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Comparative Advantage and Economies of Scale under Uncertainty

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Keywords: trade, uncertainty, economies of scale, specialization
JEL Classification: F1, F2

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1 Introduction

Trade creates a fundamental tension between the efficiency gains from sectoral specialization and the insurance benefits of cross-country diversification. On the one hand, openness encourages countries and regions to specialize in the activities where they hold a comparative advantage, raising aggregate productivity and welfare. On the other hand, specialization can make economies more vulnerable to volatility by exposing them to sector- or country–specific shocks (Koren and Tenreyro 2007, 2013, Carvalho and Gabaix 2013). At the same time, international trade also provides access to diversified sources of supply and demand, which can mitigate domestic risk (Backus, Kehoe and Kydland 1992, Fitzgerald 2012, Caselli, Koren, Lisicky and Tenreyro 2020). This tension has crystalized in recent debates on the resilience of global value chains, and the vulnerability of specialized economies to aggregate shocks, such as the COVID-19 pandemic and the U.S. tariff war.

We study how uncertainty interacts with comparative advantage, economies of scale, and worker heterogeneity, to shape spatial specialization across and within countries, and hence, the gains from trade. We use the model to quantitatively analyze the type of globalization shocks studied by Autor, Dorn and Hanson (2013), and propose an extension of hat-algebra methods to calibrate the spatial model under uncertainty. Uncertainty shifts employment away from riskier sectors and locations, relative to a deterministic benchmark, lowering the U.S. aggregate gains from trade by at least a third. Some regions, however, lose from trade, an effect mitigated by the presence of economies of scale, and worker heterogeneity.

We start by building a multi-sector multi-region multi-country model where uncertainty comes from a country-sector specific productivity shock, which affects all regions in a country. Sectors differ in their degree of economies of scale, and regions differ in their geography and fundamental productivities. Consumers are risk averse and face financial autarky, so that trade is balanced each period. The model has two periods: before and after uncertainty is realized. Irreversible decisions are made in terms of labor mobility: labor allocations are decided before uncertainty is realized by workers with heterogeneous productivities, modeled as a standard Roy-Fréchet (Lagakos and Waugh 2013). Production, consumption, and international trade happen once uncertainty is realized, and both factor and good prices adjust to clear all markets.

The gains from trade, and with it, labor reallocation, are shaped by the interaction of comparative advantage, uncertainty, economies of scale, and the degree of workers' heterogeneity. With only the forces of comparative advantage, we recover the multi-sector ACR formula (Arkolakis, Costinot and Rodríguez-Clare 2012). If we add workers' heterogeneity, our gains from trade are as in Galle, Rodríguez-Clare and Yi (2023), while if we add economies of scale, we get an expression for the gains from trade similar to Kucheryavyy, Lyn and Rodríguez-Clare (2020).[1] The inclusion of uncertainty modifies the expression for the gains from trade relative to the models in the papers above.

Our analytical results, which we derive for the special case of frictionless trade and independent shocks across locations, show that labor tends to move away from riskier locations, and the effect is amplified the stronger the economies of scale and the higher the exposure to trade. These reallocation effects occur when there is a trade shock. In particular, a move from autarky can create loses, not gains, from trade for some locations: This occurs if labor moves to sectors with lower economies of scale. Worker heterogeneity could offset the effects as it creates less elastic responses of labor supply to shocks.

Before moving to our quantitative exercises, we test the predictions of the model using data for the United States on commuting zones (CZ) and industries, over the period 1990-2010. We document that CZs where industries became less riskier did not lose — or lost less — employment than riskier CZs. However, the negative effect of sectoral uncertainty on employment growth in an industry and CZ is driven by industries with strong economies of scale.

To calibrate the model, we use data on 26 countries, plus an aggregate for the rest of the world, and 20 sectors, spanning at least 20 years over the period 1970-2019. For the United States, we construct 721 CZs, following Autor et al. (2013), with 20 sectors each, and a time series that spans several decades all the way till 2017. For that year, we use the Commodity Flow Survey to construct sectoral trade flows between CZs, following procedures in the literature (e.g. Allen and Arkolakis 2014).

We first set some parameters externally, more saliently the sectoral trade elasticities and sectoral economies of scale, using estimates from Bartelme, Costinot, Donaldson and Rodríguez-Clare (2025), and the migration elasticity, from Fajgelbaum, Morales, Serrato and Zidar (2019).

We use a simulated method of moment procedure to jointly estimate the risk aversion parameter and the volatility of the country-industry shocks. To such end, we target the

[1] We restrict the model's parameters to the case of a unique equilibrium. When we compare results with Kucheryavyy et al. (2020), we refer to their results derived under that restriction.

responses of employment to uncertainty across U.S. CZs, given by our reduced-form estimates, and the country-sector standard deviation of log value-added per worker observed in the data in 1970-2019. Our procedure adapts hat-algebra methods (Dekle, Eaton and Kortum 2008) to accommodate the presence of uncertainty.

Concretely, we treat the cross-sectional data for 2017 as representing the average economy in the model. Because employment shares are chosen ex-ante, they do not vary with the realization of shocks. In a first step, conditional on these observed shares, we simulate the economy under many draws of shocks and compute equilibrium changes relative to the average economy. This procedure generates a distribution of outcomes that allows us to calculate the variance of real value added per worker at the country–sector level and compare it with its empirical counterpart.

This first step of the algorithm does not involve the risk-aversion parameter. We calibrate this parameter in the second step, where ex-ante employment shares are now used. To discipline risk aversion, we exploit the fact that changes in employment shares must reflect variation in shock volatility—precisely what our reduced-form regressions capture. Accordingly, we hold fixed the volatility of shocks in the rest of the world (as estimated in step one), and vary U.S. volatility across sub-periods 1970–1990 and 1990–2010 to match the observed volatilities of real value added per worker. Using hat-algebra methods, we then compute the implied changes in region–sector employment shares between the U.S. economy with volatility as observed in 1970-2019 and volatility observed in each sub-period (one at the time).[2] This strategy enables us to run the same regression in the model as in the data, relating changes in the volatility of real value added per worker to changes in employment shares across U.S. regions and industries.

Our quantitative results suggest that introducing both uncertainty and sectoral economies of scale substantially alters the size and distribution of the gains from trade. At the aggregate level, the United States gains about 1.5 percent from opening up to trade in the baseline model, while eliminating uncertainty raises gains more than 50 percent to over 2.3 percent. Removing both uncertainty and scale effects increases gains further to 2.4 percent. These averages, however, mask large regional heterogeneity. Some CZs lose from trade in the baseline calibration, while under a no-uncertainty scenario all regions gain. Moreover, scale economies are key for ensuring positive gains from trade everywhere: without them, certain high-volatility regions experience losses. Employment reallocation

[2]Conditional on a volatility guess for the United States in a given sub-period, we apply the same hat-algebra approach as in step one, computing equilibrium changes from the average economy (2017) to each realization of shocks.

responses mirror these patterns. From autarky to the baseline equilibrium, employment shifts away from high-volatility sectors and regions, with average losses of around 0.8 percent, but these changes decrease when uncertainty or scale economies are removed. Overall, the results highlight that uncertainty and scale effects jointly cut U.S. trade gains by around half, while also shaping the uneven regional distribution of those gains.

Literature Review. Our work is closest to research on multi-sector spatial models with economies of scale (Kucheryavyy et al. 2020), worker heterogeneity (Galle et al. 2023), and uncertainty (Caselli et al. 2020). Relative to the first two papers, our innovation is to introduce uncertainty together with an irreversible investment choice, a feature shared with several papers discussed below. Relative to Caselli et al. (2020), our distinct contribution is to embed the irreversible decision under uncertainty into a spatial model with sub-national geography, sectoral scale economies, and heterogeneous workers. The interaction of uncertainty and sectoral scale economies generates new effects, which are quantitatively relevant, while worker heterogeneity shapes the distributional implications of volatility across regions and industries.

A central element of our model is the treatment of trade under uncertainty in combination with sunk or irreversible factor allocations. Several models adopt this approach, including Caselli et al. (2020), Allen and Atkin (n.d.), Castro-Vincenzi, Khanna, Morales and Pandalai-Nayar (2024), and Fan and Luo (2025), among others.[3] These papers emphasize that producers must commit to allocations before uncertainty is realized, and therefore trade openness changes outcomes. For example, Allen and Atkin (n.d.) show that openness reduces the negative correlation between productivity and prices, lowering revenue volatility, with farmers adjusting by shifting toward less volatile crops. In their setting, as in ours, trade raises wage volatility, reduces price volatility, and generates ambiguous effects on real wage volatility. More broadly, this literature highlights the central tension that trade can increase volatility through specialization while reducing it through diversification—a point emphasized by Caselli et al. (2020), who, using a general equilibrium model based on Eaton and Kortum (2002) and Caliendo and Parro (2015), show that the net effect depends on the source of shocks.[4] Our contribution is to embed this mechanism

[3] A related literature studies how volatility affects exporters' decisions, such as Handley and Limão (2017), *Brexit and the macroeconomic impact of trade policy uncertainty* (2019), Alessandria, Arkolakis and Ruhl (2021), Alessandria, Choi and Ruhl (2021), Grossman, Helpman and Lhuillier (2023), and Alessandria, Khan, Khederlarian, Ruhl and Steinberg (2025).

[4] Caselli et al. (2020) shows that when country specific shocks are dominant, in contrast to sector specific shocks, trade can decrease volatility. We also show analytically that with sector-country specific shocks, trade openness can decrease volatility.

into a rich spatial setting where sectors differ in their degree of economies of scale and workers have heterogeneous comparative advantages across sectors and regions.

Our solution method builds on the "exact hat algebra" approach widely used in quantitative trade models (e.g., Dekle et al. 2008). We extend this methodology to accommodate uncertainty by computing equilibrium changes not only across deterministic and average counterfactual scenarios but also across different realizations of stochastic shocks. In practice, this requires combining hat algebra with simulation techniques, generating a distribution of equilibrium outcomes for each realization of the shocks, which can be used to build moments to compare with data. In this sense, our work relates to recent advances in dynamic hat algebra methods in spatial economics (Caliendo, Dvorkin and Parro 2019, Kleinman, Liu and Redding 2025, Fan and Luo 2025, Caliendo, Kortum and Parro 2025), which adapt the hat algebra framework to richer environments with dynamics, regional mobility, and aggregate shocks. Our approach further expands this toolkit.

Finally, our paper contributes to the trade literature on regional economies. Following Autor et al. (2013), many empirical studies have examined the effects of a large aggregate shock—most notably the "China shock"—on local labor markets (see Autor, Dorn and Hanson 2016, for a review). This work has inspired a growing quantitative literature integrating regional economies into international trade models to study the local impact of aggregate shocks (e.g., Caliendo, Parro, Rossi-Hansberg and Sarte 2019, Galle et al. 2023, Lyon and Waugh 2019, Caliendo, Dvorkin and Parro 2019). Our contribution is to analyze how aggregate volatility—rather than a one-time trade shock—shapes workers' regional and sectoral choices, and how this margin affects the gains from trade.

Our paper is structured as follows: Section 2 presents the model. In Section 3, we derive analytical results, and we provide empirical evidence on the model's mechanisms in Section 4. Section 5 presents the model's calibration, followed by the quantitative analysis in Section 6. Finally, Section 7 concludes.

2 Model

In this section, we present a multi-sector spatial equilibrium model with aggregate uncertainty, economies of scale, and workers' region and sector choices.

2.1 Preliminaries

The world consists of M regions, $m = 1, \ldots, M$, belonging to N countries, $i = 1, \ldots, N$. The number of regions belonging to country i is denoted by the set M_i, with $\cup_i M_i = M$. Each country is endowed with a measure L_i of workers. Workers are immobile across countries but mobile across regions of a country.

Each region has J sectors, $j = 1, \ldots, J$. Each sector is subject to a country-wide productivity shock, $A_{i,j}(s)$, where s is a possible realization of the shock, $s = \{1, 2, \ldots, S\}$, characterized by the probability $\mathbb{P}(s)$. Without loss of generality, we normalize the average shock to one for each country and sector, $\sum_s \mathbb{P}(s) A_{i,j}(s) = 1$, for all i, j, so that the variance is given by $\sigma_{i,j}^2 = \sum_s \mathbb{P}(s) \left(A_{i,j}(s) - 1\right)^2$.

2.2 Workers

Workers make location and sector choices *before* uncertainty is realized. Once shocks are observed, they supply labor, produce, and consume. Workers are heterogeneous in their ability to work in different sectors and regions within a country i. The variable x_{mj} represents the efficiency units of labor in sector j of region m, is random, and drawn independently from a unit-Fréchet distribution with shape parameter $\kappa > 1$.

We abstract from savings, so that workers' real consumption in region m and sector j is given by their real income, $x_{mj} W_{mj}(s)/P_m(s)$, where $W_{mj}(s)$ is the wage rate per efficiency unit in region m, sector j, and state s, and $P_m(s)$ is the final-good price in region m and state s.

Workers have Constant-Relative-Risk-Aversion (CRRA) utility functions with relative risk-aversion coefficient $\gamma > 1$. For a worker in region m and sector j, the expected utility per efficiency unit (i.e. $x_{mj} = 1$) is given by

$$U_{mj} = \frac{1}{1-\gamma} \sum_s \mathbb{P}(s) \left(\frac{W_{mj}(s)}{P_m(s)} \right)^{1-\gamma}. \tag{1}$$

In equilibrium, each worker chooses the region-sector pair that delivers the highest utility. With the Fréchet distributional assumption, the probability of a worker in country i

choosing to locate and work region m and sector j is

$$\lambda_{mj} = \frac{U_{mj}^{\frac{\kappa}{1-\gamma}}}{\sum_{j'} \sum_{m' \in M_i} U_{m'j'}^{\frac{\kappa}{1-\gamma}}}. \tag{2}$$

Total labor units in each region-sector pair are then given by

$$L_{mj} = C_1 L_i \lambda_{mj}^{1-\frac{1}{\kappa}}, \tag{3}$$

where $C_1 = \Gamma\left(1 - \frac{1}{\kappa}\right) > 0$, and $\Gamma(\cdot)$ is the Gamma function.

Workers cannot move after the shock is realized. But because the wage in local labor markets adjusts after uncertainty is realized, there is no unemployment in the model.

2.3 Final Goods

The final non-tradable good in region m is produced using goods from all sectors in the region,

$$Q_m(s) = \prod_j Q_{mj}(s)^{\alpha_j}, \tag{4}$$

where $\alpha_j \in [0, 1]$ is the share of expenditure in intermediate goods from sector j, and $\sum_j \alpha_j = 1$. The price index for the final good in region m is

$$P_m(s) = \prod_j \left(\frac{P_{mj}(s)}{\alpha_j}\right)^{\alpha_j}, \tag{5}$$

where $P_{mj}(s)$ is the price index for the intermediate good from sector j.

2.4 Intermediate Goods

Each sector j, region $m \in M_i$, produces a non-tradable intermediate good under perfect competition. The sectoral intermediate good is composed of a continuum of tradable varieties $\omega \in [0, 1]$. The production technology is Constant-Elasticity-of-Substitution (CES) with elasticity of substitution $\eta_j > 1$,

$$Q_{mj}(s) = \left(\int_0^1 q_{mj}(\omega, s)^{\frac{\eta_j - 1}{\eta_j}} d\omega\right)^{\frac{\eta_j}{\eta_j - 1}}, \tag{6}$$

where $q_{mj}(\omega, s)$ is the quantity demanded of variety ω in state s.

Production of variety ω belonging to sector j in region m is given by

$$y_{mj}(\omega, s) = A_{mj}(s) z_m(\omega) l_{mj}(\omega, s), \tag{7}$$

where $y_{mj}(\omega, s)$ is output of variety ω in region m, sector j, and state s, $A_{mj}(s)$ is aggregate productivity of sector j in region m and state s, $z_m(\omega)$ is variety-level productivity, and $l_{mj}(\omega, s)$ is the number of efficiency units used in production.

We make the following assumptions. First, $z_m(\omega)$ is randomly drawn independently across regions and varieties from a unit- Fréchet distribution with shape parameter $\theta_j > 0$. Second, aggregate productivity is given by

$$A_{mj}(s) \equiv \bar{A}_{mj} A_{i,j}(s), \tag{8}$$

where $\bar{A}_{mj}$ is the fundamental productivity of sector j in region m, and $A_{i,j}(s)$ is the country-sector specific shock. Hence, the volatility of a regional-industry shock is just given by $v_{mj} = \bar{A}_{mj}\sigma_{i,j}$.

We introduce economies of scale through $\bar{A}_{mj}$, as specified next.

Assumption 1 (Economies of scale). $\bar{A}_{mj} = C_{mj} L_{mj}^{\rho_j}$ *where* $\rho_j \geq 0$.

Fundamental region-sector productivity is log-proportional to the amount of labor in that region and sector, L_{mj}. The parameter ρ_j governs the degree of economies of scale in the sector. When $\rho_j = 0$ there is no economies of scale and fundamental productivity is not endogenous to the amount of labor allocated to industry j. When $\rho_j > 0$ productivity increases with the amount of labor in the industry, reflecting industry-level external economies of scale. The region-sector fixed effect, C_{mj}, captures any other factor affecting region-sector fundamental productivity.

We keep track of the following combination of parameters,

$$\tilde{\rho}_j \equiv \frac{\kappa - 1}{\kappa} \rho_j \theta_j,$$

which delivers the total effect of economies of scale.

Finally, we assume iceberg-type trade costs from region l to m, $\tau_{lm}^j \geq 1$, with $\tau_{mm}^j = 1$.

Producers of the composite intermediate good in sector j and region m source each variety ω from the cheapest world-wide supplier,

$$p_{mj}(\omega, s) = \min_{l \in M} \frac{\tau_{lm}^j W_{lj}(s)}{A_{lj}(s) z_l(\omega)}. \tag{9}$$

The share of expenditure in sector j by region m devoted to goods from region l, in state s, is given by

$$\pi_{lm,i}^j(s) = \frac{(\tau_{lm}^j W_{lj}(s)/A_{i,j}(s))^{-\theta_j} \tilde{C}_{lj} \lambda_{lj}^{\tilde{\rho}_j} L_i^{\rho_j \theta_j}}{\sum_{i'} \sum_{l' \in M_{i'}} (\tau_{l'm}^j W_{l'j}(s)/A_{i',j}(s))^{-\theta_j} \tilde{C}_{l'j} \lambda_{l'j}^{\tilde{\rho}_j} L_{i'}^{\rho_j \theta_j}}. \tag{10}$$

where $\tilde{C}_{lj} \equiv C_{lj}^{\theta_j} C_1^{\rho_j \theta_j}$.

