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IDIOSYNCRATIC RISK AND ACYCLICALLY INCREASING PUBLIC DEBT

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This paper proposes a politico-economic theory of public debt dynamics that a policy maker decides on fiscal policies for being elected by voters of overlapping generations who face idiosyncratic risk on their disposable incomes. The Markov perfect equilibrium evolution of public debt, driven by intergenerational conflict on financing public goods provision, is composed of three distinct phases so that optimal public debt can behave acyclically as well as countercyclically. Moreover, a rise in the idiosyncratic risk can change public debt behaviour from countercyclical to acyclically increasing, which may explain recent rises in public debt of developed economies like the US.

Key words: government debt, cyclical property, idiosyncratic risk, political economy, public debt dynamics
JEL classification: H63, E62, D72

1. Introduction

Public debt of developed economies such as the US, Canada, and France has risen significantly since the early 1980s. One of the major causes of the rises in public debt is that public debt started increasing even in booming years over which it had decreased before the early 1980s; i.e., public debt behaviour changed *from* countercyclical (i.e., increasing in recessions and decreasing in booms) *to* acyclically increasing (both in booms and recessions) over the business cycle.

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This change resulted in a rapid build-up of public debt, even though macroeconomic volatility was moderated and recessions were more infrequent and lasted shorter than booms. To date, studies on public debt dynamics have not adequately explained this change in public debt dynamics which is crucial in accounting for the recent rises in public debt. This paper offers a politico-economic positive theory that rationalizes not only countercyclical public debt behaviour but also acyclical public debt behaviour over the business cycle.

Because public debt issue, as a fiscal policy, is a politico-economic decision, in the theoretical model of this paper, in each period, an office-seeking policy maker decides on fiscal policies to win over his electorate that is composed of young and old voters. Moreover, individual voters' after-tax labor incomes are exposed to uninsurable idiosyncratic risk. In characterising optimal fiscal policies from the model, the Markov perfect politico-economic equilibrium turns out to differ from the social-planner equilibrium as follows. First, politically-motivated policy makers provide more public goods than a social planner would. Second, in stark contrast to the social-planner equilibrium public debt that behaves only countercyclically, the Markov perfect politico-economic equilibrium public debt can behave acyclically as well as countercyclically over the business cycle. In particular, the Markov perfect politico-economic equilibrium evolution of public debt is composed of three phases which feature different cyclical properties of public debt and are demarcated by two thresholds. In detail, (i) when the level of public debt (which is inherited from the previous period and thus given at the beginning of the current period) is lower than the first threshold, public debt increases acyclically; (ii) when the level lies between the first and second thresholds, public debt behaves countercyclically; and (iii) when the level is higher than the second threshold, public debt decreases acyclically. This three-phase evolution of public debt is led by intergenerational conflict between young and old voters, as young voters (workers) bear a heavier burden in paying for public goods provision than old voters (retirees).

Importantly, this paper also finds that a rise in idiosyncratic risk on individual voters' disposable incomes raises the first threshold to change optimal public debt behaviour from countercyclical restraint to acyclical increase *without* a change in the level of public debt inherited, which in turn leads to a rapid build-up of public debt. The greater uncertainty is on voters' incomes disposable for their private goods consumptions, the more valuable public goods (which are certainly provided) are to voters, causing acyclical increase in public debt to become politically acceptable. Whereas the effect of a rise in the idiosyncratic risk on the change in public debt dynamics is clearly proven, the effect of an ageing population turns out to be ambiguous.

In addition, the theoretical analysis is applied to data on the US economy. Despite the Great Moderation, the US public debt has risen since 1981, after which its behaviour changed from countercyclical to acyclically increasing. At the same time, idiosyncratic risk on individuals' incomes and population share of the elderly in the US increased together. For identifying separate effects of the idiosyncratic risk and the population ageing, the model is calibrated to the US data. The simulation result shows that the increase in the idiosyncratic risk is the main driving force behind the after-1981 rise in the US public debt.

The rest of this paper unfolds as follows. Section 2 reviews related literature. Section 3 describes a theoretical model where a policy maker decides on fiscal policies to court voters. From the model, the social-planner equilibrium and the Markov perfect politico-economic equilibrium evolutions of public debt are characterised and analysed. With data of the US economy, Section 4 conducts a simulation analysis. The last section concludes the paper.

2. Literature Review

Various research has been conducted on dynamics of public debt since Barro's seminal paper (1979) that explained optimal behaviour of public debt as an outcome of tax-smoothing. In particular, Barro (1979) claimed that optimal public debt behaves countercyclically, and he verified this claim with the US data between 1922 and 1976. However, observed acyclical increase in public debt of many developed economies, which is not explainable by the public debt theory of Barro (1979), gave rise to a variety of new alternative theories. Among them, some made the improvement of incorporating the reality that issuing public debt (i.e., borrowing public funds) is decided by politically-motivated policy makers and thus is affected by preferences of voting citizens in the electorate. Such politico-economic theories that are pertinent to this paper can be summarised into the following three lines.[1]

First, studies like Alesina and Tabellini (1990) and Alesina and Drazen (1991) argued that a rapid accumulation of public debt is caused by political polarization between heterogeneous groups of voters who take turns in political decisions regarding public debt issue. However, since voters are heterogeneous over multiple dimensions, instead of one single dimension, measuring political polarizations and verifying causality on public debt increases of the polarization with data are problematic. Observing obvious increases in the elderly population share of developed economies, Tabellini (1991) and Song, Storesletten, and Zilibotti (2012) analysed a case where the intergenerational divide (young versus old) is the source of heterogeneity. However, these theoretical analyses failed to find an unambiguous effect of an ageing population on public debt.

Second, some scholars focused on preference shock (or change) as the main driver of a rise in public debt. For instance, Battaglini and Coate (2008a) showed that optimal strategy of policy makers oscillates between excessively issuing public debt ("business-as-usual") and controlling public debt ("responsible policymaking") depending on the realised preference of how valuable public

[1] For a more general and broader review of the literature on political economy of public debt, see Alesina and Passalacqua (2015).

goods are to voters. On the other hand, instead of a change in preference for public goods, some studies highlighted a change in time-inconsistent preference. For example, Bisin, Lizzeri, and Yariv (2015) attributed a large accumulation of public debt to a self-control problem of voters, whereas Halac and Yared (2014) argued that a present bias of the government facing persistent shock on social value of public goods leads to a maximal accumulation of public debt. These studies, however, did not provide evidence that changes in preference for public goods and/or in time-consistency caused the rises in public debt after the early 1980s, although such preference changes are neither self-evident nor clearly observable.

Third, other scholars overcame the limitation of all the aforementioned politico-economic theories that they allow no macroeconomic fluctuations.[2] Studies such as Battaglini and Coate (2008b), Barseghyan, Battaglini, and Coate (2013), and Müller, Storesletten, and Zilibotti (2016) introduced a shock on total factor productivity to generate business cycles. Nevertheless, these studies only showed that optimal public debt behaves countercyclically, failing to rationalize acyclical increase in public debt. On the other hand, Azzimonti, Francisco, and Quadrini (2014) introduced a shock on productivity of entrepreneurs which generates inequality; however, this study neither incorporated political processes of deciding fiscal policy nor investigated cyclical property of public debt. More fundamentally, all of the previous studies failed in explaining acyclical public debt behaviour and countercyclical public debt behaviour under the same model, although both behaviours have been observed in many developed economies.

[2] In fact, Alesina, Campante, and Tabellini (2008) also introduced aggregate income shocks; however, Alesina, Campante, and Tabellini (2008) described developing economies, instead of developed economies, by assuming that rent-seeking policy makers do not immediately release private information of public debt issue to voters, and this study did not analyze dynamics of public debt over the business cycle.

3. Theoretical Analysis

3.1. The Model

Consider a small open economy where overlapping generations of voters reside.[3] Each individual voter lives for two periods, earning labour income in the first period and dissaving the income as a retiree in the second period. The total population is normalized to one and the share of old individuals is $n_o \in (0,1)$. In each period, individual voters face uninsurable idiosyncratic shock on their after-tax labour incomes disposable for private goods consumptions. Specifically, with the probability of $\phi \in [0,1)$, after-tax labour income of an individual falls by half. Due to this independent and identically distributed negative shock, even after paying labour income tax, there is uncertainty on the resources usable for private goods consumption. Thus, the parameter ϕ captures the degree of idiosyncratic risk on individuals' disposable incomes. The utility that a young voter maximises in period *t* is $\log(c_t) + H\log(g_t) - \frac{l_t^{(1+1/\eta)}}{(1/\eta)+1} + \beta E[\log(c_{t+1}) + H\log(g_{t+1})]$, where c_t and g_t refer to private and public goods consumption, respectively, in period *t*; l_t is labour supplied in period *t*; *H* is relative preference for public goods; η is Frisch elasticity of labour supply; and, β is time preference parameter. At the same time, the utility that an old voter maximises in period *t* is $\log(c_t) + H\log(g_t)$. As a consequence, the kernels of the indirect utility functions for young and old voters, $u_Y(\tau_t, g_t, g_{t+1})$ and $u_O(\tau_{t-1}, g_t)$, respectively, which include only variables that are relevant to fiscal policies, are as follows:

(1) $$u_Y = (1+\beta)\log(I(\tau_t)) - \zeta_l(\tau_t) I(\tau_t)^{-(\eta+1)} + H\log(g_t) + \beta E[H\log(g_{t+1})],$$

(2) $$u_O = \log(I(\tau_{t-1})) + H\log(g_t),$$

[3] The assumption that this economy is small and open with perfectly mobile capital is only for the purpose of describing an economy that *takes* the price of public debt rather than sets it; hence, this paper is applicable to many developed economies such as the US, Japan, and France.

where τ_t is income tax rate in period *t*; $\zeta_l(\tau_t) = \dfrac{[(\beta+1)(1-\tau_t)w_\theta]^{\eta+1}}{[(1/\eta)+1](1-0.5\phi)^{\eta+1}}$; $I(\tau_t) = (1-\tau_t)w_\theta l_t = I_t$ is before-idiosyncratic-shock after-tax labour income earned in period *t*; and, w_θ is market wage rate that is detailed below.

