that start exports after their first year. In 4.3, first affiliate size advantage refers to the ratio of the size of the first foreign affiliate of an MNE (relative to GDP in the affiliate host market) to the size of subsequent foreign affiliates of the same MNE (relative to GDP in the affiliate host market), an average across MNEs and years. For moments in 4., size refers to horizontal affiliate sales. The elasticities in 5. are computed by OLS, aggregating the firm-level observations of MNEs that enter the same number of countries. (†): Median values.

Table 4: Calibrated MNE costs: shares of sales and monetary values. Average across host countries.

	As % of sales			In thousands $
	5th	50th	95th	
Sunk MNE cost $\tilde{F}^h_j$, as % of US parent sales	0.52	0.26	0.05	1,900
Fixed MNE cost $\tilde{f}^h_j$, as % of horizontal sales	96.3	23.3	3.33	2,822
Sunk export cost $\tilde{F}^e_{jk}$, as % of horizontal sales				
To United States	109.2	16.2	4.59	1,119
To other destinations	152.7	28.8	7.11	2,644
Fixed export cost $\tilde{f}^e_{jk}$, as % of exports sales				
To United States	10.6	5.32	0.76	60.5
To other destinations	8.15	4.12	1.31	196.0

Notes: $\tilde{x}(t) \equiv W(t)x$. US parent sales are evaluated in the year of affiliate entry. Horizontal sales are evaluated in the year the affiliate first exports to the destination, for $\tilde{F}^e_{jk}$, and averaged across years, for $\tilde{f}^h_j$. Export sales to a destination are averaged across years. Percentiles are with respect to affiliate sales in the calibrated model. Cost shares in the model are converted into thousands of US dollars using sales values for the median affiliate in each of the top-ten host countries included in the calibration, averaged across host countries. For confidentiality purposes, reported median sales are an average of the 9 observations around the median.

The costs of MNE expansion. We now evaluate the magnitude of the costs to MNE expansion in time and space. Table 4 shows the calibrated MNE costs as shares of firm revenues and in monetary values.

On average, opening an affiliate involves spending a low share of the US parent revenues (0.26 percent for the median affiliate). An affiliate's fixed operating costs range from about 3 percent of the affiliate's horizontal sales, for the largest affiliates, to about 96 percent, for the smallest affiliates; for the median affiliate, these costs amount to almost 23 percent of their horizontal sales. On average, starting to export from an existing affiliate involves spending a large share of the affiliate revenues in the host country, 28.8 percent for exports to destinations other than the United States. Intuitively, starting exporting to the US —the home country of the MNEs– is associated with comparatively lower sunk costs, equal to 16.2 percent of host-country affiliate revenues. Once affiliate export operations are in place, their maintenance involves relatively low operating costs, equal to 5.32 (4.12) percent of affiliate export sales for exports to the United States (other countries).

The right panel of Table 4 shows the costs for the median affiliate expressed in thousands of current US dollars. Starting MNE export operations is more costly in monetary terms than starting affiliate horizontal operations, the exception being exports to the United States. However, the fixed costs

Table 5: Calibrated MNE costs: by type and destination. Selected host countries.

as % of sales	Sunk MNE cost	Fixed MNE cost	Sunk export cost		Fixed export costs	
			to US	to other	to US	to other
	$\tilde{F}^h_j$	$\tilde{f}^h_j$	$\tilde{F}^e_{j,US}$	$\tilde{F}^e_{jk}$	$\tilde{f}^e_{j,US}$	$\tilde{f}^e_{jk}$
Brazil	0.57	19.0	15.6	30.6	4.73	1.21
France	0.11	21.1	1.70	28.1	9.68	1.99
Ireland	0.10	34.6	34.3	87.8	5.56	0.31
Japan	1.29	14.7	8.21	31.2	2.06	0.37
United States	–	–	–	0.46	–	10.3

Notes: $\tilde{x}(t) \equiv W(t)x$. Costs are reported as percentage of sales, using sales values for the median affiliate in each of the top-ten host countries included in the calibration. For confidentiality purposes, median sales are an average of the 9 observations around the median. Sunk MNE costs, $\tilde{F}^h_j$, are reported as a share of US parent sales in the year of affiliate entry. Fixed MNE costs, $\tilde{f}^h_j$, are reported as a share of average horizontal sales across years. Sunk export costs, $\tilde{F}^e_{jk}$ are reported as a share of horizontal affiliate sales in the year the affiliate first exports to the destination. Fixed export costs, $\tilde{f}^e_{jk}$, are reported as a share of average affiliate export sales to the destination across years.

of affiliate exports are much smaller than the fixed costs of horizontal operations.[26]

Table 5 highlights the heterogeneity in the calibrated frictions across selected host countries. Favorable "tax-haven"-like policies that attract FDI make opening an affiliate in Ireland less costly compared to other countries, as reflected in the values of sunk MNE costs as a share of US parent revenues. The operating costs of maintaining affiliates, as a share of affiliate revenues, range from 15 percent in Japan to almost 35 percent in Ireland. The sunk export costs to the United States, as a share of the affiliate horizontal sales in the year of export entry, are lower than the cost of entering other export destinations. The fixed export costs, both to the United States and to third countries, as a share of the affiliate exports to those destinations, display less variation across host countries than the sunk export costs.

The proximity-concentration tradeoff. Our quantitative model reproduces the observation, first documented by Brainard (1997), that the ratio of export to horizontal affiliate sales to a foreign country is decreasing in trade costs and increasing in the fixed cost of MNE activities. To show that our model generates a proximity-concentration tradeoff, we regress the ratio of US exports to country j to the horizontal sales in country j of affiliates of US MNEs, as implied by the calibrated model, on variable trade costs from the United States to country j, relative to the efficiency loss of opening an affiliate in j, fixed costs incurred in country j, and aggregate expenditure in country j

[26]To put our numbers in perspective, Das et al. (2007) estimate the export entry costs of Colombian exporters at around 400,000 1986 US dollars. Despite similar entry rates, our estimates for affiliate exports to non-US destinations are much larger than theirs.

Table 6: The proximity-concentration tradeoff, United States.

	$\log\left(\frac{\tau^s_{US,j}}{\tau^s_j}\right)$	$\log\left(W^s_j(t)f^{h,s}_j\right)$	$\log\ X^s_j(t)$
$\log\left(\frac{X^s_{US,US,j}(t)}{X^s_{US,j,j}(t)}\right)$	-2.84***	4.29***	0.06***
	(0.002)	(0.001)	(0.001)

Notes: $s = 1, \ldots, S$ denote simulations. OLS estimates of (31). Number of observations: 29,035. R-squared: 0.99. Model-simulated data and calibrated parameters. All specifications include time and simulation fixed effects. Robust standard errors in parenthesis. Levels of significance are denoted by $^{***}p < 0.01$, $^{**}p < 0.05$, and $^{*}p < 0.1$.

at time t,

$$\log\left(\frac{X_{US,US,j}(t)}{X_{US,j,j}(t)}\right) = \beta_\tau \log\left(\frac{\tau_{US,j}}{\tau_j}\right) + \beta_f \log\left(W_j(t)f^h_j\right) + \beta_x \log X_j(t). \tag{31}$$

Table 6 shows that our calibrated model delivers the proximity-concentration tradeoff: $\beta_\tau < 0$ and $\beta_f > 0$.[27] This substitution effects are the result of general equilibrium effects on aggregate prices and wages.

5 Quantitative Analysis

Armed with the calibrated model, we evaluate quantitatively a key feature of our theory: the role played by the extensive margin of MNE expansion, both over time and across space. To such end, we evaluate the impact on the extensive margin of: export platforms (i.e. the compound-option structure); sunk costs (i.e. dynamics); various types of trade-cost shocks (i.e. variable, fixed, sunk); and general equilibrium effects (i.e. wage adjustments). Since our sample includes the United Kingdom, Ireland, Germany, and France, and since about half of US MNEs have UK-based affiliates (see Online Appendix Table O.23), we perform exercises that change costs related to operating in—or exporting from and to—the United Kingdom, loosely mimicking the United Kingdom abandoning the European Union (EU), "Brexit." In all the exercises, we simulate the model for 30 periods and impose a permanent change in parameters related to the operations of MNEs in a host country or to export costs at t = 15.

[27] Results are virtually unchanged if we use annualized sunk MNE costs, $W_j(t)(f^h_j + \rho F^h_j)$. We do not include the fixed cost of exporting from United States to country j because it is common across destinations, $f^e_{US,j} = f^e_{US}$.

