Working Paper No. 2020001

PHBS
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Idiosyncratic Risk and Acyclically Increasing Public Debt

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Abstract: Observed public debt of developed economies is not only countercyclical but also acyclically increasing for fairly long peacetime. This paper proposes a politico-economic theory of public debt which coherently rationalizes both acyclical and countercyclical behaviors of public debt. An office-seeking policymaker decides fiscal policies to win over finitely-lived voters who face uninsurable idiosyncratic risk on their disposable incomes. The equilibrium public debt is (i) acyclically increasing, or (ii) countercyclical, or (iii) acyclically decreasing. An increase in the idiosyncratic risk can change public debt behavior to acyclically increasing from countercyclical, entailing rises in public debt.

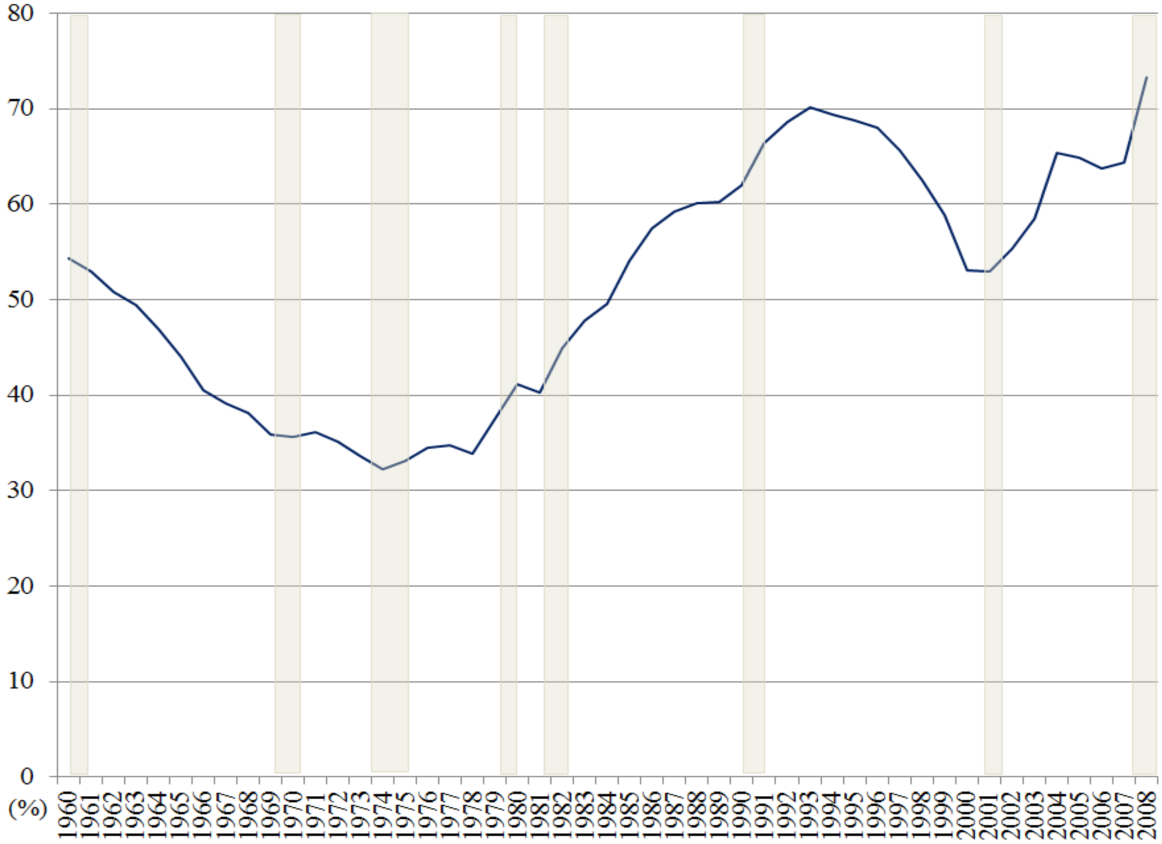
Keywords: government debt, cyclicality of public debt, idiosyncratic risk; acyclical public debt
JEL Codes: H63, E62, D72

I. Introduction

Understanding observed cyclical properties of public debt is important. The existing theories predict that public debt is only countercyclical (i.e., increasing in recessions and decreasing in booms). In fact, however, refuting this prediction, public debt of developed economies is not always countercyclical. Rather, real data shows that public debt of many developed economies increased acyclically for fairly long peacetime. For example of the United States, as shown in **Figure 1**, the US government debt (as % of GDP) increased during years of booming periods after the early 1980s. Thus, **Table 1** reports multi-decade-long acyclical increases in public debt of the US over 1983 – 2008. As a matter of fact, acyclically increasing public debt is observed in other developed economies like Japan, Germany, and Italy (See **Figure A1**, **A2**, and **A3** in Appendix A).[1] This paper offers a politico-economic theory that coherently rationalizes both countercyclical and acyclical public debt behaviors over the business cycle as well as explains changes in cyclicality of public debt.

[1] The estimated cyclicality of public debt of these three countries over the years when public debt (as % of GDP) increases even during booming periods turns out to be statistically insignificant, indicating acyclical increases of public debt of Japan, Germnay, and Italy. In particular, the estimated cyclicality of public debt of Japan over 1976 – 2008 is −0.135 with the standard error of 0.178 which is statistically insignificant. Similarly, the estimated cyclicality of public debt of Germany over 1976 – 2000 and that of Italy over 1964 – 1991 alike are statistically insignificant as well.

Figure 1] Public Debt to GDP Ratio of the US over the Business Cycle



Note: The shaded periods indicate recession according to NBER's Business Cycle Dating Committee. The data of the ratio of public debt to GDP is from Historical Public Debt Database of IMF.

Table 1] Acyclically Increasing and Countercyclical Public Debt of the US

periods (year)	Cyclicality of Public Debt	
1961 - 1982	-0.411^{**}	p-value
	(0.204)	0.057
1983 - 2008	-0.194	p-value
	(0.200)	0.343

Note: The cyclicality of public debt is estimated as correlation coefficient between cyclical part of growth rate of real GDP per capita and growth rate of public debt to GDP ratio. The data of real GDP per capita is from World Bank database and that of the ratio of public debt to GDP is from Historical Public Debt Database of IMF. Using Hodrick-Prescott filter, the cyclical part of growth rate of real GDP per capita is obtained with smoothing parameter of 6.25, according to Ravn and Uhlig (2002). The standard errors of the correlation coefficients are reported in the parenthesis. Moreover, ** refers to being statistically significant at 6% significance level. The p-value is calculated for two-tailed test to identify whether estimated cyclicality is different from zero or not. While statistically insignificant correlation coefficient means acyclical public debt, it does not identify whether public debt acyclically increases or decreases. Based on the raw data in **Figure 1**, the statistically insignificant correlation coefficient indicates acyclical increase in public debt of the US over 1983-2008.

For incorporating the fact that public debt issue is determined not by a benevolent social planner but by politically motivated policymakers, the model of this paper has an office-seeking policymaker decide fiscal policies to win over voters of overlapping generations. Moreover, individual voters' disposable incomes are exposed to uninsurable idiosyncratic risk. Under this model, the politico-economic equilibrium public debt behaves acyclically as well as countercyclically over the business cycle. This stems from voters' failure to internalize the cost from increasing public debt liability which is borne by the unborn future generations.

In particular, the politico-economic equilibrium public debt behavior changes over the two thresholds that are endogenously determined and divide the entire support of public debt levels into three ranges. In detail, (i) if the given level of inherited public debt is lower than the first threshold, optimal public debt increases acyclically ("enjoy-while-it-lasts" phase); (ii) if the given level of inherited public debt lies between the first and second thresholds, optimal public debt behaves countercyclically ("held-responsible" phase); and (iii) if the given level of inherited public debt is higher than the second threshold, optimal public debt decreases acyclically ("getting-out-of-the-crisis" phase). More importantly, this paper finds the role of the idiosyncratic risk on voters' disposable incomes in changing the cyclicality of public debt to entail rises in public debt. In particular, an increase in the idiosyncratic risk elevates the first and second thresholds so that public debt behavior changes *to* acyclically increasing *from* countercyclical or continues increasing acyclically, entailing rises in public debt. As an increase in the uncertainty on voters' incomes for their private consumption makes (*certainly* provided) public goods more valuable to risk-averse voters, acyclical increase of public debt (for financing public goods provision) becomes more politically acceptable. As a result, an increase in the idiosyncratic risk on voters' disposable incomes leads to a rise in public debt.

In short, the main contributions that this paper makes are as follows. First, this paper shows that not only countercyclical but also acyclical behavior of public debt is optimal over the business cycle, rationalizing observed acyclical increases in public debt of developed economies for multi-decade-long peacetime, which is puzzling to the existing theories. Second, it shows that an increase in idiosyncratic risk on individuals' disposable incomes can lead to a rise in public debt by changing cyclicality of public debt to acyclical increase from countercyclical constraint. Third, the theoretical model of this paper also can coherently explain various changes in cyclicality of public debt (*from* countercyclical *to* acyclical as well as *from* acyclical *to* countercyclical).

The rest of this paper unfolds as follows. Section II reviews related literature. Section III elaborates on a theoretical model, from which politico-economic equilibrium is characterized in Section IV. Section V analyzes the effect of idiosyncratic risk. The last section concludes.

II. Literature Review

To date, various researches have been conducted on public debt behavior over the business cycle since Barro (1979) which claimed that social-planner equilibrium public debt follows random walk to achieve tax-smoothing. However, many scholars found various contrary evidence (e.g., Trehan and Walsh, 1991; Bohn, 1998; Antonini, Lee, and Pires, 2013) showing rapid rises in public debt. In particular, Bizer and Durlauf (1990) found that changes in ruling political party cause changes in tax rates instead of smoothing tax rates.

Then, politico-economic theories that have policymakers decide public debt issue emerged for explaining public debt dynamics.[2] In this line, Alesina, Campante, and Tabellini (2008) presented a politico-economic model that voters try to discipline rent-seeking policymaker while voters have no information on public debt issued by the incumbent policymaker or on the rent appropriated by him from tax revenues. From this model, they showed that optimal public debt does not change at all over the business cycle after it instantly jumps to the borrowing limit at the

[2] At the same time, various politico-economic theories on public debt which do not allow business-cycle fluctuations and allow shocks only on preference for public goods (reflecting government expenditure shocks like war or natural disasters) were put forth as well (e.g., Battaglini and Coate, 2008a; Halac and Yared, 2014). However, the massive build-up of public debt in many developed economies occurred during peacetime (i.e., with no apparent shock on government expenditure).

very initial period. Thus, Alesina, Campante, and Tabellini (2008) cannot explain acyclical increases in public debt for a fairly long peacetime. However, they stated that their model intends to account for developing economies.

More suited for developed economies, Müller, Storesletten, and Zilibotti (2016) proposed a model where office-seeking policymakers choose public debt issue to win over voters whose political preference is subject to shocks. By introducing one-time recession to their model, Müller, Storesletten, and Zilibotti (2016) showed that optimal public debt behaves countercyclically. In addition, Battaglini and Coate (2008b) and Barseghyan, Battaglini, and Coate (2013) presented a politico-economic model[3] where entire economy is subject to total factor productivity shocks. Furthermore, in their model, infinitely-lived voters of the same population size reside in each of *n* districts whose representative is randomly selected from the resident voters; and, in the legislative bargaining, each representative wants to favor their own district. From this model, Battaglini and Coate (2008b) and Barseghyan, Battaglini, and Coate (2013) showed that optimal public debt behaves only countercyclically over the entire support of equilibrium public debt distribution. In their derivation of optimal public debt behavior, Battaglini and Coate (2008b) and Barseghyan, Battaglini, and Coate (2013) mentioned an exceptional one-time case where public debt increases for one period of boom if initial level of public debt happens to be strictly lower than the minimum value of the support of equilibrium public debt distribution and initial state of economy is recession. Even when these exceptional two initial conditions are met, Battaglini and Coate (2008b) and Barseghyan, Battaglini, and Coate (2013) stated that public debt never increases during a boom again, since public debt immediately jumps into the support range over which public debt always behaves

[3] Although Battaglini and Coate (2008b) and Barseghyan, Battaglini, and Coate (2013) are separate papers, the model that they used is the same one.

countercyclically. Thus, according to Battaglini and Coate (2008b) and Barseghyan, Battaglini, and Coate (2013), acyclical increase in public debt, if any, cannot last more than one period of boom. In the view of the existing politico-economic theories on public debt, multi-decade-long acyclical increases in actual public debt of developed economies (shown in **Figure 1**, **A1**, **A2**, and **A3**) are puzzling.