In this economy, a representative firm produces goods which can be used for both private and public consumption. In each period, the firm produces Y_t with inputs of aggregate capital K_t and labour L_t, following Cobb-Douglas technology; that is, $Y_t = z_\theta K_t^\alpha L_t^{1-\alpha}$ where z_θ is total factor productivity (TFP). In each period, the TFP is subject to a shock, which generates fluctuations of this economy. In particular, TFP follows a first-order Markov process with two states $\theta \in \{H, L\}$ and $z_H = H > L = z_L$, indicating that the state of this economy is a boom (recession) when $\theta = H$ ($\theta = L$). The probability of transitioning from θ to θ' in the next period is $\mathrm{p}_{\theta\theta'} = \Pr(\mathrm{z}_{t+1} = \theta' | \mathrm{z}_t = \theta) \in (0,1)$. Moreover, capital is perfectly mobile across different countries while labour is not. With profit maximisation of the representative firm, this implies that $K_t = (\frac{\alpha z_\theta}{r+\delta})^{\frac{1}{1-\alpha}} L_t$, where r is a given world-wide interest rate and δ is capital depreciation rate, and that $w_\theta = (1-\alpha)(\frac{\alpha}{r+\delta})^{\frac{\alpha}{1-\alpha}} z_\theta^{\frac{1}{1-\alpha}}$ which in turn implies that $w_H > w_L$.

The government of this economy can finance public goods provision with taxation and public debt issue. In period *t*, given level of public debt, b_t, inherited from the previous period, the government official who is elected by voters decides on public debt issue d_t, public goods provision g_t, and income tax rate τ_t, after learning that the realised current state of the economy is θ, to meet the following budget constraint:

(3) $$d_t = g_t + (1+r)b_t - (1-n_o)\tau_t w_\theta l_t \, .$$

The government issues public debt (i.e., borrows funds) by selling risk-free one-period bonds. Furthermore, the government is committed to paying the debt back, so it does not borrow more than the maximal tax revenue collectable at the worst possible state of the economy in the next period. This commitment defines the upper limit of public debt $\bar{b}$ by $\bar{b} = \frac{(1-n_o)\bar{\tau} w_L l(\bar{\tau})}{r}$ where $\bar{\tau} = \arg\max_{\tau}(1-n_o)\tau w_L l(\tau)$. On the other hand, when the government purchases bonds, it does not buy them more than necessary to efficiently provide public goods according to the Samuelson condition, which defines the lower limit of public debt $\underline{b}$ by $\underline{b} = -\frac{g^{sm}}{r}$ where $Hg_{sm}^{-1} = 1$.

In each period, the government policy maker is voted into office. To run for office, two candidates simultaneously announce fiscal policy proposals on public debt issue, public goods provision, and income tax rate, after the state of the economy is realised. Then, voters decide whom to vote for, based on both policy proposal and personal appeal of each of the candidates. Personal appeal of a candidate is nation-wide popularity which is *not* related to any policy proposal but is based on the personality of the candidate. Some surprising aspects of a candidate's personality can be disclosed during the course of an election race. Thus, the country-wide personal appeal of each candidate is not known to the two candidates when they announce their own fiscal policy proposals, whereas it is known to voters when they cast their votes. After the election, winner's fiscal policy proposal is implemented as announced. In order to maximise the winning probability, each of the two office-seeking candidates maximises the population-weighted indirect utility function $(1-n_o)u_Y(\tau_t, g_t, g_{t+1}) + n_o u_O(\tau_{t-1}, g_t)$ meeting (3) and $d_t \in [\underline{b}, \bar{b}]$ for $\forall t$. (For details, see Appendix A.) As such, the fiscal policy proposal of an elected candidate reflects *all* voters' preferences in each period. In addition, after an elected policy maker implements his fiscal policy proposal, young voters choose their labour supply, which in turn determines total output Y_t with aggregate labour supply $L_t = (1-n_o)l_t$.

3.2. Social-Planner Equilibrium

At first, a social-planner equilibrium is characterised, as a benchmark, since it does not suffer from political distortions. Without an election, a social planner directly chooses public debt issue to maximise social welfare that is the sum of utilities of *all* the present and future generations. In contrast, an office-seeking policy maker issues public debt without considering unborn future generations who should pay the debt; thus, he fails to internalize the public finance cost on the unborn future generations who are not in his electorate. In particular, the social planner chooses fiscal policies $\{d_t, g_t, \tau_t\}_{t=1}^{\infty}$ by solving the following problem:

$$\max_{\{d_t, g_t, \tau_t\}_{t=1}^{\infty}} (1-n_o)u_Y(\tau_t, g_t, g_{t+1}) + n_o u_O(\tau_{t-1}, g_t) + \sum_{s=1}^{\infty} \beta^s E[(1-n_o)u_Y(\tau_{t+s}, g_{t+s}, g_{t+s+1}) + n_o u_O(\tau_{t+s-1}, g_{t+s})] \text{ s.t. (3) and } d_t \in [\underline{b}, \bar{b}] \text{ for } \forall t.$$

This problem for the social-planner is re-stated in a recursive way as follows. For any given t and $\theta \in \{H, L\}$,

$$(4) \quad v_\theta(b_t) = \max_{\{d_t, g_t, \tau_t\}} \{(1-n_o+\beta)\log(I(\tau_t)) - (1-n_o)\zeta_l(\tau_t)[I(\tau_t)]^{-(\eta+1)} + H\log(g_t) + \beta E[\mathrm{p}_{\theta H}\, v_H(d_t) + \mathrm{p}_{\theta L}\, v_L(d_t)] \text{ s.t. (3) and } d_t \in [\underline{b}, \bar{b}] \text{ for } \forall t\,\}$$

where $v_\theta(b)$ is the value function of the social welfare when the current state of the economy is θ and the inherited public debt is b.

The optimal policy functions that solve the functional equation (4), denoted by $\{d_\theta^s(b_t), g_\theta^s(b_t), \tau_\theta^s(b_t)\}_{\theta=H,L}$, are defined by the following optimality conditions on public debt issue, public goods provision, and income tax rate. Firstly, public goods provision is set to equalize its marginal benefit with marginal cost of financing the provision; that is, for $\forall b_t \in [\underline{b}, \bar{b}]$ and $\forall \theta \in \{H, L\}$,

(5) $$H[g_\theta^s(b_t)]^{-1} = \frac{(1-n_o+\beta)}{(1-n_o)[(1-\phi)I_t + \phi \frac{I_t}{2}]} \{\frac{1-\tau_\theta^s(b_t)}{1-\tau_\theta^s(b_t)(1+\eta)}\}.$$

Notice that in assessing the marginal cost of public finance (MCPF), the *expected* value of disposable income $(1-\phi)I_t + \phi\frac{I_t}{2}$ is used, since idiosyncratic uncertainty on after-tax income I_t is not resolved when a fiscal policy is chosen. Secondly, the marginal cost of taxation to finance current public expenditures equals the present value of the expected marginal cost of public debt on all future generations. For $\forall b_t \in [\underline{b}, \bar{b}]$ and $\forall \theta \in \{H, L\}$,

(6) $$\frac{(1-n_o+\beta)}{(1-n_o)(1-0.5\phi)I_t} \{\frac{1-\tau_\theta^s(b_t)}{1-\tau_\theta^s(b_t)(1+\eta)}\} = -\beta E[\mathrm{p}_{\theta H}\, v_H'(d_\theta^s(b_t)) + \mathrm{p}_{\theta L}\, v_L'(d_\theta^s(b_t))].$$

Thirdly, the marginal benefit of public debt meets the following Euler equation. For $\forall b_t \in [\underline{b}, \bar{b}]$ and $\forall \theta \in \{H, L\}$,

(7) $$v_\theta'(b_t) = \beta(1+r)E[\mathrm{p}_{\theta H}\, v_H'(d_\theta^s(b_t)) + \mathrm{p}_{\theta L}\, v_L'(d_\theta^s(b_t))].$$

Taking (5), (6), and (7) together yields the evolution of marginal cost of public finance, which determines the social-planner equilibrium behaviour of public debt, as follows. For $\forall b_t \in [\underline{b}, \bar{b}]$ and $\forall \theta \in \{H, L\}$,

(8) $$\frac{1-\tau_\theta^s(b_t)}{I_t\{1-\tau_\theta^s(b_t)(1+\eta)\}} = \beta(1+r)\,\mathrm{E}[\frac{\mathrm{p}_{\theta H}\{1-\tau_H^s(d_\theta^s(b_t))\}}{I_{t+1}\{1-\tau_H^s(d_\theta^s(b_t))(1+\eta)\}} + \frac{\mathrm{p}_{\theta L}\{1-\tau_L^s(d_\theta^s(b_t))\}}{I_{t+1}\{1-\tau_L^s(d_\theta^s(b_t))(1+\eta)\}}].$$

That is, the current MCPF is equal to the present value of the expected MCPF of the next period. Moreover, $\beta(1+r)=1$ when time preference parameter in other economies is not different from β. Then, the MCPF obeys a martingale process, causing the social planner to do stochastic tax-smoothing over time. As a consequence, public debt behaves countercyclically while public goods provision is procyclical; that is, the social planner decreases public debt and increases public goods provision in booms whereas he increases public debt and decreases public goods provision in recessions.