5.1 The role of export platforms

We analyze the effects of an increase in the barriers to MNE activities in a model with and without export platforms —i.e. we allow for direct exports by domestic firms in each country, including US MNE parents, but not by foreign US MNE affiliates. We calibrate the model with no export platforms (i.e. no compound option) by targeting the moments in our baseline calibration that do not involve US MNE affiliate exports (moments 1.1, 2.1, and 3.1 in Table 3).[28]

The calibrated model without export platforms delivers estimates of sunk MNE costs that are virtually zero for all sales percentiles, while estimates of fixed costs for small firms (median and large firms) are less than half (around half) the magnitude of the estimates for the baseline model (see Online Appendix Table O.28). Not surprisingly, when we include the option of exporting from the host market, the value of opening MNE affiliates abroad increases, and hence, to match the shares of US MNE affiliates observed in the data, sunk (fixed) costs must be higher.

The results in Table 7 make this point clear by comparing the share of US MNE affiliates in host country j with and without the option of exporting to other markets. The comparison uses the baseline calibration with and without export platforms (i.e. no recalibration of the model with no export platforms). The results indicate that the share of US MNE affiliates in the average host country is twice as high when export platforms are allowed. US MNE presence in countries such as Canada and Ireland would decrease by more than half if affiliates there were not allowed to export to other markets; the incentives to open an MNE affiliate in a country coming from the possibility of exporting are absent. Removing the possibility of export platforms decreases horizontal sales in some host countries, like Canada, while it increases horizontal sales in others, like Ireland. These substitution effects operate through the general-equilibrium effects of the model, a channel that we further explore below.

In our next exercise, we increase the per-period fixed cost and the one-time sunk cost of horizontal MNE activities in the United Kingdom, f^h_{UK} and F^h_{UK}, by an amount such that, in either case, $f^h_{UK} + \rho F^h_{UK}$ increases by 20 percent. Figure 5 compares the dynamics of the share of US MNE affiliates in the United Kingdom after each of those changes, using the calibrated models with and without export platforms.

After an increase in f^h_{UK}, on impact, the share of US MNE affiliates declines by about 15-percentage points in both models. However, over time, the predictions of the two models differ. After five years from the cost change, the share of US MNE affiliates in the United Kingdom is almost 15-

[28]For domestic exporters, we keep the baseline calibration values of τ_{jk}, F^e_j, and f^e_j. See Online Appendix Table O.20.

Table 7: The role of export platforms in MNE affiliate entry.

	Share of US MNE affiliates in host country j		% change in sales to host country j
Model with:	export platforms	no export platforms	export- relative to no export-platform
Brazil	0.187	0.088	5.73
Canada	0.544	0.232	7.99
China	0.184	0.134	-3.56
France	0.313	0.167	-2.85
Great Britain	0.556	0.214	-3.86
Germany	0.369	0.180	-4.62
Ireland	0.122	0.042	-6.52
Japan	0.145	0.105	0.57
Mexico	0.303	0.151	-0.96
Singapore	0.125	0.047	4.56
Average	0.285	0.136	-0.35

Notes: The model with export platforms is the baseline model, while the model with no export platforms is recalibrated to the data assuming that the value of the export option for US MNE affiliates in each host country j equals to zero. Averages over 30 periods.

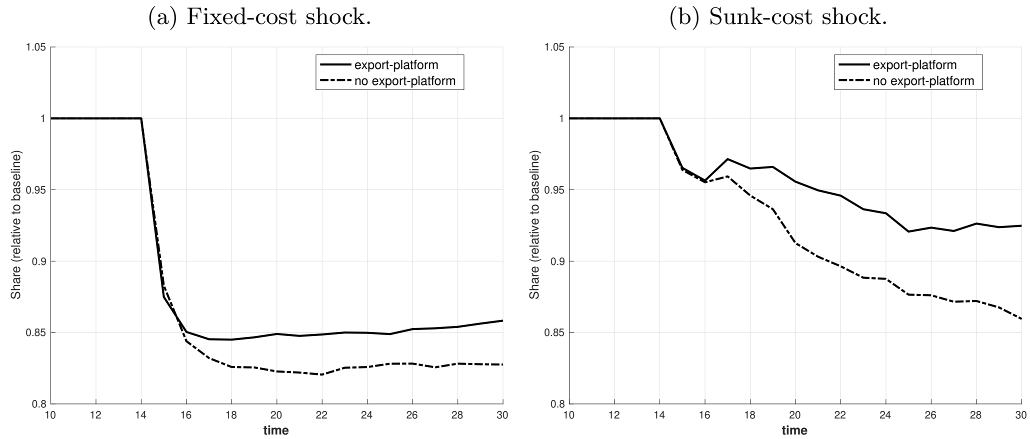
percentage points lower in the model with no export platforms compared with the baseline model. This result is due to the fact that, over time, incentives to use affiliates as export platforms in the baseline model drive MNEs to open more affiliates, despite the increase in operating costs. An increase in F^h_{UK} widens the affiliates' band of inaction, so that fewer MNEs open new affiliates and fewer incumbent affiliates exit. In both models, the reduced entry margin dominates, so that the share of US MNEs in the United Kingdom declines. But this decline is less sharp in the baseline model–of around eight-percentage points lower than in the model without export platforms. Also in this case, the baseline model displays more entry than the model without export platforms, due to the additional incentives that the option of export activities provide.

Overall, these results indicate the importance of including a dynamic and spatial structure when evaluating the expansion strategy of MNEs. Additionally, the results point to the importance of distinguishing between per-period and one-time costs, which cannot be done in the context of static models, a point to which we turn in more detail next.

5.2 The role of sunk costs

We now explore the role of sunk costs in shaping the dynamics of MNE expansion. To such end, we perform two exercises. First, we show the effects of including dynamics in the model by comparing

Figure 5: The role of export platforms: share of US MNE affiliates in the United Kingdom.



Notes: Increase in fixed cost f^h_{UK} and sunk cost F^h_{UK}. Results are shown as deviations from the calibrated models with and without export-platforms.

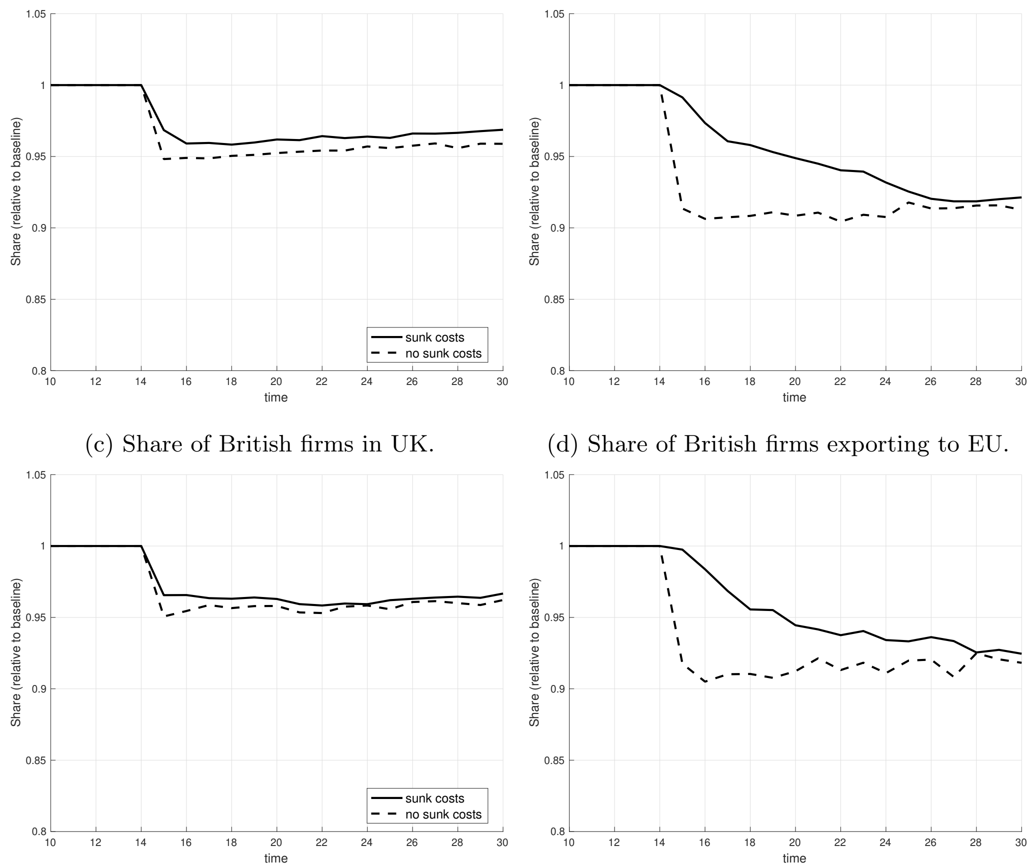
the evolution of affiliate activities in a model with and without sunk costs (i.e. a repeated static model). Second, we show the effects of changing sunk and fixed costs on firm dynamics.