III. The Model

Consider a small open economy[4] populated by overlapping generations of voters. Each voter may live up to L periods so that L generations coexist in each period. For each period, new voters are born with no endowment and the total population is normalized to one. Moreover, for any given $i \in \{1, \cdots, L\}$, i-period-old individuals survive the next period with the probability of $s_i \in [0,1]$ so that the population share of i-period-old voters constantly is n_i. For each period, $s_L = 0$ and $\sum_{i=1}^{L} n_i = 1$ with $n_i \in (0,1)$ for $\forall i$. Each voter works for the first mr periods of his life and then stays retired for the remaining lifetime (up to for $L - mr$ periods). For any given t, within-period utility of an i-period-old worker is $\log(c_{t,i}) + H\log(g_t) - (1+\frac{1}{\eta})^{-1} l_{t,i}^{1+\frac{1}{\eta}}$ while that of an i-period-old retiree is $\log(c_{t,i}) + H\log(g_t)$, where $c_{t,i}$ is private consumption of an i-period-old individual in period t; g_t is public goods provided in period t; H is preference for public goods; $l_{t,i} \in (0,1)$ is labor supplied by an i-period-old worker in period t; η is Frisch elasticity of labor supply.

[4] The assumption that economy is small and open with perfectly mobile capital is only for the purpose of describing an economy that *takes* the price of public debt rather than sets the price; hence, this paper is applicable to many developed economies such as the US, Germany, Italy, and Japan. This assumption is also adopted by the previous studies such as Müller, Storesletten, and Zilibotti (2016) and Barseghyan, Battaglini, and Coate (2013).

Although individuals do not have bequest motive,[5] the uncertainty on surviving the next period entails *accidental* bequest left. To deal with this bequest in an innocuous way,[6] let the total sum of accidental bequest from the previous period be automatically distributed to all living individuals equally via lump-sum transfer q_t. Thus, in period *t*, disposable income of an *i*-period-old worker is $(1-\tau_t)w_t l_{t,i}+(1+r)k_{t-1,i}+q_t$ while disposable income of an *i*-period-old retiree is $(1+r)k_{t-1,i}+q_t$, where $\tau_t\in[0,1)$ is labor income tax rate in period *t*; w_t is market wage rate in period *t*; r is interest rate; $k_{t-1,i}$ is capital in period *t* which is invested in the previous period. Moreover, each individual voter faces uninsurable idiosyncratic risk on his disposable income. Specifically, in each period, after receiving post-tax labor income and capital incomes of accidental bequest and investment return, the disposable income falls by $100(1-\sigma)$% for a given $\sigma\in(0,1)$, with the probability of $\phi\in[0,1)$. That is, disposable income of an *i*-period-old worker (retiree) remains intact with the probability of $1-\phi$ or decreases to $\sigma\{(1-\tau_t)w_t l_{t,i}+(1+r)$ $k_{t-1,i}+q_t\}$ ($\sigma\{(1+r)$ $k_{t-1,i}+q_t\}$ for an *i*-period-old retiree) with the probability of ϕ. This negative shock on the disposable incomes is independent for each voter. The parameter ϕ represents the degree of idiosyncratic risk on individual voters in a tractable way.

In each period, each voter chooses his own private consumption, labor supply (if he is a worker), and investment from maximizing the present value of the utility of his remaining lifetime, given the government policies, prices and the state of economy. The present value of remaining-lifetime utility of an *i*-period-old worker in period *t* is

[5] Obviously, this paper adopts the standard approach of no altruism like all the previous studies on public debt behavior. In fact, real data shows that majority of individuals actually leave *no* bequest.
[6] As the automatically-determined amount of accidental bequest is distributed uniformly to each individual as lump-sum transfer, it does not affect decision makings of individuals or policymakers in a meaningful way.

(1) $(1-\phi)\log(c_{t,i}^{NS})+\phi\log(c_{t,i}^{S})+H\log(g_t)-(1+\frac{1}{\eta})^{-1}l_{t,i}^{1+\frac{1}{\eta}}+\sum_{h=1}^{mr-i}\beta^h(\prod_{e=0}^{h-1}s_{i+e})E[\log(c_{t+h,i+h})+H\log($ $g_{t+h})-(1+\frac{1}{\eta})^{-1}l_{t+h,i+h}^{1+\frac{1}{\eta}}]+\sum_{h=mr-i+1}^{L-i}\beta^h(\prod_{e=0}^{h-1}s_{i+e})E[\log(c_{t+h,i+h})+H\log(g_{t+h})]$

while that of an *i*-period-old retiree in period *t* is

(2) $(1-\phi)\log(c_{t,i}^{NS})+\phi\log(c_{t,i}^{S})+H\log(g_t)+\sum_{h=1}^{L-i}\beta^h(\prod_{e=0}^{h-1}s_{i+e})E[\log(c_{t+h,i+h})+H\log(g_{t+h})]$,

where $c_{t,i}^{NS}$ refers to private consumption of an *i*-period-old individual in period *t* if no negative income shock occurs to him and $c_{t,i}^{S}$ refers to his private consumption if negative income shock occurs to him; $\beta\in(0,1)$ is time discount rate. Unlike $c_{t,i}\in\{c_{t,i}^{NS},c_{t,i}^{S}\}$, $l_{t,i}$ does not have the contingency superscript of *NS* or *S*, since labor is supplied before the shock is realized. Moreover, from maximizing (1) or (2) subject to the relevant budget constraints, it is straightforward that $c_{t,i}^{S*}=\sigma c_{t,i}^{NS*}$ at the maximum, which implies that $(1-\phi)\log(c_{t,i}^{NS*})+\phi\log$ $(c_{t,i}^{S*})=\log(c_{t,i}^{NS*})+\phi\log(\sigma)$.

In this economy, there exists a representative firm that produces output which can be used for both private and public goods consumption. In period *t*, the firm produces total output Y_t with inputs of aggregate labor L_t and aggregate capital K_t, following Cobb-Douglas technology; that is, $Y_t = z_t K_t^{\alpha} L_t^{1-\alpha}$ where z_t is total factor productivity (TFP) and $\alpha\in(0,1)$. Capital is completely mobile across different economies while labor is immobile. In each period, the TFP is subject to a shock which generates macroeconomic fluctuations. The TFP follows a first-order Markov stochastic process with two states $z_H > z_L$ indicating that the state of this economy is in boom if $z_t = z_H$ and is in recession if $z_t = z_L$. For any given current state $\theta_i \in \Theta\ L\}$, the probability of transitioning from θ to θ' in the next period is $\mathrm{p}_{\theta\theta'}=\Pr(\mathrm{z}_{t+1}=\mathrm{z}_{\theta'}\ |\mathrm{z}_t=\mathrm{z}_{\theta})\in(0,1)$. This aggregate TFP shock is not related to the above-described idiosyncratic shock.

Taking the government policies and state of economy given, the competitive equilibrium of this economy is defined as a set of all individuals' decision rules of labor supply, private consumption and savings, and factor prices satisfying the following conditions: (i) With the government policies, state of economy, and prices given, all individuals' decision rules solve the problem of maximizing utility of their own remaining lifetime subject to their budget constraints. (ii) The representative firm maximizes its own profit with labor factor market being cleared by $L_t = \sum_{i=1}^{mr} l_{t,i} n_i$. (iii) The aggregate resource constraint of $Y_t = \sum_{i=1}^{L} c_{t,i} n_i + K_t - (1-\delta)K_{t-1} + g_t + NX_t$ where δ is capital depreciation rate and NX_t is net export is met. According to Walras' law, at the competitive equilibrium, the aggregate resource constraint is automatically met once the government budget constraint is met.

Since capital is perfectly mobile across different economies, let a world-wide equilibrium interest rate r be given to this economy. The demand for capital of the representative firm, which is $K_t = (\frac{\alpha z_t}{r+\delta})^{\frac{1}{1-\alpha}} L_t$ from its profit maximization that equates marginal capital product with the given interest rate, is always met by capital supply from domestic and/or foreign investors. On the other hand, from the above condition (ii) for competitive equilibrium, it is clear that for any given $z_t \in \{z_H, z_L\}$, $w_t = (1-\alpha)(\frac{\alpha}{r+\delta})^{\frac{\alpha}{1-\alpha}} z_\theta^{\frac{1}{1-\alpha}} = w_\theta$ which implies that $w_H > w_L$. Thus, the same amount of labor yields larger incomes in booms than in recessions. Moreover, this implies that steady-state wage rate of $w_S = (1-\alpha)(\frac{\alpha}{r+\delta})^{\frac{\alpha}{1-\alpha}} z_S^{\frac{1}{1-\alpha}}$ and $w_H > w_S > w_L$.

The government of this economy finances public goods provision with taxation and public debt issue. In period *t*, given level of public debt, b_t, inherited from the previous period, a policymaker decides public debt issue d_t, income tax rate τ_t, and public goods provision g_t, after learning that the realized current state of the economy is θ, with meeting the following budget constraint:

(3) $$d_t = g_t + (1+r)b_t - \tau_t w_\theta \sum_{i=1}^{mr} l_{t,i} n_i .$$

The government borrows funds (i.e., issues public debt) by selling risk-free one-period bonds. As public debt issued in the current period should be paid in the next period, $d_t = b_{t+1}$. Hence, public debt serves as state variable linking adjacent periods. Moreover, the government is committed to paying the debt back in the next period, so it does not borrow more than the maximal tax revenue collectable at the worst possible state of economy. This commitment defines the upper limit of public debt $\overline{b}$ by $\overline{b} = \frac{\overline{\tau} w_L}{r} \sum_{i=1}^{mr} l_{t,i}(\overline{\tau}) n_i$ where $\overline{\tau} = \arg\max_{\tau} \; \tau w_L \sum_{i=1}^{mr} l_{t,i}(\tau) n_i$. On the other hand, when the government purchases risk-free one-period bonds, it does not buy them more than necessary for providing socially optimal level of public goods g^{sm} only with interest earnings from the bonds (no taxation), where g^{sm} is defined by the Samuelson condition that equates the sum of individual's marginal benefit of public goods consumption with marginal cost of providing public goods. This defines the lower limit of public debt $\underline{b}$ by $\underline{b} = -\frac{g^{sm}}{r}$ where $Hg_{sm}^{-1} = 1$. The initial level of public debt of this economy b_0 is randomly given from $(\underline{b}, \overline{b})$.