Proposition 1. When $\beta(1+r)=1$, social-planner equilibrium fiscal policies $\{d_\theta^s(b_t),\ g_\theta^s(b_t)\}_{\theta=H,L}$ evolve over the business cycle as follows. For $\forall b_t \in [\underline{b}, \bar{b}]$ and $\forall \theta \in \{H, L\}$,

(i) public debt is countercyclical: i.e., $d_H^s(b_t) < b_t = d_\theta^s(b_{t-1})$ and $d_L^s(b_t) > b_t$,

(ii) public goods provision is procyclical: i.e., $g_H^s(b_t) > g_\theta^s(b_{t-1})$ and $g_L^s(b_t) < g_\theta^s(b_{t-1})$.

Proof. See Appendix B.

When this economy enters a boom (recession), the social planner lowers (raises) public debt issue since he expects less (more) output available for paying the debt back in the next period. On the other hand, the social planner increases (decreases) public goods provision in a boom (recession) with more (less) resources available in the current period. Most importantly, the countercyclicality of public debt prevents the level of public debt from rising rapidly, with alternating booms and recessions over time.

3.3. Politico-Economic Equilibrium

The dynamics of public debt set by elected policy makers, instead of the social planner, is characterised by Markov perfect politico-economic equilibrium where office-seeking candidates and individual voters choose their own strategies, conditional on payoff-relevant state variables, from maximising the winning probability and the utilities for the remaining lifetime, respectively. As mentioned above, in each period, office-seeking candidates seek to win over voters by proposing fiscal policies $\{d_t, g_t, \tau_t\}$ that maximise the population-weighted indirect utility of the electorate. Unlike the social planner, each candidate cannot credibly propose future fiscal policies, because the term of elected policy makers is limited to one period only. Thus, optimal fiscal policies arising from political competition in each period are obtained by solving the following problem. For any given t and $\theta \in \{H, L\}$,

$$(9) \qquad \max_{\{d_t, g_t, \tau_t\}} (1-n_o)\{(1+\beta)\log(I(\tau_t)) - \zeta_l(\tau_t)[I(\tau_t)]^{-(\eta+1)} + \beta E[H\log(g_{t+1}(d_t))]\} + H\log(g_t) \text{ s.t. } d_t = g_t + (1+r)b_t - (1-n_o)\tau_t w_\theta l_t \text{ and } d_t \in [\underline{b}, \bar{b}].$$

The optimal policy functions of the Markov perfect politico-economic equilibrium that solves (9), denoted by $\{d_\theta(b_t), g_\theta(b_t), \tau_\theta(b_t)\}_{\theta=H,L}$, are defined by the following optimality conditions regarding marginal benefit and cost of public goods provision and public finance. For $\forall b_t \in [\underline{b}, \bar{b}]$ and $\forall \theta \in \{H, L\}$,

$$(10) \qquad H[g_\theta(b_t)]^{-1} = \frac{(1+\beta)}{(1-0.5\phi)I_t}\{\frac{1-\tau_\theta(b_t)}{1-\tau_\theta(b_t)(1+\eta)}\},$$

$$(11) \qquad \frac{(1+\beta)}{(1-0.5\phi)I_t}\{\frac{1-\tau_\theta(b_t)}{1-\tau_\theta(b_t)(1+\eta)}\} = -\beta E[\frac{H}{g_\theta(d_t)}\frac{\partial g_\theta(d_t)}{\partial d_t}]$$

To begin with, the right-hand side of (10), marginal cost of financing public goods provision, is smaller than that of (5), indicating *over-provision* of public goods by a politically-motivated policy maker (i.e., $g_\theta(b_t) > g_\theta^s(b_t)$ for any given b_t and θ). Notably, the policy maker favors his electorate with more public goods by issuing more public debt because the cost of paying the debt is born mostly by unborn future generations who are *not* in his electorate. Nevertheless, the policy maker does not fully exploit the future non-electorate by issuing public debt maximally, although he could have. While old voters prefer more public goods by raising public debt as much as possible, young voters are against it, since it will deplete future resources for providing public goods that the young voters can consume after retirement in the next period. Such an intergenerational conflict appears in $\frac{\partial g_\theta(d_t)}{\partial d_t} < 0$ with $d_t = b_{t+1}$ implied by (10) and (11).

According to (10) and (11), even when $\beta(1+r)=1$, MCPF obeys neither a martingale process, as (8) does, nor a super-martingale (sub-martingale) process. Nevertheless, it is possible to specify the optimal policy functions of the Markov perfect politico-economic equilibrium as below.

Lemma 1. Markov perfect politico-economic equilibrium fiscal policies $\{d_\theta(b_t), g_\theta(b_t), \tau_\theta(b_t)\}_{\theta=H,L}$ are defined as follows. For $\forall b_t \in [\underline{b}, \bar{b}]$ and $\forall \theta \in \{H, L\}$,

(12) $$\bar{b} - d_\theta(b_t) = \rho_\theta(\bar{b} - b_t),$$

(13) $$g_\theta(b_t) = \frac{1}{\beta}\rho_\theta(\bar{b} - b_t),$$

(14) $$\tau_\theta(b_t) = \frac{1}{(1+\eta)} - \frac{(1+\beta)}{\beta H w_\theta l_t (1-0.5\phi)(1+\eta)}\rho_\theta(\bar{b} - b_t),$$

where ρ_θ is defined by

(15) $$\rho_\theta(b_t) = \frac{\beta H(1-0.5\phi)\{(1-n_o)w_\theta l_t + (1+\eta)[\bar{b} - (1+r)b_t]\}}{(\bar{b} - b_t)(1+\beta)\{H(1-0.5\phi)(1+\eta) + (1-n_o)\}} > 0.$$

Proof. See Appendix C.

At first, whether in booms or recessions, policy makers can win over their voters by pushing up public debt level. However, growth of inherited public debt level draws greater tax revenue for paying the debt (issued in the previous period) to leave fewer resources for public goods provision in the current period; hence, the upward force is eventually counteracted. As public debt level approaches the upper limit $\bar{b}$, policy makers face downward pressure to curb public debt. Therefore, policy makers reach a threshold at which these two forces, moving in the opposite directions, are of equal magnitude so that public debt level does not move (i.e., $d_\theta(b_t) = b_t = b_{t+1}$ for any given θ). In light of (12) and (15), such a threshold of the upward pressure on public debt level, denoted by $\ddot{b}_\theta$, is defined as follows; for each $\theta \in \{H, L\}$,

(16) $$\rho_\theta(\ddot{b}_\theta) = 1 = \frac{\beta H(1-0.5\phi)\{(1-n_o)w_\theta l_\theta + (1+\eta)[\bar{b}-(1+r)\ddot{b}_\theta]\}}{(\bar{b}-\ddot{b}_\theta)(1+\beta)\{H(1-0.5\phi)(1+\eta)+(1-n_o)\}}$$

where l_θ is defined by (3), (10), and (11) under $d_\theta(b_t) = b_t$.

Lemma 2. Over the Markov perfect politico-economic equilibrium evolution of public debt, the threshold at which public debt level does not move for booms is lower than that for recessions: i.e., $\underline{b} < \ddot{b}_H < \ddot{b}_L < \bar{b}$.

Proof. See Appendix D.

As total output in booms is greater than in recessions, more resources are available for public goods provision as well as private goods consumption to necessitate *less* issue of public debt to cater voters in booms than in recessions. Hence, the public debt threshold level, from which no new additional issue of public debt is demanded from voters, is lower for booms than for recessions. To reflect this (**Lemma 2**), the two thresholds are re-labelled with $\ddot{b}_H = \ddot{b}_1$ and $\ddot{b}_L = \ddot{b}_2$. In fact, $\ddot{b}_1$ and $\ddot{b}_2$ are critical, as they serve the key thresholds over which behaviour of public debt changes from countercyclical to acyclical and vice versa.

Proposition 2. Markov perfect politico-economic equilibrium public debt $\{d_\theta(b_t)\}_{\theta=H,L}$ evolves over the business cycle as follows. (i) If $\underline{b} \le b_t \le \ddot{b}_1$, public debt increases acyclically until it reaches $\ddot{b}_1$ in booms: i.e., $d_H(b_t) > b_t = d_\theta(b_{t-1})$ and $d_L(b_t) > b_t$ for $\forall b_t \in [\underline{b}, \ddot{b}_1)$ and $\forall \theta \in \{H, L\}$, while $d_H(\ddot{b}_1) = \ddot{b}_1$ and $d_L(\ddot{b}_1) > \ddot{b}_1$. (ii) If $\ddot{b}_1 < b_t \le \ddot{b}_2$, public debt behaves countercyclically until it reaches $\ddot{b}_2$ in recessions: i.e., $d_H(b_t) < b_t$ and $d_L(b_t) > b_t$ for $\forall b_t \in (\ddot{b}_1, \ddot{b}_2)$, while $d_H(\ddot{b}_2) < \ddot{b}_2$ and $d_L(\ddot{b}_2) = \ddot{b}_2$. (iii) If $\ddot{b}_2 < b_t \le \bar{b}$, public debt decreases acyclically: i.e., $d_H(b_t) < b_t$ and $d_L(b_t) < b_t$ for $\forall b_t \in (\ddot{b}_2, \bar{b}]$.

Proof. See Appendix E.

In addition, the dynamic behaviour of public debt, described in **Proposition 2**, entails that of public goods provision and income tax rate over the business cycle as below.

Corollary 1. Markov perfect politico-economic equilibrium public goods provision and income tax rate $\{g_\theta(b_t), \tau_\theta(b_t)\}_{\theta=H,L}$ evolve over the business cycle as follows. (i) If $\underline{b} \le b_t \le \ddot{b}_1$, public goods provision decreases acyclically while income tax rate increases acyclically, until public debt reaches $\ddot{b}_1$ in booms. (ii) If $\ddot{b}_1 < b_t \le \ddot{b}_2$, public goods provision behaves procyclically while income tax rate behaves countercyclically, until public debt reaches $\ddot{b}_2$ in recessions. (iii) If $\ddot{b}_2 < b_t < \overline{b}$, public goods provision increases acyclically while income tax rate decreases acyclically.

Proof. See Appendix F.