We start by comparing the evolution of the extensive margin of US MNE affiliates in the United Kingdom and of British firms after a 20 percent increase in variable trade costs between the United Kingdom and EU countries, using our calibrated model and a model without any sunk costs. Figures 6a and 6c show that the dynamics of the share of US MNE affiliates and British firms operating in the United Kingdom are not very different between the two models — this is not surprising since barriers to horizontal sales are unchanged, and hence, the effects are only indirect. In contrast, the dynamics of exporters are *quantitatively* very different between the two models. An increase in variable trade costs decreases the shares of active US MNE affiliates and British firms. However, in the static model, this decrease happens at impact, while our dynamic model delivers a slow fifteen-year decline. The models' predicted evolutions converge only fifteen years after the shock.

Next, we evaluate the effects of changing the frictions driving firm dynamics. Figure 7 shows the effects of increasing the sunk and fixed costs of exports from the United Kingdom for all firms located in that country, as well as the sunk entry costs and fixed operating costs in that country, for US MNEs, one, five, ten, and fifteen years after the shock. Naturally, for small changes, dynamic effects are small, but long-run effects become much larger than short-run ones when the size of the shock increases.[29] It is worth noting the large difference in the dynamic responses of MNE activities

[29]Online Appendix Figure E.1 shows the changes in the share of US MNE affiliates operating in each host market in our sample.

Figure 6: The role of sunk costs.



(a) Share of US MNE affiliates in UK.

(b) Share of US MNE affiliates in UK exporting to EU.

(c) Share of British firms in UK.

(d) Share of British firms exporting to EU.

Notes: Changes in $\tau_{UK,k}$ and $\tau_{k,UK}$ where k refers to Ireland, Germany, and France.

between fixed and sunk costs changes — a comparison not possible to make in static setups.

5.3 The effects of trade-cost shocks

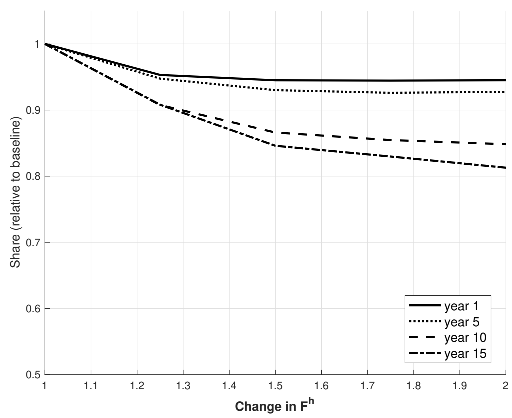
Our last set of counterfactual exercises compares the effects of different trade-cost shocks on firm dynamics. To this end, we increase, one at a time and permanently, the barriers to export between the United Kingdom and the EU countries in our sample: $\tau_{UK,k}$ and $\tau_{k,UK}$, $f^e_{UK,k}$, and $F^e_{UK,k}$, for $k =$ Ireland, Germany, and France. For comparability, we increase each friction by an amount equivalent to a 20-percent increase in the total per-period cost of exporting, $\left(f^e_{j,k} + \rho F^e_{j,k}\right) \tau^{\eta-1}_{j,k}$.

Increasing trade barriers has three main effects. First, when exporting from the United Kingdom to the EU becomes more costly, the incentive to open affiliates (and domestic firms) in the United

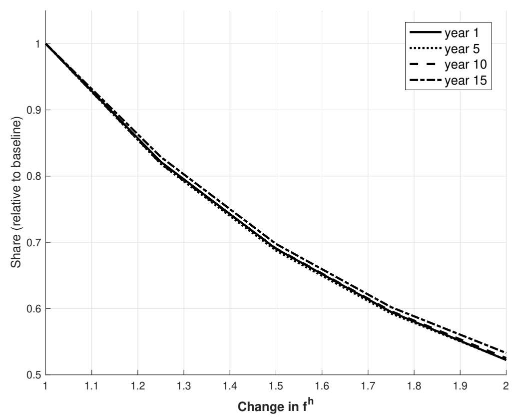
Figure 7: The dynamic effects of sunk and fixed costs.

Share of US MNE affiliates in UK.

(a) Changes in UK sunk costs.

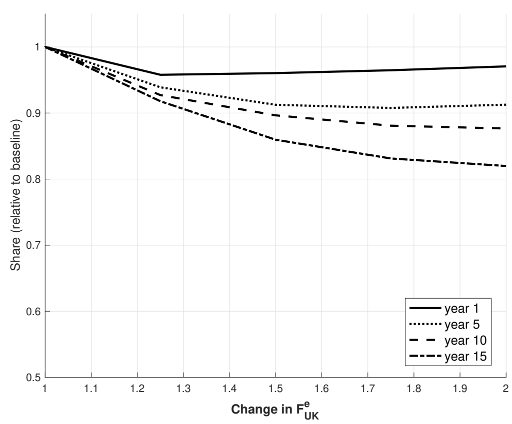


(b) Changes in UK fixed costs.

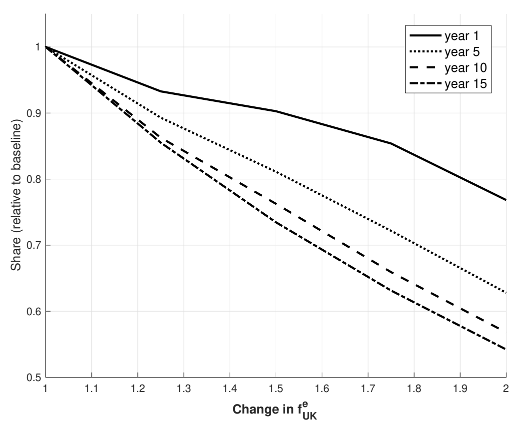


Share of US MNE affiliates in UK exporting to EU.

(c) Changes in UK sunk export costs.



(d) Changes in UK fixed export costs.



Notes: Changes in $f^e_{UK,k}$ and $F^e_{k,UK}$ where k refers to Ireland, Germany, and France, and changes in f^h_{UK} and F^h_{UK}. Years 1, 5, 10, and 15, after the shock.

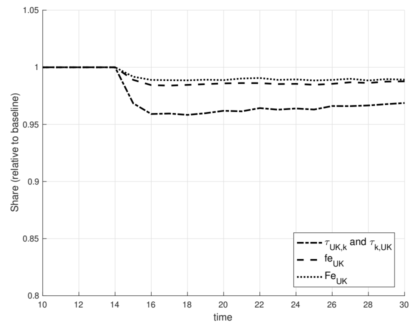
Kingdom decreases due to the smaller, and more costly, available network of export destinations. Analogously, exporting from the EU to the United Kingdom also becomes more costly, decreasing the incentive to open affiliates in those countries. Second, increases in trade costs affect the firms' export band of inaction, which affects firms' export entry and exit decisions—an effect only present in dynamic models. Finally, the increase in trade frictions changes aggregate prices and wages through general equilibrium effects.

Results in Figure 8 combine these three effects. Since trade frictions affect horizontal activities only

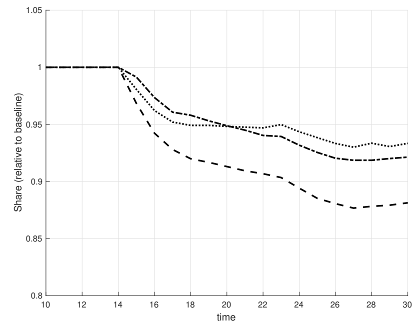
Figure 8: UK trade shock: Firms in the United Kingdom and France.

Firms in the United Kingdom.