The price index for the sector j's intermediate good is

$$P_{mj,i}(s) = C_j \left(\sum_{i'} \sum_{l \in M_{i'}} \left(\tau_{lm}^j \frac{W_{lj}(s)}{A_{i',j}(s)} \right)^{-\theta_j} \tilde{C}_{lj} \lambda_{lj}^{\tilde{\rho}_j} L_{i'}^{\rho_j \theta_j} \right)^{-\frac{1}{\theta_j}}, \tag{11}$$

with $C_j \equiv \Gamma \left(1 - \frac{\eta_j - 1}{\theta_j}\right)^{\frac{1}{1-\eta_j}}$ and $1 + \theta_j - \eta_j > 0$, for all j.

2.5 Equilibrium

We can solve the model's equilibrium backwards. Once aggregate uncertainty is realized, given the region-sector choices of workers, the competitive equilibrium consists of a collection of state-dependent wages, prices, and quantities such that in each state s: (1) workers maximize their utility given prices and income; (2) firms maximize profits taking wages and prices as given; and (3) goods' and labor markets clear, while trade is balanced. Before uncertainty is realized, workers make location-sector decisions to maximize their expected utility.

Assumption 2 (Unique and Interior Equilibrium). *For all j, $\tilde{\rho}_j - \frac{1}{\kappa} \in [0, 1)$.*

Kucheryavyy et al. (2020) show that in the deterministic model with heterogeneous workers the condition above entails that the equilibrium is unique and each sector (and region) has positive employment. When $\kappa \to \infty$, so that workers are homogeneous, the condition collapses to $\theta_j \rho_j \in [0, 1)$, for all j.We keep this assumption for the remaining of the paper.

3 Analytical results

In this section, we analyze the effects of uncertainty and economies of scale on regional and sectoral specialization. Additionally, we derive the gains from trade. All proofs are in Appendix A.

We denote by: $\bar{X}$ the expected value of $X(s)$ across states s, $\bar{X} \equiv \sum_s \mathbb{P}(s)X(s)$; X^* the value of $X(s)$ in the deterministic equilibrium; $\hat{X}(s) \equiv dX(s)/X^*$ deviations around X^* for state-dependent variables ($\hat{X} \equiv dX/X^*$ for non-state dependent variables); and $\hat{\bar{X}} \equiv \sum_s \mathbb{P}(s)\hat{X}(s)$. We normalize global expenditure in each state s to one, $\sum_i X_i(s) = 1$.

3.1 The Effects of Uncertainty: Autarky vs Frictionless Trade

We start by comparing the economy under autarky and the open economy. We first calculate the volatility of real wages. We derive the results under frictionless trade across regions and countries so that prices are equalized.

Lemma 1 (Volatility of the real wage). *Assume that productivity shocks are independent and identically distributed, and that trade is frictionless across regions worldwide, $\tau^j_{mk} = 1$ for all $m \neq k$. The volatility of the real wage in country i and sector j is*

$$\Xi_{i,j} \equiv var\left(\hat{W}_{i,j}(s) - \hat{P}_i(s)\right) = \left(\frac{\theta_j}{1+\theta_j}(1-\pi^*_{i,j}) + \alpha_j \pi^*_{i,j}\right)^2 \sigma^2_{i,j} + \left(\frac{\theta_j}{1+\theta_j} - \alpha_j\right)^2 \sum_{i' \neq i} (\pi^*_{i',j})^2 \sigma^2_{i',j} + \sum_{k \neq j} \sum_{i'} (\pi^*_{i',k})^2 \alpha^2_k \sigma^2_{i',k} \tag{12}$$

where $\pi^*_{i,j} = \sum_{m \in M_i} \pi^*_{mj}$, is the share of expenditure in each region of country i in goods from sector j from all other regions, including self, in country i, in the deterministic equilibrium. Under autarky, $\pi^*_{i,j} = 1$ for all j, and (12) *collapses to*

$$\Xi^A_{i,j} \equiv var\left(\hat{W}^A_{i,j}(s) - \hat{P}^A_i(s)\right) = \sum_j \alpha^2_j \sigma^2_{i,j}. \tag{13}$$

The lemma shows that the effect of trade openness on real wage volatility is ambiguous. Because under autarky wages are constant across states, $W^A_{i,j} = \alpha_j / L^A_{i,j}$, the only source of volatility in real wages is due to prices. Hence, while trade openness unambiguously increases wage volatility —and the effect is larger, the weaker the effect of comparative advantage (higher θ_j) —its effect on price volatility is ambiguous. For instance, if sectoral volatility were the same for all countries, $\sigma_{i,j} = \sigma_j$ for all i, j, openness would reduce price volatility due to the diversification of expenditure sources. The offsetting effects of trade openness on wages and prices have also been discussed by Caselli et al. (2020), which point out to the importance of the structure of shocks (whether sector or country specific) to conclude that trade increases or decreases volatility. In the context of agricultural markets, Allen and Atkin (n.d.) also show that openness increases wage volatility, decreases prices volatility, and has an ambiguous effect on the real wage volatility.

A clear condition can be found if we assume that countries are identical (except for the realization of shocks). In that case, (12) becomes

$$\Xi_j = \sigma_j^2 \frac{\theta_j^2}{(1+\theta_j)^2} \frac{N-1}{N} + \frac{1}{N} \sum_k \alpha_k^2 \sigma_k^2. \tag{14}$$

If $1/\theta_j > (1-\alpha_j)/\alpha_j$, then real-wage volatility in sector j would be lower in the open economy. While nominal wage volatility is always larger in the open economy (it is fixed under autarky)—and increases with the trade elasticity θ—price volatility is always $1/N$ lower in the open economy. The condition is sufficient to ensure that the price effect is larger than the wage effect.

What happens with ex-ante labor allocations? Using (2) and a second order approximation of the expected utility in (1) yields

$$\lambda_{i,j} = \left(\frac{\bar{W}_{i,j}}{\bar{P}_i}\right)^{\kappa} \frac{\left(1 - \frac{\gamma(1-\gamma)}{2} \Xi_{i,j}\right)^{\frac{\kappa}{1-\gamma}}}{\sum_{j'} \left(\frac{\bar{W}_{i,j'}}{\bar{P}_i}\right)^{\kappa} \left(1 - \frac{\gamma(1-\gamma)}{2} \Xi_{i,j'}\right)^{\frac{\kappa}{1-\gamma}}}. \tag{15}$$

Further assuming symmetric countries, we can solve for the labor share in the open economy,

$$\lambda_j = \frac{\alpha_j \mathbb{E}_j \left(1 - \frac{\gamma(1-\gamma)}{2} \Xi_j\right)^{\frac{1}{1-\gamma}}}{\sum_k \alpha_k \mathbb{E}_k \left(1 - \frac{\gamma(1-\gamma)}{2} \Xi_k\right)^{\frac{1}{1-\gamma}}},$$

where $\mathbb{E}_j \equiv \sum_s \mathbb{P}(s) \frac{A_{i,j}(s)^{\frac{\theta_j}{1+\theta_j}}}{\sum_{i'} A_{i',j}(s)^{\frac{\theta_j}{1+\theta_j}}}$, and Ξ_j is given by (14).[5] It is immediate to see that with no uncertainty, the labor share in a sector is pinned down by the expenditure share, $\lambda_j^* = \alpha_j$. Under the same conditions of Lemma 1, this is also the autarkic labor share in

[5]In this case, expected prices $\bar{P}$ (equalized across regions worldwide) cancel out and the expected wage is $\bar{W}_j = \alpha_j C_1^{-1} (\lambda_j)^{\frac{1}{\kappa}-1} L^{-1} \mathbb{E}_j$.

sector j, $\lambda_j^A = \alpha_j$.

Hence, with uncertainty, sectors with low volatility in real wages (low Ξ_j) have employment shares above the autarky levels, as well as their deterministic levels, $\lambda_j > \alpha_j$, while the opposite is true for sectors with high real wage volatility. Next, we dive deeper into the effects of uncertainty on labor reallocation, as well as its interaction with economies of scale.

3.2 Economies of scale, uncertainty, and sectoral reallocation

Uncertainty affects workers' decisions on which sector and region to work. It also changes the effects of trade openness on specialization patterns across sectors with different degrees of scale economies. In turn, these effects will shape the gains from trade. We consider a country i, which is small enough to not affect foreign wages and prices.

Proposition 1 (**Employment reallocation**). *For a small country i, suppose that shocks are independent across industries and countries. Assume that trade is frictionless across regions worldwide, $\tau_{mk}^j = 1$ for all $m \neq k$. Then relative to the deterministic labor shares:*
(1) Sector j's employment in region m decreases the higher the industry level volatility,

$$\frac{\partial \lambda_{mj}/\lambda_{mj}^*}{\partial \sigma_{i,j}} < 0;$$

(2) The effect is amplified with stronger industry-level economies of scale, or higher relative risk aversion,

$$\frac{\partial^2 \lambda_{mj}/\lambda_{mj}^*}{\partial \sigma_{ij} \partial \rho_j} < 0, \qquad \frac{\partial^2 \lambda_{mj}/\lambda_{mj}^*}{\partial \sigma_{i,j} \partial \gamma} < 0;$$

and (3) If $1/\theta_j < \frac{1-\alpha_j}{\alpha_j}$, the effect is dampened by a higher country-level exposure to trade,

$$\frac{\partial^2 \lambda_{mj}/\lambda_{mj}^*}{\partial \sigma_{i,j} \partial \pi_{i,j}^*} > 0,$$

with $\pi_{i,j}^* = \sum_{m \in M_i} \pi_{mj}^*$.

Ex-ante, workers avoid sectors with higher volatility, which are sectors with higher volatility in real wages (Lemma 1), and reallocate towards less-volatile sectors, relative to the deterministic equilibrium. This effect is similar to the one in Allen and Atkin (n.d.) in the context of farmers choosing less volatile crops to farm.

The reallocation towards less volatile sectors is stronger in sectors with stronger economies of scale. Moreover, opening up to trade reinforces reallocation away from volatile sectors as long as specialization forces are weak relative to expenditure diversification — a case that tends to increase the volatility of real wages in the open economy.

3.3 The gains from trade

Next, we calculate the gains from trade and analyze how they change with uncertainty and the strength of economies of scale. We define the gains from trade as the ratio of the average expected utility between the current equilibrium and the counterfactual equilibrium with no trade ($\tau^j_{mk} \to \infty$, for all $m \in M_i$ $k \in M_{i'}$, $i \neq i'$), across workers with different efficiencies in region m and industry j.

Due to selection, the average expected utility is the same for workers in any region and sector of country i, so that $\mathbb{U}_{mj} = \mathbb{U}_i$ for all $m \in M_i$ and all j (see Appendix for derivations),

$$\mathbb{U}_i = \Gamma\left(\frac{\kappa+\gamma-1}{\kappa}\right)\left(\sum_{m\in M_i}\sum_j (U_{mj})^{\frac{\kappa}{1-\gamma}}\right)^{\frac{1-\gamma}{\kappa}}, \tag{16}$$

where $\Gamma(\cdot)$ is the Gamma function, and $\kappa + \gamma - 1 > 0$.

The gains from trade are then given by

$$GT_i \equiv \left(\frac{\mathbb{U}_i}{\mathbb{U}_i^A}\right)^{\frac{1}{1-\gamma}} = \left(\sum_{m\in M_i}\sum_j \lambda_{mj}\left(\frac{U_{mj}}{U^A_{mj}}\right)^{-\frac{\kappa}{1-\gamma}}\right)^{-\frac{1}{\kappa}}, \tag{17}$$

where U^A_{mj} is the autarky value of expected utility per efficiency unit in region m and industry j. Since CRRA utility is negative and smaller in absolute value as it gets larger, gains from trade are positive when $\mathbb{U}_i/\mathbb{U}_i^A < 1$. In order to have positive gains when $GT_i > 1$, we just bring the exponent $1-\gamma$ to the left-hand side.

To gain intuition behind the forces shaping the gains from trade, using (1), we take a second-order approximation of the change in the value of expected utility per efficiency unit,

$$\left(\frac{U_{mj}}{U^A_{mj}}\right)^{\frac{1}{1-\gamma}} \approx \underbrace{\frac{\bar{W}_{mj}/\bar{P}_m}{\bar{W}^A_{mj}/\bar{P}^A_m}}_{\text{Change in average real wage}} \times \underbrace{\frac{1-\frac{\gamma}{2}\Xi_{mj}}{1-\frac{\gamma}{2}\Xi^A_{mj}}}_{\text{Change in volatility of real wage}}. \tag{18}$$

We can further solve for the change in the average real wage to get

$$\frac{\bar{W}_{mj}/\bar{P}_m}{\bar{W}^A_{mj}/\bar{P}^A_m} = \underbrace{\frac{\bar{W}_{mj}/\prod_j \bar{W}^{\alpha_j}_{mj}}{\bar{W}^A_{mj}/\prod_j \bar{W}^{A\alpha_j}_{mj}}}_{\text{Workers Heterogeneity}} \times \underbrace{\prod_j \left(\bar{\pi}^j_{mm}\right)^{-\frac{\alpha_j}{\theta_j}}}_{\text{Multi-sector ACR}} \times \underbrace{\prod_j \left(\frac{\lambda_{mj}}{\lambda^A_{mj}}\right)^{\alpha_j \rho_j}}_{\text{Economies of scale}} \times \underbrace{\prod_j \left(\frac{\lambda_{mj}}{\lambda^A_{mj}}\right)^{-\frac{\alpha_j \rho_j}{\kappa}}}_{\text{Interaction: ES and Heterogeneity}} . \tag{19}$$

The first term on the right-hand side captures changes in relative wages when labor is imperfectly mobile (see Arkolakis et al. 2012, Galle et al. 2023), while the second represents the multi-sector version of the standard ACR formula. The third term appears because of the presence of economies of scale, as in Kucheryavyy et al. (2020), and the fourth term captures the interaction between this force and worker heterogeneity.

To make a sharp statement on the effects of uncertainty, economies of scale, and worker heterogeneity on the gains from trade, the next proposition specifies the formula in the case of frictionless trade, and i.i.d. shocks.

Proposition 2 (**The gains from trade**). *Assume that shocks are independent across industries and countries and trade is frictionless across and within countries. The gains from trade are given by*

$$GT_i \approx \prod_j \left(\frac{\lambda_{i,j}}{\lambda^A_{i,j}}\right)^{-\frac{\alpha_j}{\kappa}} \prod_j \left(\bar{\pi}^j_{i,i}\right)^{-\frac{\alpha_j}{\theta_j}} \prod_j \left(\frac{\lambda_{i,j}}{\lambda^A_{i,j}}\right)^{\alpha_j \rho_j} \prod_j \left(\frac{\lambda_{i,j}}{\lambda^A_{i,j}}\right)^{-\frac{\alpha_j \rho_j}{\kappa}} \prod_j \left(\frac{1 - \frac{(1-\gamma)\gamma}{2}\Xi_{i,j}}{1 - \frac{(1-\gamma)\gamma}{2}\Xi^A_{i,j}}\right)^{\frac{\alpha_j}{1-\gamma}} . \tag{20}$$

where $\lambda^A_{i,j} = \alpha_j$, *and* $\Xi_{i,j}, \Xi^A_{i,j}$ *are given by* (12) *and* (13), *respectively.*

With no uncertainty, $\Xi^A_{i,j} = \Xi_{i,j} = 0$, and no worker heterogeneity, $\kappa \to \infty$, the gains from trade would collapse to the second and third term in (20), which is the same expression as in Kucheryavyy et al. (2020).[6] Conditional on $\bar{\pi}^j_{ii}$, the gains from trade are higher when labor is allocated to sectors with higher economies of scale, $\sum_j \alpha_j \rho_j \log \lambda_{i,j}/\alpha_j > 0$. In turn, with no uncertainty and no economies of scale, $\rho_j = 0$ for all j, only the first and second term in (20) survive. We then recover the expression for the gains from trade in Galle et al. (2023), which feature a model with worker heterogeneity, but neither scale economies nor uncertainty. In this case, conditional on $\bar{\pi}^j_{ii}$, the gains from trade are higher when labor shares moves away from sectors with high expenditure shares, $-(1/\kappa)\sum_j \alpha_j \log \lambda_{i,j}/\alpha_j > 0$. When worker heterogeneity is coupled with economies of scale, there is an additional

[6] The online appendix of Kucheryavyy et al. (2020) presents a version of their baseline model with worker heterogeneity. Their goal, however, is to show conditions under which a multi-sector model of trade with scale economies delivers a unique and interior solution for employment allocations.

effect on the gains from trade captured by the fourth term in (20). Here, gains from trade increase as long as $-(1/\kappa)\sum_j \rho_j\alpha_j \log\lambda_{i,j}/\alpha_j > 0$, meaning that labor moves away from sectors with high economies of scale. On net, $(1-1/\kappa)\sum_j \rho_j\alpha_j \log\lambda_{i,j}/\alpha_j > 0$, with worker heterogeneity offsetting the effect of economies of scale on sectors that gain employment. Which are those sectors? In an economy with uncertainty, these are the sectors for which the volatility of real wages decreases with openness. As we show in Section 3.1, we can unambiguously say that volatility decreases in a sector when the economy opens up to trade, countries are identical, and $(1-\alpha_j)/\alpha_j < 1/\theta_j$. Once we condition on $\lambda_{i,j}$ and $\bar{\pi}^j_{ii}$, there is no interaction between scale economies and uncertainty. However, we know that $\lambda_{i,j}$ decreases with volatility from its autarkic levels. Hence, conditional only on $\bar{\pi}^j_{ii}$, the gains from trade decrease with real-wage volatility, but this effect gets offset if those sectors are the ones with weak scale economies.

Overall, losses from trade are possible: due to reallocation of labor towards sectors with lower economies of scale; and due to increases in real-wage volatility because of increased wage volatility and insufficient diversification of expenditure sources.

These analytical results, derived under special conditions, shed light on the mechanics of the model and guide the understanding of counterfactual exercises performed with the general version of the model. Next, we first document data patterns that are qualitatively in line with the model's predictions. We then calibrate a richer version of the model (one with input-output linkages) and use it for counterfactual analysis.

4 Empirical Evidence

In this section, we present suggestive evidence about our model predictions using data for community zones (CZ) across the United States. We first describe the data and the construction of the variables used for the analysis. Next, we document that uncertainty affects negatively more the employment growth of CZs with a larger share of employment in industries with strong economies of scale. Additionally, we document that more openness of a CZ to international trade mitigates the effects of uncertainty. We use some of the reduced-form evidence moments as targets for our calibration procedure in Section 5.

