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Keywords: Credit Market Deregulation, Credit Market Friction, Macroeconomic Impact, Heterogenous Firm, New Keynesian, Firm Dynamics
JEL Classification: E3, E5, G2.

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1 Introduction

It is widely accepted that a tightening or loosening of credit conditions can have significant effects on the real economy through a number of different channels; however, the relative macroeconomic importance of these different channels is less clear. A loosening of credit conditions, for example, may reduce the cost of borrowing for both households and firms. This can directly increase consumption to some degree, as households borrow more and increase investment, as firms borrow more. The resultant increase in demand for goods might increase employment, consumption, and investment further. It is not immediately clear, however, whether the effects of the looser credit conditions on household consumption or firm investment are more quantitatively important in driving the resultant expansion. For instance, one might assume that loosening credit conditions for consumers is most important, and that any increase in investment is primarily a response to greater consumer demand. Alternatively, one might instead assume that a loosening for firms' investing is more important, and that any observed increase in consumption is primarily driven by the greater realized and expected future earnings of households as a consequence of higher investment and capital accumulation. Assessing the relative magnitude of these channels is key to understanding how changing credit conditions affect the wider economy in the short and long run, and how and whether the policies of governments and central banks should respond to changing credit conditions.

To address these questions, our paper aims to infer the quantitative role played by different channels through which loosening credit conditions affect the real economy. To do so, we combine a Heterogeneous Agent New Keynesian (HANK) model with empirical evidence from the quasi-natural experiment of US banking sector deregulation, which a large literature has argued induced a credit expansion in US states in the 1980s.[1] The HANK model consists of a small open economy with heterogeneous households and firms, as well as both a tradable and non-tradable goods sector. This economy, which is assumed to represent a US state, is hit by an expansionary credit shock, modeled as a shock to external borrowing costs. Our model is sufficiently rich enough to incorporate a number of different forces through which this credit shock may impact the economy, including effects on consumer demand, labor supply, investment, and firm entry. The magnitudes of these various channels depend, unsurprisingly, on the parametrization of the model. To empirically discipline these parameters, and therefore

[1]This literature begins with the seminal work of Jayaratne and Strahan (1996). For more recent references, see Rice and Strahan (2010), Kroszner and Strahan (2014), and citations therein.

infer the size of these channels, we rely on the credit expansion related to US bank deregulation in the 1980s. This deregulation happened in different states at different times, in a way that the literature has argued is quasi-random conditional on controls. Consequently, we can estimate the effects of this deregulation on state-level financial and macroeconomic variables by using a local projection approach. We find that the deregulation was indeed followed by a decrease in borrowing costs for households and firms, as well as followed by an increase in aggregate output and employment that persisted for more than 10 years. We set key parameters of our HANK model to match the estimated impulse responses of financial and macroeconomic variables to this empirical credit shock; our resultant parameterized model can successfully match these estimated impulse responses quantitatively.

Using the parameterized model, we quantitatively decompose the effect of the expansionary credit shock on the economy into two parts: the effect of lower borrowing costs for households (and therefore on consumer demand), and the effect of lower borrowing costs for firms (which directly impacts investment and therefore the supply of goods). We refer to these as the "demand-side" and "supply-side" effects of the credit expansion. For parameter values that best fit our estimated impulse responses, we find that the demand-side channel accounts for around 75% of the cumulative effect of the credit expansion on most variables during the first 10 years after deregulation, while the supply-side channel accounts for around 25%. The data favors parameterizations that put a stronger weight on the demand-side channel because the increase in output and employment occurs quite rapidly after deregulation. This fits the profile of the demand-side channel in the model, whereas a strong supply-side channel would instead imply a more gradual increase in output driven by the gradual accumulation of capital. As such, we find that the demand-side channel plays an even bigger role immediately after deregulation in the model, and that the relative importance of the supply-side channel grows over time. The supply-side channel does, however, play a dominant role in movements in investment and firm exit after deregulation, and accounts for over 85% of the cumulative rise in investment during the first 10 years after the deregulation.

The importance of the demand channel in the parameterized model might suggest that "Keynesian" forces related to wage and price rigidity play a key role. However, we find this not to be the case. While the model features nominal rigidity in both wages and prices, we find that these only impact the impulse responses of macroeconomic variables in the first few years after deregulation, while the increase in output overall persists for more than 10 years. Wage and price rigidity can play only a limited role here since, as is typical of the New

Keynesian literature, we consider calibrations of nominal rigidity that imply wages and prices are reset more often than every 2 years on average, and this level of rigidity is too small to significantly impact the impulse responses of variables much beyond 2 years. Instead, the key mechanism for the demand channel operates through the complementarity between demand for tradable and non-tradable goods. When borrowing costs fall after the expansionary credit shock, households front-load their consumption and borrow more from abroad. However, their convex preferences imply that they would then like to consume both more tradables and more non-tradables in quantities. Tradable goods prices are mostly not affected by home state, and the demand for tradables increases; as such, demand for non-tradables grows. Since non-tradables have to be produced in the home state, the demand for local labor as a result. In turn, local wages rise in response, which raises labor supply. As such, output and employment rise locally. Ultimately, our parameterized model suggests that this is the principal mechanism through which bank deregulation raised output and employment.

We study the sensitivity of our results to a number of parameter choices. For instance, we find that the relative importance of the supply-side channel is larger when capital adjustment costs are smaller, since lower adjustment costs imply a more rapid growth in the capital stock when firms' borrowing costs fall, and therefore a more rapid rise in output through the supply-side channel. We also experiment with different borrowing behaviors for households and firms; in general, we show that if firms can benefit relatively more from reduced borrowing costs (e.g., larger initial debt for firms or households, different dividend distribution rules for firms), then its importance can increase. We also find that a lower elasticity of intertemporal substitution reduces the effect of lower borrowing costs on consumer demand, and thus reduces the importance of the demand-side channel as well.

To our knowledge, our paper is the first to integrate a quantitative HANK model with causal evidence from a natural experiment of bank deregulations. Overall, our results suggest that the effects of credit shocks on consumer demand may be the most important, but not the only way, in which these shocks affect economic activity. At the same time, nominal rigidities may be less central to these demand-side effects than often assumed. At the same time, although we focus on the effects of changes in interest rates that come from bank deregulation, our quantitative model suggests that other shocks affecting credit supply or interest rates could impact the economy similarly, with effects on consumer demand being especially important. As such, this may be informative about the economic effects of financial crises and other negative shocks to the banking system, as well as about the effects of monetary policy, and how small open economies respond to global interest rate changes, among others.

Related Literature

Our paper builds on and contributes to several strands of literature. Closest to our paper is other work that seeks to evaluate the relative importance of different channels through which financial market changes impact real economic activity. Mian, Sufi, and Verner (2020) leverage the same natural experiment in bank deregulation that we study, and show that a number of empirical facts are more consistent with deregulation mainly impacting the economy via demand-side rather than supply-side channels. They consider a very different empirical specification to ours, but find qualitatively similar effects of the deregulation on household debt, GDP and employment.[2] Apart from the empirical specification, and some of the empirical variables considered, the principal difference between our paper and Mian, Sufi, and Verner (2020) is that they focus on qualitative comparisons between the data and predicted effects of demand-side channels and supply-side channels in a stylized static model, whereas we compare quantitatively our estimated impulse responses with the predictions of a dynamic structural model, so we can produce a detailed quantitative assessment of the different role played by various channels.

Similar to us, Kehoe et al. (2020) also quantitatively investigate the relative importance of household-side and firm-side channels for the transmission of a financial shock. Unlike us, they focus on the tightening of financial markets associated with the US Great Recession. Apart from the different financial shock that is considered, the main difference between our study and Kehoe et al. (2020) is that we use a small open economy HANK model, which is closer to models more commonly used in the literature, whereas Kehoe et al. (2020) consider these questions by using a search and matching model with human capital accumulation and no nominal rigidities, in which hiring is a risky investment activity that is affected by financial shocks. In spite of the differences on modelling and shocks, they likewise find that the direct effect of the credit shock on households is more quantitatively important than its direct effect on firms. In this sense, our findings complement Kehoe et al. (2020) and suggest that a relatively greater role for financial shocks on households rather than firms is a robust prediction of a variety of models.

More broadly, our paper relates to the large literature on the effects of financial shocks.

[2]We also provide estimates for dynamic responses, which are useful for disciplining macroeconomic, dynamic responses from a model.

With respect to this literature, our theoretical approach is closest to papers that model credit shocks in an open economy context as a rise in the cost of external borrowing, such as Neumeyer and Perri (2005) and Cugat (2022). In the US context, Kehoe et al. (2020) review the extensive literature studying transmission channels from the 2007-08 financial crisis to the real economy. Papers in this literature typically focus on a single channel through which a financial shock affects real activity, rather than comparing the quantitative importance of multiple channels as we do in our paper. As noted before, our paper is also related to the empirical literature (e.g., Jayaratne and Strahan (1996), Rice and Strahan (2010) Kroszner and Strahan (2014) among others) that investigates the effects of banking sector deregulation during the 1980s on corporate finance and economic variables.

Our modeling framework builds upon the extensive literature on heterogeneous agent New Keynesian (HANK) models, particularly those that consider open economies. Our model features heterogeneity on both the household and firm side in a small open economy framework. This is in contrast with most of the HANK literature, which incorporates rich household (but not firm) heterogeneity in a closed economy. On the household side, our model uses the TANK framework, which distinguishes between liquidity constrained and unconstrained households in a way that is simpler than most HANK models. The TANK framework for DSGE models originates with the work of Galí, López-Salido, and Vallés (2007) and Bilbiie (2008), and has subsequently been studied in an open economy context by Boerma (2014) and Buffie and Zanna (2018).[3] Rich firm-side heterogeneity of the kind we consider has only recently been introduced into New Keynesian DSGE models (e.g., Reiter, Sveen, and Weinke (2013), Ottonello and Winberry (2020), and Koby and Wolf (2020)). This literature has almost exclusively studied closed economies, although Caldara et al. (2020) incorporate firm-side heterogeneity in an open economy New Keynesian model. To the best of our knowledge, ours is the only New Keynesian model in the literature incorporating both household and rich firm-side heterogeneity in an open economy context. Our modeling of firm-side heterogeneity closely follows Ottonello and Winberry (2020), and our modeling of the open economy features (including the tradable and non-tradable sectors) mainly follows Gali and Monacelli (2005) and Nakamura and Steinsson (2014).