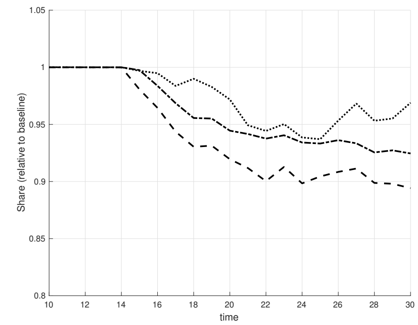
(a) US MNE affiliates.



(b) US MNE affiliates exporting to EU.

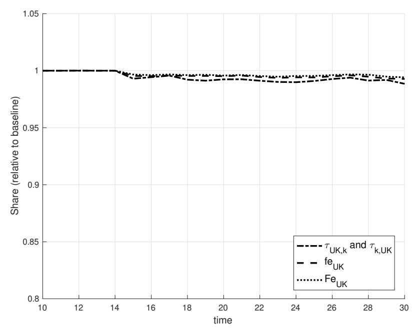


(c) British firms exporting to EU.

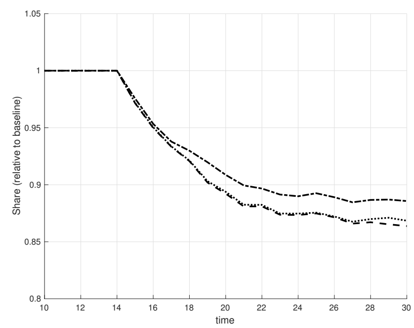


Firms in France.

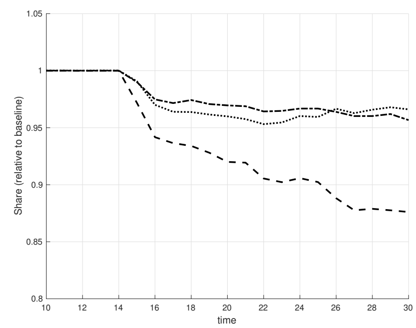
(d) US MNE affiliates.



(e) US MNE affiliates exporting to UK.



(f) French firms exporting to UK.



Notes: Increase in F^e_{UK}, f^e_{UK}, and $\tau_{UK,k}$ and $\tau_{k,UK}$, where k refers to Ireland, Germany, and France.

indirectly through the compound option, Figure 8a shows that effects are small on the share of UK-based affiliates of US MNEs: Increasing sunk or fixed export costs causes a permanent decrease in this share of around two percent, while the increase of variable trade frictions drives a decrease of almost five percent. [30] Figures 8b and 8c show that an export cost shock substantially reduces the share of firms that export, both local and US MNE affiliates. Shocks to different trade frictions have different quantitative effects on export participation rates, even if the changes in those frictions are associated with the same increase in the per-period cost of exporting. The increase in fixed export costs produces the largest decline in export participation because this cost is intimately related to a firm's decision to exit a market.

Results for firms located in France present some quantitative differences from the results for the

[30] Online Appendix Figure E.2 shows the effects of trade costs changes on sales.

United Kingdom.[31] The lower panel of Figure 8 shows higher trade costs between France and the United Kingdom reduce the incentives to locate in France, both for French firms and US MNEs.

Summing up, once again, including dynamics in evaluating the response of MNE expansion is particularly important for distinguishing shocks to variable, fixed, or sunk trade costs, given the different *quantitative* effects on aggregate firm dynamics.

5.4 General equilibrium effects and welfare

We conclude our analysis by quantifying the model's general equilibrium effects on firms' reallocation over time and space, and presenting some welfare calculations. Table 8 reports the dynamics of MNE sales and export participation after an increase in variable trade costs (equivalent to a 20-percent increase in total per-period cost of exporting) between the United Kingdom and EU countries, both under fixed and flexible price indices and wages.

The price and wage adjustments act as buffers to the decrease in sales of UK-based affiliates of US MNEs, both to the United Kingdom and export markets. The effect for UK sales is strong enough to reverse the partial-equilibrium pattern (-0.38 vs 4.21 percent). Decreases in the share of US MNE affiliates operating in the United Kingdom and their export participation are lower when prices and wages are allowed to adjust (-4.1 vs -6.7 percent).

Having a general equilibrium model allows us to compute the welfare consequences of the increase in trade costs for the United Kingdom, calculated as the change in real income per capita. Both in the short and long run, the United Kingdom experiences a decline in welfare of 0.8 percent.

Finally, export and affiliate sales are alternative ways of serving foreign markets. In our model, this substitution effect operates through general equilibrium responses. As the cost of intra-Europe exports increases, the lower panel of Table 8 shows that US parent exports increase, both on the extensive and intensive margins. This result illustrates the forces of the proximity-concentration trade-off: US exports increase when the cost of multinational activity (here, the cost of export platforms) increases. General equilibrium effects affect the relative costs of production across countries, making US domestic production and exports a cheaper way to serve the British and EU markets. The effects differ in the short and long run.

[31] Online Appendix Figures E.3 and E.4 show results for Ireland and Germany, respectively.

Table 8: The role of general equilibrium effects and welfare.

% change relative to baseline:	One period after shock General eq	 Partial eq	Fifteen periods after shock General eq	 Partial eq
US MNE horizontal affiliate sales in UK	4.21	-0.38	3.68	-0.33
Share of US MNE affiliates in UK	-4.09	-6.67	-3.13	-5.34
US MNE affiliate exports from UK to EU	-14.6	-17.1	-14.4	-17.4
Share of US MNE affiliates in UK exporting to EU	-2.66	-1.60	-7.87	-8.71
Real income in UK	-0.80	–	-0.80	–
US parent sales to UK	2.43	0.00	1.59	0.00
Share of US parents exporting to UK	0.98	0.00	0.00	0.00
US parent sales to EU	0.91	0.00	1.07	0.00
Share of US parents exporting to EU	0.45	0.00	0.65	0.00

Notes: The shock refers to increasing the variable trade costs $\tau_{UK,k}$ and $\tau_{k,UK}$, where k refers to Ireland, Germany, and France.

6 Conclusions

This paper studies the expansion patterns of multinational enterprises (MNEs) over time and across space. Using a long panel of US MNEs, we document a set of facts that guide the construction of a dynamic model of MNEs, which is tractable yet rich enough to capture the spatial complexity of MNE activities observed in the data. The model includes heterogeneity in firm productivity, persistent aggregate shocks, and a realistic structure of MNE costs. We introduce a compound-option formulation for the dynamic problem of the MNE, which is novel to the literature, and allows us to simultaneously decouple the firm's locations of production and sales, as well as to incorporate entry and exit decisions both into and from host and export markets. Our quantitative exercises reveal that the compound option structure is important for understanding the reallocation of MNE activity in time and space after a globalization shock. These exercises also reveal that the nature of the frictions to MNE activities (variable, fixed, or sunk) is important for understanding aggregate firm dynamics.

The problem of the MNE is a natural environment to apply a compound-option structure, since MNE location and export decisions happen over time and they are likely to be affected by uncertainty in demand and other market characteristics. This structure, however, can be useful in other contexts. Problems related to global sourcing decisions, which are likely to occur over time and under uncertainty, are good candidates and an avenue for future research.

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Appendix

A Model's Solution and Proofs

A.1 The dynamic firm's problem

The compound option structure of the model implies that it can be solved backwards. We start from the problem of a firm from country i that already has an affiliate in country j and has to decide whether to export to any country $k \neq j$. Once this problem is solved, the value of an affiliate in j is determined. Then we can solve the problem of a firm from country i that has to decide whether to open an affiliate in each country j. For each step of the solution, we follow the method outlined in Dixit and Pindyck (1994).

To solve the affiliate export problem, we start by solving for the value functions $V^o_{ijk}(z, \mathbf{Y})$ and $V^e_{ijk}(z, \mathbf{Y})$ in their continuation region.