In each period, collected public funds are diverted *from* providing public goods *to* paying back public debt inherited from the previous period (sum of interest and principal amount). Hence, the cost of raising public funds per unit of public goods is dependent on the level of public debt inherited from the previous period. On the other hand, although all voters benefit from public goods provision, young voters bear the burden of public funds by paying labour income taxes, while old voters do not. For resolving this intergenerational conflict with compromise, when policy makers raise more (less) public funds, they decrease (increase) public goods provision. Moreover, when raising public funds, policy makers utilise *both* policy tools (income tax and public debt issue) together, since voters prefer consumption diversification over time (the present and the future) and across types of goods (private and public goods).

Firstly, when the level of public debt inherited from the previous period is low ($\underline{b} \le b_t \le \ddot{b}_1$), raising more public funds costs relatively low. As a result, increasing public debt is politically supported, independent of the state of the economy. Thus, the level of public debt incessantly increases even with alternating booms and recessions over time, until it reaches the first threshold $\ddot{b}_1$. This acyclical behaviour of public debt contrasts to countercyclical behaviour of public debt over the *same* range ($\underline{b} \leq b_t \leq \ddot{b}_1$) under the social-planner equilibrium (**Proposition 1**). Secondly, when the inherited public debt level enters into the middle range ($\ddot{b}_1 < b_t \leq \ddot{b}_2$), increasing public funds becomes costly so that political support for it becomes dependent on the state of the economy. In particular, in booms (recessions), policy makers decrease (increase) public debt issue, expecting a smaller (greater) income tax base from which public goods are provided for the current young voters after the debt is paid in the next period. Thus, public debt behaves countercyclically, as long as it is below the alerting threshold $\ddot{b}_2$. Lastly, when the level of inherited public debt is high ($\ddot{b}_2 < b_t < \bar{b}$), issuing new additional public debt costs too much, regardless of the state of the economy. Thus, public debt acyclically decreases, converging to the second threshold $\ddot{b}_2$.

Taking these three phases together, Markov perfect politico-economic equilibrium fiscal policies over the business cycle are characterised, which are uniquely defined under a given set of parameters (**Lemma 1**, **Proposition 2** and **Corollary 1**). In contrast to previous studies on public debt dynamics, this study shows that optimal behaviour of public debt can be not only countercyclical but also acyclical. This theoretical finding is consistent with observed various behaviours of public debt in developed economies, as discussed below. Moreover, the attribute that Markov perfect politico-economic equilibrium evolution of fiscal policies is of phases with different cyclical properties stems from conflicting political interests between two generations such that young voters bear a heavier financial burden to provide public goods than old voters.

3.4. Change in Public Debt Dynamics and Idiosyncratic Risk

In reality, public debt of some economies behaves countercyclically (e.g., Sweden between 1980 and 2013) while public debt of other economies increases acyclically to rapidly accumulate (e.g., Germany and Japan between 1980 and 2013). The former economies take advantage of many booming years for reducing their public debt, while the latter economies do not. This paper can provide an explanation for this cross-sectional variation across different economies. The public debt level of the former economies is higher than their first threshold $\ddot{b}_1$ while that of the latter economies is lower than their own first threshold $\ddot{b}_1$ which may differ from $\ddot{b}_1$ of the former. On the other hand, we also observe that public debt behaviour of a given economy changes from acyclically increasing (e.g., Spain from 1965 to 1997) to countercyclical (e.g., Spain from 1998 to 2015) over time, as the level of inherited public debt increases. This observed change in the dynamics of public debt also can be explained by the model of this paper. According to **Proposition 2**, this longitudinal change in public debt behaviour is brought by public debt's surpassing the first threshold $\ddot{b}_1$ of the economy from which voters start demanding to curb rising public debt during booms.

Another important observed change in public debt dynamics is that public debt of a given economy increases acyclically at a given level of inherited public debt where it used to behave countercyclically before. For example, the US public debt behaved countercyclically from 1950 to 1981 with its level moving between 32.24 % and 87.45% of the US GDP which included the range from 40% to 65% of the US GDP (See **Figure 1** in the next section). After 1981, however, the US public debt increased acyclically over the very same range (from 40% to 65% of the US GDP). As a matter of fact, this change in public debt dynamics, which may seem puzzling, occurred in many other developed economies, such as the UK, Canada, France, and so forth, leading to the recent rises in their public debt.

To rationalize this seemingly puzzling change in public debt dynamics, let's allow for parameters of the model to change for reflecting actual economic changes over the last few decades. In particular, let's examine whether a change in the economic parameter raises the value of the first threshold $\ddot{b}_1$ below which optimal behaviour of public debt is acyclical increase *and* above which optimal behaviour of public debt is countercyclical restraint (**Proposition 2**). If after-change $\ddot{b}_1$ is higher than before-change $\ddot{b}_1$, countercyclical behaviour of public debt is switched to acyclically increasing behaviour, at a *given* level between before-change $\ddot{b}_1$ and after-change $\ddot{b}_1$, due to the change in $\ddot{b}_1$. For actual economic changes to be reflected on the parameters in (16), let's consider a change in idiosyncratic risk on individuals' incomes and in population share of the elderly (old voters), respectively, as the former is captured by parameter ϕ and the latter by n_o in the model.

First, the effect on the first threshold $\ddot{b}_1$ of ϕ, the parameter of idiosyncratic risk on individuals' disposable incomes, is examined. It turns out that a rise in ϕ raises the value of the first threshold $\ddot{b}_1$ ($\frac{\partial \ddot{b}_1}{\partial \phi} > 0$) causing a change in public debt dynamics from countercyclical restraint to acyclical increase *without* a change in the level of public debt inherited.

Proposition 3. A rise in idiosyncratic risk on individual voters' disposable incomes can change optimal public debt behaviour from countercyclical to acyclically increasing without a change in the level of public debt (b_t).

Proof. See Appendix G.

Intuitively, as individual voters face greater uncertainty on their incomes usable for their private goods consumptions, public goods become more valuable to voters, as public goods are *certainly* provided. As a result, acyclical increase in public debt becomes politically acceptable with the first threshold $\ddot{b}_1$ being elevated. Thus, even when the level of inherited public debt does not change at all, a rise in idiosyncratic risk on voters' disposable incomes can change the optimal strategy of public debt from countercyclical restraint to acyclical increase, causing public debt to rise rapidly.

In addition, as shown in the proof for **Proposition 3**, the mechanism by which idiosyncratic risk on voters' disposable incomes affects change in public debt dynamics does not depend on the degree of macroeconomic volatility. This is consistent with observed change in public debt behaviour of developed economies (such as the US) from countercyclical restraint to acyclical increase despite their improved macroeconomic stability.

As a corollary of **Proposition 3** that shows the positive effect of ϕ on $\ddot{b}_1$, a rise in idiosyncratic risk on individuals' disposable incomes causes public debt to keep increasing acyclically and delay starting to behave countercyclically. After a rise in the idiosyncratic risk raises the first threshold $\ddot{b}_1$, public debt continues to increase acyclically, although it could have decreased in booms by surpassing before-change $\ddot{b}_1$ without the rise. As another corollary of **Proposition 3**, it is straightforward that $\frac{\partial \ddot{b}_2}{\partial \phi} > 0$: thus, a rise in idiosyncratic risk on voters' disposable incomes also raises the alerting threshold $\ddot{b}_2$ to deter economies, whose public debt level is right below after-change $\ddot{b}_2$, from decreasing public debt acyclically. After all, both cases entail a rise in public debt.

Second, the effect on the first threshold $\ddot{b}_1$ of the economic parameter n_o is examined. As a rise in n_o reflects an ageing population, which has kept drawing attentions of policy makers and researchers, a number of previous studies have investigated its effect on the evolution of public debt; however, they found different results.[4] Following the same logic by which **Proposition 3** proves the effect of ϕ on change in the dynamics of public debt, whether a rise in n_o raises the value of the first threshold $\ddot{b}_1$ or not is investigated. In the end, it turns out that the effect of an ageing population on change in public debt dynamics is *ambiguous* with the current level of generality. For detailed proof, see Appendix H. Intuitively, as the population share of retirees (old voters) increases, their demand for increasing the current public goods provision with issuing more public debt may increase. At the same time, however, this demand becomes more costly to and less politically acceptable by young voters, since they now need more after-tax income facing a longer retirement in their future.

4. Quantitative Analysis

The above theoretical findings on change in public debt dynamics from countercyclical restraint to acyclical increase are worthy of an empirical investigation. To this end, data on the US economy are analysed. First of all, similar to the time trend of public debt of OECD economies (on average), the US public debt rose substantially after the early 1980s before which it had previously been curbed since World War 2. In particular, as shown in **Figure 1**, the US public debt behaved countercyclically before 1981; however, after 1981, it increased acyclically.

Moreover, as noted above, over the course of this change in public debt

[4] For example, Tabellini (1991) and Song, Storesletten, and Zilibotti (2012) also incorporated a parameter of the population share of old voters in their models. While Tabellini (1991) theoretically proved that the effect of an ageing population on public debt is ambiguous, Song, Storesletten, and Zilibotti (2012) did not examine the effect theoretically. Nonetheless, Song, Storesletten, and Zilibotti (2012) conducted a simulation with a set of specific values of parameters calibrated to OECD countries' average data to find that the ageing population positively affects the public debt level.

dynamics of the US, there occurred an overlapping range of public debt levels (between 40% and 65% of the US GDP) where public debt behaviour was countercyclical before 1981 and then changed to acyclically increasing after 1981. This is attributable to an increase in idiosyncratic uncertainty on individuals' incomes, according to **Proposition 3**.