4.1 Data and Variables' Construction

We use employment data for the United States, by county and industry, from the County Business Patterns, for 1967-2017. We follow Autor et al. (2013) to clear the data and aggregate them into commuting zones (CZs). Data on U.S. sectoral employment and U.S. sectoral value of shipments are from the NBER Manufacturing Database, for 1969-2011. To be consistent with the sector classification in our quantitative analysis in Section 5, we aggregate the data into 2-digit ISIC industries for our baseline empirical results, and we report robustness analysis based on 3-digit SIC industries. We use as our variable of interest changes in employment at the CZ, or alternately, at the CZ-sector level, between 1990 and 2010. We remove CZ-sector pairs with zero employment either in 1990 or 2010. Our sample includes 721 CZs within the United States, and 17 manufacturing industries for which we can construct volatility measures as described below. See Appendix C for details.

To obtain a measure of uncertainty, we first compute growth in average real value-added per worker by industry and year,

$$g_{j,t} = \log\left(\frac{VA_{j,t}}{Emp_{j,t}}\right) - \log\left(\frac{VA_{j,t-1}}{Emp_{j,t-1}}\right). \tag{21}$$

We then compute the change in uncertainty between 1970-1990 and 1990-2010 for industry j based on changes in the variance of real value-added per worker growth,

$$\Delta v_j = var(\underbrace{g_{j,t}}_{t=1990,\ldots,2010}) - var(\underbrace{g_{j,t}}_{t=1970,\ldots,1990}). \tag{22}$$

Next, we compute a region's change in uncertainty as a Bartik-type variable,

$$\Delta v_m \equiv \sum_j \frac{L_{mj,0}}{L_{m,0}} \Delta v_j, \tag{23}$$

where $L_{mj,0}$ is CZ m's employment in industry j, and $L_{m,0}$ is CZ m's total employment, in 1990.

We use as the measure of sectoral economies of scale the parameter $\tilde{\rho}_j \equiv \frac{\kappa-1}{\kappa}\rho_j\theta_j$ where we take both trade elasticity θ_j and the degree of economies of scale regarding local sectoral employment $\frac{\kappa-1}{\kappa}\rho_j$ from Bartelme et al. (2025).[7] Their estimates span 16 2-digit ISIC sec-

[7] In Bartelme et al. (2025), the degree of economies of scale is estimated using local sectoral employment.

tors.[8] We set the scale parameter for a more disaggregate sector when it belongs to more than one aggregate industry to an employment-based weighted average of $\tilde{\rho}_j$ across the aggregate industries. This assignment is particularly relevant for our robustness analysis that uses 3-digit SIC industries.

For some of our empirical evidence, we would need trade exposure at the CZ level. Because there are no available export and import data at the CZ geographic level, we measure each CZ's exposure to trade using its industry structure combined with the national openness of the industry. We first compute each industry's exposure to international trade by using the ratio of imports plus exports to total production. We then aggregate industries' exposure to trade in each CZ's using the CZ's industry structure as weights, following Autor et al. (2013) and Caliendo, Dvorkin and Parro (2019), and exclude the own employment of the corresponding CZ-industry pair.

4.2 Results

We start by documenting how the correlation between sectoral employment and uncertainty is shaped by scale economies.

Figure 1a plots changes in log-employment between the 1990 and 2010, $\Delta \log L_m$, against changes in uncertainty, Δv_m, across CZs. The figure suggests that more uncertainty is associated to lower employment growth.

Are scale economies amplifying this negative effect? In Figures 1b and 1c, we divide industries into two groups based on whether their scale-economy parameters are above or below the median value. We then compute CZ-level changes in employment and uncertainty, for each group of industries. We find that industries with above-median economies of scale respond more to changes in uncertainty, as suggested by the theory.

Formally, we estimate the following equation by Ordinary-Least-Squares (OLS),

$$\Delta \log L_m = \beta_0 + \beta_1 \Delta v_m + \beta_2 X_m + \epsilon_m, \tag{24}$$

where $\Delta \log L_m$ denotes changes in manufacturing employment for CZ m between 1990

In our model, according to Assumption 1, we defined economies of scale ρ_j based on labor units. Given the relationship between labor units and employment in (3), the degree of economies of scale regarding local sectoral employment is $\frac{\kappa-1}{\kappa}\rho_j$ in our model, which corresponds to the estimate in Bartelme et al. (2025).

[8] We use the concordance between ISIC and SIC industries at 2 digits in https://www.freit.org/TradeResources/TradeConcordances.php.

Figure 1: Employment growth, economies of scale, and uncertainty.

(a) All industries (b) Strong economies of scale (c) Weak economies of scale

Note: The circles are proportional to the region employment level in 1990. Strong (weak) economies of scale refers to sectors with above (below) median values of the economies of scale parameters in Bartelme et al. (2025).

and 2010, Δv_m is the change in volatility for CZ m computed as in (23), X_m denotes CZ-level controls, and ϵ_m is the error term.

Columns (1)-(3) of Table 1 corroborates the negative correlation documented in Figure 1: larger uncertainty is correlated with lower manufacturing employment growth, more so in industries with strong economies of scale, after controlling for the China shock, structural transformation, labor productivity growth, and demographic controls.

To use all the industries, rather than pooling them in two groups with high and low scale parameters, we use CZ-industry variables and estimate the following equation by OLS:

$$\Delta \log(L_{mj}) = \beta_1 \Delta v_j + \beta_2 \Delta v_j \times \tilde{\rho}_j + \beta_3 X_{mj} + \epsilon_{mj}.$$

Here, $\Delta \log(L_{mj})$ denotes the change in industry j's employment for CZ m between 1990 and 2010, Δv_j denotes changes in volatility in industry j as defined in (22), and $\tilde{\rho}_j$ is the parameter capturing scale economies in industry j. The variable X_{mj} denotes CZ-industry level controls.

Columns (4) and (5) of Table 1 presents the results. The negative effect of sectoral uncertainty on employment growth in an industry and CZ is driven by industries with larger scale parameters. The standard deviation of the industry-level scale parameters is 0.07, suggesting that that one standard-deviation increase in the scale parameter would increase the magnitude of the (negative) response of employment growth to uncertainty by 7–12 percent.

Next, we present some evidence that supports the model's predictions that trade exposure

Table 1: Employment growth, scale economies, trade exposure, and uncertainty. OLS.

	$\Delta \log L_m$			$\Delta \log L_{mj}$		$\Delta \log \frac{L_{mj}}{L_j}$
	all industries	high $\tilde{\rho}_j$	low $\tilde{\rho}_j$	all industries		all industries
	(1)	(2)	(3)	(4)	(5)	(6)
Δv_m	-4.597*	-26.33*	-13.01***			
	(2.539)	(15.56)	(2.503)			
Δv_j				-17.71***	57.93***	
				(1.837)	(17.50)	
$\Delta v_j \times \tilde{\rho}_j$					-80.60***	
					(20.36)	
$\Delta v_j \times$ regional trade exposure						-98.20***
						(24.14)
Controls	yes	yes	yes	yes	yes	no
CZ fixed effects	no	no	no	yes	yes	yes
Industry fixed effects	no	no	no	no	no	yes
Obs	720	706	675	11,038	11,038	11,327
R-squared	0.110	0.123	0.116	0.421	0.518	0.424

Note: The regressions are weighted by initial manufacturing employment in the corresponding industry group. We examine changes in region-level employment in Columns (1)–(3), changes in region-industry-level employment in Columns (4) and (5), and changes in the region's share of employment within each industry in Columns (6). Changes in industry and CZ-level uncertainty are defined as in (22) and (23), respectively. In Columns (2)–(3), manufacturing industries are divided into two groups based on economies of scale: those with above-median (high) and below-median (low) economies of scale, where $\tilde{\rho}_j \equiv \theta_j \times \rho_j$. In Columns (1)–(3), the controls include region-level China shock and labor productivity growth (between 1990 and 2010), an employment-weighted average across the specified industries, as well as the share of manufacturing employment. In Columns (4)–(5), the controls are extended to include a measure of industry-level China shock and labor productivity growth. Robust standard errors are in parentheses. * 10% ** 5% *** 1%.

may mitigate the response of employment growth to uncertainty. First, we show the results graphically in Figure 2. The pattern is consistent with the model's prediction: more openness amplifies the effect of uncertainty on employment growth at the CZ level. The effect, however, is not very sharp, most likely due to the fact that we had to impute the data on trade exposure for each CZ.

Formally, we estimate the following equation by OLS:

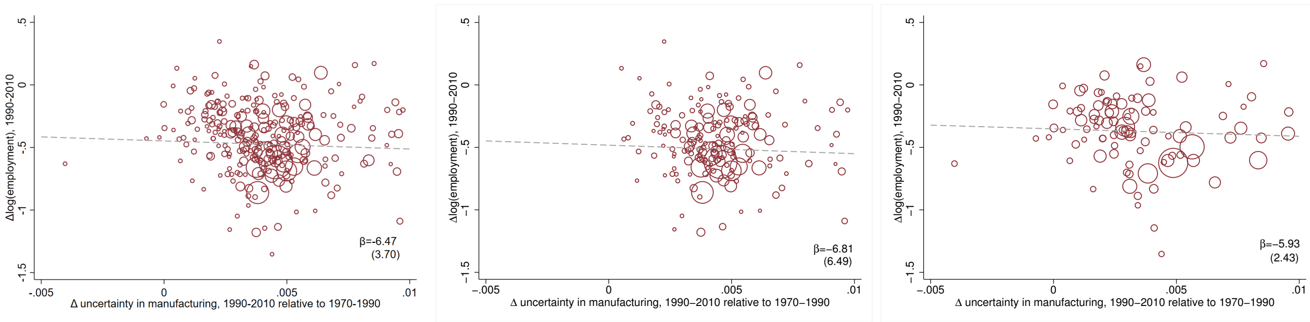
$$\Delta \log \frac{L_{mj}}{L_j} = \beta \Delta v_j \times EX_m + CZ_m + I_j + \epsilon_{mj},$$

where $\Delta \log(L_{mj}/L_j)$ is the change in the share of industry j's employment in CZ m, and EX_m denotes the exposure to international trade of CZ m. CZ fixed effects are CZ_m and industry fixed effects are I_j.

Column (6) of Table 1 corroborates that more sectoral uncertainty tends to reallocate sectoral employment out from regions with larger trade exposure.

Figure 2: Employment growth, trade exposure, and uncertainty.

(a) All industries (b) More trade exposure (c) Less trade exposure



Note: The circles are proportional to the region employment level in 1990. More (less) trade exposure refers to regions with above (below) median values of the trade exposure across CZs.

4.3 Robustness

We perform two robustness exercises.

First, we use an alternative measure for volatility from Alfaro, Bloom and Lin (2018) (ABL). Their measure is at the firm level, and reports the annual change in the volatility of the firm stock market returns. We calculate the firm-level average of annual changes between 1990–2020, assign firms to industries according to their main SIC code, and aggregate these averages across firms to get an industry-level measure of changes in volatility.

Columns (1)-(3) of Table 2 report results using ABL's uncertainty measure. Consistent with our previous results, we still find that uncertainty has negative effects on regional employment growth, and that the effect of uncertainty is larger for industries with stronger economies of scale. However, with this uncertainty measure, the negative impact of uncertainty on local employment tends to decline with regional trade exposure, but the effect is not significant.

Our second robustness exercise uses more disaggregated industries, at 3 digit SIC rather than 2 digits ISIC. Our baseline results still hold. However, the trade-exposure measure has a positive effect but it is not significant.

Finally, Appendix D presents cross-country evidence on the effects of uncertainty and scale effects on employment growth. The appendix also describes the data used. Appendix Table D.1 shows a negative correlation between changes in employment and changes in uncertainty, as well as a negative interaction between the scale parameter and changes

Table 2: Robustness: Uncertainty Measure and Industry Aggregation. OLS

	Alternative uncertainty measure (ABL), 2-digit ISIC Industries,			Baseline uncertainty measure, 3-digit SIC Industries		
	Δ log L_m	Δ log L_{mj}	Δ log $\frac{L_{mj}}{L_j}$	Δ log L_{mj}	Δ log L_{mj}	Δ log $\frac{L_{mj}}{L_j}$
	(1)	(2)	(3)	(4)	(5)	(6)
Δv_m	-18.06**					
	(3.592)					
Δv_j		88.73***		-11.40***	59.89**	
		(22.85)		(1.050)	(26.82)	
$\Delta v_j \times \tilde{\rho}_j$		-112.81***			-76.43***	
		(24.91)			(29.24)	
$\Delta v_j \times$ regional			28.08			4.831
trade exposure			(19.11)			(21.05)
Controls	yes	yes	no	yes	yes	no
CZ FE	no	yes	yes	yes	yes	yes
IND FE	no	no	yes	no	no	yes
Obs	720	11,038	11,327	25,919	25,099	27,900
R-squared	0.181	0.538	0.422	0.210	0.293	0.208

Note: The regressions are weighted by initial manufacturing employment in the corresponding industry group. We examine changes in region-level employment in Columns (1), changes in region-industry-level employment in Columns (2) and (4)–(5), and changes in the region's share of employment within each industry in Columns (3) and (6). Changes in industry and CZ-level uncertainty are defined as in (22) and (23), respectively. In Columns (1), the controls include region-level China shock and labor productivity growth (between 1990 and 2010), an employment-weighted average across all industries, as well as the share of manufacturing employment. In Columns (2), (4), and (5), the controls are extended to include a measure of industry-level China shock and labor productivity growth. Robust standard errors are in parentheses. * 10% ** 5% *** 1%.

in uncertainty. these results are consistent with our empirical evidence for the United States, though estimates are noisier due to a limited number of cross-country observations.

5 Calibration

In this section, we take the model to the data. We first describe the data sources used, and then present the computation strategy. Finally, we discuss the calibration procedure.

We have rich sub-national data for the United States only. Hence, we focus on a version of the model where all countries except the United States have one region only.

Additionally, we assume that country-industry specific shocks $A_{i,j}(s)$ follow a log-normal distribution with mean one and variance $\sigma_{i,j}$. For now, we assume that shocks are independent across countries and sectors.

Finally, we extend the model to include sectoral input-output linkages as explained below.

Sectoral Input-Output Linkages. For our quantitative analysis, we extend the model to incorporate sectoral input-output linkages. We assume that production combines labor and goods from other sectors as inputs, as in (Caliendo and Parro 2015). Formally, the expenditure cost share of region m and sector j on inputs from sector k is δ_{mj}^{k}, while the labor share is δ_{mj}^{L}, with $\sum_k \delta_{mj}^{k} + \delta_{mj}^{L} = 1$. We define ζ_{mj}^{k} as the (j,k) element of the Leontief inverse matrix $[\mathbf{I} - \mathbf{\Phi}_m]^{-1}$, where $\mathbf{I}$ is an $J \times J$ identity matrix, and the (j,k) element of $J \times J$ matrix $\mathbf{\Phi}_m$ is δ_{mj}^{k}. The equilibrium of the model with sectoral input-output linkages is relegated to Appendix B.

The gains from trade follow the expression in (17), with changes in utility per efficiency unit in (18) adjusted to take into account sectoral linkages,

$$\left(\frac{U_{mj}}{U_{mj}^{A}}\right)^{\frac{1}{1-\gamma}} = \underbrace{\frac{\bar{\omega}_{mj}}{\bar{\omega}_{mj}^{A}}}_{\text{Worker Heterogeneity}} \times \underbrace{\prod_k \left(\bar{\pi}_{mm}^{k}\right)^{-\sum_j \frac{\alpha_j \zeta_{mj}^{k}}{\theta_k}}}_{\text{ACR}} \times \underbrace{\prod_k \left(\frac{\lambda_{mk}}{\lambda_{mk}^{A}}\right)^{\sum_j \alpha_j \zeta_{mj}^{k} \rho_k}}_{\text{Economies of scale}} \times \underbrace{\prod_k \left(\frac{\lambda_{mk}}{\lambda_{mk}^{A}}\right)^{-\frac{1}{\kappa}\sum_j \alpha_j \zeta_{mj}^{k} \rho_k}}_{\text{Interaction ES \& worker heterogeneity}}$$

$$\times \underbrace{\frac{1 - \frac{\gamma}{2}\text{var}\left(\hat{W}_{mj}(s) - \hat{P}_m(s)\right)}{1 - \frac{\gamma}{2}\text{var}\left(\hat{W}_{mj}^{A}(s) - \hat{P}_m^{A}(s)\right)}}_{\text{Volatility of real wage}}, \tag{25}$$

with $\bar{\omega}_{mj} \equiv \bar{W}_{mj} / \prod_k \bar{W}_{mk}^{\sum_j \alpha_j \zeta_{mj}^{k} \delta_{mk}^{L}}$.

5.1 Data

For our quantitative analysis, we need data on other countries in addition to the United States. Our main data sources are the OECD databases. To be as close as possible to the structure of the data in Section 4, we group industries into 20 aggregated sectors (see Appendix Table C.1), which include the 17 manufacturing sectors used in our empirical analysis, as well as agricultural, mining, and service sectors. Because computing the variance for country-sector-level productivity requires a relatively long time series data at the country-sector level, we only keep countries with at least 20 years of country-sector-level data on output and employment.[9] We are left with 26 countries, along with a constructed rest of the world. See Appendix C.1 for details.

[9] An exception is China, which is not covered by the OECD databases on employment and value added. Given China's significant role in global trade, we calculate the variance of sectoral productivity growth for manufacturing sectors using data from the Annual Survey of Industry for the period 1998–2007; for other sectors, we use data from China's National Bureau of Statistics over the same period.

Output, Value-added, and Employment. We obtain sectoral output, value-added, and employment for each country-sector in our sample from the OECD STAN Database.[10] The data span from 1970 to 2019 with shorter periods (at least 20 years) for some countries. Using this time series, we construct a measure of volatility of country-sector productivity for all the available years. Specifically, for each country and sector, we first deflate the value-added by CPI indicators from the Penn World Table (10.0) and then compute the standard deviation of yearly growth in value-added per worker, computed similarly to (21). As described below, we calibrate the volatility of country-sector-specific productivity in the model to match the standard deviation of value-added per worker observed in the data.

For subnational data for the United States, we use, as in Section 4, output and employment aggregated into 721 the community zone (CZ) and 20 industries, from the U.S. County Business Patterns, for the year 2017.

International Trade Flows. We obtain sectoral trade flows between countries from the OECD Inter-Country Input-Output Tables, for 2017. Due to the lack of data on U.S. CZs' exports and imports, we make the following proportionality assumption. We assign U.S. sectoral exports to each country to each CZ using the share of each CZ's output in the U.S. total sectoral output. Similarly, we assign U.S. sectoral imports from each country to each CZ using the share of each CZ's expenditures in the U.S. total sectoral expenditures.