In terms of methodology, our paper is in line with the recent emerging literature that uses carefully identified estimates of causal effects to discipline structural macroeconomic

[3] Other recent work making use of open economy TANK models includes Iyer (2016), Motyovszki (2021), Cugat (2022), and Ida (2023).

models (e.g., evidence across different regions or across different firms). Examples from this recent literature include Nakamura and Steinsson (2014), Beraja, Hurst, and Ospina (2019), Ottonello and Winberry (2020) and the work surveyed by Nakamura and Steinsson (2018).

2 Model

There are four types of agents in the domestic economy: households, intermediate goods firms, final goods firms, and capital-producing firms. Each intermediate goods firm operates in one of two sectors: a domestic tradable goods sector (indexed by H) or a non-tradable goods sector (indexed by N). The final goods firms combine goods from the firms in the tradable and non-tradable sectors with foreign goods (indexed by F) to produce a non-differentiated final good. Households and capital-producing firms use final goods for consumption and investment purposes.

Households. There are two types of households: 'spenders' and 'savers' indexed by L (for 'limited asset participation') and S (for 'saver'), respectively. We use χ to measure spenders and $(1-\chi)$ to measure savers. As savers and spenders are all identical within their respective groups, there is a representative household of each. In each period t, households choose final goods for consumption. Savers can also choose their next period financial assets B_t.

We assume that labor markets are affected by nominal wage rigidity, which we model using the approach of Galí (2015, Chapter 6). Each household is made up of a continuum of workers who each produce a differentiated labor supply indexed by $j \in [0, 1]$. Each worker j sets a nominal wage at which he or she will work, and firms choose how much of the worker's labor to demand at this wage. Worker nominal wage setting is subject to a Calvo-type rigidity; that is, at the start of period t, the worker can set a new nominal wage with probability $1 - \theta_w$, and with probability θ_w must continue to work at the same nominal wage as in the previous period.

Households seek to maximize:

$$E_0 \sum_{t \geq 0} \beta^t U(C_t, \{N_{j,t}\}), \tag{1}$$

subject to their budget constraints. The budget constraint of a spender is:

$$P_t C_t^L = \int_{j=0}^{1} w_{j,t} N_{j,t} \mathrm{d}j, \tag{2}$$

and the budget constraint of a saver is:

$$\frac{B_t}{1-\chi} + P_t C_t^S = \frac{\Pi_t}{1-\chi} + \int_{j=0}^{1} w_{j,t} N_{j,t} \mathrm{d}j + \frac{B_{t-1}}{1-\chi}\left(1 + r_{t-1}^H\right), \tag{3}$$

where P_t denotes the price of the final good. Π_t is the net dividend paid by all firms in the domestic economy, which accrues to savers because they are assumed to own all the equity in the domestic economy.[4] $\frac{B_{t-1}}{1-\chi}$ is each saver's net financial assets from the previous period, and B_t is each saver's financial position in the next period. Thus B_t is the total net financial assets of the domestic economy in the present period.

We assume that period utility for all households takes the GHH form:

$$U(C_t, N_t) = \Gamma \frac{\left[C_t - \phi \int_{j=0}^{1} \frac{N_{j,t}^{1+\upsilon}}{1+\upsilon} \mathrm{d}j\right]^{1-\sigma}}{1-\sigma}.$$

If a worker j is free to set a new wage in period t, then he or she is assumed to set this wage to maximize the present discounted value of the worker's contribution to the household's utility. Therefore, the new wage $w_{j,t}^{\star}$ is set to maximize:

$$\mathbb{E}_t \sum_{k=0}^{\infty} (\beta\theta_w)^k \left(\frac{w_{j,t}^{\star} N_{j,t+k}}{P_{t+k}} - \phi \frac{N_{j,t+k}^{1+\upsilon}}{1+\upsilon}\right), \tag{4}$$

where $N_{j,t+k}$ is the demand for j's labor in period $t+k$, given the wage rate $w_{j,t}^{\star}$.[5]

We assume that the interest rate faced by savers is:

$$r_t^H = r + \psi_t^H, \tag{5}$$

where r is the risk-free interest rate, and we assume that there is an interest spread ψ_t^H, which we vary in the experiments when we study credit market liberalization.[6] In mapping the model to the data, we think of r as the interest rate that is common to all US states; thus,

[4] We do not model equity markets trading explicitly, but if savers were allowed to trade a share in the equity of all firms in the domestic economy, as in standard models with a representative households, it would have no effect on allocations.

[5] We will not introduce notations to specify if worker j is a member of a spender or saver household because, as we will show later, a worker j will choose the same $w_{j,t}$ and $N_{j,t}$ regardless of which of the two household types the worker belongs to.

[6] Alternatively, a more complicated specification could allow for the interest rate faced by households decreasing in B_t (or increasing in the debt level), similar to the modelling approach in Schmitt-Grohé and Uribe (2003) and Aguiar and Gopinath (2007).

the model is that of a small open economy (i.e., a US state).

Capital Goods Producer.

Capital in the local economy must be produced locally by perfectly competitive capital goods producers who own the capital stock and lease it to wholesale firms. The representative capital goods producer has the following production function:

$$K_{t+1} = (1-\delta_K)K_t + I_t - \kappa\left(\frac{K_{t+1}}{K_t} - 1\right)^2 K_t, \tag{6}$$

where these firms buy final goods at price P_t for current investment (and any additional costs).

Capital is leased at (nominal) rate $P_t(r_t^K + \delta_K)$, which is endogenously determined. Per period nominal profit then is:

$$\Pi_t^K = P_t(r_t^K + \delta_K)K_t - P_t I_t. \tag{7}$$

All firm owners act to maximize the present value of their nominal profits, with a discount factor given by:

$$M_t = \frac{P_{t+1}}{P_t(1+r_t^F)}, \tag{8}$$

In our baseline case, we assume that $r_t^F = r_t^H$. It is convenient doing so to define the stochastic discount factor in terms of units of the final good, so that (equivalently) firm owners maximize the present value of their profits evaluated in units of final goods.[7] Later on in our counterfactual experiments, we will deviate from $r_t^F = r_t^H$.

Since the capital good's producer has constant returns to scale, its value at the start of the period is proportional to its capital stock. Denote the representative capital producer's value by $\mathcal{Q}_t K_t$ in units of the final good for which $\mathcal{Q}_t \equiv \mathcal{Q}(\mathbf{X}_t)$. As a result, the Bellman equation for the representative capital goods producer is:

$$\mathcal{Q}_t K_t = \max_{I_t}(r_t^K + \delta_K)(K_t) - I_t + M_t E_t\left[\mathcal{Q}_{t+1}\right] K_{t+1}. \tag{9}$$

Final Goods Firms. There is a representative, perfectly competitive, final goods firm with two constant returns to scale technologies. The first technology is that the final goods

[7] Given the household's Euler equation, and the condition $r_t^F = r_t^H$, this assumption is equivalent to setting $\beta\frac{u'(C_{t+1})}{u'(C_t)}$ as the firm's discount factor for real units in the next period.

firm aggregates output purchased from retailers in each of the two domestic sectors N and H into a composite good in that sector and then integrates the output of retailers in the foreign economy into a composite imported good. Let I_S denote the set of retailers in sector $S \in \{N; H; F\}$, where F denotes the foreign economy. Each retailer $i \in I_S$ produces a separate variety i, sold at price p_i.

Specifically, the final goods firm uses the first technology to produce output Y_S^d of the composite good of sector $S \in \{N; H; F\}$. This technology uses the following production function:[8]

$$Y_S^d = \left[\int_{i \in I_S} \left(y_i^d \right)^{\frac{\eta-1}{\eta}} di \right]^{\frac{\eta}{\eta-1}}, \tag{10}$$

where $\{y_i\}_{i \in I_S}$ is the quantity used of the variety produced by retailer $i \in I_S$. We assume the elasticity across different varieties, $\eta > 1$.[9]

The second technology is that the final goods firm aggregates some amount of domestically produced tradable goods Y_H^d with imported tradable goods Y_F^d and non-tradable goods Y_N^d to produce a final good, based on the following production function:

$$Y_t^d = \left(\gamma^\gamma (1-\gamma)^{1-\gamma}\right)^{-1} \left[\left(\omega_H^{\frac{1}{\zeta}} \left(Y_{H,t}^d\right)^{\frac{\zeta-1}{\zeta}} + (1-\omega_H)^{\frac{1}{\zeta}} \left(Y_{F,t}^d\right)^{\frac{\zeta-1}{\zeta}} \right)^{\frac{\zeta}{\zeta-1}} \right]^\gamma \left(Y_{N,t}^d\right)^{1-\gamma}.$$

This production function entails that tradable goods from the home and foreign economy can be substituted for one another with a constant elasticity of substitution given by ζ. Tradable and non-tradable goods can be substituted for one another with a constant elasticity of substitution of 1. Here, $\omega_H \in (0,1)$ measures the degree of home bias in demand, and $\left(\gamma^\gamma (1-\gamma)^{1-\gamma}\right)^{-1}$ is a normalization factor that ensures the price of final goods satisfies $P = 1$ in the steady state.

Final goods firms trade composite goods at price P_S, for $S \in \{N; H; F\}$ and trade final goods at price P. We adopt the normalization that the price of imported composite goods (which depends on conditions in the foreign economy) satisfies $P_{F,t} = 1$ for all t.

Final goods firms choose their input of each variety i and their output of each composite good, to maximize their profits from the first technology, given by:

$$P_S Y_S^d - \int_{i \in I_S} p_i y_i^d di.$$

[8] For the moment, we omit the time index.

[9] Here, the notation d mainly stands for demand.

for each $S \in \{N; H; F\}$.