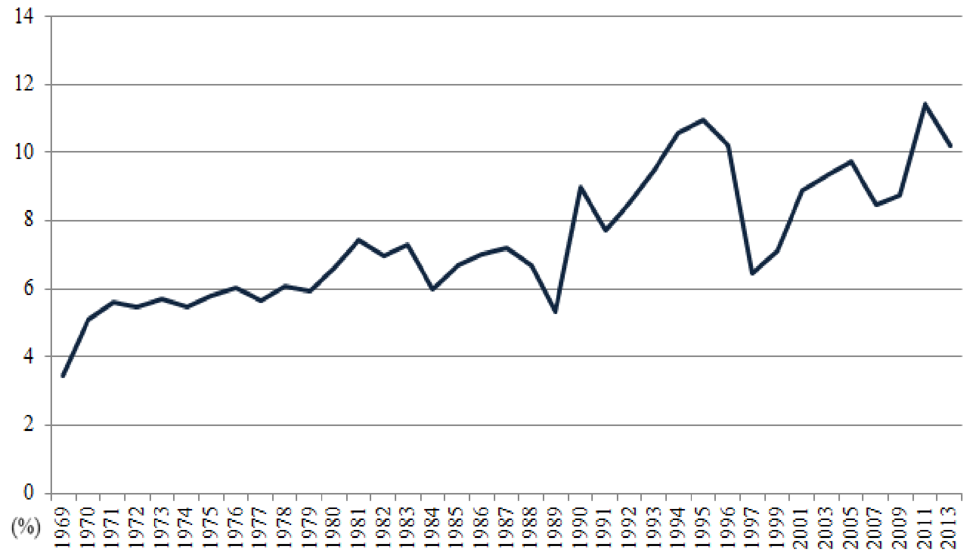
Figure1] Public Debt to GDP Ratio of the US

Note: The shaded areas indicate recession periods according to NBER's Business Cycle Dating Committee. The data on the ratio of gross government debt to GDP of the US are from Historical Public Debt Database of IMF.

Based on aggregate-level data of the US, macroeconomic volatility decreased during the middle of the 1980s (the Great Moderation). However, this does not necessarily mean that idiosyncratic risk on individuals' incomes decreased, which should be examined with disaggregate micro-level data. As a matter of fact, Haider (2001) and Gottschalk and Moffitt (1994) found that volatility of earnings of the male household head in the US rose between the 1970s and 1980s, based on a nationally representative micro-survey data of Panel Study of Income Dynamics

(PSID) of the US which are the longest-running. Similarly, with the same panel data, Dynan, Elmendorf, and Sichel (2012) found that the share of households experiencing a severe income drop increased by 1.7 times between the early 1970s and the early 2000s.

Figure2] Idiosyncratic Risk on Individuals' Incomes of the US



Note: Idiosyncratic risk on individuals' incomes is estimated by the portion of individual household head whose income (converted into the 2010 US dollars) fell by 50% or more compared to the previous survey time of the PSID. All the available waves of the PSID, from 1968 to 2013, are used.

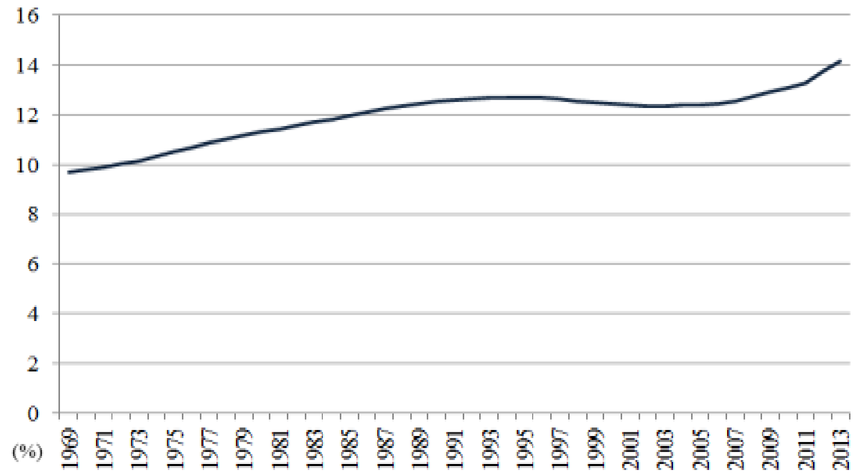
Besides these empirical studies which indicated that idiosyncratic risk on individuals' incomes rose in the US, ϕ is estimated[5] utilising all the available waves of the PSID data (from 1968 to 2013) with adding the latest wave.[6] To this end, at first, annual incomes of household headed by males and females aged

[5] Although the previous studies (Haider 2001; Gottschalk and Moffitt 1994; Dynan, Elmendorf, and Sichel 2012) examined the volatility of individuals' incomes with the micro-level survey panel data of the PSID, their estimates are not formulated exactly fitting to the definition of the parameter of this paper ϕ. They measured the volatility by imposing their own parametric assumptions on the evolution of incomes over time and restricted the sample based on gender, labor status, or age.

[6] To date, there are, in total, 38 waves of the survey panel data of the PSID available. Whereas the survey was conducted every year from 1968 to 1997, it was conducted every other year from 1999 to 2013. The PSID data have been produced and distributed by the Survey Research Center, Institute for Social Research, University of Michigan. The collection of the PSID data was partly supported by the National Institutes of Health under grant number R01 HD069609 and the National Science Foundation under award number 1157698.

between 23 and 65 are converted into the 2010 US dollars. Then, the portion of household head respondents whose income dropped by 50% or more compared to the previous wave is calculated.[7] As shown in **Figure 2**, from 1969 to 2013, the estimated probability of the negative idiosyncratic shock on individuals' incomes rose by about three times from 3.47% to 10.23%, while the US public debt behaviour changed from countercyclical restraint to acyclical increase, leading to the rise in the US public debt to 104.78% of the GDP from 38.13% (**Figure 1**). This is consistent with **Proposition 3**.

Figure3] Population Share of the Elderly in the US



Note: The data on the share of population ages 65 and above are from the OECD database.

On the other hand, as reported in **Figure 3**, from 1969 to 2013, while the idiosyncratic risk on individuals' incomes rose, the population share of the elderly (those age 65 and above) also increased from 9.71% to 14.13%. That is, both economic parameters ϕ and n_o increased concurrently, although ϕ increased by a larger margin than n_o did. Thus, for identifying the effect of ϕ accurately, it is necessary to distill out the effect of n_o on the change in the US public debt

[7] In spite of numerous changes in the definitions of survey income variables from 1968 to 2013, the variable of 'total money income' has remained consistently for all the waves. Thus, this variable is adopted for the estimation, as the previous studies. As the raw data are in nominal dollars, they are all converted into the 2010 US dollars, using the CPI calculator provided by the US Bureau of Labor Statistics.

dynamics. As noted above, the effect of an increase in n_o on change in public debt dynamics is ambiguous. To learn whether the effect is positive or negative needs to calculate $\frac{\partial \ddot{b}_1}{\partial n_o}$ with the parameters of the model calibrated to the US economy data.

Table 1] Calibrated Parameters (the US Economy)

Capital share of output	α	0.310	TFP of booms	z_H	3.600
Depreciation rate of capital	δ	0.050	TFP of recessions	z_L	1.000
Time preference (annualized)	β	0.952	Frisch elasticity of labour	η	0.667
Real interest rate to the government bonds	r	0.050	Relative preference for public goods	H	300.0
Idiosyncratic risk on individuals' (annual) incomes	ϕ	0.067	Population share of the elderly	n_o	0.110

For calibration, relevant data of the US are averaged over 1953 and 2015.[8] In particular, data on GDP and the capital share of total output are obtained from the US Bureau of Economic Analysis; data for the idiosyncratic risk on individuals' incomes from the PSID; and, data of the elderly population share and long-term interest rates to government bonds from the OECD database. One period in the model corresponds to 30 years in real time. To be consistent with the real long-term interest rate[9] of 5%, (annualized) time preference β is chosen as 0.952. With z_L being normalized to one, z_H is set as 3.6 to match the standard deviation of the real GDP of the US (4.3).[10] From Trabandt and Uhlig (2011), the revenue-maximising tax rate $\bar{\tau}$ is adopted as 60%, implying that η is 2/3. As the ratio of

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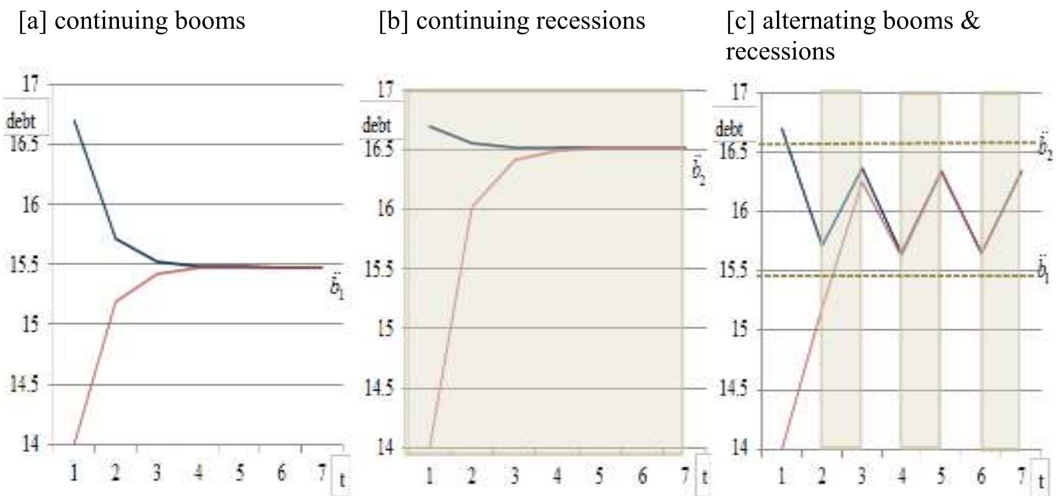
[8] The earliest year from which most of data are publicly available is 1953 while World War 2 ended in 1945.