US Subnational Trade Flows. We use the Commodity Flow Survey (CFS) for 2017 to construct sectoral trade flows between CZs in the United States. One issue is that the trade flows in the survey are not recorded at the CZ geographic level. We adopt a similar procedure as in the literature (Allen and Arkolakis 2014, Monte, Redding and Rossi-Hansberg 2018, Fajgelbaum and Gaubert 2020) to assign trade flows between CZs. Specifically, we first parameterize the elasticity of trade costs to distance and then solve for the trade flows between CZs using the model structure. The details are provided in Appendix C.2.

[10] For consistency, we also use OECD STAN for manufacturing sectors in the United States, rather than data from the NBER-CES Manufacturing Dataset, as in Section 4. Volatility of valued added per worker in manufacturing calculated from the OECD data is strongly correlated with the measure of volatility calculated using the NBER-CES Manufacturing Data (0.8). The OECD data also allow us add the agricultural, mining, and service sectors, in the United States.

5.2 Calibration Procedure

We match the deterministic equilibrium of our model to the observed economy in 2017. Solving for counterfactual changes requires values for several sets of parameters: input-output linkages $\{\delta^k_{mj,i}, \delta^L_{mj,i}\}$; sectoral expenditure shares $\{\alpha_{i,j}\}$, which we allow to be country specific; standard deviation of country-sector productivity shocks $\{\sigma_{i,j}\}$; trade elasticities $\{\theta_j\}$; economies of scale $\{\rho_j\}$; migration elasticity κ; and risk aversion coefficient γ. We calibrate these parameters as follows. Table 3 summarizes the parameter values and sources.

Externally Calibrated Parameters. We calibrate several parameters from the literature and directly using the data. We set the migration elasticity to $\kappa = 3$, which is between the cross-region migration elasticity estimated in Fajgelbaum et al. (2019) and the within-region cross-industry elasticity of substitution in Berger, Herkenhoff and Mongey (2022). As in Section 4, we follow Bartelme et al. (2025) and set the degree of scale of economies $\frac{\kappa-1}{\kappa}\rho_j$ and the sectoral trade elasticities θ_j to match their estimates. With $\kappa = 3$, we recover ρ_j for each manufacturing sector j. For the agriculture, mining, and services sectors, we set $\rho_j = 0$ and $\theta_j = 4.5$ (from the aggregate elasticity estimates in Simonovska and Waugh 2013). We obtain the input-output parameters, for each country and sector, and sectoral expenditure shares, from the OECD Input-Output Tables for 2017.[11] We assume that for U.S. regions, the input-output parameters are equal to the national parameters, for each sector: $\delta^k_{mj,US} = \delta^k_{US,j}$ and $\delta^L_{mj,US} = \delta^L_{US,j}$, for all m.

Internally Calibrated Parameters. We use a simulated method of moments to calibrate the volatility parameters $\{\sigma_{i,j}\}$ and risk aversion γ.

We calibrate the country-sector standard deviation of productivity shocks $\{\sigma_{i,j}\}$ to match the country-sector standard deviation of log value-added per worker observed in the data, for the period 1970-2019. To discipline the risk aversion coefficient, γ, we target the responses of employment to uncertainty across U.S. regions given by the OLS coefficient in Column (1) of Table 1. To such end, we need to separately match the U.S. sectoral standard deviation of value-added per worker in the periods 1970–1990 and 1990–2010, respectively, and compute the change in regional employment between 1990 and 2010, to

[11] Expenditure shares $\{\alpha_{i,j}\}$ are computed as $\frac{\text{Output}_{i,j} + \text{M}_{i,j} - \text{E}_{i,j} - \text{INTER}_{i,j}}{\sum_j \text{Output}_{i,j} + \text{M}_{i,j} - \text{X}_{i,j} - \text{INTER}_{i,j}}$, where $\text{Output}_{i,j}$, $\text{M}_{i,j}$, $\text{X}_{i,j}$ and $\text{INTER}_{i,j}$ are output, imports, exports and intermediate-input use of goods from sector j, respectively, in each country i.

Table 3: Parameter Values

Notation	Description	Value	Source/Moment
Panel A: Externally Calibrated Parameters			
κ	Migration elasticity	3	(Fajgelbaum et al. 2019)-(Berger et al. 2022)
$\{\theta_j\}$	Sectoral trade elasticities	5.66 (2.61)	Bartelme et al. (2025)
$\{\rho_j\}$	Sectoral economies of scale	0.25 (0.16)	Bartelme et al. (2025)
$\{\delta^k_{i,j}, \delta^L_{i,j}\}$	Sectoral input-output linkages	–	OECD Input-Output Tables (year 2017)
$\{\alpha_{i,j}\}$	Expenditure shares	–	OECD Input-Output Tables (year 2017)
Panel B: Internally Calibrated Parameters			
$\{\sigma_{i,j}\}$	S.D. of productivity shocks	–	S.D. of value-added per worker, OECD STAN (SMM)
γ	Risk aversion coefficient	7.2	OLS coefficient in Column (1) of Table 4 (SMM)

Note: For θ_j and ρ_j, we report the average and standard deviations across sectors.

perform the same regression as in (24).

We pair the average economy in the model, with data for 2017. The main restriction for this choice is the availability of intra-national trade flows for the United States, which are available for the year 2017.

We describe the algorithm next.

Algorithm for SMM Procedure. We use hat-algebra methods (Dekle et al. 2008) adapted to accommodate the impact of uncertainty. Given the input-output linkages $\{\delta^k_{i,j}, \delta^L_{i,j}\}$, expenditure shares $\{\alpha_{i,j}\}$, economies of scale $\{\rho_j\}$, and trade elasticities $\{\theta_j\}$, we pair the observed economy in 2017 with the model's average economy. The calibration proceeds in two steps.[12]

1. Calibration of volatility of productivity shocks, $\{\sigma_{i,j}\}$.
    (a) We guess a matrix $\{\sigma_{i,j}\}$ and simulate 100 realizations for the matrix of shocks drawn from a log-normal distribution with mean one and variance $\sigma^2_{i,j}$.
    (b) Conditional on the country-sector employment shares observed in the data in 2017, denoted by λ^{17}_{mj}, and our guess of $\{\sigma_{i,j}\}$, for each realization of the matrix of shock, we compute the model's equilibrium in changes (from the average

[12] Admittedly, the model's parameters are restricted so that it does not generate corner solutions in sectoral employment in any location. We match zero employment in the data by assuming that fundamental productivity is zero in those regions and sectors. If we assumed that $\tilde{\rho}_j = 1$, for some j, the possibility of corner solutions arises, as shown by Kucheryavyy et al. (2020), and slackness conditions should be considered. We decided to stay away from that possibility since the calibration algorithm is already computational complex.

to the state s economy) applying standard hat algebra methods (see Appendix B).[13]

(c) We calculate the standard deviation of real value added per worker using the 100 realizations of the shock, for a given matrix of $\sigma_{i,j}$, and compare with the same variable in the data. For the United States, in the model, we aggregate real value added per worker in each region using the employment shares in the data in 2017.

(d) We then iterate over $\{\sigma_{i,j}\}$ until the model-generated standard deviation of real value added per worker matches the equivalent data moments. We denote this volatilities as $\{\sigma_{i,j}^{70-19}\}$, since they are calibrated to match the data on country-sector real value added per worker over the period 1970-2019.

(e) We calculate the mean and standard deviation of real wages for each region and industry in the United States using the 100 draws from the last iteration. We denote these variables by $\bar{W}_{m,j}^{70-19}/\bar{P}_{m,j}^{70-19}$ and $\Xi_{m,j}^{70-19}$. They will be used in Step 2.

Step 1 of the algorithm does not use the risk aversion parameter γ. We proceed to calibrate this parameter in our second step, using the ex-ante labor allocation decisions implied by the model, and as target moment, the OLS estimate of the coefficient β_1 in (24), reported in Column (1) of Table 1.

2. Calibration of the risk aversion parameter, γ.

   (a) Given $\{\sigma_{i,j}^{70-19}\}$ calibrated in Step 1, we change the parameters capturing volatility for the United States, so that we move from $\sigma_{US,j}^{70-19}$ to an equilibrium with a different level of $\sigma_{US,j}^{'}$, for each j.

   2.1 For a given γ, employment shares change with the volatility of the shock. We guess a change in employment share from λ_{mj}^{17} to $\lambda_{mj}^{'}$. We draw 100 realizations of the matrix of shocks using $\{\sigma_{i\neq US,j}^{70-19}, \sigma_{US,j}^{'}\}$, and compute the hat-changes between the average economy, which we pair with the observed economy in 2017, and each state s, conditional on our guess for changes in employment shares, as we did in Step 1.

[13]Conditional on location-sector level employment, for each state s, our model is a standard multi-sector Eaton-Kortum model with sectoral input-output linkages (e.g., Costinot and Rodríguez-Clare 2014, Caliendo and Parro 2015).

2.2 We solve for the change in employment shares between the economy with $\{\sigma_{i,j}^{70-19}\}$ and the economy with $\{\sigma_{i\neq US,j}^{70-19}, \sigma_{US,j}^{'}\}$,

$$\hat{\lambda}_{mj} = \frac{\lambda_{mj}'}{\lambda_{mj}^{70-19}} \approx \left(\frac{\hat{\bar{W}}_{mj}}{\hat{\bar{P}}_m}\right)^{\kappa} \frac{\left(\frac{1-\frac{\gamma(1-\gamma)}{2}\Xi_{mj}'}{1-\frac{\gamma(1-\gamma)}{2}\Xi_{mj}^{70-19}}\right)^{\frac{\kappa}{1-\gamma}}}{\sum_{m'\in M_i}\sum_{j'}\lambda_{m'j'}^{70-19}\left(\frac{\hat{\bar{W}}_{m'j'}}{\hat{\bar{P}}_{m'}}\right)^{\kappa}\left(\frac{1-\frac{\gamma(1-\gamma)}{2}\Xi_{m'j'}'}{1-\frac{\gamma(1-\gamma)}{2}\Xi_{m'j'}^{70-19}}\right)^{\frac{\kappa}{1-\gamma}}},$$

where $\Xi_{mj}^{'}$ denotes the variance of the real wage in region m and sector j in the economy with U.S. volatily equal to $\sigma_{US,j}^{'}$.

2.3 We iterate until our guess for $\hat{\lambda}_{mj}$ in 2.1 coincides with the one in 2.2.

(b) We repeat the steps in (a) above twice. First, we change volatility from $\sigma_{US,j}^{70-19}$ to $\sigma_{US,j}^{70-90}$ until we match the observed standard deviation of log real value added per worker across regions and manufacturing sectors in the United States, in the period 1970-1990. We denote the resulting real wage volatility by $\sigma_{US,j}^{70-90}$ and the employment share as λ_{mj}^{90}. Second, we repeat the same procedure considering a change from $\sigma_{US,j}^{70-19}$ to $\sigma_{US,j}^{90-10}$ until we match the observed standard deviation of log real value added per worker across regions and manufacturing sectors in the United States, in the period 1990–2010. We denote the resulting real wage volatility by $\sigma_{US,j}^{90-10}$ and employment shares by λ_{mj}^{10}.

(c) We use the model-generate data to estimate (24) by OLS,

$$\Delta L_m = \beta_0 + \beta_1 \Delta v_m + \beta_d X_m + \epsilon_m.$$

We construct $\Delta \log L_m = \log L_m^{10} - \log L_m^{90}$, where $L_m^{10} = L_{US}^{10}\sum_j \lambda_{mj}^{10}$ and $L_m^{90} = L_{US}^{90}\sum_j \lambda_{mj}^{90}$, with L_{US}^{10} and L_{US}^{90} denoting total U.S. employment in 2010 and 1990, respectively. We use the model-generated employment weights for 1990 to construct the volatility measure Δv_m as in (23). We include as controls the model-generated (change in) log average real wage in industry j and region m, the share of workers in manufacturing sectors in region m, and the share of trade from China.

(d) We iterate on γ until we match the estimate of the coefficient β_1 in Column (1) of Table 1.

5.3 Calibration Results

Table 4 compares the predictions of the effect of changes in uncertainty on regional employment levels between the model and the data, for CZs in the United States. We find that with a risk aversion coefficient of 7.2, the model-generated data matches the observed response of CZ-level employment to uncertainty, as shown in Columns (1) and (2). Our estimate of relative risk aversion is within the ballpark of estimates in the finance literature (see Elminejad, Havránek and Irsova 2022, for a meta-analysis). Columns (3)-(6) report the un-targeted responses of CZ-sector level employment to uncertainty and how this effect varies with sectoral economies of scale, and CZ trade exposure. Our model captures fairly well the pattern observed in the data, even though the magnitude of the effects are smaller.

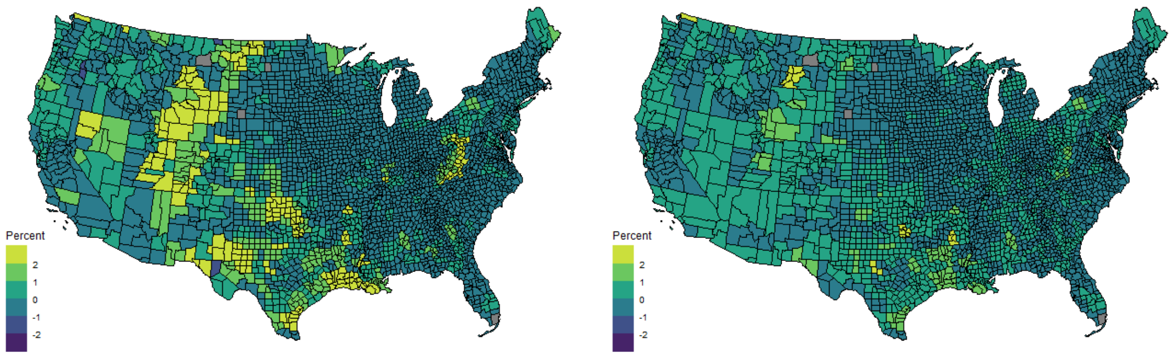
Table 4: Employment Growth and Uncertainty, by CZs and Industries.

	Targeted		Untargeted			
	$\Delta \log L_m$		$\Delta \log L_{mj}$		$\Delta \log \frac{L_{mj}}{L_j}$	
	(1) data	(2) model	(3) data	(4) model	(5) data	(6) model
Δv_m	-4.597*	-4.577***				
	(2.539)	(0.845)				
Δv_j			57.93***	5.218***		
			(17.50)	(0.861)		
$\Delta v_j \times \tilde{\rho}_j$			-80.60***	-16.42***		
			(20.36)	(0.686)		
$\Delta v_j \times$ regional trade exposure					-98.20***	-3.193***
					(24.14)	(1.072)
CZ FE	no	no	yes	yes	yes	yes
IND FE	no	no	no	no	yes	yes
Obs	720	720	11,038	11,439	11,327	11,438
R-squared	0.110	0.849	0.421	0.928	0.424	0.414

Note: The regressions are weighted by initial manufacturing employment in the corresponding industry group. We examine changes in region-level employment in Columns (1) and (2), changes in region-industry-level employment in Columns (3) and (4), and changes in the region's share of employment within each industry in Columns (5) and (6). Changes in industry and CZ-level uncertainty are defined as in equations (22) and (23), respectively. In Columns (1)–(2), the controls include region-level China shock and labor productivity growth, an employment-weighted average across the specified industries, as well as the share of manufacturing employment. In Columns (3)–(6), the controls are extended to include a measure of industry-level China shock and labor productivity growth. Robust standard errors are in parenthesis. * 10% ** 5% *** 1%.

Figure 3: The Effects of Uncertainty on Employment Reallocation, By CZ.

(a) With Scale Effects (b) Without Scale Effects



	mean	50th pc	5th pc	95th pc	minimum	maximum
With scale effects	0.19%	-0.19%	-0.59%	2.24%	-1.39%	4.22%
Without scale effects	0.07%	-0.07%	-0.34%	0.92%	-0.66%	2.52%

Note: Percentage change in CZ employment from the calibrated economy to an economy without uncertainty ($\sigma_{i,j} = 0$ for all i, j), with scale effects (calibrated ρ_j) in Panel (a), and without scale effects ($\rho_j = 0$ for all j) in Panel (b).

6 Quantitative Analysis

Armed with the calibrated model, we now perform several counterfactual exercises to shed light on the role of uncertainty and scale economies in shaping specialization patterns and the gains from trade. Each exercise simulates the economy for 100 realizations of the matrix of shocks.

6.1 Economies of Scale, Uncertainty, and Regional Specialization

To understand the role of uncertainty in shaping workers' specialization patterns in the United States, we simulate the model with no uncertainty ($\sigma_{i,j} = 0$). In this case, uncertainty does not affect the location and sectoral choices of workers. To analyze the interaction between uncertainty and scale economies, we repeat the exercise for an economy with no scale effects, $\rho_j = 0$ for all j. Results are shown in Figure 3.

Table 5: Smallest and Largest Employment Responses, Baseline to No Uncertainty.

Largest Employment Losses	With ES	Without ES	Largest Employment Gains	With ES	Without ES
Van Horn (TX)	-1.39%	-0.42%	Beaumont (TX)	2.99%	1.95%
Condon (OR)	-1.39%	-0.40%	Corpus Christi (TX)	3.05%	1.89%
Scobey (MT)	-1.09%	-0.32%	Buckhannon (WV)	3.24%	1.57%
Clayton (NM)	-0.99%	-0.29%	Dickinson city (ND)	3.32%	1.29%
Rosebud (SD)	-0.87%	-0.64%	Bellingham (WA)	3.45%	2.18%
Seymour (TX)	-0.87%	-0.26%	El Dorado (AR)	3.46%	2.52%
Matador (TX)	-0.85%	-0.66%	Billings (MT)	3.50%	2.04%
Socorro (NM)	-0.84%	-0.61%	Ardmore (OK)	3.76%	2.23%
Lakeview (OR)	-0.81%	-0.47%	Graham (TX)	3.80%	1.65%
Trenton (MO)	-0.80%	-0.36%	Brady (TX)	4.22%	2.16%

Note: Ten CZs with the largest employment losses (gains) from moving from the calibrated economy to one with no uncertainty ($\sigma_{i,j} = 0$), with and without economies of scale (ES), respectively. We list the largest location in the CZ in terms of population (data from USDA).

The analytical case in Proposition 1 suggests that uncertainty in productivity should decrease employment growth, and the effect should be amplified with stronger scale economies. Quantitatively, the effect is relevant.