Moreover, each policymaker faces one-period term limit and follows Markovian strategy so that d_t, τ_t, and g_t depend only on the current state variables of b_t and θ; i.e., $d_{t,\theta}(b_t), \tau_{t,\theta}(b_t)$, $g_{t,\theta}(b_t)$ for any given *t* and the state of economy θ. In terms of policy choice variables (d_t, τ_t, and g_t) of the current policymaker, utility of individual voters can be succinctly restated as follows. Since $d_t = b_{t+1}$, the present value of remaining-lifetime utility of an *i*-period-old worker in period *t* is

(4) $$\phi \log(\sigma) + \log(c_{t,i}^{NS*}(\tau_t)) + H \log(g_t) - (1+\frac{1}{\eta})^{-1} l_{t,i}^{*}(\tau_t)^{1+\frac{1}{\eta}} + \beta E[wv_i(d_{t+1,\theta'}(d_t), \tau_{t+1,\theta'}(d_t), g_{t+1,\theta'}(d_t))]$$

where $E[wv_i(d_{t+1,\theta'}(d_t), \tau_{t+1,\theta'}(d_t), g_{t+1,\theta'}(d_t))]$ is the expected present value of utility of the worker for his remaining life time after the next period that can be affected by the current policy choice of d_t via subsequent Markovian fiscal policies. That is, $E[wv_i(d_{t+1,\theta'}(d_t), \tau_{t+1,\theta'}(d_t),$

$$g_{t+1,\theta'}(d_t))] = \sum_{h=1}^{mr-i} \beta^{h-1}(\prod_{e=0}^{h-1} s_{i+e}) E[\phi \log(\sigma) + \log(c_{t+h,i+h}^{NS*}(\tau_{t+h})) + H \log(g_{t+h}) - (1+\frac{1}{\eta})^{-1} l_{t+h,i+h}^{*}(\tau_{t+h})^{1+\frac{1}{\eta}}]$$
$$+ \sum_{h=mr-i+1}^{L-i} \beta^{h-1}(\prod_{e=0}^{h-1} s_{i+e}) E[\phi \log(\sigma) + \log(c_{t+h,i+h}^{NS*}) + H \log(g_{t+h})].$$

Clearly, $l_{t,i}^{*}(\tau_t)$ and $c_{t,i}^{NS*}(\tau_t)$ should not be misunderstood that workers' labor supply and private consumption depend only on the current income tax rate. These are for reflecting that whenever a policymaker changes income tax rate, the change in the income tax rate directly affects post-tax return to workers' labor supply and their private consumption (purchased with post-tax income). Likewise, the present value of remaining-lifetime utility of an *i*-period-old retiree in period *t* is restated as

(5) $$\phi \log(\sigma) + \log(c_{t,i}^{NS*}(\tau_t)) + H \log(g_t) + \beta E[rv_i(d_{t+1,\theta'}(d_t), \tau_{t+1,\theta'}(d_t), g_{t+1,\theta'}(d_t))]$$

where $E[rv_i(d_{t+1,\theta'}(d_t), \tau_{t+1,\theta'}(d_t), g_{t+1,\theta'}(d_t))] = \sum_{h=1}^{L-i} \beta^{h-1}(\prod_{e=0}^{h-1} s_{i+e}) E[\phi \log(\sigma) + \log(c_{t+h,i+h}^{NS*}) + H \log(g_{t+h})]$ is the expected present value of utility of the retiree for his remaining life time.

Above all, policymaker of this economy is elected in every period and subject to one-period term limit.[7] In any given period, right after the state of this economy is realized and publicly known at the very beginning of the period, two candidates run for office and both candidates care only about their own probability of winning the election. These candidates simultaneously announce their own policy proposals on public debt issue, tax rate, and public goods provision. Then, voters decide whom to vote for, based on both policy proposal and personal appeal of each of the candidates. Personal appeal of a candidate is nation-wide popularity of his personality and it is not related to any policy proposal. Moreover, personal appeal of each candidate is known to none of the two candidates when they announce their own policy proposals, whereas it is known to all voters when they cast their own votes. After the election, winner's policy proposal is implemented as announced. In order to maximize their own winning probability, *each* of the two office-seeking candidates chooses to propose a set of policies that maximizes the population-weighted sum of utility function of all voters subject to the government budget constraint of (3) and $d_t \in [\underline{b}, \overline{b}]$ for any given θ. (For details, see Appendix B1.)[8] After an elected policymaker implements his policy proposal, labor supplies are chosen by workers before the uninsurable idiosyncratic risk is resolved. After the risk is resolved, all voters consume private and public goods with reaching the competitive equilibrium of this economy. The level of public debt d_t issued in the current period serves as certain state variable b_{t+1} for the next period, while the TFP of the next period is currently uncertain.

IV. Politico-Economic Equilibrium Public Debt

Now, this section characterizes politico-economic Markov-perfect equilibrium where each office-seeking one-period-term policymaker chooses policies that depend only on the current

[7] In fact, the one-period term limit of policymakers is standard assumption adopted by many of the previous studies such as Battaglini and Coate (2008b), Barseghyan, Battaglini, and Coate (2013) and Müller, Storesletten, and Zilibotti (2016) as well as by most of political economy literature. Even though actual term limit of policymakers may be longer than a period (one year), it can be translated into repetition of one-period term in a theoretical model.

[8] This is a *variant* (not an original form) of probabilistic voting model of Lindbeck and Weibull (1987). Hence, the details need to be elaborated in Appendix B1.

state variables for maximizing his own probability of winning the election. As shown in Section III and Appendix B1, the maximum winning probability of each policymaker is attained when he chooses policies that maximize the population-weighted sum of utility of his electorate. Because the term of elected policymakers is limited to one period, each policymaker cannot credibly commit to any future policy after his own term. When each policymaker decides public debt issue, tax rate, and public goods provision for his own term, he takes what all the other policymakers (of the past and the future) would do as given. Furthermore, let us characterize stable politico-economic Markov-perfect equilibrium[9] by finding the optimal policy functions, denoted by $\{d_\theta(b), g_\theta(b), \tau_\theta(b)\}_{\theta=H,L}$, that are not time variant. The optimal Markovian strategies $\{d_\theta(b), g_\theta(b), \tau_\theta(b)\}_{\theta=H,L}$ specify what policies each policymaker will implement if realized state variables of this economy are (b,θ) that not constant but variables which can generate distribution of public debt. Since $\{d_\theta(b)\}_{\theta=H,L}$ defines transition of public debt variable over time, it also can indicate whether distribution of public debt variable converges to a (unique) time-invariant distribution, not only can it describe optimal public debt responses to business-cycle shocks.

Formally, each policymaker chooses policies from solving the following maximization problem: for any given $b \in [\underline{b}, \overline{b}]$ and $\theta \in \{H, L\}$,

$$(6)\quad \max_{\{d,\tau,g\}} \sum_{i=1}^{mr} n_i \{\log(c_i^{NS*}(\tau)) + H\log(g) - (1+\frac{1}{\eta})^{-1} l_i^*(\tau)^{1+\frac{1}{\eta}} + \beta E[wv_i(d'(d), g'(d), \tau'(d))]\} + \sum_{i=mr+1}^{L} n_i \{\log(c_i^{NS*}) + H\log(g) + \beta E[rv_i(d'(d), g'(d), \tau'(d))]\} \text{ s.t. } d = g + (1+r)b - \tau w_\theta \sum_{i=1}^{mr} l_i^* n_i \text{ and } d \in [\underline{b}, \overline{b}]$$

[9] Since this paper analyzes public debt response to macroeconomic fluctuations, we focus on differentiable equilibrium, since non-differentiable equilibrium entails discontinuous jumps or falls of public debt and no public debt response to macroeconomic fluctuations (i.e., neither increase nor decrease in public debt for responding to TFP shocks) which are out of interest of this paper.

where $c_{t,i}^{NS*}$ and $l_{t,i}^{*}$ are private consumption and labor supply of an *i*-period-old individual at the competitive equilibrium of this economy.[10] From solving (6), the optimal Markovian strategies $\{d_\theta(b), g_\theta(b), \tau_\theta(b)\}_{\theta=H,L}$ are defined by the optimality conditions regarding marginal benefit and cost of public goods provision and public funds. In particular, each policymaker equalizes marginal benefit of public goods provision and marginal cost of financing the provision; i.e., for $\forall b \in [\underline{b}, \bar{b}]$ and $\forall \theta \in \{H, L\}$,

$$(7) \qquad H[g_\theta(b)]^{-1} = \{\frac{1}{1-\phi+\sigma\phi} - \eta\}\{\frac{1-\tau_\theta(b)}{1-\tau_\theta(b)(1+\eta)}\}\zeta^\theta,$$

where $\zeta^\theta = \{\sum_{i=1}^{mr}(1-\tau)w_\theta l_i^* n_i\}^{-1}\sum_{i=1}^{mr}(l_i^*)^{1+\frac{1}{\eta}}n_i$. For allocating public finance cost over tax and public debt, the policymaker equalizes the marginal cost of taxation and the present value of marginal disutility of his electorate from an increase in public debt (decided in the current period) which reduces the available resources for public goods provision in the next period. That is, for $\forall b$ $\in [\underline{b}, \bar{b}]$ and $\forall \theta \in \{H, L\}$,

$$(8) \qquad \{\frac{1}{1-\phi+\sigma\phi} - \eta\}\{\frac{1-\tau_\theta(b)}{1-\tau_\theta(b)(1+\eta)}\}\zeta^\theta = -\tilde{\beta}E[\frac{H}{g_{\theta'}(d_\theta(b))}\frac{\partial g_{\theta'}(d_\theta(b))}{\partial d_\theta(b)}]$$

where $\tilde{\beta} \equiv \beta\sum_{i=1}^{L} n_i s_i = \beta(1-n_L) < \beta$. Based on these FOCs, the optimal policy functions of the politico-economic Markov-perfect equilibrium, which are nonlinear, can be identified.

Lemma 1. The politico-economic Markov-perfect equilibrium policy functions $\{d_\theta(b), \tau_\theta(b),$ $g_\theta(b)\}_{\theta=H,L}$ are defined as follows. For $\forall b \in [\underline{b}, \bar{b}]$ and $\forall \theta \in \{H, L\}$,

$$(9) \qquad d_\theta(b) = \bar{b} - \rho_\theta(\bar{b} - b),$$

[10] Recall that the survival rates are set to maintain population share of each age group constantly over time. Thus, s_i does not show up when the social planner aggregates individual's utility over the entire population.

(10) $$\tau_\theta(b) = \frac{1}{(1+\eta)} - \frac{\xi([1-\phi+\sigma\phi]^{-1}-\eta)}{w_\theta \tilde{\beta} H(1+\eta)} \rho_\theta(\bar{b}-b)$$

(11) $$g_\theta(b) = \frac{1}{\tilde{\beta}} \rho_\theta(\bar{b}-b),$$

where $\xi = \{\sum_{i=1}^{mr} l_i^* n_i\}^{-1} \sum_{i=1}^{mr} (l_i^*)^{1+\frac{1}{\eta}} n_i$ and ρ_θ is defined by

(12) $$\rho_\theta(b) = \{1 + \frac{1}{\tilde{\beta}} + \frac{([1-\phi+\sigma\phi]^{-1}-\eta)\sum_{i=1}^{mr}(l_i^*)^{1+\frac{1}{\eta}} n_i}{\tilde{\beta} H(1+\eta)}\}^{-1} \frac{\bar{b} - (1+r)b + w_\theta(1+\eta)^{-1}\sum_{i=1}^{mr} l_i^* n_i}{(\bar{b}-b)} > 0.$$

Proof. See Appendix B2.

In the first place, voters' demand for more public goods provision exerts upward pressure on public debt issue of office-seeking policymakers. As this upward pressure raises the level of public debt liability, larger amount of the government revenue is diverted to paying the inherited public debt liability and less is available for public goods provision. In the end, the concern that no public goods can be provided when public debt level reaches $\bar{b}$ eventually counteracts the upward force on public debt. As this downward pressure grows, a policymaker reaches a threshold at which the upward and downward forces on public debt are of equal magnitude so that optimal public debt neither increases nor decreases (i.e., $d_\theta(b) = b = b'$ for any given θ). In light of (9) and (12), such a threshold, denoted by $\ddot{b}_\theta$, at which public debt level no longer increases is defined as follows; for each $\theta \in \{H, L\}$,

(13) $$\rho_\theta(\ddot{b}_\theta) = 1 = \{1 + \frac{1}{\tilde{\beta}} + \frac{([1-\phi+\sigma\phi]^{-1}-\eta)\sum_{i=1}^{mr}(l_i^*)^{1+\frac{1}{\eta}} n_i}{\tilde{\beta} H(1+\eta)}\}^{-1} \frac{\bar{b} - (1+r)\ddot{b}_\theta + w_\theta(1+\eta)^{-1}\sum_{i=1}^{mr} l_i^* n_i}{(\bar{b}-\ddot{b}_\theta)}.$$

Lemma 2. The threshold at which the politico-economic Markov-perfect equilibrium public debt neither increases nor decreases is lower for boom state than for recession state. That is, $\underline{b} < \ddot{b}_H < \ddot{b}_L < \bar{b}$.

Proof. See Appendix B3.