Let $V^{s\prime}_{ijk}(z, Y_{ik})$ and $V^{s\prime\prime}_{ijk}(z, Y_{ik})$, $s = o, e, h$ denote the first and second derivative of a value function with respect to the composite shock Y_{ik}. Writing the Bellman equation in (16) in the continuation region, taking the limit as $\Delta t \rightarrow 0$, and applying Ito's Lemma yield the no-arbitrage condition

$$\rho V^o_{ijk}(z, Y_{ik}) = \mu_{ik} Y_{ik} {V'}^o_{ijk}(z, Y_{ik}) + \frac{\sigma^2_{ik}}{2} Y^2_{ik} {V''}^o_{ijk}(z, Y_{ik}). \tag{A.1}$$

Guessing a solution for the value function and applying the method of undetermined coefficients, the value of the option of exporting to country k for an affiliate in country j of a firm from country i has general solution given by

$$V^o_{ijk}(z, Y_{ik}) = A^o_{ijk}(z) Y^{\alpha_{ik}}_{ik} + B^o_{ijk}(z) Y^{\beta_{ik}}_{ik}, \tag{A.2}$$

where $\alpha_{ik} < 0$ and $\beta_{ik} > 1$ are the roots of $\frac{1}{2}\sigma^2_{ik}\xi^2 + \left(\mu_{ik} - \frac{\sigma^2_{ik}}{2}\right)\xi - \rho = 0$. As $Y_{ik} \rightarrow 0$, the option of exporting becomes worthless, so it must be that $A^o_{ijk}(z) = 0$. Conversely, the option of exporting becomes more attractive as Y_{ik} increases, so it must be that $B^o_{ijk}(z) > 0$.

Similarly, writing the Bellman equation in (17) in the continuation region, taking the limit as $\Delta t \rightarrow 0$, and applying Ito's Lemma, yield the no-arbitrage condition

$$\rho V^e_{ijk}(z, Y_{ik}) = \pi_{ijk}(z, Y_{ik}) + \mu_{ik} {V'}^e_{ijk}(z, Y_{ik}) + \frac{\sigma^2_{ik}}{2} Y^2_{ik} {V''}^e_{ijk}(z, Y_{ik}). \tag{A.3}$$

The value of the option of exporting to country k for an affiliate in country j of a firm from country

i has general solution given by

$$V_{ijk}^e(z, Y_{ik}) = \frac{\tilde{\pi}_{ijk}(z)}{\rho - \mu_{ik}} Y_{ik} - \frac{W_j f_{ijk}}{\rho} + A_{ijk}^e(z) Y_{ik}^{\alpha_{ik}} + B_{ijk}^e(z) Y_{ik}^{\beta_{ik}}, \tag{A.4}$$

where $\tilde{\pi}_{ijk}(z, Y_{ik})$ is defined in (20). As $Y_{ik} \to 0$, there is value from the possibility of endogenously stopping to export, so it must be that $A_{ijk}^e(z) > 0$. Also, as Y_{ik} increases, the value of exports converges to the discounted profit flow (i.e., there is no further expansion option), so it must be that $B_{ijk}^e(z) = 0$.

To completely solve the affiliate export problem, we need to solve for the policy functions, which are thresholds for realizations of the composite shock that induce the affiliate to start and stop exporting. For each country triplet (i, j, k) and for each firm with productivity z, the terms $B_{ijk}^o(z) > 0$, $A_{ijk}^e(z) > 0$, and the export entry and exit thresholds, denoted by $\bar{Y}_{ijk}^e(z)$ and $\underline{Y}_{ijk}^e(z)$, respectively, can be recovered from the following system of value-matching conditions,

$$V_{ijk}^o(z, \bar{Y}_{ijk}^e) = V_{ijk}^e(z, \bar{Y}_{ijk}^e) - W_j F_{ijk} \quad \text{and} \quad V_{ijk}^o(z, \underline{Y}_{ijk}^e) = V_{ijk}^e(z, \underline{Y}_{ijk}^e), \tag{A.5}$$

and smooth-pasting conditions,

$$V'^o_{ijk}(z, \bar{Y}_{ijk}^e) = V'^e_{ijk}(z, \bar{Y}_{ijk}^e) \quad \text{and} \quad V'^o_{ijk}(z, \underline{Y}_{ijk}^e) = V'^e_{ijk}(z, \underline{Y}_{ijk}^e). \tag{A.6}$$

To determine the value of an affiliate in j of a firm from i, we still need to solve for the value of horizontal sales. Writing the Bellman equation in (15) in the continuation region, taking the limit as $\Delta t \to 0$, and applying Ito's Lemma,

$$\rho V_{ij}^h(z, Y_{ij}) = \pi_{ijj}(z, Y_{ij}) + \mu_{ij} Y_{ij} V'^h_{ij}(z, Y_{ij}) + \frac{\sigma_{ij}^2}{2} Y_{ij}^2 V''^h_{ij}(z, Y_{ij}). \tag{A.7}$$

The value of horizontal sales for an affiliate in country j has general solution given by

$$V_{ij}^h(z, Y_{ij}) = \frac{\tilde{\pi}_{ijj}(z)}{\rho - \mu_{ij}} Y_{ij} - \frac{W_j f_{ijj}}{\rho} + A_{ij}^h(z) Y_{ij}^{\alpha_{ij}} + B_{ij}^h(z) Y_{ij}^{\beta_{ij}}. \tag{A.8}$$

As $Y_{ij} \to 0$, there is value from the possibility of shutting down the affiliate, so it must be that $A_{ij}^h(z) > 0$. As Y_{ij} increases, the value of horizontal sales converges to the discounted profit flow, so it must be that $B_{ij}^h(z) = 0$.

Up to this point, the value of an affiliate in country j of a firm from country i, $V_{ij}^a(z, \mathbf{Y})$ is completely characterized up to the option value term $A^h_{ij}(z)$,

$$V^a_{ij}(z,\mathbf{Y}) = \frac{\tilde{\pi}_{ijj}(z)}{\rho-\mu_{ij}}Y_{ij} - \frac{W_j f_{ijj}}{\rho} + A^h_{ij}(z)Y_{ij}^{\alpha_{ij}} + \sum_{k\in\mathcal{E}_{ij}(z)}\left[\frac{\tilde{\pi}_{ijk}(z)}{\rho-\mu_{ijk}}Y_{ik} - \frac{W_j f_{ijk}}{\rho} + A^e_{ijk}(z)Y_{ik}^{\alpha_{ik}}\right] + \sum_{k\notin\mathcal{E}_{ij}(z)}\left[B^o_{ijk}(z)Y_{ik}^{\beta_{ik}}\right] \tag{A.9}$$

where $\mathcal{E}_{ij}(z)$ denotes the set of markets in which a country j affiliate of a firm from country i with productivity z exports.

To solve the affiliate entry problem, we still need to solve for the option value of opening an affiliate, and for the policy functions. Writing the Bellman equation in (14) in the continuation region, taking the limit for $\Delta t \to 0$, and applying Ito's Lemma yield

$$\rho V^o_{ij}(z,Y_{ij}) = \mu_{ij}Y_{ij}V'^o_{ij}(z,Y_{ij}) + \frac{\sigma^2_{ij}}{2}Y^2_{ij}V''^o_{ij}(z,Y_{ij}). \tag{A.10}$$

Hence, the value of the option of opening an affiliate in country j for a firm from country i has general solution given by

$$V^o_{ij}(z,Y_{ij}) = A^o_{ij}(z)Y_{ij}^{\alpha_{ij}} + B^o_{ij}(z)Y_{ij}^{\beta_{ij}}. \tag{A.11}$$

As $Y_{ij} \to 0$, the option of opening an affiliate becomes worthless, so it must be that $A^o_{ij}(z) = 0$. Conversely, the option of opening an affiliate becomes more attractive as Y_{ij} increases, so it must be that $B^o_{ij}(z) > 0$.

Let $\bar{Y}^h_{ij}(z)$ and $\underline{Y}^h_{ij}(z)$ denote the thresholds for the realization of the composite shock that induce a firm from country i to open or shut down an affiliate in country j, respectively. The value-matching and smooth-pasting conditions that deliver the terms $A^h_{ij}(z)$, $B^o_{ij}(z)$, and the policy functions $\bar{Y}^h_{ij}(z)$ and $\underline{Y}^h_{ij}(z)$, are given by

$$V^o_{ij}(z,\bar{Y}^h_{ij}) = V^a_{ij}(z,\bar{Y}^h_{ij},\mathbf{Y}_{i,-j}) - W_jF_{ijj} \quad , \quad V^o_{ij}(z,\underline{Y}^h_{ij}) = V^a_{ij}(z,\underline{Y}^h_{ij},\mathbf{Y}_{i,-j}), \tag{A.12}$$

$$V'^o_{ij}(z,\bar{Y}^h_{ij}) = V'^a_{ij}(z,\bar{Y}^h_{ij},\mathbf{Y}_{i,-j}) \quad , \quad V'^o_{ij}(z,\underline{Y}^h_{ij},\mathbf{Y}_{i,-j}) = V'^a_{ij}(z,\underline{Y}^h_{i,-j}), \tag{A.13}$$

where $\mathbf{Y}_{i,-j}$ denotes the vector of composite shocks for origin country i and destination countries other than j.