It is straightforward to derive the first-order condition for a final goods firm's optimal choice of each y_i^d:

$$P_S \left(Y_S^d\right)^{\frac{1}{\eta}} \left(y_i^d\right)^{\frac{-1}{\eta}} = p_i. \tag{11}$$

Furthermore, final goods firms choose their input Y_S^d of each composite good S and output of final goods to maximize their profit from the second technology, which is $PY^d - \sum_S P_S Y_S^d$. Profit maximization from the second technology implies (after some rearrangement) that:

$$\begin{aligned}
P_N Y_N^d &= (1-\gamma) P Y^d, \\
P_H Y_H^d &= \omega_H \gamma Y^d P \left(P_H^\gamma P_N^{1-\gamma} P^{-1}\right)^{\frac{1-\zeta}{\gamma}}, \\
P_F Y_F^d &= (1-\omega_H)\gamma Y^d P \left(P_F^\gamma P_N^{1-\gamma} P^{-1}\right)^{\frac{1-\zeta}{\gamma}}.
\end{aligned}$$

where the final goods price satisfies:

$$P = P_N^{1-\gamma} \left(\left[\omega_H P_H^{1-\zeta} + (1-\omega_H) P_F^{1-\zeta}\right]^{\frac{1}{1-\zeta}} \right)^\gamma$$

As discussed above, we normalize the price of foreign goods to 1, so these conditions now become:

$$\begin{aligned}
P_N Y_N^d &= (1-\gamma) P Y^d, \\
P_H Y_H^d &= \omega_H \gamma Y^d P \left(P_H^\gamma P_N^{1-\gamma} P^{-1}\right)^{\frac{1-\zeta}{\gamma}}, \\
Y_F^d &= (1-\omega_H)\gamma Y^d P \left(P_N^{1-\gamma} P^{-1}\right)^{\frac{1-\zeta}{\gamma}}.
\end{aligned}$$

where the final goods price satisfies:

$$P = P_N^{1-\gamma} \left(\left[\omega_H P_H^{1-\zeta} + (1-\omega_H)\right]^{\frac{1}{1-\zeta}} \right)^\gamma$$

We let $Y_{S,t}$ denote the total real value of the output of retailers in sector $S \in \{N, H\}$. In the non-tradable sector, market clearing requires that value equals the real input of the final goods firm, so that $Y_{N,t}^d = Y_{N,t}$.

Exports. We assume that final goods firms in the foreign economy behave similarly to those in the domestic economy, and the foreign demand y_i^* for each tradable domestic variety i satisfies:

$$P_H \left(Y_H^*\right)^{\frac{1}{\eta}} \left(y_i^*\right)^{\frac{-1}{\eta}} = p_i. \tag{12}$$

where the aggregate foreign demand for domestic tradable goods Y_H^* satisfies:

$$P_H Y_H^* = (1-\omega_F)\gamma Y^* P^* \left(P_H^{\gamma} \left(P_N^*\right)^{1-\gamma} \left(P^*\right)^{-1}\right)^{\frac{1-\zeta}{\gamma}}.$$

We assume that the foreign economy, representing all other states of the US, is large compared to the domestic economy. Therefore, aggregate foreign expenditure Y^*, foreign price levels P^*, and P_N^* are all assumed to be exogenous and constant. We can then simplify the previous expression to:

$$Y_H^* = \overline{X} P_H^{-\zeta}.$$

where $\overline{X}$ is a constant. We will calibrate the model so that, in the steady state, $P_H = 1$ and $\overline{X}$ is the steady state level of exports.

The home economy's level of (nominal) net exports is given by:

$$NX_t = P_H Y_H^* - Y_{F,t}^d = \overline{X} P_H^{1-\zeta} - (1-\omega_H)\gamma Y^d P \left(P_N^{1-\gamma} P^{-1}\right)^{\frac{1-\zeta}{\gamma}}. \tag{13}$$

Market clearing in the home economy's tradable sector implies that the total output of retailers in this sector is equal to the demand of the final goods firms at home and abroad, so that $Y_{H,t}^d + Y_{H,t}^* = Y_{H,t}$.

Retailers. There is a continuum of measure 1 of retailers in each sector S. Retailers do not enter or exit. For each period, each retailer pays a fixed cost $\frac{c_R}{2}$ units of final goods (so that steady state profit is 0). Each retailer has the capacity to use one unit of the wholesale good in its sector to produce one unit of its variety $i \in [0,1]$. The wholesale price in sector S is $P_{W,S,t}$.

Retailers set their price p_i to maximize present discounted profits, given domestic and foreign final goods firms' respective demand function for their variety. However, they face Calvo pricing frictions; only a fraction $1-\vartheta_P$ of retailers can change their price each period. It is also straightforward to show that profit maximization would imply $p_i = \frac{\eta}{\eta-1} P_{W,S,t}$ when there were no pricing frictions.

As shown in Galí (2015, Chapter 3), these assumptions, combined with the first-order conditions of the final goods firm, imply that the price of each composite good Y_S, $S \in \{N, H\}$, satisfies:

$$\left(\frac{P_{S,t}}{P_{S,t-1}}\right)^{1-\eta} = \vartheta_P + (1-\vartheta_P)\left(\frac{P^\star_{S,t}}{P_{S,t-1}}\right)^{1-\eta},$$

where $P^\star_{S,t}$ is the price chosen by firms that reset prices in period t. This is given by the first-order condition:

$$\sum_{k=0} \vartheta_P^k \mathbb{E}_t\left[\prod_{j=0}^{k} M_{t+j}\left(\frac{P^\star_{S,t}}{P_{S,t+k}}\right)^{-\eta} Y_{S,t+k}\left(\frac{1}{P_{t+k}}\right)\left(P^\star_{S,t} - \frac{\eta}{\eta-1}P_{W,S,t+k}\right)\right] = 0.$$

To simplify this, we let:

$$M_t\mathcal{A}_{P,S,t} = \frac{\eta}{\eta-1}\sum_{k=0}\vartheta_P^k\mathbb{E}_t\left[\prod_{j=0}^{k} M_{t+j}P^{\eta}_{S,t+k}\left(\frac{1}{P_{t+k}}\right)Y_{S,t+k}P_{W,S,t+k}\right]$$

$$M_t\mathcal{B}_{P,S,t} = \sum_{k=0}\vartheta_P^k\mathbb{E}_t\left[\prod_{j=0}^{k} M_{t+j}P^{\eta}_{S,t+k}\left(\frac{1}{P_{t+k}}\right)Y_{S,t+k}\right].$$

Then, we can write the first-order condition as:

$$\mathcal{B}_{P,S,t}P^\star_{S,t} - \mathcal{A}_{P,S,t} = 0$$

or

$$P^\star_{S,t} = \frac{\mathcal{A}_{P,S,t}}{\mathcal{B}_{P,S,t}}.$$

Substituting this into the equation for the evolution of aggregate composite good prices, we find that these prices must satisfy:

$$\left(\frac{P_{S,t}}{P_{S,t-1}}\right)^{1-\eta} = \vartheta_P + (1-\vartheta_P)\left(\frac{\mathcal{A}_{P,S,t}}{\mathcal{B}_{P,S,t}P_{S,t-1}}\right)^{1-\eta}. \tag{14}$$

That is, for any given t, when adjusting prices, all retailers in sector S choose the same price, $P^\star_{S,t} = \frac{\mathcal{A}_{P,S,t}}{\mathcal{B}_{P,S,t}}$.

The equations above also imply that $\mathcal{A}_{P,S,t}$ and $\mathcal{B}_{P,S,t}$ satisfy:

$$\mathcal{A}_{P,S,t} = \left(\frac{\eta}{\eta-1}\right)P^{\eta}_{S,t}P_t^{-1}Y_{S,t}P_{W,S,t} + \vartheta_P M_{t+1}\mathbb{E}_t\mathcal{A}_{P,,S,t+1}$$

$$\mathcal{B}_{P,S,t} = P^{\eta}_{S,t}P_t^{-1}Y_{S,t} + \vartheta_P M_{t+1}\mathbb{E}_t\mathcal{B}_{P,S,t+1}.$$

Total nominal profits of the retailers Π_t^R are as follows:

$$\Pi_t^R = \left(\sum_{S \in \{N,H\}} P_{S,t} Y_{S,t} - P_{W,S,t} Y_{W,S,t} \right) - c^R P_t. \tag{15}$$

where $Y_{W,S,t} = \int_{I \in S} y_i di$ is the wholesale output of sector $S \in \{N; H\}$, which is used as an input for the retailers in that sector.

By integrating the demand curves for retailers we find that:

$$Y_{W,S,t} = Y_{S,t} \left(\mathcal{D}_{S,t} \right)^{-\eta} \tag{16}$$

where $\mathcal{D}_{S,t}$ is a measure of price dispersion, given by:

$$\mathcal{D}_{S,t} = \frac{\left(\int p_i^{-\eta} di \right)^{\frac{-1}{\eta}}}{P_{S,t}}$$

Then, analogously with $P_{S,t}$, $\mathcal{D}_{S,t}$ evolves according to the following expression:

$$\left(\frac{P_{S,t} \mathcal{D}_{S,t}}{P_{S,t-1} \mathcal{D}_{S,t-1}} \right)^{-\eta} = \vartheta_P + (1 - \vartheta_P) \left(\frac{\mathcal{A}_{P,S,t}}{\mathcal{B}_{P,S,t} P_{S,t-1} \mathcal{D}_{S,t-1}} \right)^{-\eta}. \tag{17}$$

Wholesalers. Following the literature (e.g., Ottonello and Winberry (2020)), we assume that there are wholesale firms have idiosyncratic productivity, rich firm dynamics, and produce homogenous products for each sector $S \in \{N, H\}$. For these firms, both the input and output markets are perfectly competitive. In particular, new wholesalers enter at the start of each period. Each new firm pays an entry cost in units of final goods before they enter. After paying an entry cost, each firm draws a TFP z from the distribution G_z, which is i.i.d. across firms. Therefore, potential new entrants have to compare the expected payoffs with the entry costs. Each new firm i produces a different variety. In each period, with probability ρ_z, a continuing firm keeps the same level of z as in the previous period; and with probability $1 - \rho_z$, a firm draws a new z from the distribution G_z. For tractability, we make the following distributional assumption.