Since more outputs are produced and available for providing public goods and paying the public debt liability in a boom than in a recession, the public debt threshold level, from which no new additional issue of public debt is demanded from voters, is lower for boom state than for recession state. To reflect **Lemma 2**, the two thresholds are re-labelled as $\ddot{b}_H = \ddot{b}_1$ and $\ddot{b}_L = \ddot{b}_2$. In fact, $\ddot{b}_1$ and $\ddot{b}_2$ are critical, since behavior of optimal public debt changes from countercyclical to acyclical and vice versa over each of these two thresholds.

Proposition 1. The politico-economic Markov-perfect equilibrium public debt $\{d_\theta(b)\}_{\theta=H,L}$ behaves over the business cycle as follows:

(i) when $\underline{b} \le b \le \ddot{b}_1$, public debt increases acyclically until it reaches $\ddot{b}_1$ in booms: i.e., $d_H(b) > b = d_\vartheta(b)$ and $d_L(b) > b$ for $\forall b \in [\underline{b}, \ddot{b}_1)$ and $\forall \vartheta \in \{H, L\}$, whereas $d_H(\ddot{b}_1) = \ddot{b}_1$ and $d_L(\ddot{b}_1) > \ddot{b}_1$;

(ii) when $\ddot{b}_1 < b \le \ddot{b}_2$, public debt behaves countercyclically until it reaches $\ddot{b}_2$ in recessions: i.e., $d_H(b) < b$ and $d_L(b) > b$ for $\forall b \in (\ddot{b}_1, \ddot{b}_2)$, whereas $d_H(\ddot{b}_2) < \ddot{b}_2$ and $d_L(\ddot{b}_2) = \ddot{b}_2$;

(iii) when $\ddot{b}_2 < b \le \overline{b}$, public debt decreases acyclically until it reaches $\ddot{b}_2$: i.e., $d_H(b) < b$ and $d_L(b) < b$ for $\forall b \in (\ddot{b}_2, \overline{b}]$.

(iv) Moreover, under the politico-economic Markov-perfect equilibrium, the distribution of public debt variable $d_\theta(b)$ converges to a unique invariant distribution with the support of $[\underline{b}, \overline{b}]$.

Proof. See Appendix B4.

In addition, the optimal responses of public debt to the aggregate productivity shocks, described in **Proposition 1**, beget those of public goods provision and income tax rate as below.

Corollary 1. The politico-economic Markov-perfect equilibrium public goods provision and tax rate $\{g_\theta(b), \tau_\theta(b)\}_{\theta=H,L}$ behave over the business cycle as follows: (i) when $\underline{b} \le b \le \ddot{b}_1$, public goods provision decreases acyclically while tax rate increases acyclically, until public debt reaches $\ddot{b}_1$ in booms; (ii) when $\ddot{b}_1 < b \le \ddot{b}_2$, public goods provision behaves procyclically while tax rate behaves countercyclically, until public debt reaches $\ddot{b}_2$ in recessions; (iii) when $\ddot{b}_2 < b \le \overline{b}$, public goods provision increases acyclically while tax rate decreases acyclically until public debt reaches $\ddot{b}_2$.

Proof. See Appendix B5.

Each office-seeking policymakers always wants to win over his electorate by providing more public goods in his term, so long as his policy for financing the provision obtains political support from his electorate. Voters' support for his policy hinges upon effective cost of public funds (sum of tax revenue and issued public debt) per unit of public goods provided, which in turn depends on the given level of public debt liability inherited from the previous period, because public funds are diverted to paying the given public debt liability before financing public goods provision.

If the given level of public debt liability is relatively low ($\underline{b} \le b \le \ddot{b}_1$), little public funds are taken away for paying the inherited public debt. Low effective cost of public funds per unit of public goods provided enables issuing new additional public debt to obtain political support from voters, regardless of whether the economy is in a boom or in a recession. Far away from the debt limit $\overline{b}$, voters can self-indulgently increase public debt even during booms to provide more public goods for themselves, as the consequent build-up of public debt will be paid off by the unborn future generations to come after they die. Voters enjoy the low effective cost of public funds per unit of public goods while it lasts. As a result, public debt acyclically increases until it reaches the first threshold $\ddot{b}_1$. Let us call this range of $\underline{b} \le b \le \ddot{b}_1$ as "enjoy-while-it-lasts" phase. On the other hand, if the given level of public debt inherited is in the middle range ($\ddot{b}_1 < b \le \ddot{b}_2$), public funds per unit of public goods provided become costly enough to hold voters fiscally responsible. Being relatively close to the debt limit $\overline{b}$, if voters do not control growing public debt, they essentially cause suffering of no public goods provision by hitting the debt limit while they are alive. Since total output is larger in booms than in recessions, voters make policymakers take advantage of booms for curbing public debt rather than recessions. Thus, public debt issue decreases (increases) in booms (recessions); that is, public debt behaves countercyclically. This range of $\ddot{b}_1 < b \le \ddot{b}_2$ is "held-responsible" phase. Lastly, if the given level of public debt liability is very high ($\ddot{b}_2 < b < \overline{b}$), issuing new additional public debt costs too much to obtain political support, regardless of the state of the economy. As servicing public debt costs so high taking up more public funds than providing public goods, the economy is in public debt crisis. Since voters want to get out of the debt crisis, they allow policymakers to reduce public debt even during recessions. Thus, public debt acyclically decreases, as long as it is above the second threshold $\ddot{b}_2$. This range of $\ddot{b}_2 < b < \overline{b}$ is "getting-out-of-the-crisis" phase.

In sum, **Lemma 1**, **Proposition 1** and **Corollary 1** together characterize the politico-economic Markov-perfect equilibrium fiscal policies over the business cycle. Since each of the two ranges, $\underline{b} \le b \le \ddot{b}_1$ and $\ddot{b}_2 < b < \overline{b}$, is of strictly positive measure[11] and can be considerably wide, acyclical public debt behavior is substantial under the politico-economic Markov-perfect equilibrium. Notice that in the "enjoy-while-it-lasts" phase ($\underline{b} \le b \le \ddot{b}_1$) public debt increases

[11] By the definition of 'support,' the density on these two ranges of acyclical behavior of public debt is strictly positive under the unique invariant distribution of equilibrium public debt.

during booms again, in contrast to Battaglini and Coate (2008b) and Barseghyan, Battaglini, and Coate (2013). According to **Proposition 1**, the observed behavior of acyclically increasing public debt for multi-decade-long peacetime in developed economies is not regarded as irrational or puzzling behavior of passing up the advantageous opportunity of many booming years for lowering their public debt level.

Fundamentally, optimal acyclical public debt behavior stems from the point that each office-seeking policymaker seeks to cater *finitely-lived* voters.[12] In each period, voters, whose utility maximizations steer the policymaker's decision as appears in (6), do not care about the utility of an infinite number of unborn future voters. As a result, part of cost from rising public debt liability which is borne by the unborn future voters is not internalized in (6) so that acyclically increasing public debt behavior can obtain political support in the "enjoy-while-it-lasts" phase ($\underline{b} \leq b \leq \ddot{b}_1$). Distinct from the existing theories, this study shows that not only countercyclical but also acyclical behavior of public debt can be optimal over the business cycle.

V. Effect of Idiosyncratic Risk on Disposable Incomes

Proposition 1 immediately can explain that cyclicality of optimal public debt changes from acyclically increasing to countercyclical or from acyclically decreasing to countercyclical. However, as shown in **Figure 1** and **Table 1**, the observed cyclicality of the US public debt changed from countercyclical to acyclically increasing. In fact, this observed change in the cyclicality of public debt can also be explained based on **Proposition 1** by allowing a change in the economic parameter for idiosyncratic risk on disposable incomes of individual voters. Specifically, a rise in ϕ (i.e., a rise in uninsurable idiosyncratic risk on individual voters'

[12] In this regard, it is noteworthy that Battaglini and Coate (2008b) and Barseghyan, Battaglini, and Coate (2013) assumed *infinitely- lived* voters and showed that optimal behavior of public debt is only countercyclical over the support of the invariant distribution of equilibrium public debt. Moreover, notice that acyclically increasing behavior of public debt is not from time inconsistency of policymakers or voters, which differentiates this paper from Halac and Yared (2014).

disposable incomes) lifts the first threshold $\ddot{b}_1$ up ($\frac{\partial \ddot{b}_1}{\partial \phi} > 0$) causing cyclicality of optimal public debt to change from countercyclical restraint to acyclical increase without a change in the level of inherited public debt liability.

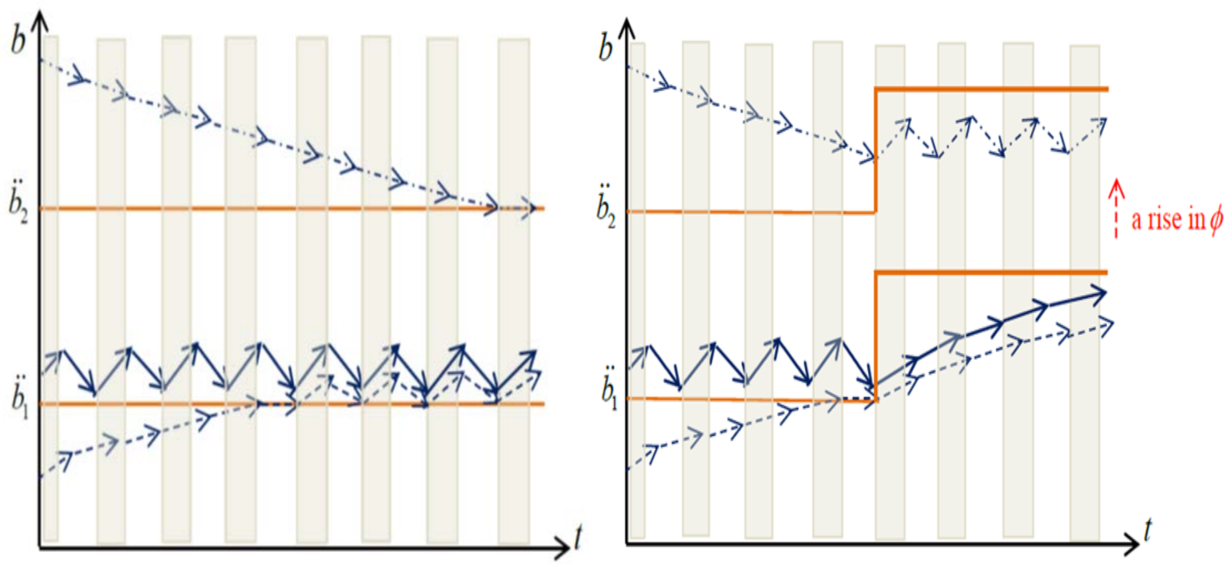
Proposition 2. A rise in idiosyncratic risk on individual voters' disposable incomes can change optimal public debt behavior from countercyclical to acyclically increasing with no change in the inherited level of public debt liability.

Proof. See Appendix B6.

Intuitively, as a rise in ϕ makes individual voters face greater uncertainty on their own resources for their private consumption, public goods become more valuable to risk-averse voters because public goods are *certainly* provided. As a result, with higher ϕ, acyclical increase of public debt (for financing more public goods provision in the current period) gains more political support from voters to elevate the first threshold $\ddot{b}_1$. As illustrated by the solid line of the panel (b) of **Figure 2**, since the rise in ϕ elevates the level of $\ddot{b}_1$, countercyclical public debt above the before-rise $\ddot{b}_1$ can end up with being below the after-rise $\ddot{b}_1$ to be converted to acyclically increasing public debt (according to **Proposition 1**) even when the level of inherited public debt does not change at all.

Explaining change in the cyclicality of public debt from countercyclical restraint to acyclical increase, **Proposition 2** is the key finding of this paper that sheds light on how idiosyncratic risk on individuals' disposable incomes can lead to rapid rises in public debt. In fact, various empirical studies found that idiosyncratic risk on individuals' disposable incomes actually rose in the US over the early 1980s when the cyclicality of the US public debt changed from countercyclical to acyclically increasing. Using Panel Study of Income Dynamics data, Gottschalk and Moffitt (1994), Haider (2001) and Dynan, Elmendorf, and Sichel (2012) found that the estimated probability of negative shocks on earnings of male household heads rose subst antially over the early 1980s. Notably, **Proposition 2** suggests that the observed rises in the risk on individuals' incomes over the early 1980s contributed the observed concurrent change in the cyclicality of the US public debt from countercyclical restraint to acyclical increase.