A.2 Proof of Theorem 1

We show that the optimal solution of the entry and exit problem defined by Equations (18)-(19) is equivalent to the solution of the problem for a myopic firm.

For clarity, we suppress the subscripts (i, j, k), and abstract from the possibility of exit. The proof for the solution to the exit problem is analogous to the one for entry.

The proof proceeds in two steps. First, we write down and characterize the problem of a social planner who chooses the optimal mass of firms in a market (efficient entry). Second, we show that the solution to the planner's problem is equivalent to the solution to the entry problem for the least productive firm in the market. Assuming –without loss of generality– that firms enter in order of decreasing productivity, the least productive firm behaves as a myopic firm. We then invoke the equivalence between the solution of the planner's problem and the firm's optimal solution under monopolistic competition with CES preferences, as shown in Dhingra and Morrow (2019).

Planner's problem. Let $U(M, Y)$ denote the value of social welfare for the planner when considering the possibility of entry of an additional firm (this could be affiliate entry, export entry, or affiliate export entry). The value of social welfare depends on the mass of firms currently in the market, M, and on the realization of the shock, Y. In the continuation region, the value of social welfare is given by

$$U(M, Y) = S(M, Y)dt + e^{-\rho dt}E[U(M, Y')|Y], \tag{A.14}$$

where $S(M, Y)$ denotes the net social surplus.

Because of the Brownian motion assumption on the shock, the value of social welfare can be expressed as the sum of two terms: the expected present discounted value of the net social surplus when the mass of entrants is held constant plus the option value of increasing the mass of entrants,

$$U(M, Y) = T(M, Y) + B(M)Y^{\beta}, \tag{A.15}$$

where

$$T(M, Y) \equiv E[\int_0^{\infty} e^{-\rho dt}S(M, Y)dt],$$

is the expected discounted value of the net social surplus, β is the positive solution of the fundamental quadratic equation, and $B(M)$ is a function to be determined.

Let $\bar{Y}$ denote the threshold such that it is optimal for the planner to add a firm to the market for any realizations of the shock $Y \geq \bar{Y}$. One can solve for $\bar{Y}$ and for the option value of expansion $B(M)$ by imposing value matching and smooth pasting conditions. At the threshold, the increase in the value of social welfare must equate the entry cost of the additional firm, given by the units of labor needed for the sunk cost $\tilde{F}$. By smooth pasting, the derivatives of the gain in the value function and of the additional cost with respect to the shock must be equal,

$$T'_M(M,Y) + B'_M(M)Y^{\beta} \;\; = \;\; \tilde{F} \tag{A.16}$$

$$T''_{M,Y}(M,Y) + \beta B'_M(M)Y^{\beta-1} \;\; = \;\; 0. \tag{A.17}$$

Solving for $B'_M(M)$ from (A.17) and substituting the resulting expression into (A.16) yield

$$-\frac{Y}{\beta}T''_{M,Y}(M,Y) + T'_M(M,Y) = \tilde{F}. \tag{A.18}$$

whose solution is the threshold $\bar{Y}$ as a function of the mass of firms M.

Firm's problem. The entry problem of a myopic firm is described by the following value functions

$$V^o(z,Y;P,Q,W) \;\; = \;\; B^o(z;P,Q,W)Y^{\beta}, \tag{A.19}$$

$$V^e(z,Y;P,Q,W) \;\; = \;\; \frac{\tilde{\pi}(z;P,Q,W)}{\rho-\mu}Y - \frac{Wf}{\rho}, \tag{A.20}$$

where we make explicit the dependence of profits and option value terms on aggregate equilibrium variables, which the myopic firm takes as given —this firm does not perceive that these variables depend on the shock Y nor does it form expectations about their evolution. These two conditions coincide with the value functions of the last (least) productive firm to enter the market, $\bar{z}$.

We solve for $\bar{Y}$ and for the option value of expansion $B^o(z;P,Q,W)$ imposing value matching and smooth pasting conditions,

$$B^o(z;P,Q,W)\bar{Y}^{\beta} \;\; = \;\; \frac{\tilde{\pi}(z;P,Q,W)}{\rho-\mu}\bar{Y} - \frac{Wf}{\rho} - WF \tag{A.21}$$

$$\beta B^o(z;P,Q,W)\bar{Y}^{\beta-1} \;\; = \;\; \frac{\tilde{\pi}(z;P,Q,W)}{\rho-\mu}. \tag{A.22}$$

Solving for $B^o(z;P,Q,W)$ from (A.22) and substituting the resulting expression into (A.21) yield

$$\frac{1}{\beta}\frac{\tilde{\pi}(z;P,Q,W)}{\rho-\mu}\bar{Y} - \left(\frac{\tilde{\pi}(z;P,Q,W)}{\rho-\mu}\bar{Y} - \frac{Wf}{\rho}\right) + WF = 0. \tag{A.23}$$

Equivalence between the solution of the planner's problem and myopic firm. To show that conditions (A.18) and (A.23) are equivalent, it suffices to show that $T'_M(M,Y)$ in (A.18) is equal to the profit flow of the myopic firm, $\frac{\tilde{\pi}(z;P,Q,W)}{\rho-\mu}Y - \frac{Wf}{\rho}$.

The net social surplus is given by the planner's utility minus the total cost of production,

$$S(M,Y) = Y\left[\int_\Omega q(z)^{\frac{\eta-1}{\eta}}dz\right]^{\frac{\eta}{\eta-1}} - \int_\Omega \left[\frac{\tau W}{z}q(z) - Wf\right]dz. \tag{A.24}$$

For each market, the set of active firms Ω is an interval $[\,\bar{z},\infty\,)$, where $\bar{z}$ denotes the productivity of the least productive active firm. Let M denote the measure of Ω. Hence,

$$T'_M(M,Y) = -\frac{\partial T(M,Y)}{\partial \bar{z}} = \left(\frac{\eta}{\eta-1}\right)q(\bar{z})^{\frac{\eta-1}{\eta}}\frac{Q^{\frac{1}{\eta}}Y}{\rho-\mu} - \frac{\tau W}{\bar{z}}\frac{q(\bar{z})}{\rho-\mu} - \frac{Wf}{\rho}, \tag{A.25}$$

and the planner chooses the optimal quantity $q(\bar{z})$ to maximize (A.25). The first order condition of that problem is

$$\left(\frac{q(\bar{z})}{Q}\right)^{-\frac{1}{\eta}}Y - \frac{\tau W}{\bar{z}} = 0, \tag{A.26}$$

which substituting back into (A.25) yields

$$-\frac{\partial T(M,Y)}{\partial \bar{z}} = \left(\frac{1}{\eta-1}\right)\frac{\tau W}{\bar{z}}\frac{q(\bar{z})}{\rho-\mu} - \frac{Wf}{\rho}. \tag{A.27}$$

Similarly, the present discounted value of the optimal profit of the least productive firm can be written as

$$\begin{aligned}\frac{\tilde{\pi}(\bar{z};P,Q,W)}{\rho-\mu}Y - \frac{Wf}{\rho} &= \frac{p(\bar{z})q(\bar{z})}{\rho-\mu} - \frac{\tau W}{\bar{z}}\frac{q(\bar{z})}{\rho-\mu} - \frac{Wf}{\rho}\\ &= \left(\frac{1}{\eta-1}\right)\frac{\tau W}{\bar{z}}\frac{q(\bar{z})}{\rho-\mu} - \frac{Wf}{\rho},\end{aligned} \tag{A.28}$$

which coincide with the profit of a myopic firm. The equivalence between (A.18) and (A.23) follows from noting that if $T'_M(M,Y) = \frac{\tilde{\pi}(z;P,Q,W)}{\rho-\mu}Y - \frac{Wf}{\rho}$, then $\frac{Y}{\beta}T''_{M,Y}(M,Y) = \frac{1}{\beta}\frac{\tilde{\pi}(z;P,Q,W)}{\rho-\mu}Y$ —the first term in (A.23), and from equating the sunk cost $\tilde{F}$ to its value in terms of labor, WF.