[9] The real interest rate is obtained by subtracting the inflation rate (whose data are secured from the US Bureau of Labor Statistics) from the nominal long-term interest rate to government bonds (whose data are from the OECD database).

[10] The real GDP is obtained by converting nominal GDP in the 2010 US dollars. Moreover, with two possible states of the economy (H and L), the actual relative frequency of booms (83%) and recessions (17%), according to NBER's Business Cycle Dating Committee, is used for the weight, when calculating the standard deviation of simulated outputs over the business cycle and when calculating the correlation between simulated output and simulated public debt over the business cycle.

capital to total output is 3, the capital depreciation rate is 0.05. Moreover, parameter H is chosen to match the correlation between the public debt and the real GDP (0.6).

Figure4] Public Debt Dynamics and Macroeconomic Fluctuations



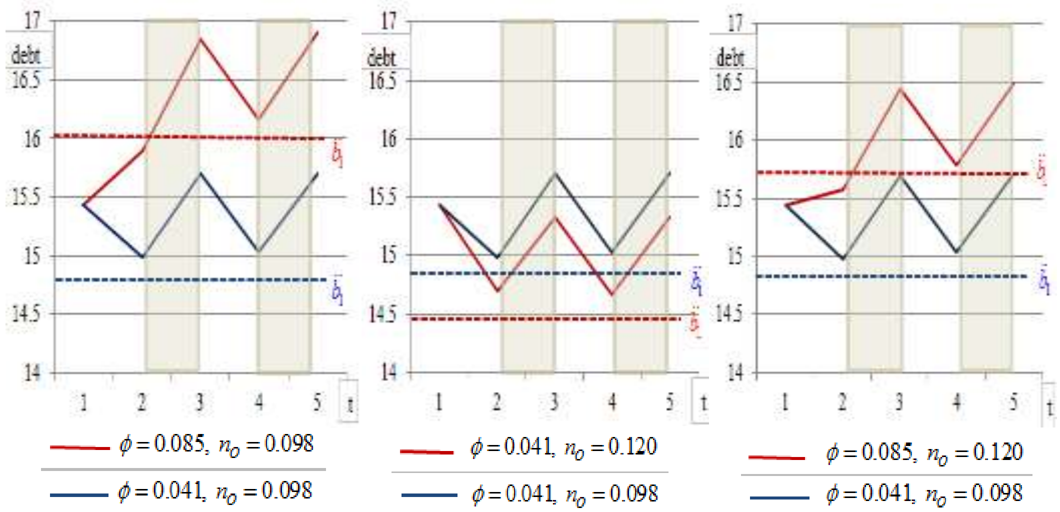
Note: The shaded areas indicate recession periods with the realised state of economy being L. On the other hand, the non-shaded areas indicate boom periods with the realised state of economy being H. Except for changes in the state of economy which engender macroeconomic fluctuations, all the other parameters remain the same as reported in **Table 1**. The dotted lines in the panel (c) locate the first threshold $\ddot{b}_1$ and the second threshold $\ddot{b}_2$ respectively.

To begin with the calibrated parameters (**Table 1**), how public debt behaves with and without a macroeconomic fluctuation is demonstrated in **Figure 4**. As shown in the panels (a) and (b), hypothetically, if there were no macroeconomic fluctuation, public debt would converge to the first threshold or the second threshold, regardless of whether the initial level of public debt is high or low. On the other hand, with macroeconomic fluctuations from changing TFPs every period, public debt behaves countercyclically between the first and second thresholds while it increases (decreases) acyclically below the first threshold (above the second threshold), as shown in the panel (c) of **Figure 4**. This is consistent with **Proposition 2**. Admittedly, the duration of a recession (a boom) in this simulation is longer than that of actual episodes. Nevertheless, this simulation reproduces the various observed dynamics of public debt such as countercyclical movement and acyclical increase.

Now, let us separately introduce changes in the parameters of ϕ and n_o, for identifying their own effects on the 1981 change in public debt dynamics of the US. To this end, a change in only one of the two parameters is simulated with the other fixed, although both changed simultaneously in reality. In particular, according to the average of the before-1981 data, before-change ϕ and n_o are 0.041 and 0.098, respectively. On the other hand, after-change ϕ and n_o are 0.085 and 0.12, respectively, from averaging the after-1981 data.

Figure 5] Change in Public Debt Dynamics, Idiosyncratic Risk, and Ageing Population

[a] increase only in idiosyncratic risk on individuals' incomes ϕ [b] increase only in population share of the elderly n_o [c] concurrent increase in both (ϕ and n_o)



Note: The shaded areas indicate recession periods with the realised state of economy being L. On the other hand, the non-shaded areas indicate boom periods with the realised state of economy being H. The dotted lines in each panel locate the first threshold $\ddot{b}_1$ before and after increases in the parameters of ϕ or n_o or both.

As displayed in the panel (a) of **Figure 5**, the rise in idiosyncratic risk on individuals' incomes ϕ from 0.041 to 0.085 with the elderly population share n_o fixed elevates the first threshold $\ddot{b}_1$ to change public debt dynamics from countercyclical (decreasing in the first boom and increasing in the first recession) to acyclically increasing (increasing in the first boom and recession alike). This simulation result is consistent with **Proposition 3**. On the contrary, as shown in the panel (b) of **Figure 5**, the increase in the elderly population share n_o from 0.098 to 0.12 with the idiosyncratic risk ϕ fixed *lowers* the first threshold $\ddot{b}_1$, which is consistent with $\frac{\partial \ddot{b}_1}{\partial n_o} = -16.5$ calculated with the calibrated parameters, and does *not* change the before-1981 (countercyclical) dynamics of public debt.

Finally, the two conflicting effects of ϕ and n_o are taken together to replicate the actual concurrent changes in both factors. As shown in the panel (c) of **Figure 5**, it turns out that the effect of the rise in idiosyncratic risk on individuals' incomes dominates so that the first threshold $\ddot{b}_1$ of the US increased after 1981, causing the US public debt dynamics to change from countercyclical restraint to acyclical increase. This indicates that the rise in the idiosyncratic risk drove the after-1981 rise in the US public debt. Moreover, a comparison of the panels (b) and (c) suggests that if there were no increase in the population share of the elderly, we could have observed a larger rise in the US public debt after 1981 than we actually have.

5. Concluding Remarks

This paper proposes a politico-economic model of public debt dynamics over the business cycle, where individual voters face uninsurable idiosyncratic risk on their after-tax labour incomes and an elected policy maker decides on fiscal policies to court young and old voters. Unlike the social-planner equilibrium public debt that behaves only countercyclically, the Markov perfect politico-economic equilibrium public debt can behave acyclically as well as countercyclically. The Markov perfect politico-economic equilibrium evolution of public debt is composed of three phases which differ by cyclical properties of public debt and are demarcated by two thresholds. In detail, (i) if the level of public debt inherited is lower than the first threshold, public debt increases acyclically; (ii) if the level is between the first and second thresholds, public debt behaves countercyclically; and, (iii) if the level is higher than the second threshold, public debt decreases acyclically. This three-phase evolution of public debt is led by intergenerational conflict between young and old voters regarding financial burden per public goods provision.

Importantly, this paper also finds that a rise in idiosyncratic risk on individual voters' disposable incomes can change public debt behaviour from countercyclical restraint to acyclical increase, *without* a change in the level of public debt inherited, causing public debt to rise rapidly even with alternating booms and recessions. Intuitively, a rise in idiosyncratic risk on voters' disposable incomes, which increases uncertainty on their private goods consumptions so that certainly provided public goods become more valuable to voters, causes acyclical increase in public debt to become politically acceptable. With calibrating the model to the US economy data, simulation analysis shows that the after-1981 rise in the US public debt is attributable to increased idiosyncratic risk on individuals' incomes, rather than increased share of the elderly population.

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Appendix

A. *Election of a Policy Maker by Voting and Choice of Fiscal Policy Proposal*

As you may notice, this is a variant of probabilistic voting model developed by Lindbeck and Weibull (1987). Let two office-seeking candidates be denoted by A and B, and their policy proposals by (d_A, g_A, τ_A) and (d_B, g_B, τ_B) respectively. The nation-wide relative popularity (personal appeal) of candidate B over A is δ which follows a uniform distribution and is not realised (known) to the two candidates when they announce their policy proposals. Specifically, δ ~Uni $[-\frac{1}{2\psi}, \frac{1}{2\psi}]$. Moreover, each voter will cast his vote for one of the two candidates who gives him higher level of indirect utility. Thus, young voters vote for candidate A if $u_Y(\tau_A, g_A, g'(d_A)) - u_Y(\tau_B, g_B, g'(d_B)) - \delta > 0$ and old voters do so if $u_O(g_A) - u_O(g_B) - \delta > 0$, where g' refers to public goods provision of the next period that is affected by public debt issue of the current period. Therefore, the winning probability of candidate A is as follows:

$$\Pr(\pi_A \geq \frac{1}{2}) = \Pr\{(1-n_o)[u_Y(\tau_A, g_A, g'(d_A)) - u_Y(\tau_B, g_B, g'(d_B))] + n_o[u_O(g_A) - u_O(g_B)]$$
$$> \delta\} = \frac{1}{2} + \psi\{(1-n_o)[u_Y(\tau_A, g_A, g'(d_A)) - u_Y(\tau_B, g_B, g'(d_B))] + n_o[u_O(g_A) - u_O(g_B)]\}$$

where π_A is the share of voting for candidate A. Since $\pi_A = 1 - \pi_B$, the winning probability of candidate B is defined symmetrically. Hence, in choosing fiscal policy proposal $\{d_t, g_t, \tau_t\}$ to maximise the winning probability, each of the two candidates maximises $(1-n_o)u_Y(\tau_t, g_t, g_{t+1}(d_t)) + n_o u_O(\tau_{t-1}, g_t)$ subject to (3) and $d_t \in [\underline{b}, \bar{b}]$. As each candidate solves the same problem of maximising the weighted sum of voters' indirect utilities, both candidates end up with announcing the same policy proposals. Then, one of the two candidates will be picked randomly with the same chance for each (i.e., at equilibrium, $\pi_A^* = \pi_B^* = \frac{1}{2}$).