Panel (a) of Figure 3 shows that eliminating uncertainty tends to reallocate employment to regions in the middle of the United States (i.e. yellow areas). However, there is considerable heterogeneity across regions, ranging to losses of 0.59 percent for the 5th percentile of CZs and 0.19 percent for the median CZ, to gains of 0.19 percent for the mean and 2.24 percent for the 95th percentile CZ. Eliminating scale effects—and leaving only the forces of comparative advantages and geography in operation–mitigates the employment changes both for gains and losses, except for the Mountain region in the West where losses are exacerbated. Employment, however, would be reallocated away from the average CZ.

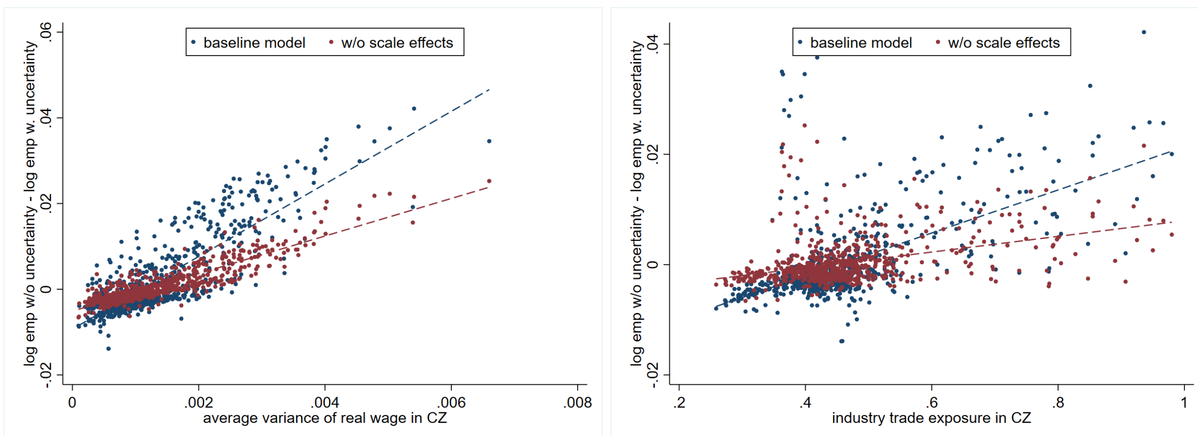
Who are the CZs gaining and losing according to our model predictions? Table 5 further lists the ten CZs with the largest employment losses and employment gains, respectively.

To better visualize how uncertainty shapes local employment levels under scale effects and different degrees of trade openness, Panel (a) of Figure 4a plots the impact of eliminating uncertainty on employment against the average variance of real wages in the calibrated economy, across CZs. Regions with initially a larger average volatility would attract more workers enjoy larger employment when uncertainty is eliminated. This result is intuitive: risk-averse workers move away from regions and sectors with larger uncertainty, which lowers their utility. In line with the results in Figure 3, a world without economies of scale would be flatter, and the benefits of eliminating uncertainty are lower and similar across CZs.

Panel (b) of Figure 4a shows that removing uncertainty would reallocate employment towards CZ more exposed to trade, likely because these regions are specialized in industries with strong scale economies — in the absence of scale economies labor allocation does not change much for these regions.

Figure 4: Employment Reallocation, Uncertainty, and Scale Effects.

(a) Reallocation and Real-wage Variance　　(b) Reallocation and Trade Exposure



Note: Employment changes from eliminating uncertainty against local volatility of real income in the calibrated equilibrium. For each CZ, trade exposure is calculated as the ratio of exports plus imports to total output across sectors, as in Section 4.

6.2 The U.S. Gains from Trade

We now apply the calibrated model to understand the forces shaping the U.S. gains from trade—comparative advantage, economies of scale, uncertainty, and workers' heterogeneity. The gains from trade are defined in (17) as the change in the average expected utility from autarky to the observed economy, and we use the model with sectoral linkages. We set international trade costs for each CZ to infinity and calculate equilibrium variables under autarky. We compare the autarky and the baseline equilibrium, and repeat the exercise for an economy with no uncertainty, $\sigma_{i,j} = 0$, for all i, j, no scale economies, $\rho_j = 0$ for all j, and lower worker heterogeneity, $\kappa = 5$.

Table 6 summarizes the results. In the first row, we find that the overall gains from trade in our baseline model are 1.53 percent. Aggregate uncertainty reduces the gains from trade from 2.34 to 1.53 percent. This result suggests that larger wage volatility after trade openness dominates the role of trade in diversifying the sources of expenditures—and hence, the ability of trade of reducing the volatility of aggregate prices. In line with Proposition 1, the larger uncertainty in real wages drives workers away from sectors with large economies of scale, limiting their role. In fact, while uncertainty reduces the gains from trade by around 30 percent, with scale economies (2.34 vs 1.53 percent) or without them (2.31 vs 1.69 percent), there is a slight increase in the gains from trade if scale economies were removed in a deterministic environment (from 2.31 to 2.34 percent), in contrast to a decrease in a risky environment (from 1.69 to 1.53 percent). That is, the interaction between uncertainty and scale economies brings down the benefits of specialization.

Table 6: Gains from Trade: Scale Effects, Uncertainty, and Worker Heterogeneity.

	US Gains from Trade.		
	Baseline	No uncertainty	$\kappa = 5$
Scale economies	1.53%	2.34%	1.41%
No scale economies	1.69%	2.31%	1.60%

Note: Gains from trade are calculated by comparing the autarkic to the observed economy, using (17). In the case of no uncertainty, we set $\sigma_{i,j} = 0$, for all i, j. In the case of no scale economies, we set $\rho_j = 0$ for all j. For the baseline case and no uncertainty case, $\kappa = 3$.

The aggregate numbers in Table 6 mask substantial heterogeneity across regions. In Figure 5, we show regional gains from trade: We aggregate over sectors the expression in (25) using region-specific sector employment shares, $\sum_j \lambda_{mj} \left(U_{mj}/U_{mj}^A\right)^{-\frac{\kappa}{1-\gamma}} / \sum_j \lambda_{mj}$. Table 7 shows summary statistics across CZs.

The mean and median CZs gain from trade in our baseline calibration, around 1.4-1.5 percent. However, some CZ loses, as indicated by the negative percent for the minimum. Eliminating uncertainty not only would produce more than 50 percent higher gains for the average CZ, but also no CZ would lose from trade. Simulating the model with both no uncertainty and no economies of scale reveals that economies of scale are also important in delivering positive gains from trade everywhere; the model without those two forces would deliver average gains 75 percent higher than in the baseline, but some locations would still experience loses from trade. Finally, more responsive labor ($\kappa = 5$) produces less gains from trade.

Figure 5 shows that workers in the Bay area, and the Northeast corridor tend to enjoy larger increases in utility from trade openness. In contrast, workers in the Mountain

Table 7: Regional Gains from Trade. Summary Statistics.

	mean	50-pc	5-pc	95-pc	minimum	maximum
Baseline	1.38%	1.49%	0.45%	1.91%	-0.76%	4.17%
No uncertainty	2.21%	2.19%	1.83%	2.66%	0.85%	4.97%
No scale economies	1.74%	1.79%	0.97%	2.36%	-1.03%	3.76%
No scale economies & No uncertainty	2.39%	2.38%	1.73%	3.00%	-0.44%	4.11%
$\kappa = 5$	1.27%	1.37%	0.39%	1.75%	-0.78%	4.71%
$\kappa = 5$ & No scale economies	1.68%	1.72%	1.07%	2.19%	-0.23%	3.29%
$\kappa = 5$ & No scale economies & No uncertainty	2.37%	2.35%	1.86%	2.87%	0.35%	3.67%

Note: Gains from trade in each CZ, $\sum_j \lambda_{mj} \hat{U}_{mj}^{-\frac{1}{1-\gamma}}$. For the model without uncertainty, we set $\sigma_{i,j} = 0$, for all i, j. For the model with no scale effects, we set $\rho_j = 0$, for all j. Statistics calculated over values across CZ's.

region, parts of Texas, and parts of the Rust Belt (darker blue) would lose from trade openness. The role of scale economies can be seen inspecting this figure: scale economies are strong enough that areas in northern Georgia, for instance, would lose from opening up to trade if we eliminated those forces, even if uncertainty were minimal.

Table 8 lists the communities with largest gains and the largest losses, both in the baseline model and the model with no uncertainty. While uncertainty decreases the gains from trade for the top winners between half and one percentage points, it turns gains into losses for locations such as El Paso (TX) and Bellingham (WA). Focusing on El Paso (TX) as an example reveals that while the ACR model predicts a positive gain from trade of 2.56 percent, our baseline model predicts an overall negative gain of -0.06 percent. This reversal is driven by substantial losses from economies of scale (-1.65 percent) and increased uncertainty (-0.92 percent). In line with the scale-related losses, the region also experiences an employment decline of 5.44 percent when moving from autarky to trade openness. Moreover, when labor mobility is more elastic (κ= 5), the trade losses deepen to -0.18 percent, driven by stronger employment responses (-9.14 percent). Conversely, when either economies of scale or uncertainty is removed, the gains from trade turn positive—0.70 and 1.56 percent, respectively. Overall, an interesting pattern emerges: While the top-ten winners remain about the same in the baseline and in a world with no uncertainty, the communities with the largest losses in the baseline change — they would not be the ones with the smallest gains if uncertainty were absent.

Further examining Figure 6, which shows labor reallocation across regions from autarky

Table 8: Smallest and Largest Responses in Regional Gains from Trade.

Baseline Smallest		Largest		No Uncertainty Smallest		Largest	
El Dorado (AR)	-0.76%	Devils Lake (ND)	2.26%	Andrews (NC)	0.85%	Miami (FL)	3.04%
Bellingham (WA)	-0.65%	Lake Providence (LA)	2.28%	Rome (GA)	0.95%	Las Vegas (NV)	3.04%
Brady (TX)	-0.14%	Orlando (FL)	2.28%	Kosciusko (MS)	1.36%	Lake Providence (LA)	3.07%
El Paso (TX)	-0.06%	Miami (FL)	2.37%	Matador (TX)	1.38%	San Francisco (CA)	3.08%
Beaumont (TX)	-0.05%	Limon (CO)	2.52%	Bonners Ferry (ID)	1.45%	Limon (CO)	3.28%
Dickinson (ND)	-0.03%	John Day (OR)	2.58%	Many (LA)	1.50%	John Day (OR)	3.32%
Bonners Ferry (ID)	-0.03%	Burns (OR)	3.23%	El Paso (TX)	1.56%	Burns (OR)	3.74%
Graham (TX)	0.02%	Murdo (SD)	3.35%	Bellingham (WA)	1.60%	Murdo (SD)	3.90%
Buckhannon (WV)	0.07%	Arlington (VA)	3.72%	Van Horn (TX)	1.60%	Arlington (VA)	4.55%
Ardmore (OK)	0.08%	New York (NY)	4.17%	Condon (OR)	1.61%	New York (NY)	4.97%

Note: Ten CZs with the largest or smallest gains from trade, with and without uncertainty, respectively. For each CZ, we list the largest location in the CZ in terms of population (data from USDA).

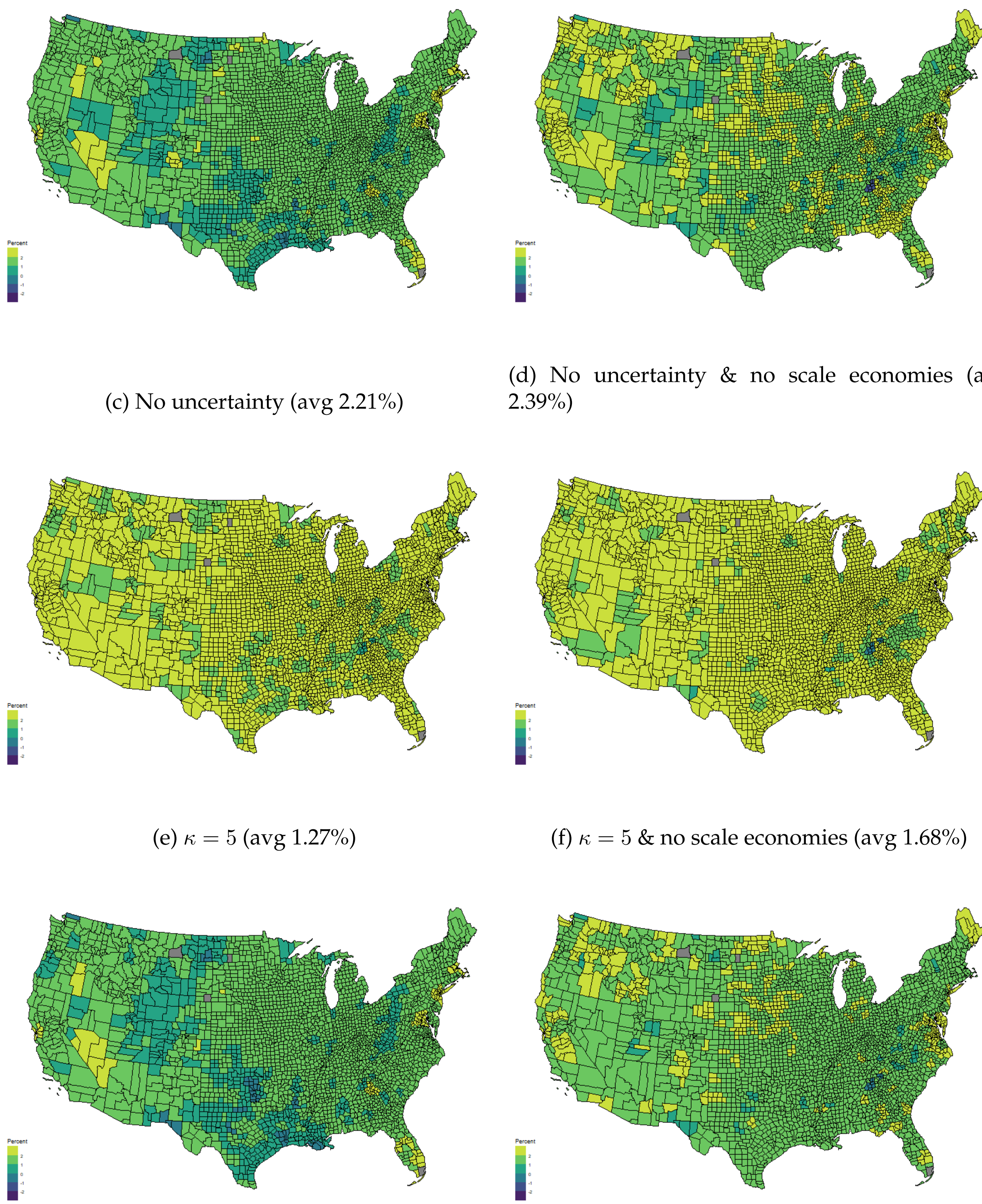
to trade openness, suggests that the regions that lose from openness to trade are the ones driving employment away. In the baseline model, the change in employment shares is -0.76 percent for the average CZ. Economies of scale, together with $\kappa = 5$, delivers the strongest employment responses with the average CZ having a 1.5 percent decrease in its employment share relative to autarky — doubling the magnitude delivered by our baseline model with a lower migration response ($\kappa = 3$). In the model with no scale economies, moving from autarky to the observed equilibrium slightly decreases the average employment share (-0.03 percent), while if uncertainty were removed, the average change in employment shares would be similar to the one in the baseline model. It is the interaction between uncertainty and economies of scale that compound to change the sign of the effect of autarky on employment shares: In the model without those two forces, the employment share of the average CZ would increase, not decrease, to 0.12 percent when the economy moves from the autarky to the observed equilibrium with trade.

7 Conclusion

In this paper, we construct a multi-sector spatial equilibrium model, featuring economies of scale, volatility in productivity, and heterogeneous workers who choose the region and sector where to work. We use the model to study how uncertainty affects specialization patterns and welfare. We provide analytical characterizations of the interactions between specialization, economies of scale, and uncertainty. We find empirical support for the model predictions using panel data for U.S. commuting zones. Quantitatively, the model calibrated to match features of subnational U.S. economies and other countries, for 2017, predicts that volatility shifts employment away from sectors and regions with high real wage volatility, and that considering economies of scale and uncertainty together cuts the gains from trade for the United States by more than half.

Our analysis highlights that volatility in fundamental productivity shapes specialization patterns within and across countries, as long as workers are risk averse. In principle, volatility can manifest in various other forms, such as trade policy uncertainty (Handley and Limão 2017, Alessandria et al. 2025) and supply chain disruptions (Grossman et al. 2023). Additionally, volatility may alter firm responses, such as increasing the inaction area of investments (Das, Roberts and Tybout 2007). Exploring the effects of these other types of volatility on specialization patterns and their interaction with economies of scale presents an intriguing area for future research.

Figure 5: Regional Gains from Trade.



(a) Baseline (avg 1.38%)

(b) No scale economies (avg 1.74%)

(c) No uncertainty (avg 2.21%)

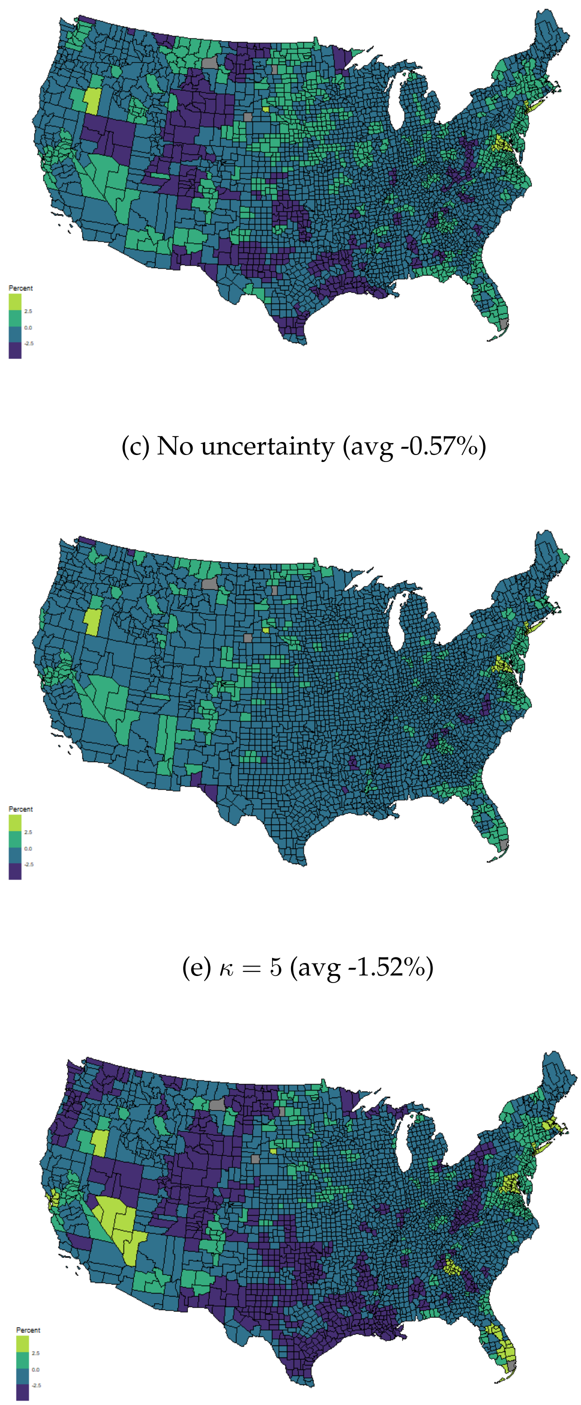
(d) No uncertainty & no scale economies (avg 2.39%)

(e) $\kappa = 5$ (avg 1.27%)

(f) $\kappa = 5$ & no scale economies (avg 1.68%)

Note: Gains from trade in each CZ, $\sum_j \lambda_{mj} \hat{U}_{mj}^{-\frac{1}{1-\gamma}}$. For the model without uncertainty, we set $\sigma_{i,j} = 0$, for all i, j. For the model with no scale effects, we set $\rho_j = 0$, for all j. For panels (a)-(d), $\kappa = 3$. Averages calculated over values across CZ's.