Assumption 1. *The distribution g is a Pareto distribution, so that $g(z) = \xi z^{-\xi-1}$ for $z \geq 1$ and $g(z) = 0$ for $z < 1$.*

A wholesale firm i produces according to the production function:

$$y_i = z_i^{1-\theta}(k_i^{1-\alpha}n_i^{\alpha})^{\theta}, \tag{18}$$

where n_i is a composite of the quantities of labor of each worker j that is used by the firm:

$$n_i = \left(\int_0^1 n_{i,j}^{\frac{\epsilon_W-1}{\epsilon_W}}\,\mathrm{d}j\right)^{\frac{\epsilon_W}{\epsilon_W-1}}$$

and $n_{i,j}$ denotes firm i's use of labor j.

Each period, if the firm chooses to stay in business, it must pay c^F units of final goods as fixed cost; or the firm exits. Exit also occurs at the start of the period, immediately after new firms' entry. We allow for the possibility that a new entrant may exit instantly.

The optimization problem for the firm at time t is described by the following Bellman equation:

$$\begin{aligned} W(z;\mathbf{X}_t,S) \;=\;& \pi^W(z;\mathbf{X}_t,S) + M_t\rho_z \max\{W(z;\mathbf{X}_{t+1},S);0\} \\ & + M_t(1-\rho_z)E_{G_z}[\max\{W(z';\mathbf{X}_{t+1},S);0\}] \end{aligned} \tag{19}$$

where $\mathbf{X}_t$ denotes the time t aggregate variables, S is the sector, $\pi^W(z;\mathbf{X}_t,S)$ is the period t optimal profit which is evaluated in units of final goods, and M_t is the firm owner's discount factor. This is the same discount factor as that for the capital goods producing firms.[10]

For the static profit $\pi^W(z;\mathbf{X}_t,S)$, we have:

$$P_t\pi^W(z;\mathbf{X}_t,S) \;\equiv \tag{20}$$

$$\max_{k,\{n_{i,j}\}}\left\{P_{W,S,t}y_i - (r_t^K+\delta_K)P_tk_i - c^FP_t - \int_0^1 w_{t,j}n_{i,j}\,\mathrm{d}j\right\}, \tag{21}$$

where y_i is given by the production function (18).

In equilibrium, the value of an intermediate goods firm $W(z;\mathbf{X}_t,S)$ is increasing in z. Therefore, there will be a unique cutoff $z^{\star}_{S,t}$ such that the firm in sector S will exit if and only if its productivity is below $z^{\star}_{S,t}$.

We assume that the entry cost is paid in units of final goods, but that the cost of entry is an increasing convex function of the number of entrants, as in a number of recent macroeconomic

[10]Note that there will be possible exits in the next period, even if the firm chooses to operate in this period; and that's why we include the maximization operator for the next period.

models of entry (see Bergin, Feng, and Lin (2018) for a list of citations). In particular, we assume that the entry cost in a sector is equal to $\nu P_t \left(\frac{M^e_{S,t}}{M^W_{S,t-1}}\right)^\Theta$, where $\Theta \geq 0$ denotes the elasticity of entry costs with respect to the number of entrants.

In equilibrium, the following conditions for free entry and free exit of intermediate goods firms must be satisfied:

$$\nu \left(\frac{M^e_{S,t}}{M^W_{S,t-1}}\right)^\Theta = E[\max\{W(z_t; \mathbf{X}_t, S); 0\}] \tag{22}$$

$$W(z^\star_{S,t}; \mathbf{X}_t, S) = 0, \text{ or } W(1; \mathbf{X}_t, S) > 0 \text{ and } z^\star_{S,t} = 1. \tag{23}$$

The latter case here corresponds to the case in which no firms exit.

Let $M^W_{S,t}$ denote the mass of active wholesale firms in period t in sector S, after both entry and exit decisions have been made. $M^e_{S,t}$ denote the mass of firms that enter, even though some of these firms will exit immediately. Also, $\mu_{S,t}(z)$ denotes the density function for active firms' productivity in period t in sector S.

Then, for the dynamics of the distribution of the firms, we have:

$$\begin{aligned}
& \forall \hat{z} \geq z^\star(\mathbf{X}_{S,t+1}), M^W_{S,t+1} \int I_{\{z' \geq z^\star(\mathbf{X}_{S,t+1})\}} I_{\{z' \leq \hat{z}\}} \mu_{S,t+1}(z') dz' \\
= \; & M^W_{S,t} \rho_z \int I_{\{z \geq z^\star(\mathbf{X}_{S,t})\}} I_{\{z \leq \hat{z}\}} I_{\{z \geq z^\star(\mathbf{X}_{S,t+1})\}} \mu_{S,t}(z) dz \\
& + \left[M^W_{S,t}(1-\rho_z) + M^e_{S,t+1}\right] \int I_{\{z' \leq \hat{z}\}} I_{\{z' \geq z^\star(\mathbf{X}_{S,t+1})\}} dG_z(z').
\end{aligned} \tag{24}$$

And for the measure of active firms and exiting firms, we have the following dynamics:

$$\begin{aligned}
M^W_{S,t+1} &= M^W_{S,t} \rho_z \int I_{\{z \geq z^\star(\mathbf{X}_{S,t})\}} I_{\{z \geq z^\star(\mathbf{X}_{S,t+1})\}} \mu_{S,t}(z) dz \\
&\quad + \left[M^W_{S,t}(1-\rho_z) + M^e_{S,t+1}\right] \times \int I_{\{z' \geq z^\star(\mathbf{X}_{S,t+1})\}} dG_z(z')
\end{aligned} \tag{25}$$

$$\begin{aligned}
M^{Exit}_{S,t+1} &= M^W_{S,t} \rho_z \int I_{\{z \geq z^\star(\mathbf{X}_{S,t})\}} I_{\{z \leq z^\star(\mathbf{X}_{S,t+1})\}} \mu_{S,t}(z) dz \\
&\quad + \left[M^W_{S,t}(1-\rho_z) + M^e_{S,t+1}\right] \times \int I_{\{z' \leq z^\star(\mathbf{X}_{S,t+1})\}} dG_z(z')
\end{aligned} \tag{26}$$

Aggregate Profits. Total nominal profits (net of entry costs) are given by:

$$\Pi_t = \int_{i \in I_H \cup I_N} P_t \pi^W_{i,t} di + \Pi^K_t + \Pi^R_t - \nu P_t \left(\frac{M^e_{H,t}}{M^W_{H,t-1}}\right)^\Theta M^e_{H,t} - \nu P_t \left(\frac{M^e_{N,t}}{M^W_{N,t-1}}\right)^\Theta M^e_{N,t}. \tag{27}$$

where $\pi_{i,t}^W$ is the period profit of the wholesaler i in units of the final good, according to the function in Eq. (21), and Π_t^K and Π_t^R are the total profits of capital goods firms and retailers given in Eq. (7) and Eq. (15). $\Pi_t = \Pi_t^R$ at the steady state since all firms apart from retailers are either competitive or face free entry; during the transition path, $\Pi_t - \Pi_t^R$ may not be exactly 0. Since final goods firms make zero profits every period, we do not need to consider their profits here.

Exogenous Dividend Rules and Firm Borrowing. The large representative household consists of all consumers and firms, and they face the same interest rates $r_{F,t} = r_{H,t}$ in our benchmark case. In our counterfactual analysis later in this paper, when firms and consumers face different interest rates, they may have different incentives to borrow but at the same time they still belong to the same extended large family. Therefore, results for the counterfactual analysis may be different if "consumers borrow for firms" or "firms borrow for consumers." This issue is irrelevant in the benchmark case since firms and consumers face the same interest rates dynamics. Thus, following the literature (e.g., Leary and Michaely (2011), Chen, Karabarbounis, and Neiman (2017)), we assume firms follow an exogenous dividend rule (say, due to some institutional reasons or other reasons), and each period's dividend distribution is a constant fraction of total output.[11]

Market Clearing Conditions.

[11] With this assumption, the model equations during the counterfactual analysis for different interest rates need to be modified slightly. First, note that the economy's total debt includes both firms'debt and consumers'debt, $B_t = B_t^H + B_t^F$. Firms have the following dynamics with respect to debt:

$$B_t^F = \Pi_t - Div_t + B_{t-1}^F(1 + r_{t-1}^F),$$

where Π_t is the total net profits at time t as defined in equation (27) in the model, and Div_t is dividend distribution to consumers at time t. The consumers' budget is then:

$$B_t^H + \left[(1-\chi)P_tC_t^S + \chi P_tC_t^L\right] = Div_t + w_tN_t + B_{t-1}^H\left(1 + r_{t-1}^H\right).$$

We can also write the budget for the consumer as:

$$\begin{aligned} B_t + \left[(1-\chi)P_tC_t^S + \chi P_tC_t^L\right] &= \Pi_t + w_tN_t + B_{t-1}^H\left(1 + r_{t-1}^H\right) \\ &\quad + B_{t-1}^F(1 + r_{t-1}^F), \\ &= \Pi_t + w_tN_t + B_{t-1}\left(1 + r_{t-1}^H\right) \\ &\quad + B_{t-1}^F(r_{t-1}^F - r_{t-1}^H). \end{aligned}$$

When compared to the budget in the benchmark model, we can observe that the only difference is $B_{t-1}^F(r_{t-1}^F - r_{t-1}^H)$. It is clear that only when r_t^F is different from r_t^H (only during the counterfactual analysis) the consumer budget changes relative to the benchmark case.