B. *Proof for Proposition 1*

[step 1] As total output in booms is more abundant than in recessions, for any given level of inherited public debt, MCPF (marginal cost of public finance) in booms is lower than MCPF in recessions. This implies that $v'_H(d^s_\theta(b_t)) > v'_\theta(b_t) > v'_L(d^s_\theta(b_t))$ under $\beta(1+r)=1$ for $\forall b_t \in [\underline{b}, \bar{b}]$ and $\forall \theta \in \{H, L\}$, because MCPF, the left-hand side of (6), is negatively related to (7) and because $p_{\theta H} \in (0,1)$ and $p_{\theta L} \in (0,1)$. Moreover, notice that the value function $v_\theta(b)$ is concave for $\forall \theta \in \{H, L\}$ because the constraints in (4) are nonempty and compact (closed and bounded) and the object function in (4) is concave. In addition, as public debt issued in the previous period is the level of inherited public debt in the current period, $b_t = d^s_\theta(b_{t-1})$ for $\forall b_t \in [\underline{b}, \bar{b}]$ and $\forall \theta \in \{H, L\}$. Due to the concavity of the value function v_θ for $\forall \theta \in \{H, L\}$, $v'_H(d^s_H(b_t)) > v'_H(b_t)$ implies that $d^s_H(b_t) < b_t = d^s_\theta(b_{t-1})$; and, $v'_L(b_t) > v'_L(d^s_L(b_t))$ implies that

$d_L^s(b_t) > b_t = d_\theta^s(b_{t-1})$ for $\forall\theta \in \{H, L\}$. Thus, public debt behaves countercyclical.

[step 2] From (5), (6), and (7), $v_\theta'(b_t) = -\frac{H(1+r)}{g_\theta^s(b_t)}$ for $\forall b_t \in [\underline{b}, \bar{b}]$ and $\forall\theta \in \{H, L\}$. Moreover, by lagging one period of the inequality found in the above step 1, $v_H'(b_t) > v_\theta'(b_{t-1}) > v_L'(b_t)$. Thus, $-\frac{H(1+r)}{g_H^s(b_t)} > -\frac{H(1+r)}{g_\theta^s(b_{t-1})} > -\frac{H(1+r)}{g_L^s(b_t)}$ for $\forall b_t \in [\underline{b}, \bar{b}]$ and $\forall\theta \in \{H, L\}$. This implies that $g_H^s(b_t) > g_\theta^s(b_{t-1})$ and $g_L^s(b_t) < g_\theta^s(b_{t-1})$ under $\beta(1+r) = 1$ for $\forall b_t \in [\underline{b}, \bar{b}]$ and $\forall\theta \in \{H, L\}$ (procyclical public goods provision). ■

C. *Proof for Lemma 1*

The optimal policy functions of the Markov perfect politico-economic equilibrium $\{d_\theta(b_t),\ g_\theta(b_t), \tau_\theta(b_t)\}_{\theta=H,L}$ are defined by (10) and (11) which are from (9). Thus, it is enough to show that (12), (13), (14), and (15) meet the defining conditions (10), (11) and (3) for $\forall b_t \in [\underline{b}, \bar{b}]$ and $\forall\theta \in \{H, L\}$. In fact, replacing $d_\theta(b_t), g_\theta(b_t)$, and $\tau_\theta(b_t)$ with (12), (13), (14), and (15) satisfies all the three equalities of (3), (10) and (11) for $\forall b_t \in [\underline{b}, \bar{b}]$ and $\forall\theta \in \{H, L\}$. Moreover, from (10) and (11), $\frac{1}{g_\theta(b_t)} = -\beta E[\frac{1}{g_\theta(d_t)}\frac{\partial g_\theta(d_t)}{\partial d_t}] = -\beta E[\frac{1}{g_\theta(b_{t+1})}\frac{\partial g_\theta(b_{t+1})}{\partial b_{t+1}}]$ for $\forall b_t \in [\underline{b}, \bar{b}]$ and $\forall\theta \in \{H, L\}$. This implies that $\frac{\partial g_\theta(b_{t+1})}{\partial b_{t+1}} < 0$, which entails $\rho_\theta > 0$ of (15) as $-\frac{\partial g_\theta(b_{t+1})}{\partial b_{t+1}} = \frac{\rho_\theta}{\beta} > 0$ from (13). ■

D. *Proof for Lemma 2*

[step 1] To begin, note that $\frac{\partial \rho_\theta(b_t)}{\partial b_t} > 0$ for $\forall b_t \in [\underline{b}, \bar{b})$ and $\forall\theta \in \{H, L\}$. To see

this, $\frac{\partial \rho_\theta(b_t)}{\partial b_t} = \frac{\{(1-n_o)w_\theta l_t + (1+\eta)[\bar{b}-(1+r)b_t] - (\bar{b}-b_t)(1+\eta)(1+r)\}}{(\bar{b}-b_t)^2(1+\beta)\{H(1-0.5\phi)(1+\eta)+(1-n_o)\}}\beta H(1-$ $0.5\phi)$ from (15). The denominator of this and $\beta H(1-0.5\phi)$ are positive; so, the sign of $\frac{\partial \rho_\theta(b_t)}{\partial b_t}$ depends on the sign of $(1-n_o)w_\theta l_t + (1+\eta)[\bar{b}-(1+r)b_t] - (\bar{b}-b_t)$ $(1+\eta)(1+r) = (1-n_o)w_\theta l_t - (1+\eta)r\bar{b} = (1-n_o)\{w_\theta l_t - (1+\eta)\bar{\tau}w_L l(\bar{\tau})\}$ as $\bar{b} = \frac{1}{r}(1-$ $n_o)\bar{\tau}w_L l(\bar{\tau})$. Then, this is further simplified into $(1-n_o)\{w_\theta l_t - (1+\eta)\bar{\tau}w_L l(\bar{\tau})\}$ $= (1-n_o)\{w_\theta l_t - w_L l(\bar{\tau})\}$, since $\bar{\tau} = \arg\max_\tau \ \tau w_L(1-n_o)l(\tau)$ implies that $1-(1+$ $\eta)\bar{\tau} = 0$. Moreover, from (14) in **Lemma 1**, $\tau_\theta(b_t) < \frac{1}{1+\eta} = \bar{\tau}$ for $\forall b_t \in [\underline{b}, \bar{b})$, which implies that $l_t(\tau_\theta(b_t)) - l(\bar{\tau}) > 0$ for $\forall b_t \in [\underline{b}, \bar{b})$, since individual labor supply is negatively affected by labor income tax. Consequently, as $w_H > w_L$, $(1-n_o)\{w_\theta l_t - w_L l(\bar{\tau})\} > 0$ for $\forall b_t \in [\underline{b}, \bar{b})$ and $\forall \theta \in \{H, L\}$. Therefore, $\frac{\partial \rho_\theta(b_t)}{\partial b_t}$ > 0 for $\forall b_t \in [\underline{b}, \bar{b})$ and $\forall \theta \in \{H, L\}$. In addition, $\frac{\partial \rho_\theta(\bar{b})}{\partial \bar{b}} = 0$ for $\forall \theta \in \{H, L\}$ since $\tau_\theta(b_t) = \bar{\tau}$ at $b_t = \bar{b}$ for $\forall \theta \in \{H, L\}$.

[step 2] Both $\ddot{b}_H$ and $\ddot{b}_L$ lie between $\underline{b}$ and $\bar{b}$. To show this, firstly, as b_t approaches $\bar{b}$, $\rho_\theta(b_t)$ approaches $+\infty$ for any given $\theta \in \{H, L\}$, due to (15) in **Lemma 1**. Secondly, by the definition of the lower limit of public debt $\underline{b}$, $\rho_\theta(b_t)$ at $b_t = \underline{b}$ is not equal to one but greater than one, since the Markov perfect politico-economic equilibrium public goods are provided more than the social-planner equilibrium public goods, as proven in the text by comparing (5) and (10). By the above step 1, these two imply that $\ddot{b}_H$ and $\ddot{b}_L$ lie between $\underline{b}$ and $\bar{b}$.

[step 3] Now, to show that $\ddot{b}_H < \ddot{b}_L$ by way of contradiction, suppose $\ddot{b}_H \geq \ddot{b}_L$. Moreover, $w_H l_H > w_L l_L$ since booms have more aggregate labor supply to yield greater total output than recessions do. This implies that, from the finding of the above step 1, $\rho_H(\ddot{b}_H) = \frac{\beta H(1-0.5\phi)\{(1-n_o)w_H l_H + (1+\eta)[\bar{b}-(1+r)\ddot{b}_H]\}}{(\bar{b}-\ddot{b}_H)(1+\beta)\{H(1-0.5\phi)(1+\eta)+(1-n_o)\}}$ is *strictly* greater than $\rho_L(\ddot{b}_L) = \frac{\beta H(1-0.5\phi)\{(1-n_o)w_L l_L + (1+\eta)[\bar{b}-(1+r)\ddot{b}_L]\}}{(\bar{b}-\ddot{b}_L)(1+\beta)\{H(1-0.5\phi)(1+\eta)+(1-n_o)\}}$, which is a contradiction to the equality of (16) that $\rho_H(\ddot{b}_H) = \rho_L(\ddot{b}_L) = 1$. This proves that $\ddot{b}_H < \ddot{b}_L$. With the finding of the above step 2, this means that $\underline{b} < \ddot{b}_H < \ddot{b}_L < \bar{b}$. ■