Figure 6: Employment Reallocation: From Autarky to Baseline Economy.



(a) Baseline (avg -0.76%)

(b) No scale economies (avg -0.03%)

(c) No uncertainty (avg -0.57%)

(d) No uncertainty & no scale economies (0.12%)

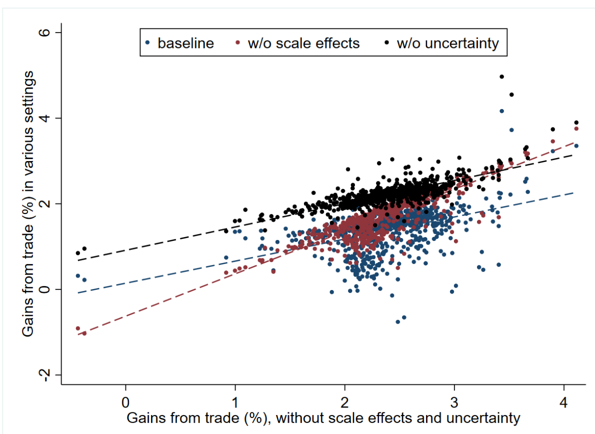
(e) $\kappa = 5$ (avg -1.52%)

(f) $\kappa = 5$ & no scale economies (avg -0.02%)

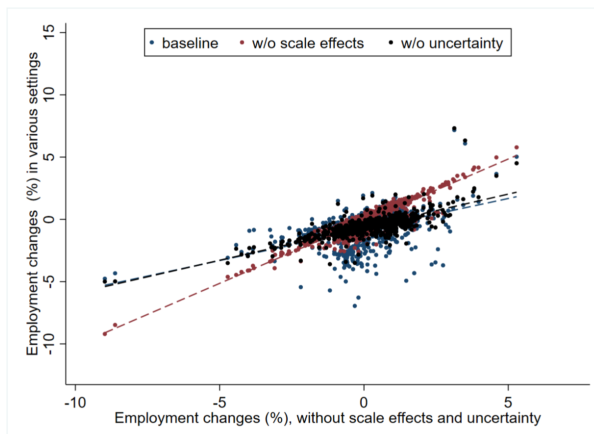
Note: Proportional changes in employment from autarky to the baseline equilibrium, in each CZ. For the model with no uncertainty, we set $\sigma_{i,j} = 0$ for all i, j. For the model with no scale effects, we set $\rho_j = 0$, for all j. For panels (a)-(d), $\kappa = 3$. Averages calculated over values across CZ's.

Figure 7: Regional Gains from Trade and Employment Reallocation: Correlations.

(a) Gains from trade



(b) Employment reallocation



Note: Regional gains from trade $\sum_j \lambda_{mj} \hat{U}_{mj}^{-\frac{1}{1-\gamma}}$ and proportional changes in employment from autarky to the baseline equilibrium calculated with baseline calibration, no scale economies ($\rho_j = 0$, for all j), no uncertainty ($\sigma_{i,j} = 0$, for all i, j), and no scale economies and uncertainty.

References

Alessandria, G., Arkolakis, C. and Ruhl, K. J. (2021), 'Firm dynamics and trade', *Annual Review of Economics* **13**(1), 253–280.

Alessandria, G., Choi, H. and Ruhl, K. (2021), 'Trade adjustment dynamics and the welfare gains from trade', *Journal of International Economics* **131**(5), 1034–58.

Alessandria, G., Khan, S., Khederlarian, A., Ruhl, K. and Steinberg, J. (2025), 'Trade-Policy Dynamics: Evidence from 60 Years of U.S.-China Trade', *Journal of Political Economy* **133**(3), 713–749.

Alfaro, L., Bloom, N. and Lin, X. (2018), 'The finance uncertainty multiplier', *The Quarterly Journal of Economics* **133**(1), 1–51.

Allen, T. and Arkolakis, C. (2014), 'Trade and the topography of the spatial economy', *Quarterly Journal of Economics* **129**(6), 1085–1140.

Allen, T. and Atkin, D. (n.d.), 'Volatility and the gains from trade', *Econometrica* **90**(5), 2053–2092.

Arkolakis, C., Costinot, A. and Rodríguez-Clare, A. (2012), 'New trade models, same old gains?', *American Economic Review* **102**(1), 94–130.

Autor, D., Dorn, D. and Hanson, G. H. (2013), 'The china syndrome: Local labor market effects of import competition in the united states', *American Economic Review* **103**(6), 2121–2168.

Autor, D. H., Dorn, D. and Hanson, G. H. (2016), 'The china shock: Learning from labor-market adjustment to large changes in trade', *Annual Review of Economics* **8**, 205–40.

Backus, D. K., Kehoe, P. J. and Kydland, F. E. (1992), 'International real business cycles', *American Economic Review* **82**(4), 864–888.

Bartelme, D. G., Costinot, A., Donaldson, D. and Rodríguez-Clare, A. (2025), 'The textbook case for industrial policy: Theory meets data', *Journal of Political Economy* **133**(5), 1527–1573.

Berger, D. W., Herkenhoff, K. F. and Mongey, S. (2022), 'Labor market power', *American Economic Review* **112**(7), 2098–2139.

Brexit and the macroeconomic impact of trade policy uncertainty (2019), *Journal of International Economics* **117**, 175–195.

Caliendo, L., Dvorkin, M. and Parro, F. (2019), 'Trade and Labor Market Dynamics: General Equilibrium Analysis of the China Trade Shock', *Econometrica* **87**(3), 741–835.

Caliendo, L., Kortum, S. S. and Parro, F. (2025), 'Tariffs and trade deficits', *NBER Working Paper 34003* .

Caliendo, L. and Parro, F. (2015), 'Estimates of the trade and welfare effects of NAFTA', *Review of Economic Studies* **82**(1), 1–44.

Caliendo, L., Parro, F., Rossi-Hansberg, E. and Sarte, P.-D. (2019), 'The impact of regional and sectoral productivity changes on the u.s. economy', *American Economic Review* **109**(10), 3678–3716.

Carvalho, V. M. and Gabaix, X. (2013), 'The great diversification and its undoing', *Econometrica* **81**(3), 883–940.

Caselli, F., Koren, M., Lisicky, M. and Tenreyro, S. (2020), 'Diversification through trade', *The Quarterly Journal of Economics* **135**(1), 449–502.

Castro-Vincenzi, J., Khanna, G., Morales, N. and Pandalai-Nayar, N. (2024), 'Weathering the storm: Supply chains and climate risk', *NBER Working Paper 32218* .

Costinot, A. and Rodríguez-Clare, A. (2014), Trade theory with numbers: Quantifying the consequences of globalization, *in* G. Gopinath, E. Helpman and K. Rogoff, eds, 'Handbook of International Economics', Vol. 4, chapter 4.

Das, S., Roberts, M. J. and Tybout, J. R. (2007), 'Market entry costs, producer heterogeneity, and export dynamics', *Econometrica* **75**(3), 837–873.

Dekle, R., Eaton, J. and Kortum, S. (2008), 'Global rebalancing with gravity: Measuring the burden of adjustment', *IMF Staff Papers* **55**(3), 511–540.

Eaton, J. and Kortum, S. (2002), 'Technology, geography and trade', *Econometrica* **70**(5), 1741–1779.

Elminejad, A., Havránek, T. and Irsova, Z. (2022), 'Relative risk aversion: A meta-analysis', *Journal of Economic Surveys* **36**(2), 498–529.

Fajgelbaum, P. D. and Gaubert, C. (2020), 'Optimal Spatial Policies, Geography, and Sorting', *The Quarterly Journal of Economics* **135**(2), 959–1036.

Fajgelbaum, P. D., Morales, E., Serrato, J. C. S. and Zidar, O. M. (2019), 'State Taxes and Spatial Misallocation', **86**(1), 333–376.

Fan, J. and Luo, W. (2025), Global production networks with global uncertainty. Mimeo, Penn State University.

Fitzgerald, D. (2012), 'Trade costs, asset market frictions, and risk sharing', *American Economic Review* **102**(6), 2700–2733.

Galle, S., Rodríguez-Clare, A. and Yi, M. (2023), 'Slicing the pie: Quantifying the aggregate and distributional effects of trade', *Review of Economic Studies* **90**(1), 411–439.

Grossman, G. M., Helpman, E. and Lhuillier, H. (2023), 'Supply chain resilience: Should policy promote international diversification or reshoring?', *Journal of Political Economy* **131**(12), 3462–3496.

Handley, K. and Limão, N. (2017), 'Policy uncertainty, trade, and welfare: Theory and evidence for china and the united states', *American Economic Review* **107**(9), 2731–2783.

Kleinman, B., Liu, E. and Redding, S. J. (2025), International trade in an uncertain world. Mimeo, Princeton University.

Koren, M. and Tenreyro, S. (2007), 'Volatility and development', *The Review of Economics and Statistics* **89**(4), 670–687.

Koren, M. and Tenreyro, S. (2013), 'Technological diversification', *The American Economic Review* **103**(1), 378–414.

Kucheryavyy, K., Lyn, G. and Rodríguez-Clare, A. (2020), 'Grounded by gravity: A well-behaved trade model with external economies', *Journal of Political Economy* **128**(6), 2344–2407.

Lagakos, D. and Waugh, M. E. (2013), 'Specialization, agriculture, and cross-country productivity differences', *American Economic Review* **103**(2), 948–980.

Lyon, S. and Waugh, M. (2019), Quantifying the Losses from International Trade. Mimeo, NYU-Stern.

Monte, F., Redding, S. J. and Rossi-Hansberg, E. (2018), 'Commuting, Migration, and Local Employment Elasticities', *American Economic Review* **108**(12), 3855–3890.

Simonovska, I. and Waugh, M. (2013), 'The elasticity of trade: Estimates and evidence', *Journal of International Economics* **92**, 34–50.

A Derivations and Proofs

We denote by: $\bar{X}$ the expected value of a state-dependent variable X, $\bar{X} \equiv \sum_s \mathbb{P}(s)X(s)$; X^* the value of X in the deterministic equilibrium; $\hat{X}(s) \equiv dX(s)/X^*$ deviations around X^* for state-dependent variables ($\hat{X} \equiv dX/X^*$ for non-state dependent variables); and $\bar{\hat{X}} \equiv \sum_s \mathbb{P}(s)\hat{X}(s)$. We normalize global expenditure in each state s to one, $\sum_i X_i(s) = 1$.

A.1 Proof of Lemma 1

Assume that trade is frictionless across regions and countries. The market clearing condition for sector j and region m is

$$W_{mj}(s)L_{mj} = \frac{(W_{mj}(s)/A_{mj}(s))^{-\theta_j}}{\sum_{m'}(W_{m'j}(s)/A_{m'j}(s))^{-\theta_j}}\alpha_j. \tag{A.1}$$

We can solve for wages analytically,

$$W_{mj}(s) = \alpha_j \frac{A_{mj}(s)^{\frac{\theta_j}{\theta_j+1}} L_{mj}^{-\frac{1}{\theta_j+1}}}{\sum_{m'} A_{m'j}(s)^{\frac{\theta_j}{\theta_j+1}} L_{m'j}^{\frac{\theta_j}{\theta_j+1}}}, \tag{A.2}$$

as well as trade shares,

$$\pi_{mj}(s) = \frac{(A_{mj}(s)L_{mj})^{\frac{\theta_j}{\theta_j+1}}}{\sum_{n'}(A_{n'j}(s)L_{n'j})^{\frac{\theta_j}{\theta_j+1}}}$$

Using (5), we can further solve for the final-good price index

$$P_m(s) = C\prod_j \left(\sum_{m'\in M} A_{m'j}(s)^{\frac{\theta_j}{\theta_j+1}} L_{m'j}^{\frac{\theta_j}{\theta_j+1}}\right)^{-\alpha_j \frac{1+\theta_j}{\theta_j}}, \tag{A.3}$$

where $C \equiv \prod_j (C_j)^{\alpha_j} > 0$ and $C_j \equiv \Gamma\left(1 - \frac{\eta_j - 1}{\theta_j}\right)^{\frac{1}{\eta_j - 1}}$. Taking logs on (A.2) and (A.3) yields

$$\log W_{mj}(s) = \log \alpha_j + \frac{\theta_j}{\theta_j+1}\log A_{mj}(s) - \frac{1}{\theta_j+1}\log L_{mj} - \log \sum_{m'} A_{m'j}(s)^{\frac{\theta_j}{\theta_j+1}} L_{m'j}^{\frac{\theta_j}{\theta_j+1}},$$

and

$$\log P_m(s) = \sum_j \alpha_j \log C_j - \sum_j \alpha_j \frac{1+\theta_j}{\theta_j}\log \sum_{m'} A_{m'j}(s)^{\frac{\theta_j}{\theta_j+1}} L_{m'j}^{\frac{\theta_j}{\theta_j+1}}.$$

We calculate fluctuations of wages and prices around the deterministic equilibrium by linearizing the two previous expressions,

$$\hat{W}_{mj}(s) = \frac{\theta_j}{\theta_j+1}\hat{A}_{mj}(s) - \frac{1}{\theta_j+1}\hat{L}_{mj} - \sum_{m'} \pi^*_{m'j}\frac{\theta_j}{\theta_j+1}(\hat{A}_{m'j}(s) + \hat{L}_{m'j}), \tag{A.4}$$

and

$$\hat{P}_m(s) = -\sum_k \alpha_k \sum_{m'} \pi^*_{m'k}(\hat{A}_{m'k}(s) + \hat{L}_{m'k}), \tag{A.5}$$

where π^*_{mj} is the trade share from region m, sector j in the deterministic equilibrium. Fluctuations of the real wage around its deterministic value are simply given by

$$\begin{aligned}\hat{W}_{mj}(s) - \hat{P}_m(s) =& \frac{\theta_j}{\theta_j+1}\hat{A}_{mj}(s) - \frac{1}{\theta_j+1}\hat{L}_{mj} \\ & - \sum_{m'}\left(\pi^*_{m'j}\frac{\theta_j}{\theta_j+1}(\hat{A}_{m'j}(s)+\hat{L}_{m'j}) - \sum_k \alpha_k \pi^*_{m'k}(\hat{A}_{m'k}(s)+\hat{L}_{m'k})\right).\end{aligned}$$

Shocks are country-industry specific so that $\hat{A}_{mj}(s) - \hat{\bar{A}}_{mj} = \hat{A}_{nj}(s) - \hat{\bar{A}}_{nj} = \hat{A}_{i,j}(s)$ for all $m, n \in M_i$. From (A.4),

$$\begin{aligned}\hat{W}_{mj}(s) &= \frac{\theta_j}{\theta_j+1}\hat{A}_{mj}(s) - \frac{1}{\theta_j+1}\hat{L}_{mj} - \sum_{m'}\pi^*_{m'j}\frac{\theta_j}{\theta_j+1}(\hat{A}_{m'j}(s)+\hat{L}_{m'j}) \\ &= \frac{\theta_j}{\theta_j+1}(\hat{\bar{A}}_{mj} + \hat{A}_{mj}(s)) - \frac{1}{\theta_j+1}\hat{L}_{mj} - \sum_{m'}\pi^*_{m'j}\frac{\theta_j}{\theta_j+1}(\hat{\bar{A}}_{m'j} + \hat{A}_{m'j}(s)+\hat{L}_{m'j}) \\ &= \frac{\theta_j}{\theta_j+1}\left(1 - \sum_{m'\in M_i}\pi^*_{m'j}\right)\hat{A}_{i,j}(s) - \frac{\theta_j}{\theta_j+1}\sum_{i'\neq i}\left(\sum_{m'\in M_{i'}}\pi^*_{m'j}\right)\hat{A}_{i',j}(s) \\ &+ \frac{\theta_j}{\theta_j+1}\hat{\bar{A}}_{mj} - \frac{1}{\theta_j+1}\hat{L}_{mj} - \sum_{m'}\pi^*_{m'j}\frac{\theta_j}{\theta_j+1}(\hat{\bar{A}}_{m'j}+\hat{L}_{m'j})\end{aligned}$$

Similarly, the price changes can be rewritten as:

$$\begin{aligned}\hat{P}_m(s) &= -\sum_k \alpha_k \sum_{m'}\pi^*_{m'k}(\hat{A}_{m'k}(s)+\hat{L}_{m'k}) \\ &= -\sum_k \alpha_k \sum_{i'}\left(\sum_{m'\in M_{i'}}\pi^*_{m'k}\right)\hat{A}_{i'k}(s) - \sum_k \alpha_k \sum_{m'}\pi^*_{m'k}(\hat{\bar{A}}_{m'k}+\hat{L}_{m'k}).\end{aligned}$$

We obtain (12) by subtracting $\hat{W}_{mj}(s)$ from $\hat{P}_m(s)$, combining the coefficients on each country-industry shock, $\hat{A}_{ik}$, and calculating the variance under the assumption that shocks are i.i.d.