- The market for labor of each type j clears:

$$\sum_{S\in\{H,N\}} M^W_{S,t} \int I_{\{z\geq z^\star(S,\mathbf{X}_t)\}} n^\star_j(S,z)\mu_{S,t}(z)dz = N_{j,t} \tag{28}$$

- The market for capital goods clears:

$$\sum_{S\in\{H,N\}} M^W_{S,t} \int I_{\{z\geq z^\star(S,\mathbf{X}_t)\}} k^\star(S,z)\mu_t(S,z)dz = K_t. \tag{29}$$

- The market for domestically produced tradable goods clears (total supply $Y_{H,t}$ equals total demand $Y^d_{H,t} + Y^*_{H,t}$):

$$Y_{H,t} = Y^d_{H,t} + Y^*_{H,t} \tag{30}$$

- The market for domestically produced non-tradable goods clears ($Y^d_{N,t}$ is the total demand):

$$Y_{N,t} \;\; = \;\; Y^d_{N,t} = \frac{(1-\gamma)P_tY^d_t}{P_{N,t}} \tag{31}$$

- The market for final goods clears (Y^d_t is the total supply):

$$Y^d_t = C_t + I_t + c^R + \sum_{S\in\{H,N\}} c^F M^W_{S,t} + \sum_{S\in\{H,N\}} \nu M^e_{S,t} \left(\frac{M^e_{S,t}}{M^W_{S,t-1}}\right)^\Theta \tag{32}$$

3 Equilibrium and Characterizations

3.1 Equilibrium Definition

Denote the total mass of continuing firms in sector S as $M^W_{S,t}$, denote the pdf for continuing firms as $\mu_{S,t}(z)$, and denote the total mass of new entrants at the end of each period t as $M^e_{S,t}$. The competitive equilibrium consists of prices $\{P_{Nt}, P_t, w_t\}^\infty_{t=0}$ and value functions of $W(z;\mathbf{X}_t, S)$ such that,

- C^L_t, C^S_t, B_t and $N_{j,t}$ for each j solve the spender's and saver's problems (1).
- The interest rate faced by savers is given by (5).

- The discount factor of firms is given by (8), and the aggregate profit of firms is given by (27), and retailers' profits are given by (15).
- For Y_t^d and $Y_{S,t}^d$ of each $S \in \{N, H\}$, final goods firms maximize their period profits.
- $P_{N,t}$, $P_{H,t}$ and w_t evolve according to the Calvo price-setting and wage-setting conditions (14) and (4).
- Aggregate investment I_t solves the optimization problem of the capital goods firm (9). The total profits of capital goods firms are given by (7). Total capital stock evolves according to the law of motion in (6).
- Wholesale firms' value function $W(\cdot)$ evolves according to the Bellman equation (19). For each wholesaler i, its profit $\pi_{i,t}^W$, output $y_{i,t}$ and input uses $k_{i,t}$, $n_{i,j,t}$ (for each j) are given by the solution of its within-period problem in (21). $z_{S,t}^\star$ and $M_{S,t}^E$ are consistent with the free entry and exit conditions (22) and (23).
- The total mass and distribution of wholesale firms evolves according to (24) and (25) .
- Labor, capital, non-tradable goods markets, and final goods markets clear, according to Eqs. (28), (29),(32) and (31).

3.2 Characterization for the Economy

Before using the model to quantitatively evaluate the macroeconomic impacts of credit market deregulations, we can first simplify the model equations considerably and characterize several optimization problems analytically. Our model has heterogenous firms with endogenous entry and exit; however, we can fully characterize the dynamics of firms in and out of steady state with analytical solutions. We next summarize these characterizations intuitively and provide more details in Section E in the Appendix.

3.2.1 Dynamics of the Number of Wholesalers, and Their Average Productivity

To better describe the dynamics of average productivity and the total measure of wholesalers, we first define the average TFP of active wholesalers as $\overline{z}_t = \int_{z>z_t^\star} z\mu_t(z)dz$. The exact dynamics of $\overline{z}_t$ and the total mass of wholesalers M_t^W are described by the following proposition.[12]

[12]For convenience, we omit the subscript S for sector in the statement of the proposition.

Proposition 1. *Suppose that Assumption 1 holds. Let*

$$j_t = \max\left[\left\{k \in \{0, 1, ..., t-1\} \middle| z_k^\star > z_t^\star \text{ or } z_k^\star \geq z_i^\star, \forall i < k\right\} \cup \{0\}\right] \tag{33}$$

Then the values of $\overline{z}_t$ and M_t^W evolve over time according to the following equations:

$$\begin{aligned}
\overline{z}_t &= \frac{\xi z_t^\star}{\xi - 1} + \frac{\rho_z^{t-j_t} M_{j_t}^W}{M_t^W}\left(\overline{z}_{j_t} - \frac{\xi \min\{z_t^\star; z_{j_t}^\star\}}{\xi - 1}\right) \\
M_t^W &= \rho_z^{t-j_t} M_{j_t}^W \min\left\{1; \left(\frac{z_t^\star}{z_{j_t}^\star}\right)^{-\xi}\right\} + (z_t^\star)^{-\xi} \sum_{i=1}^{t-j_t} \rho_z^{i-1}((1-\rho_z)M_{t-i}^W + M_{t-i+1}^e)
\end{aligned}$$

Proof. See Appendix E.1. □

Intuitively, there are two facts are important for the analytical results. First, we assume that a firm's productivity does not change in the next period with probability ρ_z, and with probability $1 - \rho_z$, the firm draws a new productivity from the same, exogenous distribution g). Second, intermediate good firms are perfectly competitive, so that the firm's optimal decision is relatively simple (maximizing static profits for a given z, and deciding on exits or not). As a result, the dynamics of optimal cutoffs and the average TFP of active firms are relatively easy and tractable. We provide more details and intuitions in Section E in the Appendix.

3.2.2 Closed-Form Solution for Entry and Exit

In our model, heterogenous firms have endogenous entry and exit behavior. We find that convergence of the computation is vastly improved when we use the following closed-form solution for the free entry and free exit conditions. In particular, we can solve for the optimal cutoffs and the mass of entrants, $z_t^\star$ and M_t^e.

Proposition 2. *$z_t^\star$ and M_t^e satisfy:*

$$\nu\left(\frac{M_{S,t}^e}{M_{S,t-1}^W}\right)^\Theta = \sum_{j=0}^{\infty} \mathcal{A}_{t,j}\left(\frac{z_{t,j}^{\star-(\xi-1)}}{\xi - 1}\right) \tag{34}$$

and

$$z_t^\star = \frac{c_F - M_t(1-\rho_z)\sum_{j=0}^{\infty}\mathcal{A}_{t+1,j}\left(\frac{z_{t+1,j}^{\star-(\xi-1)}}{\xi-1}\right)}{(1-\rho_Z M_t)A_t} - \left(\frac{1}{(1-\rho_z M_t)A_t}\right)\sum_{j=1}^{\infty}\mathcal{A}_{t,j}\left(z_{t,j}^\star - z_{t+1,j-1}^\star\right) \tag{35}$$

where

$$A_{S,t} = \frac{\overline{\pi}_{S,t}^{W}}{1-\rho_z M_t} \tag{36}$$

$$z_{S,t,j}^\star = \max\{z_{S,\tau}^\star\}_{\tau=t}^{\tau=t+j} \tag{37}$$

$$\mathcal{A}_{t,j} = (1-\rho_Z M_{t+j})A_{t+j}\frac{\Pi_{k=0}^{k=j}(M_{t+k}\rho_z)}{M_{t+j}\rho_z} \tag{38}$$

Proof. See Appendix E.2 for proofs and more details. □

3.3 Equilibrium Conditions for the Economy

Lastly, we can also simplify the optimization problems for households since we essentially have representative households within each type (see Section B.1 in the Appendix). We provide details for optimal wage setting (Section B.2 in the Appendix), for the capital goods producers' optimization (Section B.3 in the Appendix), and for wholesalers' optimization (Section B.4 in the Appendix). Thus, we can summarize the economy's equilibrium conditions in Section D and offer a further simplified version in Section D.2.

For given paths on equilibrium variables $\bar{\pi}_{H,t}^F$, $\bar{\pi}_{N,t}^F$, r_t^H and M_t, we can have simple characterizations for the whole economy by a few blocks: the firm-side optimization problem, the Phillips curve equations for goods prices, the Phillips curve equations for wages, the first-order condition for capital goods producers, the resource constraints, and first-order conditions for the household. Our numerical exercises are based on these characterizations.

4 Calibration

Parameters Set Exogenously. We first set a few parameters exogenously. Our model period is one year. On the household side of the economy, the discount factor β is 0.96, the risk aversion parameter σ equals 2.0, and the labor supply elasticity $1/\upsilon$ is set to 2.5; all of these values are typically used in the literature (e.g., Arellano, Bai, and Kehoe, 2019). We also normalize the dis-utility of labor supply parameter ϕ to 1.0. The fraction of households that are credit constrained and hence are hand-to-mouth, χ, is set to be 0.25 (Aguiar, Bils, and Boar, 2020) in our benchmark model. r is fixed as a constant of $1/\beta - 1$.

On the production side of the economy, we set the depreciation for physical capital δ_K to 0.1 and $\alpha = 0.64$ so that the share of payment to workers relative to capital payment is consistent with standard values. We discipline θ, the scale of return for production firms or entrepreneurs, with the share of rents going to entrepreneurs: as pointed out by Quadrini (2000) and Cagetti, De Nardi et al. (2006), this number should fall between 0.15 and 0.1. In our setting, absent fixed costs, the share corresponds to $1 - \theta$; we therefore simply choose $\theta = 0.85$. We calibrate parameter γ so that the share of tradable goods is about 0.7 in the steady state (e.g., Nakamura and Steinsson, 2014). Similarly, for the degree of home bias in the demand of tradable goods, ω_H, we set it to 0.17. We assume that the distribution G_z is Pareto with pdf as $g_z(z) = z_m^{\xi} \frac{\xi}{z^{\xi+1}}$. z_m is normalized to 1, and we use data on the US distribution of firm size to discipline our choice of ξ.[13] Figure 10 in the Appendix shows that our model matches the firm size distribution in the data reasonably well. To calibrate parameters related to sticky prices and wages, we follow the literature (Galí, 2015, Chapter 3, Nakamura and Steinsson, 2013, Basu and House, 2016) and set the elasticity for type-specific labor variety ϵ_w to 21.0, θ_w to 0.3895 so that the probability for nominal wage changes in a quarter is 21%, and also set ϑ_P to 0.2470 so that the probability for nominal price changes in a month is 11%. Lastly, we assume that the dividend distributed to households is always a constant ratio of all firms' output in and out of steady state. We calibrate this ratio so that the initial firm-side debt is roughly about 62% of total debt, consistent of the data in 1980. The implied ratio for dividend is about 0.245. The parameters are listed in Table 1.