Figure 2] Change in Public Debt Cyclicality and a Rise in Idiosyncratic Risk on Income
(a) alternating booms and recessions (b) same booms and recessions with a rise in ϕ



Note: The shaded periods indicate recessions with θ = *L*, whereas the non-shaded periods indicate booms with θ = *H*. In each panel, there are three different kinds of lines (dotted, solid, and dashed lines), depending on the relation of the initially given level of public debt to the first and second thresholds ($\ddot{b}_1$.and $\ddot{b}_2$) while all the lines remain within [$\underline{b}$, $\bar{b}$]. Moreover, the three starting points of the given level of public debt are equal for both panels.

In addition, as demonstrated by the dashed line in the panel (b) of **Figure 2**, a rise in the idiosyncratic risk on individual voters' disposable incomes causes optimal public debt to keep in creasing acyclically longer, with no change in the inherited level of public debt. Consider an economy whose public debt is right below its first threshold $\ddot{b}_1$. With no rise in ϕ, its public debt is about to change into countercyclical from acyclically increasing by surpassing the before-rise $\ddot{b}_1$. However, as a rise in the idiosyncratic risk on individuals' disposable incomes elevates the first threshold $\ddot{b}_1$, public debt of the economy ends up with being far below the after-rise $\ddot{b}_1$ so that it continues acyclically increasing for quite a long time. This corollary of **Proposition 2** may provide an explanation for multi-decade-long acyclical increases in public debt of Japan, Germany, and Italy (**Figure A1**, **A2**, and **A3** in Appendix A).

This paper suggests that even when output volatility decreases, public debt can rise due to an increase in idiosyncratic risk on individuals' disposable incomes, which is contrast to Azzimonti, Francisco, and Quadrini (2014) that attributed an increase in steady-state public debt to a rise in the output volatility. In particular, Azzimonti, Francisco, and Quadrini (2014) assumed that individual workers face no risk while firm owners (who are the only producer in their model and thus correspond to the representative firm in the model of this paper) confront idiosyncratic risk on the productivity for output, showing that a rise in the idiosyncratic risk on the firms' productivity, which is translated as GDP volatility, causes steady-state public debt to increase.

VI. Concluding Remarks

This paper proposes a politico-economic model of public debt behavior over the business cycle, where voters face uninsurable idiosyncratic risk on their disposable incomes and an office-seeking policymaker decides fiscal policies. From this model, the politico-economic Markov-perfect equilibrium public debt exhibits one of the following three cyclicalities: (i) acyclical increase, (ii) countercyclical restraint, and (iii) acyclical decrease. Basically, an office-seeking policymaker chooses to acyclically increase public debt because voters do not internalize the cost from rising public debt liability borne by unborn future voters who are not his electorate. Importantly, this paper also finds that a rise in idiosyncratic risk on individual voters' disposable incomes can change public debt behavior to acyclically increasing from countercyclical or make public debt continue increasing acyclically, entailing rises in public debt. These theoretical findings are consistent with observed multi-decade-long acyclical increases in public debt of developed economies like the US, Japan, Germany, and Italy.

Appendix A

Figure A1] Public Debt to GDP Ratio of Japan over the Business Cycle

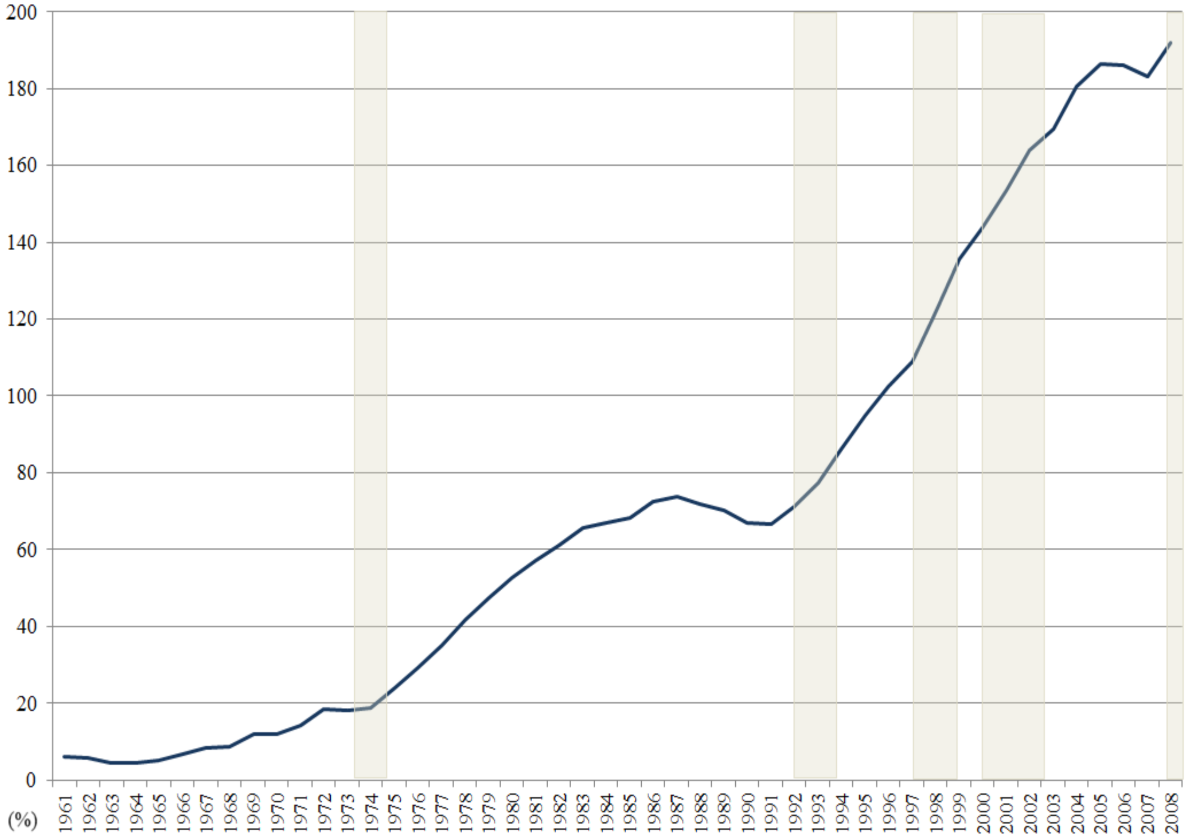


Figure A2] Public Debt to GDP Ratio of Germany over the Business Cycle

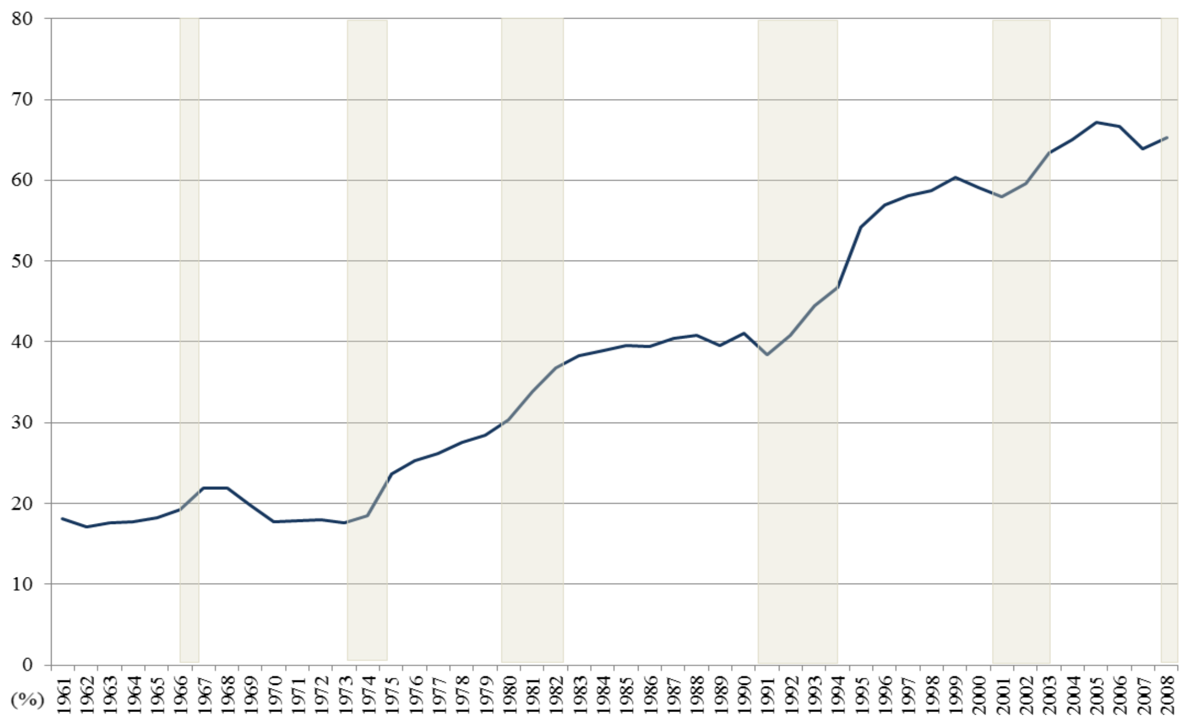
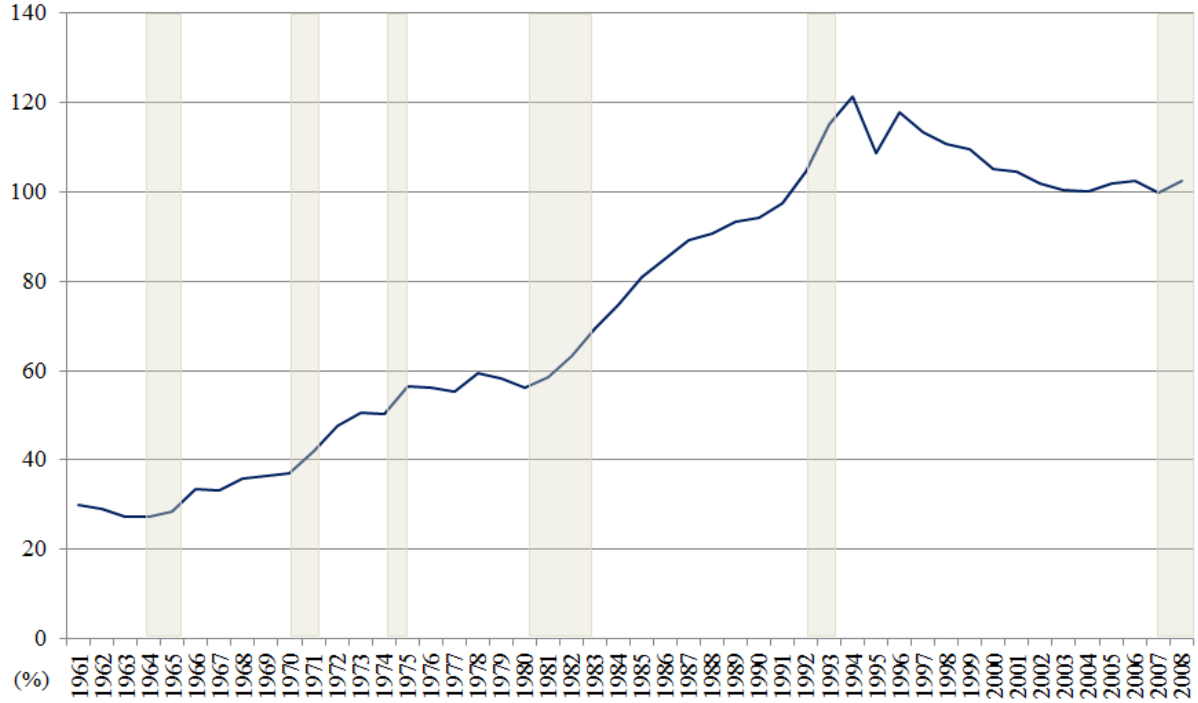


Figure A3] Public Debt to GDP Ratio of Italy over the Business Cycle



Note: The shaded periods indicate recession according to Economic Cycle Research Institute. The data of the ratio of public debt to GDP is from Historical Public Debt Database of IMF.