A.2 Autarky

Using (A.1) and (A.3) under international trade autarky, but frictionless trade across regions within a country, the autarky wage is given by $W_{i,j}^A = \alpha_j / L_{i,j}^A$, while the price index is $P_i^A(s) = C \prod_j \left(A_{i,j}(s) L_{i,j}^A\right)^{-\alpha_j}$. Hence, the average real wage is given by

$$\frac{\bar{W}_{i,j}^A}{\bar{P}_i^A} = \frac{\alpha_j}{C L_{i,j}^A} \mathbb{E} \prod_{j'} (L_{i,j'}^A A_{i,j'}(s))^{\alpha_{j'}},$$

Using (2), autarkic labor shares in country i and sector j are

$$\lambda_{i,j}^A = \frac{\left(\bar{W}_{i,j}^A / \bar{P}_i^A\right)^\kappa}{\sum_{j'} \left(\bar{W}_{i,j'}^A / \bar{P}_i^A\right)^\kappa}$$

With $L_{i,j}^A = C_1 (\lambda_{i,j}^A)^{\frac{\kappa-1}{\kappa}} L_i$, we get that $\lambda_{i,j}^A = \alpha_j$, and $W_{i,j}^A = \alpha_j^{1/\kappa} / C_1 L_i$.

A.3 Proof of Proposition 1

A second-order approximation of expected utility in (1) yields

$$U_{mj} \approx \frac{1}{1-\gamma} \left(\frac{\bar{W}_{mj}}{\bar{P}_m}\right)^{1-\gamma} \left(1 - \frac{\gamma(1-\gamma)}{2} \text{var}(\hat{W}_{mj}(s) - \hat{P}_m(s))\right), \tag{A.6}$$

while further considering fluctuations around the deterministic equilibrium yields

$$\hat{U}_{mj} \approx (1-\gamma)(\hat{\bar{W}}_{mj} - \hat{\bar{P}}_m) - \frac{\gamma(1-\gamma)}{2} d\text{var}\left(\hat{W}_{mj}(s) - \hat{P}_m(s)\right), \tag{A.7}$$

where

$$\hat{\bar{W}}_{mj} = -\frac{1}{\theta_j + 1}(1 - \rho_j \theta_j) \hat{L}_{mj} - \frac{\theta_j}{\theta_j + 1}(\rho_j + 1) \sum_n \pi_{nj}^* \hat{L}_{nj} \tag{A.8}$$

$$\hat{\bar{P}}_m = -\sum_k \alpha_k \sum_n \pi_{nk}^* (1 + \rho_k) \hat{L}_{nk} \tag{A.9}$$

and we use Assumption 1 to calculate $\hat{\bar{A}}_{mj} = \rho_j \hat{L}_{mj}$.

Using the expression for employment shares in (2), and the approximation in (A.6), we can calculate

$$\lambda_{mj} = \left(\frac{\bar{W}_{mj}}{\bar{P}_m}\right)^\kappa \frac{\left(1 - \frac{\gamma(1-\gamma)}{2} \text{var}(\hat{W}_{mj}(s) - \hat{P}_m(s))\right)^{\frac{\kappa}{1-\gamma}}}{\sum_{j'} \sum_{m' \in M_i} \left(\frac{\bar{W}_{m'j'}}{\bar{P}_{m'}}\right)^\kappa \left(1 - \frac{\gamma(1-\gamma)}{2} \text{var}(\hat{W}_{m'j'}(s) - \hat{P}_{m'}(s))\right)^{\frac{\kappa}{1-\gamma}}}.$$

Fluctuations of the employment share around the deterministic value are given by

$$\hat{\lambda}_{mj} = \frac{\kappa}{1-\gamma}\hat{U}_{mj} - \sum_{n\in M_i}\sum_{k}\lambda^*_{nk}\frac{\kappa}{1-\gamma}\hat{U}_{nk}, \tag{A.10}$$

where λ^*_{nk} is the employment share of region n, sector k, in the deterministic equilibrium, and $\sum_{n\in M_i,k}\lambda^*_{nk} = 1$.

Using (3), we get that

$$\hat{L}_{mj} = \frac{\kappa-1}{\kappa}\hat{\lambda}_{mj}. \tag{A.11}$$

Because the shock is common to all regions and sectors, and trade is frictionless, $\hat{L}_{mj} = \hat{L}_{nj}$, for all $m, n \in M_i$, implying that $\hat{\lambda}_{mj} = \hat{\lambda}_{nj}$.[14]

Substituting (A.7) into (A.10) yields

$$\frac{1}{\kappa}\hat{\lambda}_{mj} = \hat{\bar{W}}_{mj} - \hat{\bar{P}}_m - \frac{\gamma}{2}d\text{var}\left(\hat{W}_{mj}(s) - \hat{P}_m(s)\right) - \sum_{n\in M_i}\sum_{k}\lambda^*_{nk}\left[\hat{\bar{W}}_{nk} - \hat{\bar{P}}_n - \frac{\gamma}{2}d\text{var}\left(\hat{W}_{nk}(s) - \hat{P}_n(s)\right)\right],$$

which can be further written as

$$\frac{1}{\kappa}\hat{\lambda}_{mj} = \hat{\bar{W}}_{mj} - \frac{\gamma}{2}d\text{var}\left(\hat{W}_{mj}(s) - \hat{P}_m(s)\right) - \sum_{n\in M_i}\sum_{k}\lambda^*_{nk}\left[\hat{\bar{W}}_{nk} - \frac{\gamma}{2}d\text{var}\left(\hat{W}_{nk}(s) - \hat{P}_n(s)\right)\right].$$

Using (A.8), $\hat{L}_{mj} = \hat{L}_{nj}$ for all $m, n \in M_i$, and (A.11) yield

$$\hat{\bar{W}}_{mj} = \left[\frac{1}{\theta_j+1}(\rho_j\theta_j - 1) - \pi^*_{i,j}\frac{\theta_j}{\theta_j+1}(\rho_j+1)\right]\frac{\kappa-1}{\kappa}\hat{\lambda}_{mj}, \tag{A.12}$$

where we further assume that country i is small relative to the rest of the world so that for $n \notin M_i$, $\hat{L}_{nj} = 0$ for all j.

Define

$$\zeta_j \equiv -(\kappa-1)\,\frac{1-\theta_j\rho_j + \pi^*_{i,j}\theta_j(\rho_j+1)}{1+\theta_j},$$

which is negative for $\theta_j\rho_j \in [0,1)$. Replacing $\hat{\bar{W}}_{mj}$ in the expression for $\hat{\lambda}_{mj}$ and noting that $\sum_{n\in M_i}\sum_k \lambda^*_{nk}\hat{\lambda}_{nk} = 0$ yield

$$\hat{\lambda}_{mj} = \frac{-\gamma\kappa\left[d\text{var}\left(\hat{W}_{mj}(s) - \hat{P}_m(s)\right) - \sum_{n\in M_i}\sum_k \lambda^*_{nk}d\text{var}\left(\hat{W}_{nk}(s) - \hat{P}_n(s)\right)\right]}{2(1-\zeta_j)}.$$

Under frictionless trade, for $m \neq n$, we have that

$$\text{var}\left(\hat{W}_{mj}(s) - \hat{P}_m(s)\right) = \text{var}\left(\hat{W}_{nj}(s) - \hat{P}_n(s)\right),$$

[14] If we guess $\hat{L}_{mj} = \hat{L}_{nj}$, then using (A.8), $\hat{\bar{W}}_{mj} = \hat{\bar{W}}_{nj}$. Plugging $\hat{\bar{W}}_{mj} = \hat{\bar{W}}_{nj}$ into (A.10) and notice that changes in the price index and real-wage volatility are the same for all $m \in M_i$ in industry j, we obtain that $\hat{L}_{mj} = \hat{L}_{nj}$ for all $m, n \in M_i$.

so that

$$\hat{\lambda}_{mj} = \frac{\gamma\kappa\left[\sum_k \lambda^*_{i,k} d\text{var}\left(\hat{W}_{mk}(s) - \hat{P}_m(s)\right) - d\text{var}\left(\hat{W}_{mj}(s) - \hat{P}_m(s)\right)\right]}{2(1-\zeta_j)}. \tag{A.13}$$

Since $d\sigma_{i',k} = 0$ for $i' \neq i$ and $k \neq j$, using (12) yields

$$d\text{var}\left(\hat{W}_{mj}(s) - \hat{P}_m(s)\right) = \left(\frac{\theta_j}{1+\theta_j}(1-\pi^*_{i,j}) + \alpha_j \pi^*_{i,j}\right)^2 d\sigma^2_{i,j},$$

and for all $k \neq j$,

$$d\text{var}\left(\hat{W}_{mk}(s) - \hat{P}_m(s)\right) = (\alpha_j \pi^*_{i,j})^2 d\sigma^2_{i,j}.$$

Replacing in (A.13) yields

$$\hat{\lambda}_{mj} = \frac{-\gamma\kappa(1-\lambda^*_{i,j})}{2(1-\zeta_j)} \frac{\theta_j}{1+\theta_j}(1-\pi^*_{i,j}) \left[\frac{\theta_j}{1+\theta_j}(1-\pi^*_{i,j}) + 2\alpha_j \pi^*_{i,j}\right] d\sigma^2_{i,j}, \tag{A.14}$$

which entails that

$$\frac{\partial \lambda_{mj}/\lambda^*_{mj}}{\partial \sigma^2_{i,j}} < 0,$$

where $\hat{\lambda}_{mj} \approx \partial\lambda_{mj}/\lambda^*_{mj}$. Further taking derivative with respect to ρ_j and γ, respectively, we obtain that $\frac{\partial^2 \lambda_{mj}/\lambda^*_{mj}}{\partial \sigma^2_{i,j} \partial \gamma} < 0$ and $\frac{\partial^2 \lambda_{mj}/\lambda^*_{mj}}{\partial \sigma^2_{i,j} \partial \rho_j} < 0$.

Finally, taking derivative with respect to $\pi^*_{i,j}$ yields

$$\begin{aligned}\frac{\partial^2 \lambda_{mj}/\lambda^*_{mj}}{\partial \sigma^2_{ij} \partial \pi^*_{i,j}} &= \frac{-\gamma\kappa(1-\lambda^*_{i,j})\left[-2\left(\frac{\theta_j}{1+\theta_j}\right)^2(1-\pi^*_{i,j}) + 2\frac{\theta_j}{1+\theta_j}\alpha_j(1-2\pi^*_{i,j})\right]}{2(1-\zeta_j)} \\ &+ \frac{\gamma\kappa(1-\lambda^*_{i,j})\left[\left(\frac{\theta_j}{1+\theta_j}(1-\pi^*_{i,j})\right)^2 + 2\alpha_j \frac{\theta_j}{1+\theta_j}(1-\pi^*_{i,j})\pi^*_{i,j}\right]}{2(1-\zeta_j)^2} \frac{\theta_j(\rho_j+1)}{1+\theta_j}(\kappa-1).\end{aligned}$$

Less trade exposure (i.e. higher $\pi^*_{i,j}$) lowers the negative impact of volatility, $\frac{\partial^2 \lambda_{mj}/\lambda^*_{mj}}{\partial \sigma^2_{i,j} \partial \pi^*_{i,j}} > 0$, as long as $\alpha_j < \frac{\theta_j}{1+\theta_j}$.

A.4 Average Expected Utility

Average expected utility for workers in region m and industry j is

$$\begin{aligned}
\mathbb{U}_{mj} &= \int_0^\infty U_{mj} x^{1-\gamma} g(x) \prod_{j'\neq j \text{ or } m'\neq m\ (m'\in M_i)} F\left(\frac{U_{mj}^{1/(1-\gamma)} x}{U_{m'j'}^{1/(1-\gamma)}}\right) dx/\lambda_{mj} \\
&= \int_0^\infty U_{mj} y^{-(1-\gamma)/\kappa} \left(\sum_{m'\in M_i, j'} \left(\frac{U_{m'j'}}{U_{mj}}\right)^{\kappa/(1-\gamma)}\right)^{(1-\gamma)/\kappa} \exp(-y) dy \qquad \text{(A.15)} \\
&= \Gamma\left(1 - \frac{1-\gamma}{\kappa}\right)\left(\sum_{m'\in M_i, j'} (U_{m'j'})^{\kappa/(1-\gamma)}\right)^{(1-\gamma)/\kappa},
\end{aligned}$$

where $\Gamma(\cdot)$ is the Gamma function, and $F(\cdot)$ is a unit- Fréchet distribution with shape parameter $\kappa > 0$. The first equality follows from the definition of average utility for workers in region m and industry j, which is the integral of utility conditional on choosing region m and industry j, adjusted by the probability of choosing region m and industry j. The second equality uses the change of variables $y = \sum_{m'\in M_i, j'} \left(\frac{U_{m'j'}}{U_{mj}}\right)^{\kappa/(1-\gamma)} x^{-\kappa}$, and the final line simplifies the formula.

A.5 Proof of Proposition 2

Assume that $M_i = 1$, for all i. A second-order approximation of average expected utility yields

$$\begin{aligned}
\mathbb{U}_i &= \Gamma\left(\frac{\kappa+\gamma-1}{\kappa}\right)\left(\sum_{j'} (U_{i,j'})^{\frac{\kappa}{1-\gamma}}\right)^{\frac{1-\gamma}{\kappa}} \\
&\approx \Gamma\left(\frac{\kappa+\gamma-1}{\kappa}\right)\left(\sum_{j'} \left(\frac{\bar{W}_{i,j'}}{\bar{P}_i}\right)^{\kappa} \left(\frac{1}{1-\gamma} - \frac{\gamma}{2}\text{var}\left(\hat{W}_{i,j'}(s) - \hat{P}_i(s)\right)\right)^{\frac{\kappa}{1-\gamma}}\right)^{\frac{1-\gamma}{\kappa}} \\
&\approx \Gamma\left(\frac{\kappa+\gamma-1}{\kappa}\right)\left(\sum_{j'} \left(\frac{\bar{W}_{i,j'}}{\prod_j \bar{W}_{i,j}^{\alpha_j}}\right)^{\kappa} \left(\frac{1}{1-\gamma} - \frac{\gamma}{2}\text{var}\left(\hat{W}_{i,j'}(s) - \hat{P}_i(s)\right)\right)^{\frac{\kappa}{1-\gamma}}\right)^{\frac{1-\gamma}{\kappa}} \left(\frac{\prod_j \bar{W}_{i,j}^{\alpha_j}}{\bar{P}_i}\right)^{1-\gamma}
\end{aligned}$$

Since

$$\lambda_{i,j} = \frac{(\bar{W}_{i,j}/\bar{P}_i)^{\kappa}\left(1 - \frac{\gamma(1-\gamma)}{2}\text{var}\left(\hat{W}_{i,j}(s) - \hat{P}_i(s)\right)\right)^{\kappa/(1-\gamma)}}{\sum_{j'} (\bar{W}_{i,j'}/\bar{P}_i)^{\kappa}\left(1 - \frac{\gamma(1-\gamma)}{2}\text{var}\left(\hat{W}_{i,j'}(s) - \hat{P}_i(s)\right)\right)^{\kappa/(1-\gamma)}},$$

we have that the second term on the right-hand side can be written as

$$\sum_{j'} \left(\frac{\bar{W}_{i,j'}}{\prod_j \bar{W}_{i,j}^{\alpha_j}} \right)^{\kappa} \left(\frac{1}{1-\gamma} - \frac{\gamma}{2} \text{var}\left(\hat{W}_{i,j'}(s) - \hat{P}_i(s) \right) \right)^{\frac{\kappa}{1-\gamma}}$$

$$= \left(\frac{1}{1-\gamma} \right)^{\frac{\kappa}{1-\gamma}} \prod_j \left(\frac{\left(1 - \frac{(1-\gamma)\gamma}{2} \text{var}\left(\hat{W}_{ij}(s) - \hat{P}_i(s) \right) \right)^{\kappa/(1-\gamma)}}{\lambda_{i,j}} \right)^{\alpha_j}$$

while the last term on the right-hand side is

$$\frac{\prod_j \bar{W}_{i,j}^{\alpha_j}}{\bar{P}_i} = C^{-1} \frac{\prod_j \bar{W}_{i,j}^{\alpha_j}}{\prod_j \left((\bar{W}_{i,j}/\bar{A}_{i,j})^{-\theta_j} / \bar{\pi}_{i,i}^j \right)^{-\frac{\alpha_j}{\theta_j}}} = C^{-1} \prod_j (\lambda_{i,j})^{\rho_j \alpha_j \frac{\kappa-1}{\kappa}} \prod_j \left(\bar{\pi}_{i,i}^j \right)^{-\frac{\alpha_j}{\theta_j}},$$

where $C > 0$ is a constant, and we use that $\bar{A}_{i,j} \propto L_{i,j}^{\rho_j}$ and $L_{i,j} \propto \lambda_{i,j}^{(\kappa-1)/\kappa}$. Hence,

$$\mathbb{U}_i^{\frac{1}{1-\gamma}} \approx C_2 \prod_j \left(1 - \frac{(1-\gamma)\gamma}{2} \text{var}\left(\hat{W}_{ij}(s) - \hat{P}_i(s) \right) \right)^{\frac{\alpha_j}{1-\gamma}} \prod_j (\lambda_{i,j})^{-\frac{\alpha_j(\rho_j+1)}{\kappa}} \prod_j (\lambda_{i,j})^{\alpha_j \rho_j} \prod_j \left(\bar{\pi}_{i,i}^j \right)^{-\frac{\alpha_j}{\theta_j}}.$$

where C_2 is a constant.

The gains from trade are

$$GT_i \equiv \left(\frac{\mathbb{U}_i}{\mathbb{U}_i^A} \right)^{\frac{1}{1-\gamma}} \approx \prod_j \left(\frac{1 - \frac{(1-\gamma)\gamma}{2} \text{var}\left(\hat{W}_{ij}(s) - \hat{P}_i(s) \right)}{1 - \frac{(1-\gamma)\gamma}{2} \text{var}\left(\hat{W}_{ij}^A(s) - \hat{P}_i^A(s) \right)} \right)^{\frac{\alpha_j}{1-\gamma}} \prod_j \left(\frac{\lambda_{i,j}}{\lambda_{i,j}^A} \right)^{-\frac{\alpha_j(\rho_j+1)}{\kappa} + \alpha_j \rho_j} \prod_j \left(\bar{\pi}_{i,i}^j \right)^{-\frac{\alpha_j}{\theta_j}}.$$

If $\kappa \to \infty$, $\lambda_{i,j}^A = \alpha_j$ and the gains from trade collapse to;

$$GT_i \equiv \left(\frac{\mathbb{U}_i}{\mathbb{U}_i^A} \right)^{\frac{1}{1-\gamma}} \approx \prod_j \left(\frac{1 - \frac{(1-\gamma)\gamma}{2} \text{var}\left(\hat{W}_{ij}(s) - \hat{P}_i(s) \right)}{1 - \frac{(1-\gamma)\gamma}{2} \text{var}\left(\hat{W}_{ij}^A(s) - \hat{P}_i^A(s) \right)} \right)^{\frac{\alpha_j}{1-\gamma}} \prod_j \left(\frac{\lambda_{i,j}}{\alpha_j} \right)^{\alpha_j \rho_j} \prod_j \left(\bar{\pi}_{i,i}^j \right)^{-\frac{\alpha_j}{\theta_j}},$$

where the variance of the real wage under autarky is given by (13) and in the equilibrium with frictionless trade by (12).