Parameters to be Estimated. The remaining parameters to be determined include: (1) c^F, flow operating cost for wholesalers, which is in units of final goods; (2) ρ_z, the persistence for firm-level productivity among wholesalers; (3) ν, the level coefficient for new firms' entry costs; (4) Θ, the elasticity of entry costs with respect to aggregate entry rates; (5) κ, the capital adjustment cost parameter for capital goods producers; (6) η, the demand elasticity for intermediate goods produced by wholesaler firms; (7) ζ, the constant elasticity of substitution between home tradable goods and foreign tradable goods; and (8) the average level of steady state debt, $\bar{B}$. In principle, these parameters are not directly observable (or

[13]We used the data from Restuccia and Rogerson (2008), which in turn was from Rossi-Hansberg and Wright (2007).

Table 1: Parameters Set Exogenously

Parameter	Description	Value
β	Discount factor	0.96
σ	Risk aversion	2.0
υ_L	Labor supply Elasticity	2.5
γ	Preference over Tradable goods	0.7
χ	Share of HHs hand-to-mouth	0.25
δ_K	Capital depreciation	0.1
α	Labor share for Wholesalers	0.64
ξ	Scale for Productivity Pareto Dist.	1.3
θ	Return to scale for Wholesalers	0.85
ω_H	Home bias within tradable goods demand	0.17
ϵ_w	Elasticity for type-specific labor variety	20.0
θ_w	Prob. for wages not changing	0.3895
ϑ_P	Prob. for prices not changing	0.2470

difficult to measure) in the data, nor are there standard values used in the literature.

To estimate these parameters, we choose the values that minimize the distance between a number of moments in the model and in the data. As we will argue below, the dynamic effect of credit market conditions on a number of economic variables in the model is informative for the values of these parameters. To estimate such dynamic effects in the data, we leverage the credit expansion induced by the deregulation of the banking sector in the US in the 1980s. This event has been studied extensively in the literature, and we follow this literature in our analysis. In the Appendix, we review the institutional details of this reform and describe all our data sources. To estimate the dynamic response of an outcome variable of interest to the credit expansion induced by the bank deregulation, we estimate specifications of the form:

$$g_{i,t+k} = \alpha_{i,k} + \bar{\alpha}_{t,k} + \sum_{j=1}^{J} \beta_{j,k} g_{i,t-j} + \sum_{j=0}^{J} \gamma_{j,k} D_{i,t-j}^{intra} + \sum_{j=0}^{k-1} \delta_{j,k} D_{i,t+k-j}^{intra} + \Gamma \mathbf{X}_{it} + \epsilon_{i,t+k} \quad (39)$$

where $g_{i,t+k}$ is the growth rate of the outcome variable that we consider in state i between $t+k-1$ and $t+k$; $D_{i,t}^{intra}$ is a dummy variable equal to 1 if state i lifted its intrastate branching restrictions in year t and zero otherwise; $\mathbf{X}_{it}$ is a vector of controls for output and employment growth, employment share of major sectors; $\alpha_{i,k}$ and $\overline{\alpha}_{t,k}$ are state and year fixed effects, respectively. We set the number of lags J to 4.[14] We then estimate equation (39) in

[14]This method is robust to the lag length choice. When we include more lags in the estimation (i.e., when

growth rates to account for non-stationarity in some of our outcome variables.[15] Additionally, we include lags of the dependent variable to allow for serial correlation in the inequality growth rate. The coefficients $\beta_{j,k}$ capture the autoregressive structure of the outcome variables growth. We cluster standard errors at the state level to account for serial correlations within state and across time.

This method of estimating impulse response functions is known to be robust to misspecification of the data-generating process (Jorda, 2005). Additionally, it directly estimates the impulse response function of the outcome variables to bank deregulation, and does so without requiring us to first specify and estimate an underlying dynamic model and then compute the response analytically.[16] In particular, the k years ahead effect on growth of an outcome variable is $IRF\left(k\right) = \gamma_{0,k}$, $k \geq 0$. The impulse response function of the level of the outcome variable is the cumulative sum of the growth responses. Although controlling for lags of the dependent variable and state fixed effects may, in principle, bias the estimation in small samples, Nickell (1981) shows that the order of bias is $1/T$, which is small for our dataset.[17]

We estimate impulse responses for lending rates, household debt, output, employment, wages, and firm entry and exit rates. Our data on lending rates and household debt are from the Call Report data of institutions regulated by the Federal Deposit Insurance Corporation (FDIC). Data on output, employment, and wages are from Bureau of Economic Analysis (BEA), and data on firm (establishment) entry and exit rates are from the Business Dynamics Statistics database.

Having obtained empirical responses, we next briefly discuss the intuition for our model parameters' identification. Given other parameters, $\bar{B}$ is mostly relevant for the steady state level of debt to output (of course, $\bar{B}$ also matters for how households respond to interest rates shocks). We use 1.2% as the debt service to output ratio in the steady state (the debt to output ratio is about 27% as in the data from around the 1980s). κ affects the speed of capital adjustment and hence output and labor productivity dynamics. The parameters c^F, ρ_z, ν, and Θ affect firms' profitability, profits, and therefore firms' decisions to enter and exit. Thus, entry rates and exit rates in the steady state and their dynamic responses can provide

$J > 4$), the results hardly change.

[15]We test for stationarity both with panel unit root tests (Im-Pesaran-Shin) and univariate unit root tests (augmented Dickey-Fuller).

[16]We find that our results are robust to this approach.

[17]Since in our dataset T is large, we can also estimate equation (39) with OLS and state dummies, rather than with the FE estimator. The results are also very similar, which confirms that the bias is extremely small.

Table 2: Endogenously Estimated Parameters

Parameter	Description	Value	Relevant Moments
c^F	Flow operating costs	4.2388	Entry and exit rates (level and dynamics)
ρ_z	Persistence for firm-level productivity	0.5349	Entry and exit rates
ν	Entry cost, linear coefficient	19.9071	Entry and exit rates
Θ	Elasticity of entry costs w.r.t. to Aggregate entry rate	0.4207	Entry and exit rates
κ	Capital adj. costs	0.5895	Overall Employment and GDP dynamics
η	Demand Elas. for intermediate goods	5.5623	Overall Employment and GDP dynamics
ζ	Elasticity between Home and Foreign tradable goods	1.8302	Tradable employment dynamics
$\bar{B}$	HH steady state debt level	-2.2724	steady state debt service to output ratio

important information about these parameters. Lee and Mukoyama (2015) report that in the Census of Manufacturers, 1972-1997, the average entry and exit rates are approximately 6.2% and 5.5%, respectively.[18] The demand elasticity for wholesaler goods η is mostly relevant for firms' demand changes, and thus output dynamics will provide important information to discipline. The elasticity of substitution between home tradable goods and foreign tradable goods, ζ, matters for foreign demand in responses to domestic price changes.

Using the estimated impulse response function for the lending rate and equation 5, we back out the sequence of underlying shocks ψ_t^H, which we then feed into the model. Since our empirical analysis only focuses on the first 10 years after the deregulation, we also focus on the first 10 years' responses in our model, and assume that the underlying shock of ψ_t^H after 11 years gradually returns to its initial level with a fixed persistence parameter of 0.95.[19] We jointly calibrate all these parameters using the simulated method of moments (SMM) to match the impulse responses to bank deregulation of real GDP, employment, (real) wages, firm entry rates and exit rates, and household debt, as well as the average level of debt to output across states (about 27%). We also use the inverse of the empirically estimated variances (for each variable and at each horizon of the impulse response function) as weights for the distance between model and data moments.[20]

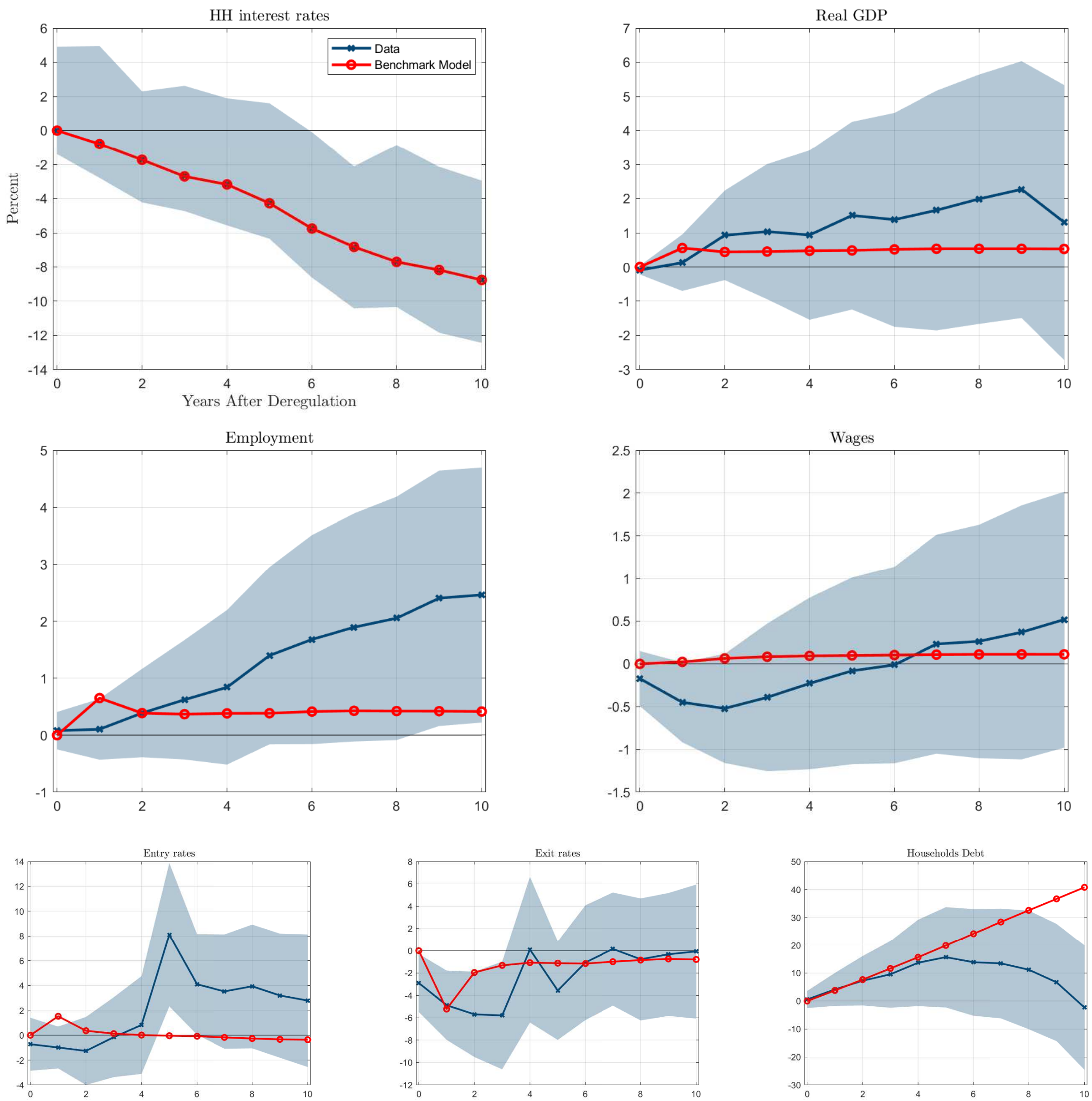
We list our calibrated and estimated parameters in Table 1 and Table 2. Figure 1 illustrates the model's ability to match targeted dynamic responses. Overall, our model can match empirical dynamic responses fairly well, given that we did not have too many free parameters to choose and our model structure is relatively standard. Some of the parameters that we