Appendix B

B1. *Election of a Policymaker and Choice of Policy Proposal*

Let two office-seeking candidates be denoted by A and B, and their policy proposals by (d_A, τ_A, g_A) and (d_B, τ_B, g_B) respectively. When the two candidates simultaneously announce their own policy proposals, none certainly knows the nation-wide relative personal appeal of candidate B over candidate A, which is denoted by γ, and each of the two candidates has to estimate γ. In this regard, let γ follow a zero-median uniform distribution of $Uni[-\frac{1}{2\psi}, \frac{1}{2\psi}]$ with $\psi > 0$. After both candidates announce their own policy proposals and then the relative personal appeal (the value of γ) is revealed, each voter casts his own vote for one of the two candidates who gives him higher level of utility. In particular, an *i*-period-old worker chooses to vote for candidate A if and only if

$$\{\phi\log(\sigma)+\log(c_{t,i}^{NS*}(\tau_A))+H\log(g_A)-(1+\eta^{-1})^{-1}l_{t,i}^{*}(\tau_A)^{1+\frac{1}{\eta}}+\beta E[wv_i(d_{t+1,\theta'}(d_A),\tau_{t+1,\theta'}(d_A),g_{t+1,\theta'}(d_A))]\}-\{\phi\log(\sigma)+\log(c_{t,i}^{NS*}(\tau_B))+H\log(g_B)-(1+\eta^{-1})^{-1}l_{t,i}^{*}(\tau_B)^{1+\frac{1}{\eta}}+\beta E[wv_i(d_{t+1,\theta'}(d_B),\tau_{t+1,\theta'}(d_B),g_{t+1,\theta'}(d_B))]\}-\gamma>0,$$

where $c_{t,i}^{NS*}$ and $l_{t,i}^{*}$ are the private consumption and labor supply of the worker at the competitive equilibrium of this economy. At the same time, an *i*-period-old retiree chooses to vote for candidate A if and only if

$$\{\phi\log(\sigma)+\log(c_{t,i}^{NS*})+H\log(g_A)+\beta E[rv_i(d_{t+1,\theta'}(d_A),\tau_{t+1,\theta'}(d_A),g_{t+1,\theta'}(d_A))]\}-\{\phi\log(\sigma)+\log(c_{t,i}^{NS*})+H\log(g_B)+\beta E[rv_i(d_{t+1,\theta'}(d_B),\tau_{t+1,\theta'}(d_B),g_{t+1,\theta'}(d_B))]\}-\gamma>0,$$

where $c_{t,i}^{NS*}$ is the private consumption of the retiree at the competitive equilibrium. Reflecting these voting decision rules, when the two candidates simultaneously announce their own policy proposals, the winning probability of candidate A is stated as

$$\Pr(\pi_A\geq\frac{1}{2})=\frac{1}{2}+\psi[\sum_{i=1}^{mr}n_i\{\log(c_{t,i}^{NS*}(\tau_A))+H\log(g_A)-(1+\eta^{-1})^{-1}l_{t,i}^{*}(\tau_A)^{1+\frac{1}{\eta}}+\beta E[wv_i(d_{t+1,\theta'}(d_A),\tau_{t+1,\theta'}(d_A),g_{t+1,\theta'}(d_A))]-(\log(c_{t,i}^{NS*}(\tau_B))+H\log(g_B)-(1+\eta^{-1})^{-1}l_{t,i}^{*}(\tau_B)^{1+\frac{1}{\eta}}+\beta E[wv_i(d_{t+1,\theta'}(d_B),\tau_{t+1,\theta'}(d_B),g_{t+1,\theta'}(d_B))])\}+\sum_{i=mr+1}^{L}n_i\{\log(c_{t,i}^{NS*})+H\log(g_A)+\beta E[rv_i(d_{t+1,\theta'}(d_A),\tau_{t+1,\theta'}(d_A),g_{t+1,\theta'}(d_A))]-(\log(c_{t,i}^{NS*})+H\log(g_B)+\beta E[rv_i(d_{t+1,\theta'}(d_B),\tau_{t+1,\theta'}(d_B),g_{t+1,\theta'}(d_B))])\}]$$

where π_A is the share of votes for candidate A. Since $\pi_A=1-\pi_B$, the winning probability of candidate B is defined symmetrically. Therefore, in any given period *t*, for choosing policy proposal $\{d_t,\tau_t,g_t\}$ that maximizes their own winning probability, each of the two candidates maximizes the population-weighted sum of utility of all voters subject to the government budget constraint of (3) and $d_t\in[\underline{b}\ ,\bar{b}]$, for any given θ. As both candidates alike only want to win the election, both basically solve the same maximization problem for choosing their own policy proposal. As a result, both candidates end up with announcing the same policy proposal. Then, one of the two candidates will be selected randomly with the same chance for each (i.e., at equilibrium, $\pi_A^*=\pi_B^*=\frac{1}{2}$).

B2. *Proof for Lemma 1*

[step 1] To begin with, guess (9), (10), and (11) for the politico-economic Markov-perfect

equilibrium policy functions $\{d_\theta(b),\ \tau_\theta(b), g_\theta(b)\}_{\theta=H,L}$ with the coefficient ρ_θ being unknown. Plug (9), (10), and (11) into the government budget constraint $d_\theta(b) = g_\theta(b) + (1+r)b$ $-\tau_\theta(b)(w_\theta \sum_{i=1}^{mr} l_i^* n_i)$ to get

$$\bar{b} - \rho_\theta(\bar{b}-b) = \frac{1}{\tilde{\beta}}\rho_\theta(\bar{b}-b) + (1+r)b - (w_\theta \sum_{i=1}^{mr} l_i^* n_i)\{\frac{1}{(1+\eta)} - [(w_\theta \sum_{i=1}^{mr} l_i^* n_i)\tilde{\beta}H(1+\eta)]^{-1}[([1-\phi+\sigma\phi]^{-1}$$
$$-\eta)\sum_{i=1}^{mr}(l_i^*)^{1+\frac{1}{\eta}} n_i]\ \rho_\theta(\bar{b}-b)\}$$

which entails the coefficient ρ_θ to be equal to (11).

[step 2] For verifying the guess of the above step 1, firstly, notice that the optimality condition of (7) is met by (9), (10), (11), and (12) because

$$\{\frac{1}{(1-\phi+\sigma\phi)} - \eta\}\{\frac{1-\tau_\theta(b)}{1-\tau_\theta(b)(1+\eta)}\}\zeta^\theta = \{\frac{1}{(1-\phi+\sigma\phi)} - \eta\}\{\frac{\tilde{\beta}H}{([1-\phi+\sigma\phi]^{-1}-\eta)(\sum_{i=1}^{mr}(l_i^*)^{1+\frac{1}{\eta}} n_i)\rho_\theta(\bar{b}-b)}\}$$
$$\sum_{i=1}^{mr}(l_i^*)^{1+\frac{1}{\eta}} n_i = \frac{H}{\tilde{\beta}^{-1}\rho_\theta(\bar{b}-b)} = \frac{H}{g_\theta(b)}$$

for any given $b \in [\underline{b}, \bar{b}]$ and $\theta \in \{H, L\}$. Secondly, the optimality condition of (8) is also met by (9), (10), (11), and (12) because

$$-\tilde{\beta}E[\frac{H}{g_{\theta'}(d_\theta(b))}\frac{\partial g_{\theta'}(d_\theta(b))}{\partial d_\theta(b)}] = -\tilde{\beta}E[\frac{H}{\tilde{\beta}^{-1}\rho_\theta(d)(\bar{b}-d_\theta(b))}(-\tilde{\beta}^{-1}\rho_\theta(d))] = \frac{\tilde{\beta}H}{\bar{b}-d_\theta(b)} = \frac{H}{\tilde{\beta}^{-1}\rho_\theta(\bar{b}-b)}$$
$$= \frac{H}{g_\theta(b)} = \{\frac{1}{(1-\phi+\sigma\phi)} - \eta\}\{\frac{1-\tau_\theta(b)}{1-\tau_\theta(b)(1+\eta)}\}\zeta^\theta$$

for any given $b \in [\underline{b}, \bar{b}]$ and $\theta \in \{H, L\}$. Therefore, the policy functions of (9), (10), (11) with (12) satisfy all the optimality conditions of the politico-economic Markov-perfect equilibrium.

[step 3] From the above step 2, $\frac{H}{\tilde{\beta}^{-1}\rho_\theta(\bar{b}-b)} = \frac{H}{g_\theta(b)}$. Notice that optimal public goods provision is strictly positive to prevent the utility of voters from plunging into $-\infty$; that is, $g_\theta(b) = \frac{1}{\tilde{\beta}}\rho_\theta(\bar{b}-b) > 0$ for $\forall b \in [\underline{b}, \bar{b})$ due to (11). This implies that $\rho_\theta(b) > 0$ for $\forall b \in [\underline{b}, \bar{b})$ since $0 < \tilde{\beta} < 1$. And, $\frac{H}{\tilde{\beta}^{-1}\rho_\theta(\bar{b}-b)} \to +\infty$ as $b \to \bar{b}$ because $g_\theta(b) \to 0$ as $b \to \bar{b}$. Therefore,

$\rho_\theta(b) > 0$ for $\forall b \in [\underline{b}, \overline{b}]$. ■

B3. *Proof for Lemma 2*

[step 1] To show that $\ddot{b}_H < \ddot{b}_L$ by way of contradiction, suppose $\ddot{b}_H \geq \ddot{b}_L$. Moreover, notice that $\ddot{b}_H$ and $\tau_H(\ddot{b}_H)$ are the steady-state solutions of (6) if the TFP is constantly set as high as z_H. Likewise, $\ddot{b}_L$ and $\tau_L(\ddot{b}_L)$ are the steady-state solutions of (6) if the TFP is constantly set as low as z_L. Therefore, by the nature of boom and recession states, income tax revenue is larger for $\theta = H$ (high TFP) than for $\theta = L$ (low TFP) since both total output and wage rate are greater for $\theta = H$ than for $\theta = L$; i.e.,

$$\tau_H(\ddot{b}_H) w_H \sum_{i=1}^{mr} l_i^*(\tau_H(\ddot{b}_H)) n_i > \tau_L(\ddot{b}_L) w_L \sum_{i=1}^{mr} l_i^*(\tau_L(\ddot{b}_L)) n_i .$$

On the other hand, in terms of (9) and (11) of **Lemma 1**, the government budget constraint is stated as $\tau_\theta(\ddot{b}_\theta) w_\theta \sum_{i=1}^{mr} l_i^*(\tau_\theta(\ddot{b}_\theta)) n_i + \ddot{b}_\theta = \frac{1}{\tilde{\beta}}(\overline{b} - \ddot{b}_\theta) + (1+r)\ddot{b}_\theta$, for any given $\theta \in \{H, L\}$, which implies that $[\tau_H(\ddot{b}_H) w_H \sum_{i=1}^{mr} l_i^*(\tau_H(\ddot{b}_H)) n_i] - [\tau_L(\ddot{b}_L) w_L \sum_{i=1}^{mr} l_i^*(\tau_L(\ddot{b}_L)) n_i] = \frac{1}{\tilde{\beta}}(\overline{b} - \ddot{b}_H) - \frac{1}{\tilde{\beta}}(\overline{b} - \ddot{b}_L) + r$ $(\ddot{b}_H - \ddot{b}_L)$. Since $\ddot{b}_H \geq \ddot{b}_L$, $r = \frac{1}{\beta} - 1$, and $\tilde{\beta} = \beta \sum_{i=1}^{L} n_i s_i = \beta(1 - n_L) < \beta$,

$$[\tau_H(\ddot{b}_H) w_H \sum_{i=1}^{mr} l_i^*(\tau_H(\ddot{b}_H)) n_i] - [\tau_L(\ddot{b}_L) w_L \sum_{i=1}^{mr} l_i^*(\tau_L(\ddot{b}_L)) n_i] = (\frac{1}{\beta} - \frac{1}{\tilde{\beta}} - 1)(\ddot{b}_H - \ddot{b}_L) \leq 0 ,$$

which is contradicting to $\tau_H(\ddot{b}_H) w_H \sum_{i=1}^{mr} l_i^*(\tau_H(\ddot{b}_H)) n_i > \tau_L(\ddot{b}_L) w_L \sum_{i=1}^{mr} l_i^*(\tau_L(\ddot{b}_L)) n_i$. This contradiction shows that $\ddot{b}_H < \ddot{b}_L$.