A.3 Propositions 1 and 2

Propositions 1 and 2 rely on assuming that the fixed costs of affiliate operations are "small," so that there is no endogenous exit. Under this assumption, the option-value terms $A^h_{ij}(z)$ and $A^e_{ijk}(z)$ in (19) and (21) are zero. Hence, we can obtain closed-form solutions for the affiliate entry and export entry thresholds,

$$\bar{Y}^h_{ij}(z) = \left(\frac{\beta_{ij}}{\beta_{ij}-1}\right)\left(\frac{W_j f_{ijj} + \rho W_j F_{ijj}}{\rho} - V^e_{ij}(z, \mathbf{Y}_{i,-j})\right)\left(\frac{\rho-\mu_{ij}}{\tilde{\pi}_{ijj}(z)}\right), \tag{A.29}$$

$$\bar{Y}^e_{ijk}(z) = \left(\frac{\beta_{ik}}{\beta_{ik}-1}\right)\left(\frac{W_j f_{ijk} + \rho W_j F_{ijk}}{\rho}\right)\left(\frac{\rho-\mu_{ik}}{\tilde{\pi}_{ijk}(z)}\right), \tag{A.30}$$

where $V^e_{ij}(z, \mathbf{Y}_{i,-j})$ denotes the total value of exports from an affiliate in j of a firm from i with productivity z,

$$V^e_{ij}(z, \mathbf{Y}_{i,-j}) = \sum_{k \in \mathcal{E}_{ij}(z)} \left[\frac{\tilde{\pi}_{ijk}(z; P, Q, W)}{\rho-\mu_{ik}} Y_{ik} - \frac{W_j f_{ijk}}{\rho}\right] + \sum_{k \notin \mathcal{E}_j(z)} \left[B^o_{jk}(z) Y_k^{\beta_k}\right]. \tag{A.31}$$

Notice that if $W_j(f_{ijj} + \rho F_{ijj})/\rho - V^e_{ij}(z, \mathbf{Y}_{i,-j}) < 0$, then $\bar{Y}^h_{ij}(z) < 0$. In this case, a firm from i with productivity z opens an affiliate in j for any realization of Y_{ij} because the value of its potential export network is larger than the cost of opening the affiliate.

Additionally, the option value terms are

$$B^o_{ij}(z) = \beta_{ij}^{-\beta_{ij}} (\beta_{ij}-1)^{\beta_{ij}-1} \left(\frac{W_j f_{ijj} + \rho W_j F_{ijj}}{\rho} - V^e_{ij}(z, \mathbf{Y}_{i,-j})\right)^{1-\beta_{ij}} \left(\frac{\tilde{\pi}_{ijj}(z)}{\rho-\mu_{ij}}\right)^{\beta_{ij}} \tag{A.32}$$

$$B^o_{ijk}(z) = \beta_{ik}^{-\beta_{ik}} (\beta_{ik}-1)^{\beta_{ik}-1} \left(\frac{W_j f_{ijk} + \rho W_j F_{ijk}}{\rho}\right)^{1-\beta_{ik}} \left(\frac{\tilde{\pi}_{ijk}(z)}{\rho-\mu_{ik}}\right)^{\beta_{ik}}. \tag{A.33}$$

Proposition 1. *For a given triplet (i, j, k), more productive firms have lower entry thresholds into a host market and lower entry thresholds into export markets: $\partial \bar{Y}^h_{ij}(z)/\partial z \leq 0$ and $\partial \bar{Y}^e_{ijk}(z)/\partial z \leq 0$.*

Proof. First, we notice that $\frac{\partial \tilde{\pi}_{ijk}(z)}{\partial z} > 0$ and $\frac{\partial V^e_{ij}(z, \mathbf{Y}_{i,-j})}{\partial z} > 0$. Taking the derivative in (A.30) yields the result,

$$\frac{\partial \bar{Y}^e_{ijk}(z)}{\partial z} = \underbrace{\left(\frac{\beta_{ik}}{\beta_{ik}-1}\right)\left(\frac{W_j f_{ijk} + \rho W_j F^e_{ijk}}{\rho}\right)}_{\geq 0} \underbrace{\left(-\frac{\rho-\mu_{ik}}{\tilde{\pi}_{ijk}(z)^2}\right)}_{<0} \underbrace{\frac{\partial \tilde{\pi}_{ijk}(z)}{\partial z}}_{>0} \leq 0.$$

For the affiliate entry threshold, taking the derivative in (A.29) yields

$$\begin{aligned}\frac{\partial \bar{Y}_{ij}^{h}(z)}{\partial z} &= \underbrace{\left(\frac{\beta_{ij}}{\beta_{ij}-1}\right)}_{>0}\left\{\underbrace{\frac{-\partial V_{ij}^{e}(z,\mathbf{Y}_{i,-j})}{\partial z}\frac{\rho-\mu_{ij}}{\tilde{\pi}_{ijj}(z)}}_{<0}+\underbrace{\left(\frac{W_j f_{ijj}+\rho W_j F_{ijj}}{\rho}-V_{ij}^{e}(z,\mathbf{Y}_{i,-j})\right)}_{\geq 0}\underbrace{\left(-\frac{\rho-\mu_{ij}}{\tilde{\pi}_{ijj}(z)^2}\right)\frac{\partial \tilde{\pi}_{ijj}(z)}{\partial z}}_{<0}\right\} \\ &\leq 0.\end{aligned}$$

Notice that we compute the derivative of (A.29) for the case of $(W_j f_{ijj}+\rho W_j F_{ijj})/\rho - V_{ij}^{e}(z,\mathbf{Y}_{i,-j}) > 0$, so that $\bar{Y}_{ij}^{h}(z) > 0$ and the entry problem is well-defined.

Corollary 1. *More productive firms enter more export markets.*

Proof. $\frac{\partial\left[prob\left\{y\geq \bar{Y}_{ij}^{h}(z)\right\},\forall j\right]}{\partial z} = \frac{\partial\left[prob\left\{y\geq \bar{Y}_{ij}^{h}(z)\right\},\forall j\right]}{\partial \bar{Y}_{ij}^{h}(z)}\frac{\partial \bar{Y}_{ij}^{h}(z)}{\partial z}$ where the first term is negative and Proposition 1 implies that the second term is weakly negative, so that $\frac{\partial\left[prob\left\{y\geq \bar{Y}_{ij}^{h}(z)\right\},\forall j\right]}{\partial z} \geq 0$.

Corollary 2. *The mass of firms having affiliates in n host markets is decreasing in n.*

Proof. Without loss of generality, let us assume that all firms only sell in country i and are considering whether and where to open affiliates ($t = 1$). Let us order the productivity thresholds needed to open an affiliate in a market in ascending order, and let $\bar{Z}_n^h$ ($\bar{Z}_{n-1}^h$) denote the maximum among the first n ($n-1$) productivity thresholds, so that $\bar{Z}_n^h \geq \bar{Z}_{n-1}^h$. Let M_{ij} denote the mass of firms that at $t = 1$ open affiliates in country j: $M_{ij} = \int_{\bar{Z}_{ij}^h}^{\infty} dG(z)$, so $\frac{\partial M_{ij}}{\partial \bar{Z}_{ij}^h} \leq 0$. Let $\bar{M}_n$ ($\bar{M}_{n-1}$) denote the mass of firms that at $t = 1$ open affiliates in the n ($n-1$) countries with the lowest productivity thresholds. Then, $\bar{Z}_n^h \geq \bar{Z}_{n-1}^h$ and $\frac{\partial M_{ij}}{\partial \bar{Z}_{ij}^h} \leq 0$ implies that $\bar{M}_n \leq \bar{M}_{n-1}, \forall n$.