[18] There numbers appear to be lower than other sources, 12% for entry rate and 8% for exit rate before 1989 (e.g., see Hathaway and Litan and Decker et al. (2014)).

[19] Numerically, we could also experiment with many other settings, such as letting the shocks of ψ_t^H remain at their lowest point for an additional 50 years and then gradually reverting to the initial level of 0. Since our empirical and quantitative analysis focuses on the first 10 years, we find that our results are not affected when we change our assumptions regarding what happens in the very long-run future in the model.

[20] Nominal GDP in our model economy is $P_tY_t^d + NX_t$ and Real GDP is nominal GDP deflated by P_t.

Figure 1: Model vs. Data



estimated are actually consistent with those in other literatures. For example, with our estimated capital adjustment cost, the implied elasticity of capital price with respect to investment capital ratio, is fairly close with that of Bernanke, Gertler, and Gilchrist (1999) (see Section 5.1 of the handbook chapter). In another example, the constant elasticity of substitution between home tradable goods and foreign tradable goods, ζ, is estimated to be about 1.83, which is fairly close to the 2.0 estimate such as Gali and Monacelli (2005) and Nakamura and Steinsson (2014). Also, our value for η is about 5.56, so the implied markup ratio would be about 21%, largely consistent with the values of 12%–15%, as in Basu and Fernald (1997), Burstein and Hellwig (2008) and Atkeson and Kehoe (2005). In addition, as pointed out by Mian, Sufi, and Verner (2020) (Table V on page 975), the dynamics of tradable employment are quite different from those of non-tradable employment: ex-post to their deregulation measures, they find that the average of tradable employment growth is not statistically significant and only weakly positive, while the average of non-tradable employment growth is statistically significant and sizable (the numbers are 0.0024 versus 0.057** in their paper). Our model's responses are qualitatively similar (-0.001647 versus 0.017814 in our model when we take the averages from periods 5 to 10).[21]

We also provide comparative statics for a few parameters in the Appendix (other parameters are not reported for the sake of space). We illustrate with higher and lower parameter values comparing to the benchmark case (we generally set these changes as 20% higher or 20% lower; for elasticities, we set 5.0 and 2.0 for υ_L, and 5.0 and 1.2 for ζ). The results reported are in Figure 14 (for different values in η), Figure 15 (for different values in Labor supply Elasticity, υ_L), Figure 16 (for different values in ζ), Figure 17 (for different values in capital adjustment costs κ), Figure 18 (for different values in flow operating costs c^F), Figure 19 (for different values in persistence for firm-level productivity ρ_z), Figure 20 (for different values in entry cost parameter ν), and Figure 21 (for different values in elasticity of entry costs Θ). At the same time, from a different perspective, we provide the elasticity of model moments to parameter changes in Figure 22 (for a given parameter, we simply compute the model's responses – the percent deviations from corresponding steady state values – averaged from

[21]Our data on tradable and non-tradable employment comes from Mian, Sufi, and Verner (2020), and we believe these data are the best available for our purposes. That said, the data only covers the post-1984 period for 27 to 31 states, depending on the year. However, by 1984, 22 states had already lifted intrastate branching restrictions, with 17 more deregulating between 1985 and 1988. Thus, it's hard to argue that we could have enough meaningful variation to run our local projections as we do in the empirical section. This is probably one of the reasons that why Mian, Sufi, and Verner (2020) aggregate those variables. Therefore, we provide a qualitative comparison on the average summary statistics across the two sectors.

periods 2 to periods 10, as a summary moment). In short, we observe that almost all of the model responses are monotonic in parameter values, which is helpful for our parameter estimations.

5 Quantitative Exercises

5.1 Understanding the Mechanisms Intuitively

An Illustration: Simple AR(1) Shocks to Interest Rates. To better understand the model mechanism, we first investigate with a very simple exercise.[22] We let the economy start from the initial steady state, and in the beginning of period 1, there is an "MIT" shock to the interest rate. That is, in Eq. (5) for households, $\psi_t^H = 0$ for all t before 0 and $\psi_1^H < 0$. The size of ψ_1^H on impact is roughly about 20% of the steady state level of r (about 0.007 in level), and it decays over time with a persistence parameter of 0.95.[23] We report the impulse responses in Figure 11 in the Appendix. All variables are denoted in percent deviations from their corresponding initial steady state values, and we only report the first 100 periods. For ψ_t^H, we use its levels, and for each real GDP component (consumption, investment, net exports), we scale the percent deviations by the corresponding steady state share so that all are comparable to GDP dynamics.

Intuitively, when interest rates are exogenously reduced due to shocks in ψ_t^H, members of households who are financially unconstrained will borrow more in household debt and increase their consumption relative to the initial steady state level. As first row of Figure 11 shows, consumption increases for about 1% of the steady state GDP. Increased consumption demand and sticky prices in the goods market induce higher firm profits in the non-tradable goods sector on impact. Thus, firms will have stronger incentives to hire more labor and capital inputs and produce more. This in turn will drive up factor prices (e.g., real wages, and capital goods renting prices r_K). For the tradable goods sector, however, the analysis is different. As there is a significant portion of domestically produced goods demanded by foreign states. Increases in tradable goods prices due to factor price increases, even if relatively small, will tend to reduce foreign demand. Overall, depending on the magnitude of foreign demand

[22]We also conduct other intuitive exercises in Section F.2 in the Appendix, including different types of AR(1) shocks to interest rates, and also the responses for shocks to aggregate productivity.

[23]Numerically, we solve for the model with a very large number of time periods (e.g., 800) periods; our results for the first 10 or 20 periods essentially remain the same when we further increase the number of periods.

elasticity, the impact on the tradable goods sector could be negative and it is in the opposite direction of non-tradable goods; this is clearly reflected in net exports' dynamics. In turn, we also see that employment in the two sectors have different responses.

Turning to investment, as capital goods producers face lower interest rates (the same as households) and thus lower costs for investing, they will supply more capital goods to the rental market. However, for two reasons, these producers cannot change capital supply quickly: (1) there are convex capital goods adjustment costs, (2) there are limited resource available to households members for consumption and investment for a given level of household debt in each period. Therefore, we observe in our responses that investment increases in the first few periods and then stays almost flat for the remaining periods.

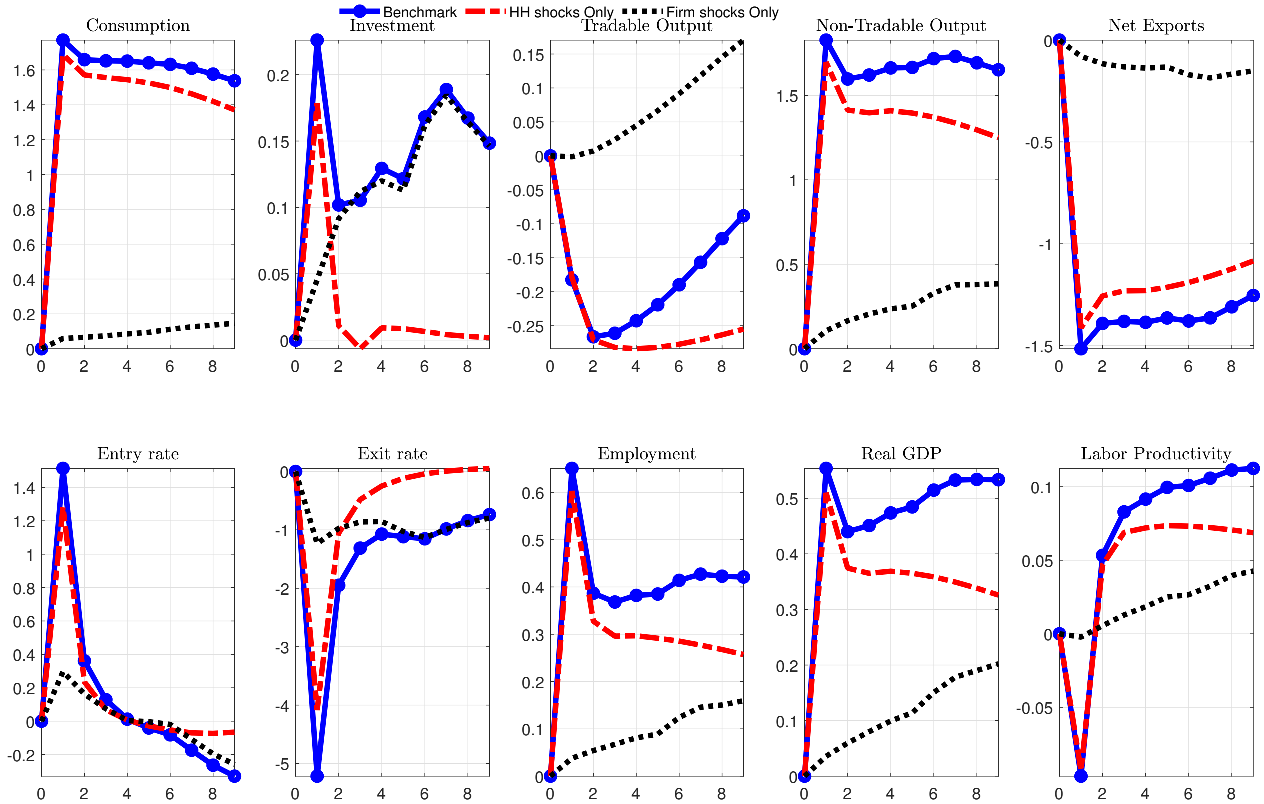
Lastly, it is worth noting that since interest rate shocks are mean reverting in nature, there are no long-run changes in technology or any other economic fundamentals. Thus, the lifetime wealth for the representative household does not experience much change. Consequently, we expect that households have to reduce consumption demand after the first few periods and gradually repay the debt burden (say, after about 10 or 15 model periods). Closely related to this point, since firms expect that there are no long-run changes in technology or other factor prices, their profits only increase temporarily for the first few periods; the incentives for long-term investment and for new firms' entry are also limited and not persistent. Numerically, after about 30 years, we observe that the small open economy gradually converges to a new equilibrium with lower consumption and non-tradable output/employment, higher tradable goods output/employment, and higher net exports.[24]