[step 2] According to (6), both $\ddot{b}_H$ and $\ddot{b}_L$ must lie in $[\underline{b}, \overline{b}]$. Having shown that $\ddot{b}_H < \ddot{b}_L$ in the above step 1, firstly, we want to show that $\ddot{b}_L < \overline{b}$. Due to (12) of **Lemma 1**, as $\ddot{b}_L$ approaches $\overline{b}$, $\rho_\theta(\ddot{b}_L)$ goes to $+\infty$ for $\forall \theta \in \{H, L\}$, which is a contradiction. Thus, $\ddot{b}_L \neq \overline{b}$ and $\ddot{b}_L < \overline{b}$. Secondly, to show that $\ddot{b}_H \neq \underline{b}$ by way of contradiction, suppose that $\ddot{b}_H = \underline{b}$. Then, the government asset (negative public debt), which is large enough to provide public goods g^{sm} for the current and unborn future generations only with the interest earnings over the infinite time

horizon, is kept (not consumed) for two consecutive boom periods. However, since the policymaker and his electorate do not take account of future voters (who will be born after the current period), they are strictly better off by consuming the government asset (i.e., by increasing public debt from $\underline{b}$) to provide strictly more public goods than g^{sm} to his current electorate. Thus, $\ddot{b}_H = \underline{b}$ cannot be a Markov-perfect equilibrium, since it is strictly dominated strategy. This shows that $\ddot{b}_H \neq \underline{b}$ at the politico-economic Markov-perfect equilibrium. Taking the step 1 and 2 together shows that $\underline{b} < \ddot{b}_H < \ddot{b}_L < \bar{b}$. ■

B4. *Proof for Proposition 1*

[step 1] To begin, it is necessary to show $\frac{\partial \rho_\theta(b)}{\partial b} > 0$ for $\forall b \in [\underline{b}, \bar{b})$ and $\forall \theta \in \{H, L\}$ while $\frac{\partial \rho_H(\bar{b})}{\partial \bar{b}} > 0$ and $\frac{\partial \rho_L(\bar{b})}{\partial \bar{b}} = 0$. To this end, from (12),

$$\frac{\partial \rho_\theta(b)}{\partial b} = [1 + \frac{1}{\tilde{\beta}} + \frac{([1-\phi+\sigma\phi]^{-1} - \eta)}{\tilde{\beta} H(1+\eta)} \sum_{i=1}^{mr} (l_i^*)^{1+\frac{1}{\eta}} n_i]^{-1} (\frac{-\bar{\tau} w_L \sum_{i=1}^{mr} l_i^*(\bar{\tau}) n_i + w_\theta (1+\eta)^{-1} \sum_{i=1}^{mr} l_i^* n_i}{(\bar{b} - b)^2})$$

as $\bar{b} = \frac{\bar{\tau} w_L}{r} \sum_{i=1}^{mr} l_i^*(\bar{\tau}) n_i$. Thus, the sign of $\frac{\partial \rho_\theta(b)}{\partial b}$ is determined by the sign of $[-\bar{\tau} w_L \sum_{i=1}^{mr} l_i^*(\bar{\tau}) n_i +$ $w_\theta (1+\eta)^{-1} \sum_{i=1}^{mr} l_i^* n_i]$. Furthermore, the FOC of $\bar{\tau} = \arg\max_{\tau} \tau w_L \sum_{i=1}^{mr} l_i^*(\tau) n_i$ implies that $\bar{\tau} = (1+\eta)^{-1}$; thus, for $\forall \theta \in \{H, L\}$,

$$-\bar{\tau} w_L \sum_{i=1}^{mr} l_i^*(\bar{\tau}) n_i + (1+\eta)^{-1} w_\theta \sum_{i=1}^{mr} l_i^* n_i = (1+\eta)^{-1} \{ w_\theta \sum_{i=1}^{mr} l_i^* n_i - w_L \sum_{i=1}^{mr} l_i^*(\bar{\tau}) n_i \} .$$

Due to (10) and (12), $(1+\eta)^{-1} > \tau_\theta(b)$ for $\forall b \in [\underline{b}, \bar{b})$ and $\forall \theta \in \{H, L\}$, while $(1+\eta)^{-1} = \tau_\theta(\bar{b})$. Firstly, when $\theta = H$, $w_H \sum_{i=1}^{mr} l_i^* n_i - w_L \sum_{i=1}^{mr} l_i^*(\bar{\tau}) n_i > 0$ for $\forall b \in [\underline{b}, \bar{b}]$ since higher wage ($w_H > w_L$) and lower tax rate entail larger labor supplies. Thus, $\frac{\partial \rho_H(b)}{\partial b} > 0$ for $\forall b \in [\underline{b}, \bar{b}]$. Secondly, when $\theta = L$, $w_L \sum_{i=1}^{mr} l_i^* n_i - w_L \sum_{i=1}^{mr} l_i^*(\bar{\tau}) n_i > 0$ for $\forall b \in [\underline{b}, \bar{b})$ since tax rate that is lower than $\bar{\tau}$ entails larger labor supplies than $\overline{\tau}$ entails with the same wage rate w_L. Moreover, $w_L \sum_{i=1}^{mr} l_i^*(\tau_\theta(\overline{b}))n_i$ $- w_L \sum_{i=1}^{mr} l_i^*(\overline{\tau})n_i = 0$ at $b = \overline{b}$ as $\overline{\tau} = (1+\eta)^{-1} = \tau_\theta(\overline{b})$. Therefore, $\frac{\partial \rho_L(b)}{\partial b} > 0$ for $\forall b \in [\underline{b}, \overline{b})$ and $\frac{\partial \rho_L(\overline{b})}{\partial \overline{b}} = 0$.

[step 2] Due to (9), for any given $\theta \in \{H, L\}$,

$$\overline{b} - d_\theta(b) > \overline{b} - b \text{ if } \rho_\theta(b) > 1;\ \overline{b} - d_\theta(b) = \overline{b} - b \text{ if } \rho_\theta(b) = 1;\ \overline{b} - d_\theta(b) < \overline{b} - b \text{ if } \rho_\theta(b) < 1.$$

This implies that, due to (13),

$$\rho_H(b) < 1 \text{ for } \forall b \in [\underline{b}, \ddot{b}_1);\ \rho_H(\ddot{b}_1) = 1; \text{ and, } \rho_H(b) > 1 \text{ for } \forall b \in (\ddot{b}_1, \overline{b}]$$

since $\frac{\partial \rho_H(b)}{\partial b} > 0$ for $\forall b \in [\underline{b}, \overline{b}]$. Moreover, in each period, public debt issued in the previous period is given public debt in the current period; i.e., $b = \grave{d} = d_{\vartheta}(\grave{b})$ for $\forall b \in [\underline{b}, \overline{b}]$ and $\forall \vartheta \in \{H, L\}$. As a result, for any given previous state $\vartheta \in \{H, L\}$, $d_H(b) > b = d_{\vartheta}(\grave{b})$ for $\forall b \in$ $[\underline{b}, \ddot{b}_1)$; $d_H(\ddot{b}_1) = \ddot{b}_1$; and, $d_H(b) < b$ for $\forall b \in (\ddot{b}_1, \overline{b}]$. By the same logic,

$$\rho_L(b) < 1 \text{ for } \forall b \in [\underline{b}, \ddot{b}_2);\ \rho_L(\ddot{b}_2) = 1;\ \rho_L(b) > 1 \text{ for } \forall b \in (\ddot{b}_2, \overline{b}].$$

This implies that for any given previous state $\vartheta \in \{H, L\}$, $d_L(b) > b = d_{\vartheta}(\grave{b})$ for $\forall b \in [\underline{b}, \ddot{b}_2)$; $d_L(\ddot{b}_2) = \ddot{b}_2$; and, $d_L(b) < b$ for $\forall b \in (\ddot{b}_2, \overline{b}]$. Since $\underline{b} < \ddot{b}_1 < \ddot{b}_2 < \overline{b}$, combining these public debt responses to the positive and negative TFP shocks completes the proof for (i), (ii), and (iii).

[step 3] In order to show that the distribution of the politico-economic Markov-perfect equilibrium public debt variable converges to a unique invariant distribution with the support of $[\underline{b}, \overline{b}]$, let us define σ-algebra $\Im$ on the set of $S = [\underline{b}, \overline{b}] \times \{H, L\}$ so that $\Im$ is the family of Borel sets of S and $(S, \Im)$ is a measurable space. Let μ_{pe} be a probability measure on $(S, \Im)$ in the current period. In this line, μ'_{pe} is a probability measure on $(S, \Im)$ in the next period. For any given state $s \in S$, let $P_{pe}(s, A)$ be transition function on $(S, \Im)$ that defines the probability of transitioning from a given state s to a state $A \in \Im$ via one step (one period), which is driven from the politico-economic Markov-perfect equilibrium public debt behavior $d_\theta(b)$ described in

the above step 2. That is, when $s=(b,\theta)$ and $A=(b^A,\theta^A)$,

$$P_{pe}(s,A)=\mathrm{p}_{\theta\theta'}\mathrm{I}\{d_\theta(b)=b^A \text{ and } \theta'=\theta^A\}$$

where I{·} is a binary indicator function that takes the value of one if the statement in the parenthesis is true and the value of zero otherwise. With this notation, $P_{pe}^N(s,A)$ refers to the probability of transitioning from state s to state A in N steps. As a result, the evolution of the distribution, from μ_{pe} to μ'_{pe}, is determined by the transition function; that is,

$$\mu'_{pe}(A)=\sum_\theta\int_{\underline{b}}^{\bar{b}}P_{pe}((b,\theta),A)\mu_{pe}(db,\theta).$$

The distribution of the politico-economic Markov-perfect equilibrium public debt variable is invariant if

$$\mu^*_{pe}(A)=\sum_\theta\int_{\underline{b}}^{\bar{b}}P_{pe}((b,\theta),A)\mu^*_{pe}(db,\theta).$$

Thus, we need to show that μ_{pe} converges to μ^*_{pe} eventually and that μ^*_{pe} is unique. To this end, we evoke Theorem 11.12 of Stokey, Lucas, and Prescott (1989), according to which it is enough to show that there exists $\varepsilon>0$ and an integer $N\geq 1$ such that for any $A\in\Im$, either $P_{pe}^N(s,A)\geq\varepsilon$ for all $s\in S$ or $P_{pe}^N(s,A^c)\geq\varepsilon$ for all $s\in S$ (*Condition M*).

[step 4] First, suppose $A=(b^A,H)$ with b^A being arbitrarily given from $[\underline{b},\ddot{b}_2)$ or $(\ddot{b}_2,\bar{b}]$; then, A^c includes L (recession arrival state) with any public debt level that is not equal to the given b^A. For any given starting state $s=(b,\theta)\in S$, there exists an integer $N\geq 1$ such that

$$N=\arg\min_m\{m:m>\frac{|\ddot{b}_2-b|}{|d_L(b)-b|}+1\}.$$

Since continuing recessions make politico-economic Markov-perfect equilibrium public debt keep increasing if $b<\ddot{b}_2$ and keep decreasing if $b>\ddot{b}_2$ until it reaches $\ddot{b}_2$ in N steps (as shown in the above step 2) such an N meets

$$P_{pe}^N(s,A^c)\geq\mathrm{p}_{\theta L}(\mathrm{p}_{LL})^{N-1}>0.$$

Then, we obtain $P_{pe}^N(s,A^c)\geq\varepsilon$ by choosing $\varepsilon\in(0,\mathrm{p}_{\theta L}(\mathrm{p}_{LL})^{N-1})$.