In the case of symmetric countries, the variance of real wages, applying Lemma 1, collapses to

$$\text{var}\left(\hat{W}_{i,j}(s) - \hat{P}_i(s) \right) = \frac{N-1}{N} \left(\frac{\theta_j}{1+\theta_j} \right)^2 \sigma_j^2 + \frac{1}{N} \sum_j \alpha_j^2 \sigma_j^2.$$

so that gains from trade are

$$GT_i \approx \prod_j (\bar{\pi}^j_{ii})^{-\frac{\alpha_j}{\theta_j}} \prod_j \left(\frac{\lambda_{i,j}}{\alpha_j}\right)^{\alpha_j \rho_j} \prod_j \left(\frac{1 - \frac{\gamma(1-\gamma)}{2} \frac{N-1}{N} \left(\frac{\theta_j}{\theta_j+1}\right)^2 \sigma_j^2 - \frac{\gamma(1-\gamma)}{2} \frac{1}{N} \sum_j \alpha_j^2 \sigma_j^2}{1 - \frac{\gamma(1-\gamma)}{2} \sum_j \alpha_j^2 \sigma_j^2} \right)^{\frac{\alpha_j}{1-\gamma}}.$$

B Model with Sectoral Input-Output Linkages

Each intermediate good ω is produced with labor and the aggregate sectoral good from each sector k, with shares δ^L_{mj} and δ^k_{mj}, respectively, with $\delta^L_{mj} + \sum_k \delta^k_{mj} = 1$. We present the equilibrium of the model with sectoral input-output links given the workers' location and sectoral choices. In this case, the unit cost of production in region m and sector j is:

$$c_{mj}(s) = \left(\frac{W_{mj}(s)}{\delta^L_{mj}}\right)^{\delta^L_{mj}} \prod_k \left(\frac{P_{mk}(s)}{\delta^k_{mj}}\right)^{\delta^k_{mj}}. \tag{B.1}$$

The price index in region m and sector j is

$$P_{mj}(s) = C_j \left(\sum_{l \in M} \left(\frac{\tau^j_{lm} c_{lj}(s)}{A_{lj}(s)} \right)^{-\theta_j} \right)^{-\frac{1}{\theta_j}}, \tag{B.2}$$

$C_j \equiv \Gamma \left(\frac{1-\eta_j+\theta_j}{\theta_j}\right)^{\frac{1}{1-\eta_j}}$, and the final-good price in region m is:

$$P_m(s) = \prod_j \left(\frac{P_{mj}(s)}{\alpha_j}\right)^{\alpha_j}. \tag{B.3}$$

The good-market clearing condition in region m and sector j requires that

$$Y_{lj}(s) = \sum_m \frac{(\tau^j_{lm} c_{lj}(s)/A_{lj}(s))^{-\theta_j}}{\sum_{l' \in M} (\tau^j_{l'm} c_{l'j}(s)/A_{l'j}(s))^{-\theta_j}} X_{mj}(s), \tag{B.4}$$

where $Y_{lj}(s)$ is the production value in region m and sector j, and $X_{mj}(s) = \sum_k \delta^j_{mk} Y_{mk}(s) + \alpha_j \left(\sum_k \delta^L_{mk} Y_{mk}(s)\right)$ is the total expenditures on goods from sector j in region m.

Finally, the labor market clearing requires that

$$\delta^L_{mj} Y_{mj}(s)/W_{mj}(s) = L_{mj}. \tag{B.5}$$

The left-hand side is the demand for labor in region m and sector j, and the right-hand side is the labor supply in region m and sector j.

We can combine (B.1)–(B.5) to solve for $\{W_{mj}(s), c_{mj}(s), Y_{mj}(s), P_{mj}(s), P_m(s)\}$. Finally, we can solve workers' location and sectoral choices λ_{mj} from (2).

With a slight abuse of notation, define $\widehat{x} \equiv \frac{x'}{x}$ as the counterfactual change in variable x. Next, we present the equilibrium system (B.1)–(B.5) in changes,

$$\widehat{c}_{mj}(s) = \left(\widehat{W}_{mj}(s)\right)^{\delta^L_{mj}} \prod_k \left(\widehat{P}_{mk}(s)\right)^{\delta^k_{mj}}, \tag{B.6}$$

$$\widehat{P}_{mj}(s) = \left(\sum_{l \in M} \pi^j_{lm}(s) \left(\frac{\hat{\tau}^j_{lm}\widehat{c}_{lj}(s)}{\widehat{A}_{lj}(s)}\right)^{-\theta_j}\right)^{-\frac{1}{\theta_j}}, \tag{B.7}$$

where $\widehat{A}_{lj}(s) = \widehat{L}^{\rho_s}_{lj}$ captures changes in economies of scale,

$$\widehat{P}_m(s) = \prod_j \left(\widehat{P}_{mj}(s)\right)^{\alpha_j}, \tag{B.8}$$

$$\widehat{Y}_{lj}(s)Y_{lj}(s) = \sum_m \frac{\pi^j_{lm}(s)(\hat{\tau}^j_{lm}\widehat{c}_{lj}(s)/\widehat{A}_{lj}(s))^{-\theta_j}}{\sum_{l' \in M} \pi^j_{l'm}(s)(\hat{\tau}^j_{l'm}\widehat{c}_{l'j}(s)/\widehat{A}_{l'j}(s))^{-\theta_j}} \widehat{X}_{mj}(s)X_{mj}(s), \tag{B.9}$$

where $\widehat{X}_{mj}(s)X_{mj}(s) = \sum_k \delta^j_{mk}\widehat{Y}_{mk}(s)Y_{mk}(s) + \beta_j \left(\sum_k \delta^L_{mk}\widehat{Y}_{mk}(s)Y_{mk}(s)\right)$, and finally

$$\frac{\widehat{Y}_{mj}(s)}{\widehat{W}_{mj}(s)} = \widehat{L}_{mj}. \tag{B.10}$$

We can combine (B.6)–(B.10) to solve for $\{\widehat{W}_{mj}(s), \widehat{c}_{mj}(s), \widehat{Y}_{mj}(s), \widehat{P}_{mj}(s), \widehat{P}_m(s)\}$.

Derivation of Equation (25)**.** Using the expression in (B.1), the average unit cost of production in region m and industry j is

$$\bar{c}_{mj} = \left(\frac{\bar{W}_{mj}}{\delta^L_{mj}}\right)^{\delta^L_{mj}} \prod_k \left(\frac{\bar{P}_{mk}}{\delta^k_{mj}}\right)^{\delta^k_{mj}},$$

which combined with the expression for domestic trade shares,

$$\bar{\pi}^j_{mm} = C_j^{-\theta} \left(\frac{\bar{c}_{mj}/\bar{A}_{mj}}{\bar{P}_{mj}}\right)^{-\theta_j},$$

delivers the price index $\bar{P}_{mj}$.

Again, with some abuse of notation, let $\hat{x} \equiv \log(x'/x)$ denote the counterfactual change in variable x. Combining the changes in $\bar{c}_{mj}$ and $\bar{\pi}^j_{mm}$ yields

$$\hat{\bar{P}}_{mj} = \frac{\hat{\bar{\pi}}^j_{mm}}{\theta_j} - \hat{\bar{A}}_{mj} + \delta^L_{mj}\hat{\bar{W}}_{mj} + \sum_k \delta^k_{mj}\hat{\bar{P}}_{mk}.$$

After some matrix calculations, we get that

$$\hat{\bar{P}}_{mj} = \sum_k \zeta_{mj}^k \left(\frac{\hat{\bar{\pi}}_{mm}^k}{\theta_k} - \hat{\bar{A}}_{mk} + \delta_{mk}^L \hat{\bar{W}}_{mk} \right),$$

where ζ_{mj}^k is the (j,k) element of the Leontief inverse matrix $\left(\mathbf{I} - \{\delta_{mj}^k\}\right)^{-1}$. Applying this formula to autarky yields

$$\frac{\bar{P}_{mj}^A}{\bar{P}_{mj}} = \prod_k (\bar{\pi}_{mm}^k)^{-\frac{\zeta_{mj}^k}{\theta_k}} \prod_k \left(\frac{\lambda_{mk}}{\lambda_{mk}^A}\right)^{\frac{\kappa-1}{\kappa}\zeta_{mj}^k \rho_k} \prod_k \left(\frac{\bar{W}_{mk}^A}{\bar{W}_{mk}}\right)^{\zeta_{mj}^k \delta_{mk}^L},$$

where we used that $\hat{A}_{mj} = \frac{\kappa-1}{\kappa}\rho_j \hat{\lambda}_{mj}$. Further using that $\bar{P}_m = \prod_j (\bar{P}_{mj}/\alpha_j)^{\alpha_j}$ yields

$$\frac{\bar{W}_{mj}/\bar{P}_m}{\bar{W}_{mj}^A/\bar{P}_m^A} = \underbrace{\frac{\bar{\omega}_{mj}}{\bar{\omega}_{mj}^A}}_{\text{Workers Heterogeneity}} \underbrace{\times \prod_k (\bar{\pi}_{mm}^k)^{-\sum_j \frac{\alpha_j \zeta_{mj}^k}{\theta_k}}}_{\text{ACR}} \underbrace{\times \prod_k \left(\frac{\lambda_{mk}}{\lambda_{mk}^A}\right)^{\sum_j \alpha_j \zeta_{mj}^k \rho_k}}_{\text{Economies of scale}} \underbrace{\times \prod_k \left(\frac{\lambda_{mk}}{\lambda_{mk}^A}\right)^{-\frac{1}{\kappa}\sum_j \alpha_j \zeta_{mj}^k \rho_k}}_{\text{Interaction}},$$

with $\bar{\omega}_{mj} \equiv \bar{W}_{mj}/\prod_k \bar{W}_{mk}^{\sum_j \alpha_j \zeta_{mj}^k \delta_{mk}^L}$. Combining the above decomposition with (18), we obtain (25).

C Data

C.1 Countries and Sectors

We consider the following countries: Austria; Belgium; Canada; China; Czech Republic; Germany; Denmark; Spain; Estonia; Finland; France; the United Kingdom; Greece; Hungary; Iceland; Italy; Lithuania; Latvia; Mexico; Netherlands; Norway; Poland; Portugal; Slovakia; Sweden; and the United States. We also construct a Rest of the World aggregating all the other countries.

Table C.1 presents the 20 aggregated sectors we consider in the empirical analysis and calibration, including agriculture, mining, 17 manufacturing sectors, and services. The industry classification is based on the International Standard Industrial Classification (ISIC) Revision 4.

C.2 Constructing Bilateral Trade Flows For Subnational Geographies

We use the Commodity Flow Survey (CFS) for 2017 to construct subnational bilateral trade flows for CZ's across the United States. One issue is that the trade flows in the survey are not based on the finer geographic levels we consider (CZs). We adopt a similar procedure in the literature (Allen and Arkolakis 2014, Monte et al. 2018, Fajgelbaum and Gaubert 2020) to deal with this issue as follows.

Table C.1: Industry Classification Based on ISIC Revision 4.

Industry Code	Description	$\tilde{\rho}_j$	θ_j
D01T03	Agriculture	0	4.5
D05T09	Mining	0	4.5
D10T12	Manuf. of food products, beverages, and tobacco products	0.24	3.6
D13T15	Manuf. of textiles, wearing apparel, leather and related products	0.14	8.1
D16	Manuf. of wood	0.15	5.9
D17T18	Manuf. of paper and paper products; printing and reproduction of recorded media	0.18	5.8
D19	Manuf. of coke and refined petroleum products	0.09	9
D20	Manuf. of chemicals and chemical products	0.27	3.1
D21	Manuf. of basic pharmaceutical products	0.27	3.1
D22	Manuf. of rubber and plastics products	0.45	1.7
D23	Manuf. of other non-metallic mineral products	0.19	5.1
D24	Manuf. of basic metals	0.11	8.9
D25	Manuf. of fabricated metal products, except machinery and equipment	0.14	7
D26	Manuf. of computer, electronic and optical products	0.09	10.8
D27	Manuf. of electrical equipment	0.10	10.8
D28	Manuf. of machinery and equipment n.e.c.	0.28	3.3
D29	Manuf. of motor vehicles, trailers and semi-trailers	0.20	4.5
D30	Manuf. of other transport equipment	0.20	4.5
D31T33	Manuf. of furniture and other manufacturing, repair	0.20	4.5
D35T99	Services	0	4.5

First, for each tradable sector j, we parameterize the trade costs between subnational regions in the United States as $\log \tau^j_{lm} = \zeta_j \log dist_{lm} + e_{lm}$, where $dist_{lm}$ is the distance between region l and region m. We follow Allen and Arkolakis (2014) to estimate ζ_j using trade flows between CFS areas, controlling for origin and destination fixed effects. We then set trade costs between CZs to $\tau^j_{lm} = dist^{\zeta_j}_{lm}$ for any two different CZs $l, m \in M_{US}$. The intra-region trade costs are $\tau^j_{ll} = 1 \; \forall \, l \in M_{US}$.

Second, the good market clearing for region l and sector j can be written as

$$Y^{US}_{lj} = \sum_{m \in M_{US}} \frac{D_{lj}(\tau^j_{lm})^{-\theta_j}}{\sum_{l' \in M_{US}} D_{l'j}(\tau^j_{l'm})^{-\theta_j}} \sum_{l \in M_{US}} X^{US}_{mj}, \tag{C.1}$$

where D_{lj} is a region-industry-specific fixed effect, capturing region-industry-specific productivity and production costs. Y^{US}_{lj} is the production value in region l and sector j, and X^{US}_{mj} is the expenditures on goods from sector j in m. We use superscript US to denote that these values are net of exports and imports. Production and expenditure values are constructed from the County Business Patterns.[15] Along with the trade costs τ^j_{lm} constructed from the first step, we can use (C.1) to solve for D_{lj} and the implied trade flows between CZs.

[15]With the production data, we can use input-output tables to construct intermediate-input expenditures on goods sourced from each sector. Then we can use labor income and workers' expenditure shares to construct final-good expenditures on goods sourced from each sector.

D Empirical Evidence: Cross-Country Results

We draw sectoral output, value-added, and employment for each country from the OECD STAN Database. The data spans from 1970 and 2019 with shorter periods (at least 20 years) for some countries. Because the data for several countries end in 2017 and spans fewer years than for the United States, we focus on the impact of uncertainty on employment changes between 1977–2017. We concentrate on the same 17 manufacturing sectors featured in our empirical analysis.

To obtain a measure of uncertainty, we first compute growth in average real value-added per worker by country, industry, and year,

$$growth_{jt} = \log\left(\frac{VA_{i,j,t}}{Emp_{i,j,t}}\right) - \log\left(\frac{VA_{i,j,t-1}}{Emp_{i,j,t-1}}\right).$$

We then compute the change in uncertainty between 1977-1997 and 1997-2017 for country i and industry j based on changes in the variance of productivity growth,

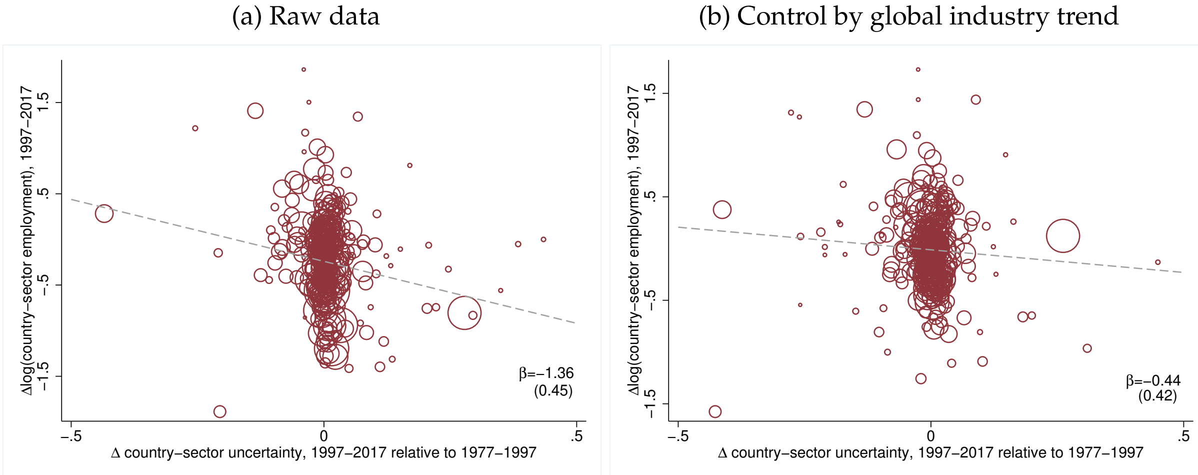
$$\Delta v_{i,j} = var(\underbrace{growth_{i,j,t}}_{t=1997,\ldots,2017}) - var(\underbrace{growth_{i,j,t}}_{t=1977,\ldots,1997}).$$

We estimate the following equation:

$$\Delta\log(L_{i,j}) = \beta_0 + \beta_1 \Delta v_{i,j} + \beta_2 \Delta v_{i,j} \times \tilde{\rho}_j + \beta_3 X_{i,j} + \epsilon_{i,j}.$$

$\Delta\log(L_{ij})$ represents changes in industry employment for country i between 1997 and 2017, $\tilde{\rho}_j$ is the scale parameter for industry j, from Bartelme et al. (2025), and $X_{i,j}$ are other country-industry controls. In some specifications, we include country fixed effects and industry fixed effects.

Figure D.1: Country-sector Employment Growth and Uncertainty.



Note: The circles are proportional to the region employment level in 1997. In Panel (b), we first demean country-sector-level employment and uncertainty changes by the global averages.

Table D.1: Changes in Employment and Uncertainty, by Country and Industry.

	Δlog(country-industry employment), 97–17			
	(1)	(2)	(3)	(4)
$\Delta v_{i,j}$, 97–17 relative to 77–97	-1.358***	-0.413	-0.485*	0.228
	(0.397)	(0.344)	(0.289)	(1.887)
$\Delta v_{i,j} \times \tilde{\rho}_j$				-0.755
				(2.026)
Controls	no	no	yes	yes
Country FE	no	yes	yes	yes
Sector FE	no	yes	yes	yes
Obs	373	373	373	373
R-squared	0.039	0.583	0.595	0.596

Note: Regressions are weighted by initial sectoral employment shares within each country. Controls include productivity growth and the China shock between 1997 and 2017. Standard errors are clustered by country. * 10% ** 5% *** 1%.