Proposition 2. *For a firm with productivity z, the affiliate entry threshold is increasing in the iceberg cost in the host market and in the sunk cost of affiliate opening: $\partial \bar{Y}_{ij}^h(z)/\partial \tau_{ijj} \geq 0$ and $\partial \bar{Y}_{ij}^h(z)/\partial F_{ijj} \geq 0$. Similarly, the affiliate export entry threshold is increasing in the iceberg cost in the destination market and in the sunk cost of affiliate export: $\partial \bar{Y}_{ijk}^e(z)/\partial \tau_{ijk} \geq 0$ and $\partial \bar{Y}_{ijk}^e(z)/\partial F_{ijk} \geq 0$.*

Proof. Affiliate's variable profits are decreasing in the MNE efficiency loss, $\frac{\partial \tilde{\pi}_{ijj}(z)}{\partial \tau_{ijj}} < 0$. Taking the derivative with respect to τ_{ijj} in (A.29) yields

$$\frac{\partial \bar{Y}_{ij}^{h}(z)}{\partial \tau_{ijj}} = \underbrace{\frac{\beta_{ij}}{\beta_{ij}-1}\left(\frac{W_j f_{ijj}+\rho W_j F_{ijj}}{\rho}-V_{ij}^{e}(z,\mathbf{Y}_{i,-j})\right)}_{\geq 0}\underbrace{\left(-\frac{\rho-\mu_{ij}}{\tilde{\pi}_{ijj}(z)^2}\right)}_{<0}\underbrace{\left(\frac{\partial \tilde{\pi}_{ijj}(z)}{\partial \tau_{ijj}}\right)}_{<0} \geq 0,$$

where we compute the derivative of (A.29) for the case that $(W_j f_{ijj}+\rho W_j F_{ijj})/\rho - V^e_{ij}(z,\mathbf{Y}_{i,-j}) > 0$, so that $\bar{Y}^h_{ij}(z) > 0$ and the entry problem is well-defined. Additionally, taking the derivative with respect to F_{ijj} in (A.29) yields

$$\frac{\partial \bar{Y}^h_{ij}(z)}{\partial F_{ijj}} = \frac{\beta_{ij}}{\beta_{ij}-1} W_j \left(\frac{\rho-\mu_{ij}}{\tilde{\pi}_{ijj}(z)}\right) > 0.$$

Similarly for the affiliate export entry threshold:

$$\begin{aligned}
\frac{\partial \bar{Y}^e_{ijk}(z)}{\partial \tau_{ijk}} &= \underbrace{\frac{\beta_{ik}}{\beta_{ik}-1}\left(\frac{W_j f_{ijk}+\rho W_j F_{ijk}}{\rho}\right)}_{\geq 0} \underbrace{\left(-\frac{\rho-\mu_{ik}}{\tilde{\pi}_{ijk}(z)^2}\right)}_{<0} \underbrace{\left(\frac{\partial \tilde{\pi}_{ijk}(z)}{\partial \tau_{ijk}}\right)}_{<0} \geq 0 \\
\frac{\partial \bar{Y}^e_{ijk}(z)}{\partial F_{ijk}} &= \frac{\beta_{ik}}{\beta_{ik}-1} W_j \left(\frac{\rho-\mu_{ik}}{\tilde{\pi}_{ijk}(z)}\right) > 0.
\end{aligned}$$

A.4 Connectedness of the sets $\Omega_{ijk}(t)$

The sets $\Omega_{ijk}(t)$ and $\Omega_{ij}(t)$ are defined as

$$\begin{aligned}
\Omega_{ijk}(t) &= \Big\{z : z \in \Big\{\Omega_{ijk}(t-\Delta t) \bigcap \left[\underline{z}^e_{ijk}(t),\infty\right)\Big\} \bigcup \\
&\quad \Big\{(\Omega_{ij}(t-\Delta t)\backslash\Omega_{ijk}(t-\Delta t)) \bigcap \left[\bar{z}^e_{ijk}(t),\infty\right)\Big\}\Big\},
\end{aligned} \tag{A.34}$$

$$\begin{aligned}
\Omega_{ij}(t) &= \Big\{z : z \in \Big\{\Omega_{ij}(t-\Delta t) \bigcap \Big[\underline{z}^h_{ij}(t),\infty\Big)\Big\} \bigcup \\
&\quad \Big\{(\Omega_{i}(t-\Delta t)\backslash\Omega_{ij}(t-\Delta t)) \bigcap \Big[\bar{z}^h_{ij}(t),\infty\Big)\Big\}\Big\},
\end{aligned} \tag{A.35}$$

with $\Omega_i(t) = [1,\infty)$.

Proposition 3. *Let the support of the distribution $G(z)$ be equal to $[1,\infty)$. Then,*

$$\Omega_{ij}(t) = \Big[\tilde{z}^h_{ij}(t),\infty\Big) \qquad \Omega_{ijk}(t) = \left[\tilde{z}^e_{ijk}(t),\infty\right),$$

where

$$\tilde{z}^h_{ij}(t) = \begin{cases} \bar{z}^h_{ij}(t) & \text{if } \tilde{z}^h_{ij}(t-1) \geq \bar{z}^h_{ij}(t) \\ \max\{\tilde{z}^h_{ij}(t-1), \underline{z}^h_{ij}(t)\} & \text{if } \tilde{z}^h_{ij}(t-1) < \bar{z}^h_{ij}(t) \end{cases},$$

and similarly,

$$\tilde{z}^e_{ijk}(t) = \begin{cases} \bar{z}^e_{ijk}(t) & \text{if } \tilde{z}^e_{ijk}(t-1) \geq \bar{z}^e_{ijk}(t) \\ \max\{\tilde{z}^e_{ijk}(t-1), \underline{z}^e_{ijk}(t)\} & \text{if } \tilde{z}^e_{ijk}(t-1) < \bar{z}^e_{ijk}(t) \end{cases}.$$

Proof. We show the proof for $\Omega_{ij}(t)$. The one for $\Omega_{ijk}(t)$ is analogous. We present the proof under discrete time to ease the notation. The proof proceeds by induction. Since the mass of firms is given, $\Omega_i(t) = [1, \infty)$, $\forall t$. At the beginning of time, all firms are domestic, so $\Omega_{ij}(0) = \emptyset$. At $t = 1$,

$$\Omega_{ij}(1) = \left\{ z : z \in \left\{ \Omega_{ij}(0) \bigcap \left[\underline{z}_{ij}^h(1), \infty \right) \right\} \bigcup \left\{ [1, \infty) \bigcap \left[\bar{z}_{ij}^h(1), \infty \right) \right\} \right\}.$$

At $t = 2$,

$$\Omega_{ij}(2) = \left\{ z : z \in \left\{ \Omega_{ij}(1) \bigcap \left[\underline{z}_{ij}^h(2), \infty \right) \right\} \bigcup \left\{ \left[1, \bar{z}_{ij}^h(1) \right] \bigcap \left[\bar{z}_{ij}^h(2), \infty \right) \right\} \right\}.$$

Let $\bar{\underline{z}}_{ij}^h(1,2) \equiv \max\{\bar{z}_{ij}^h(1), \underline{z}_{ij}^h(2)\}$. Then,

$$\Omega_{ij}(2) = \left\{ z : z \in \left[\bar{\underline{z}}_{ij}^h(1,2), \infty \right) \bigcup \left\{ \left[1, \bar{z}_{ij}^h(1) \right] \bigcap \left[\bar{z}_{ij}^h(2), \infty \right) \right\} \right\},$$

which leads to

$$\Omega_{ij}(2) = \begin{cases} \left[\bar{z}_{ij}^h(2), \infty \right) & \text{if } \bar{z}_{ij}^h(1) \geq \bar{z}_{ij}^h(2) \\ \left[\bar{\underline{z}}_{ij}^h(1,2), \infty \right) & \text{if } \bar{z}_{ij}^h(1) < \bar{z}_{ij}^h(2) \end{cases}.$$

Let $\tilde{z}_{ij}^h(2)$ denote the lower bound of $\Omega_{ij}(2)$. Then, at $t = 3$,

$$\Omega_{ij}(3) = \left\{ z : z \in \left\{ \Omega_{ij}(2) \bigcap \left[\underline{z}_{ij}^h(3), \infty \right) \right\} \bigcup \left\{ \left[1, \tilde{z}_{ij}^h(2) \right] \bigcap \left[\bar{z}_{ij}^h(3), \infty \right) \right\} \right\}.$$

Let $\bar{\underline{z}}_{ij}^h(2,3) \equiv \max\{\tilde{z}_{ij}^h(2), \underline{z}_{ij}^h(3)\}$. Then, following the same procedure as above,

$$\Omega_{ij}(3) = \left\{ z : z \in \left[\bar{\underline{z}}_{ij}^h(2,3), \infty \right) \bigcup \left\{ \left[1, \tilde{z}_{ij}^h(2) \right] \bigcap \left[\bar{z}_{ij}^h(3), \infty \right) \right\} \right\},$$

which leads to

$$\Omega_{ij}(3) = \begin{cases} \left[\bar{z}_{ij}^h(3), \infty \right) & \text{if } \tilde{z}_{ij}^h(2) \geq \bar{z}_{ij}^h(3) \\ \left[\bar{\underline{z}}_{ij}^h(2,3), \infty \right) & \text{if } \tilde{z}_{ij}^h(2) < \bar{z}_{ij}^h(3) \end{cases}.$$

Since $\tilde{z}_{ij}^h(1) = \bar{z}_{ij}^h(1)$, one obtains the result.