Second, suppose $A=(b^A,H)$ with $b^A=\ddot{b}_2$; then, A^c includes L (recession arrival state) with any public debt level that is not equal to the given b^A. For any given starting state $s=(b,\theta)\in S$, there exists an integer $N\geq 1$ such that

$$N=\arg\min_m\{m: m>\frac{|\ddot{b}_1-b|}{|d_H(b)-b|}+1\}.$$

Since continuing booms make politico-economic Markov-perfect equilibrium public debt keep increasing if $b<\ddot{b}_1$ and keep decreasing if $b>\ddot{b}_1$ until it reaches $\ddot{b}_1$, as shown in the above step 2, and $\ddot{b}_1\neq b^A=\ddot{b}_2$ (**Lemma 2**), such an N meets

$$P_{pe}^N(s,A^c)\geq \mathrm{p}_{\theta H}(\mathrm{p}_{HH})^{N-1}>0.$$

Then, we meet $P_{pe}^N(s,A^c)\geq\varepsilon$ by choosing $\varepsilon\in(0,\mathrm{p}_{\theta H}(\mathrm{p}_{HH})^{N-1})$.

Third, suppose $A=(b^A,L)$ with b^A being arbitrarily given from $[\underline{b},\ddot{b}_1)$ or $(\ddot{b}_1,\overline{b}]$; then, A^c includes H (boom arrival state) with any public debt level that is not equal to the given b^A. For any given starting state $s=(b,\theta)\in S$, there exists an integer $N\geq 1$ such that

$$N=\arg\min_m\{m: m>\frac{|\ddot{b}_1-b|}{|d_H(b)-b|}+1\}.$$

Since continuing booms make politico-economic Markov-perfect equilibrium public debt keep increasing if $b<\ddot{b}_1$ and keep decreasing if $b>\ddot{b}_1$ until it reaches $\ddot{b}_1$, such an N meets

$$P_{pe}^N(s,A^c)\geq \mathrm{p}_{\theta H}(\mathrm{p}_{HH})^{N-1}>0.$$

Then, $P_{pe}^N(s,A^c)\geq\varepsilon$ is met by choosing $\varepsilon\in(0,\mathrm{p}_{\theta H}(\mathrm{p}_{HH})^{N-1})$.

Fourth, suppose $A=(b^A,L)$ with $b^A=\ddot{b}_1$; then, A^c includes H (boom arrival state) with any public debt level that is not equal to the given b^A. For any given starting state $s=(b,\theta)\in S$, there exists an integer $N\geq 1$ such that

$$N=\arg\min_m\{m: m>\frac{|\ddot{b}_2-b|}{|d_L(b)-b|}+1\}.$$

According to the above step 2, continuing recessions make politico-economic Markov-perfect equilibrium public debt keep increasing if $b<\ddot{b}_2$ and keep decreasing if $b>\ddot{b}_2$ until it reaches $\ddot{b}_2$

and $\ddot{b}_2 \neq b^A = \ddot{b}_1$ (**Lemma 2**). Thus, such an N meets

$$P_{pe}^N(s, A^c) \geq \mathrm{p}_{\theta L}\ (\mathrm{p}_{LL})^{N-1} > 0 .$$

Then, we obtain $P_{pe}^N(s, A^c) \geq \varepsilon$ by choosing $\varepsilon \in (0, \mathrm{p}_{\theta L}(\mathrm{p}_{LL})^{N-1})$.

Notice that the above four cases constitute all the possible cases that an arbitrarily given state $A = (b^A, \theta^A) \in \Im$ can take. Putting these cases altogether implies that *Condition M* is satisfied for all $s \in S$ for an arbitrarily given state $A = (b^A, \theta^A) \in \Im$. Therefore, the distribution of the politico-economic Markov-perfect equilibrium public debt variable converges to a unique invariant distribution with the support of $[\underline{b}, \bar{b}]$. ■

B5. *Proof for Corollary 1*

[step 1] At first, from (9) and (11), for $\forall \theta \in \{H, L\}$ and $\forall b \in [\underline{b}, \bar{b}]$, $g_\theta(b) = \dfrac{1}{\tilde{\beta}}(\bar{b} - d_\theta(b))$

which implies that for $\forall \theta \in \{H, L\}$ and $\forall b \in [\underline{b}, \bar{b}]$,

$$\frac{\partial g_\theta(d_\theta(b))}{\partial d_\theta(b)} = \frac{\partial g_\theta(d)}{\partial d} = -\frac{1}{\tilde{\beta}} < 0 .$$

Similarly, from (9) and (10), we get $\tau_\theta(b) = \dfrac{1}{(1+\eta)} - \dfrac{\xi([1-\phi+\sigma\phi]^{-1}-\eta)}{w_\theta \tilde{\beta} H(1+\eta)}(\bar{b} - d_\theta(b))$ for $\forall \theta \in \{H, L\}$ and $\forall b \in [\underline{b}, \bar{b}]$, which implies that for $\forall \theta \in \{H, L\}$ and $\forall b \in [\underline{b}, \bar{b}]$,

$$\frac{\partial \tau_\theta(d_\theta(b))}{\partial d_\theta(b)} = \frac{\partial \tau_\theta(d)}{\partial d} = \frac{\xi([1-\phi+\sigma\phi]^{-1}-\eta)}{w_\theta \tilde{\beta} H(1+\eta)} > 0 .$$

[step 2] Since $b = \grave{d} = d_{\grave{\theta}}(\grave{b})$, the politico-economic Markov-perfect equilibrium public debt described in the step 2 of the proof for **Proposition 1** can be restated as follows. For any given previous state $\grave{\theta} \in \{H, L\}$, $d_H(b) = d > b = \grave{d}$ for $\forall b \in [\underline{b}, \ddot{b}_1)$; $d_H(\ddot{b}_1) = \ddot{b}_1$; and, $d_H(b) = d < b = \grave{d}$ for $\forall b \in (\ddot{b}_1, \bar{b}]$. Moreover, $g_\theta(b) = g_\theta(\grave{d}) = g_\theta(d_{\grave{\theta}}(\grave{b}))$ and $\grave{g} = g_{\grave{\theta}}(\grave{b})$ for $\forall \grave{\theta}$ and $\theta \in \{H, L\}$. Due to the above step 1 on the negative relationship between optimal public goods provision and public debt, this restatement implies that for any given previous state $\grave{\theta} \in \{H, L\}$,

$$g_H(b) < \grave{g} = g_{\grave{\theta}}(\grave{b}) \text{ if } \forall b \in [\underline{b}, \ddot{b}_1);\ g_H(b) = \grave{g} \text{ if } b = \ddot{b}_1;\ g_H(b) > \grave{g} \text{ if } \forall b \in (\ddot{b}_1, \bar{b}].$$

Likewise, $\tau_\theta(b) = \tau_\theta(\grave{d}) = \tau_\theta(d_{\grave{\theta}}(\grave{b}))$ and $\grave{\tau} = \tau_{\grave{\theta}}(\grave{b})$ for $\forall \grave{\theta}$ and $\theta \in \{H, L\}$. Hence, due to the above step 1 on the positive relationship between optimal tax rate and public debt, for any given previous state $\grave{\vartheta} \in \{H, L\}$,

$$\tau_H(b) > \grave{\tau} = \tau_{\vartheta}(\grave{b}) \text{ if } \forall b \in [\underline{b}, \ddot{b}_1)\text{; } \tau_H(b) = \grave{\tau} \text{ if } b = \ddot{b}_1\text{; } \tau_H(b) < \grave{\tau} \text{ if } \forall b \in (\ddot{b}_1, \overline{b}]\,.$$

For the same reason, by restating the politico-economic Markov-perfect equilibrium public debt response to negative TFP shocks as $d_L(b) = d > b = \grave{d}$ for $\forall b \in [\underline{b}, \ddot{b}_2)$; $d_L(\ddot{b}_2) = \ddot{b}_2$; and, $d_L(b) = d < b = \grave{d}$ for $\forall b \in (\ddot{b}_2, \overline{b}]$. The above step 1 implies optimal responses of public goods provision and tax rate to negative TFP shocks as follows. For any given previous state $\grave{\vartheta} \in \{H, L\}$,

$$g_L(b) < \grave{g} = g_{\vartheta}(\grave{b}) \text{ if } \forall b \in [\underline{b}, \ddot{b}_2)\text{; } g_L(b) = \grave{g} \text{ if } b = \ddot{b}_2\text{; } g_L(b) > \grave{g} \text{ if } \forall b \in (\ddot{b}_2, \overline{b}]$$

and

$$\tau_L(b) > \grave{\tau} = \tau_{\vartheta}(\grave{b}) \text{ if } \forall b \in [\underline{b}, \ddot{b}_2)\text{; } \tau_L(b) = \grave{\tau} \text{ if } b = \ddot{b}_2\text{; } \tau_L(b) < \grave{\tau} \text{ if } \forall b \in (\ddot{b}_2, \overline{b}]\,.$$

Combining these two rules of politico-economic Markov-perfect equilibrium public goods provision and tax rate together completes the proof. ■

B6. *Proof for Proposition 2*

[step 1] Notice that a rise in idiosyncratic risk on individual voters' disposable incomes means a rise in ϕ of the model. Moreover, from **Proposition 1**, it is the first threshold $\ddot{b}_1$ at which optimal public debt behavior changes from countercyclical to acyclically increasing. Hence, it is enough to see the effect of a rise in ϕ on the first threshold. For the notational efficiency, let $\ddot{b}_1^{before}$ denote the first threshold before a rise in ϕ and $\ddot{b}_1^{after}$ denote the first threshold after the rise in ϕ. According to **Proposition 1**, countercyclical behavior of public debt before the rise in ϕ means that the level of public debt b before the rise is higher than $\ddot{b}_1^{before}$ (i.e., $\ddot{b}_1^{before} < b$).

[step 2] Next, it is necessary to show that a rise in idiosyncratic risk on individual voters' disposable incomes (a rise in ϕ) elevates the first threshold so that $\ddot{b}_1^{before} < \ddot{b}_1^{after}$. To this end, it is sufficient to show that $\frac{\partial \ddot{b}_1}{\partial \phi} > 0$. Applying Implicit Function Theorem to (13) under $\theta = H$, $\frac{\partial \ddot{b}_1}{\partial \phi} = -\frac{\partial \rho_H}{\partial \phi}[\frac{\partial \rho_H}{\partial \ddot{b}_1}]^{-1}$. Firstly, $\frac{\partial \rho_H}{\partial \phi}$ is equal to

$$\frac{\bar{b}-(1+r)\ddot{b}_1+\frac{w_H}{1+\eta}\sum_{i=1}^{mr} l_i^* n_i}{(\bar{b}-\ddot{b}_1)}[1+\frac{1}{\tilde{\beta}}+\frac{([1-\phi+\sigma\phi]^{-1}-\eta)\sum_{i=1}^{mr}(l_i^*)^{1+\frac{1}{\eta}} n_i}{\tilde{\beta}H(1+\eta)}]^{-2}\frac{\{-(1-\sigma)\sum_{i=1}^{mr}(l_i^*)^{1+\frac{1}{\eta}} n_i\}}{\tilde{\beta}H(1+\eta)[1-\phi+\sigma\phi]^2}$$

which is strictly negative, due to (12) and $\sigma \in (0,1)$. Secondly, $\frac{\partial \rho_H}{\partial \ddot{b}_1} > 0$ since $\frac{\partial \rho_H(b)}{\partial b} > 0$ for $\forall b \in [\underline{b}, \bar{b}]$ as shown in the proof for **Proposition 1** and $\underline{b} < \ddot{b}_1 < \bar{b}$. Therefore, $\frac{\partial \ddot{b}_1}{\partial \phi} > 0$.

[step 3] Due to the above step 2 ($\frac{\partial \ddot{b}_1}{\partial \phi} > 0$), a rise in ϕ elevates the first threshold *without* a change in the level of public debt b so that b lies between $\ddot{b}_1^{before}$ and $\ddot{b}_1^{after}$. Consequently, as $\ddot{b}_1^{before} < b < \ddot{b}_1^{after}$, a rise in idiosyncratic risk on individual voters' disposable incomes (i.e., a rise in ϕ) changes optimal behavior of public debt *from* countercyclical (as $\ddot{b}_1^{before} < b$ before the rise in ϕ due to **Proposition 1**) *to* acyclically increasing (as $b < \ddot{b}_1^{after}$ after the rise in ϕ due to **Proposition 1**) with *no* change in the level of public debt b. ■

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