E. *Proof for Proposition 2*

To begin, due to (12), for any given $\theta \in \{H, L\}$, $\bar{b} - d_\theta(b_t) > \bar{b} - b_t$ whenever $\rho_\theta(b_t) > 1$; $\bar{b} - d_\theta(b_t) < \bar{b} - b_t$ whenever $\rho_\theta(b_t) < 1$; and, $\bar{b} - d_\theta(b_t) = \bar{b} - b_t$ if $\rho_\theta(b_t) = 1$. Moreover, for any given $\theta \in \{H, L\}$, $\frac{\partial \rho_\theta(b_t)}{\partial b_t} > 0$ for $\forall b_t \in [\underline{b}, \bar{b})$, and, $\frac{\partial \rho_\theta(\bar{b})}{\partial \bar{b}} = 0$ from the step 1 in the proof for **Lemma 2**. This implies that, in light of (16), (i) $\rho_H(b_t) < 1$ for $\forall b_t \in [\underline{b}, \ddot{b}_1)$, (ii) $\rho_H(\ddot{b}_1) = 1$, and (iii) $\rho_H(b_t) > 1$ for $\forall b_t \in (\ddot{b}_1, \bar{b}]$. In addition, in each period, public debt issued in the previous period is the level of inherited public debt in the current period; thus, $b_t = d_\theta(b_{t-1})$ for $\forall b_t \in [\underline{b}, \bar{b}]$ and $\forall \theta \in \{H, L\}$. Therefore, (i) $d_H(b_t) > b_t = d_\theta(b_{t-1})$ for $\forall b_t \in$ $[\underline{b}, \ddot{b}_1)$ and $\forall \theta \in \{H, L\}$, (ii) $d_H(\ddot{b}_1) = \ddot{b}_1$, and (iii) $d_H(b_t) < b_t$ for $\forall b_t \in (\ddot{b}_1, \bar{b}]$. By the same logic, (i) $\rho_L(b_t) < 1$ for $\forall b_t \in [\underline{b}, \ddot{b}_2)$, (ii) $\rho_L(\ddot{b}_2) = 1$, and (iii) $\rho_L(b_t)$

>1 for $\forall b_t \in (\ddot{b}_2, \bar{b}]$. This implies that (i) $d_L(b_t) > b_t = d_\theta(b_{t-1})$ for $\forall b_t \in [\underline{b}, \ddot{b}_2)$ and $\forall \theta \in \{H, L\}$, (ii) $d_L(\ddot{b}_2) = \ddot{b}_2$, and (iii) $d_L(b_t) < b_t$ for $\forall b_t \in (\ddot{b}_2, \bar{b}]$. Combining these evolutions of public debt under the two states (H and L) completes the proof, as $\underline{b} < \ddot{b}_1 < \ddot{b}_2 < \bar{b}$ (**Lemma 2**). ■

F. *Proof for Corollary 1*

At first, notice that $d_t = d_\theta(b_t)$ and $d_{t-1} = d_\theta(b_{t-1})$ for $\forall b_t \in [\underline{b}, \bar{b}]$ and $\forall \theta \in \{H, L\}$. Moreover, putting (12), (13), and (15) of **Lemma 1** together, $\frac{\partial g_\theta(d_t)}{\partial d_t} < 0$ for $\forall d_t \in [\underline{b}, \bar{b}]$ and $\forall \theta \in \{H, L\}$. Likewise, putting (12), (14), and (15) of **Lemma 1** together, $\frac{\partial \tau_\theta(d_t)}{\partial d_t} > 0$ for $\forall d_t \in [\underline{b}, \bar{b}]$ and $\forall \theta \in \{H, L\}$. Then, **Proposition 2** implies what follows. For $\forall \theta \in \{H, L\}$, (i) if $\underline{b} \le b_t < \ddot{b}_1$, $g_H(b_t) < g_\theta(b_{t-1})$ and $g_L(b_t) < g_\theta(b_{t-1})$, while $\tau_H(b_t) > \tau_\theta(b_{t-1})$ and $\tau_L(b_t) > \tau_\theta(b_{t-1})$; (ii) if $b_t = \ddot{b}_1 = d_\theta(b_{t-1})$, $g_H(b_t) = g_\theta(b_{t-1})$ and $g_L(b_t) < g_\theta(b_{t-1})$, while $\tau_H(b_t) = \tau_\theta(b_{t-1})$ and $\tau_L(b_t) > \tau_\theta(b_{t-1})$; (iii) if $\ddot{b}_1 < b_t < \ddot{b}_2$, $g_H(b_t) > g_\theta(b_{t-1})$ and $g_L(b_t) < g_\theta(b_{t-1})$, while $\tau_H(b_t) < \tau_\theta(b_{t-1})$ and $\tau_L(b_t) > \tau_\theta(b_{t-1})$; (iv) if $b_t = \ddot{b}_2 = d_\theta(b_{t-1})$, $g_H(b_t) > g_\theta(b_{t-1})$ and $g_L(b_t) = g_\theta(b_{t-1})$, while $\tau_H(b_t) = \tau_\theta(b_{t-1})$ and $\tau_L(b_t) = \tau_\theta(b_{t-1})$; and (v) if $\ddot{b}_2 < b_t \le \bar{b}$, $g_H(b_t) > g_\theta(b_{t-1})$ and $g_L(b_t) > g_\theta(b_{t-1})$, while $\tau_H(b_t) < \tau_\theta(b_{t-1})$ and $\tau_L(b_t) < \tau_\theta(b_{t-1})$. ■

G. *Proof for Proposition 3*

[step 1] At the outset, for the notational simplicity, let $\ddot{b}_1^{before}$ denote the first threshold before a rise in ϕ and $\ddot{b}_1^{after}$ denote the first threshold after the rise in ϕ. Moreover, according to **Proposition 2**, that optimal public debt behaviour is

countercyclical before a rise in ϕ means that the level of public debt inherited b_t before the rise is greater than $\ddot{b}_1^{before}$ (i.e., $\ddot{b}_1^{before} < b_t$).

[step 2] Next, a rise in idiosyncratic risk on individual voters' disposable incomes, which is described as a rise in ϕ, raises the value of the first threshold so that $\ddot{b}_1^{before} < \ddot{b}_1^{after}$. To prove, it is sufficient to show that $\frac{\partial \ddot{b}_1}{\partial \phi} > 0$. Applying Implicit Function Theorem to (16) under $\theta = H$, $\frac{\partial \ddot{b}_1}{\partial \phi} = -\frac{\partial \rho_H}{\partial \phi}[\frac{\partial \rho_H}{\partial \ddot{b}_1}]^{-1}$. Firstly, $\frac{\partial \rho_H}{\partial \phi} =$ $-0.5(1-n_o)[\frac{\beta H\{(1-n_o)w_H l_H + (1+\eta)[\bar{b} - (1+r)\ddot{b}_1]\}}{(\bar{b} - \ddot{b}_1)(1+\beta)\{H(1-0.5\phi)(1+\eta) + (1-n_o)\}^2}] < 0$ because $n_o \in (0,1)$ and (15) of **Lemma 1** implies that $\frac{\beta H\{(1-n_o)w_H l_H + (1+\eta)[\bar{b} - (1+r)\ddot{b}_1]\}}{(\bar{b} - \ddot{b}_1)(1+\beta)\{H(1-0.5\phi)(1+\eta) + (1-n_o)\}}$ > 0. Secondly, $\frac{\partial \rho_H}{\partial \ddot{b}_1} > 0$ as $\frac{\partial \rho_\theta(b_t)}{\partial b_t} > 0$ for $\forall b_t \in [\underline{b}, \bar{b})$ and $\forall \theta \in \{H, L\}$ (the step 1 in the proof for **Lemma 2**) and $\underline{b} < \ddot{b}_1 < \bar{b}$ (**Lemma 2**). Taking these together, it is proven that $\frac{\partial \ddot{b}_1}{\partial \phi} > 0$.

[step 3] Due to the finding of the above step 2 ($\frac{\partial \ddot{b}_1}{\partial \phi} > 0$), a rise in ϕ elevates the value of the first threshold, without a change in the level of public debt inherited b_t, so that b_t can lie between $\ddot{b}_1^{before}$ and $\ddot{b}_1^{after}$ (i.e., $\ddot{b}_1^{before} < b_t < \ddot{b}_1^{after}$). Consider this case ($\ddot{b}_1^{before} < b_t < \ddot{b}_1^{after}$) to show whether a rise in ϕ (i.e., a rise in idiosyncratic risk on individuals' disposable incomes) can change optimal behaviour of public debt from countercyclical to acyclically increasing. As mentioned in the above step 1, before a rise in ϕ, optimal public debt behaves countercyclically. However, after the rise in ϕ without a change in the level of

public debt b_t, optimal behaviour of public debt is no longer countercyclical but changed to acyclical increasing because of $b_t < \ddot{b}_1^{after}$ and **Proposition 2**. ■

H. *Proof for Ambiguous Effect of an Ageing Population*

Following the same logic of the proof for **Proposition 3**, the sign of $\frac{\partial \ddot{b}_1}{\partial n_o} = -\frac{\partial \rho_H}{\partial n_o}[\frac{\partial \rho_H}{\partial \ddot{b}_1}]^{-1}$ is examined. Applying Implicit Function Theorem to (16) under $\theta = H$, $\frac{\partial \rho_H}{\partial n_o} = [\frac{\beta H(1-0.5\phi)(1+\eta)\{(-w_H l_H)H(1-0.5\phi)+[\bar{b}-(1+r)\ddot{b}_1]\}}{(\bar{b}-\ddot{b}_1)(1+\beta)\{H(1-0.5\phi)(1+\eta)+(1-n_o)\}^2}]$. While it is clear that $\frac{\partial \rho_H}{\partial \ddot{b}_1} > 0$ (as shown in the step 3 of the proof for **Proposition 3**), the sign of $\frac{\partial \ddot{b}_1}{\partial n_o}$ is ambiguous, since the sign of $\frac{\partial \rho_H}{\partial n_o}$ cannot be clearly determined with the current level of generality. ■