5.2 Quantifying the Relative Importance of Different Channels

Shocks to Households Only or Shocks to Firms Only. To further investigate the impact of credit market deregulations, we consider counterfactual experiments using the benchmark model. Specifically, we consider the case in which there are interest rate shocks ψ_t only to households or only to firms in the economy. This experiment can help us understand the relative importance of the two different channels: whether deregulation mainly acts through the firm's side or through the household's side (i.e., supply vs. demand channel). The large representative household in the benchmark model consists of consumer (and worker) members, capital goods producer members, and wholesaler members (they are all involved in

[24]Our main focus is not the economy's long-run dynamics, although these dynamics do comprise one part of the model economy that we feature.

Figure 2: Shocks to Firms vs. Households Only



dynamic decisions), and all of them face the same interest rates, $r_{F,t} = r_{H,t}$. Here in these counterfactual experiments, $r_{F,t}$ are different from $r_{H,t}$.

Figure 2 plots the responses for these three different situations: (1) the benchmark model which we label "Benchmark" in the figure; (2) only shocks to interest rates that consumers face, $r_{H,t}$, and $r_{F,t}$ is fixed at the constant initial level, which we label "HH shocks Only" in the figure; and (3) only shocks to the interest rates faced by the producers, $r_{F,t}$, and $r_{H,t}$ is fixed at the constant initial level, which we label "Firm shocks Only." All variables are denoted by percentages that deviate from their corresponding steady state values, and for GDP components, the deviations are already scaled by the share of that component relative to GDP.

By comparing the three different responses and inspecting the dynamics, we find a few patterns. First, all three cases can generate an economic boom by shocking interest rates in some way. With respect to the effects of interest rates on the demand side, the intuition is as follows. When we reduce the interest rates faced by consumers, they will have incentives to consume more today relative to tomorrow, holding all other prices and labor supply not changed. Thus, the nominal demand for final consumption goods in both sectors increases.

Since there are frictions for the producers to adjust the nominal goods prices in the economy, the nominal and real demand for each individual firm increases, and they find it profitable to increase the production scale by hiring more labor and renting more capital. Equilibrium wages, like nominal goods prices, are also sticky but increasing slowly. Thus, even if there are no changes to firms' productivity, firms have higher profits; in turn, fewer incumbent firms exit and more new potential firms enter (the investment of new firms thus increases).

Overall, the demand-side effect is closely related to the effects of monetary policy shocks in a standard New Keynesian Model but different in several aspects. In our small open economy, interest rates (real) are exogenous and do not respond further to economic conditions in equilibrium, whereas in NK models they are endogenously determined eventually (lower nominal interest rates due to some monetary policy shocks will bring higher nominal consumption demand and higher inflation, and further lower real interest rates). Also, in our small open economy, tradable goods prices from outside are exogenous and do not respond to the increases in local consumption demand for tradable goods. In our model, when borrowing costs fall after the expansionary credit shock, households front-load their consumption and borrow more from abroad. A key force of our demand channel operates through the complementarity between demand for tradable and non-tradable goods. Their convex preferences imply that they would then like to consume both more tradables and more non-tradables (in quantities). As such, demand for non-tradables grows, and since non-tradables have to be produced in the home state, this increases the demand for labor locally. Local wages rise in response, which raises the labor supply. As such, output and employment rise locally. Ultimately, our parameterized model suggests that this is the principal mechanism through which the bank deregulation raised output and employment.

When there are favorable shocks to producers, however, the mechanism works differently. To see this, first consider the capital goods producer's optimization problem. The first-order condition for K_{t+1} is given by:[25]

$$E_t\left[M_t \mathcal{Q}_{t+1}\right] - 1 - 2\kappa\left(\frac{K_{t+1}}{K_t} - 1\right) = 0$$

where firm discount is $M_t = \frac{P_{t+1}}{P_t(1+r_{F,t})}$. Ignoring any uncertainty, we have a simplified expression:

$$\frac{P_{t+1}}{P_t}\mathcal{Q}_{t+1} = (1 + r_{F,t})\left[1 + 2\kappa(\frac{K_{t+1}}{K_t} - 1)\right]$$

[25]We omit the S subscript for sectors for the moment; and also see B.3 in the Appendix for more details.

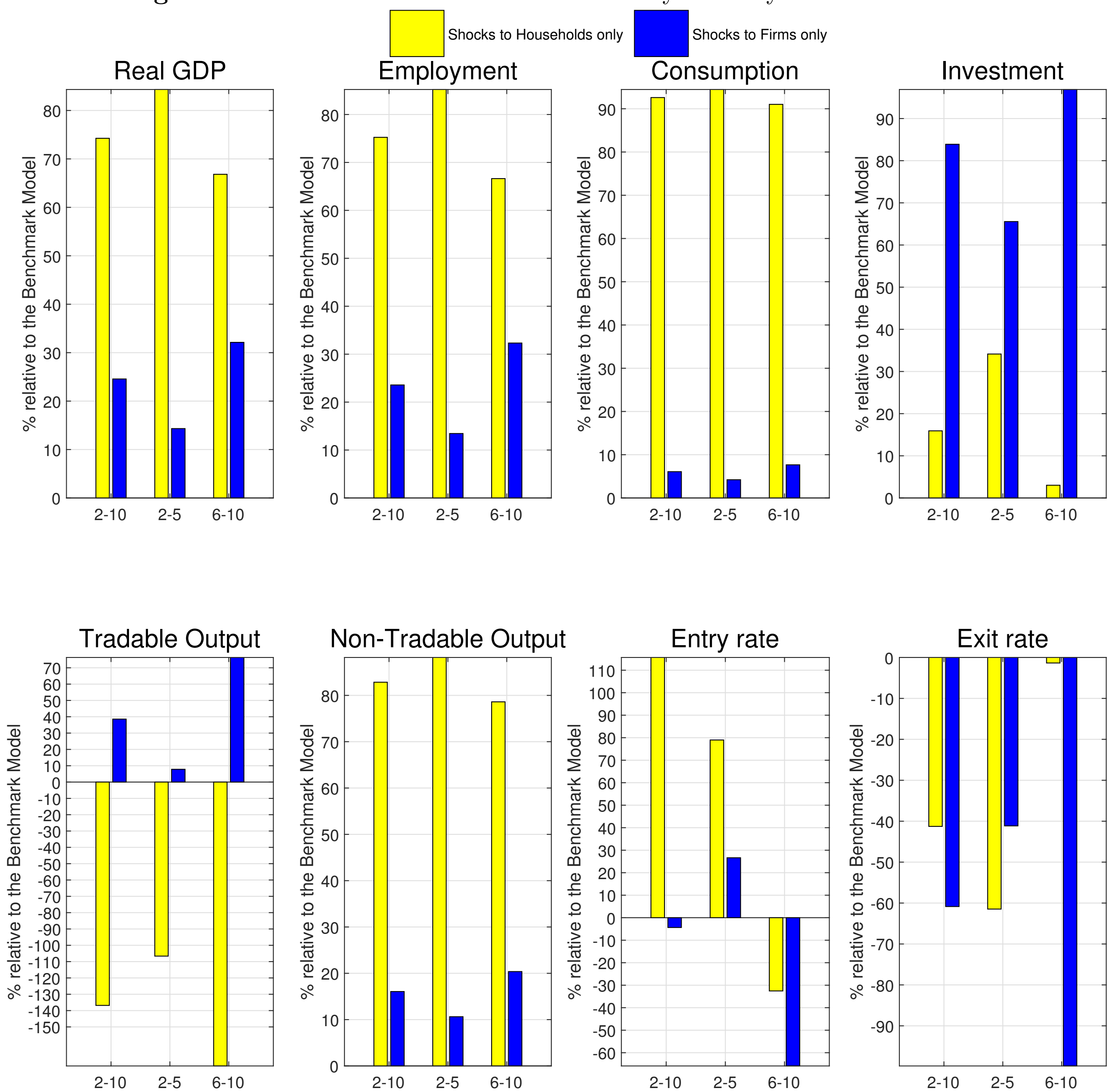
Thus, from the optimality condition, we can see that, for a given initial level of K_t and holding constant for all other prices (P_t, P_{t+1}, and $\mathcal{Q}_{t+1}$), when capital goods producers have lower interest rates in $r_{F,t}$, effectively the marginal cost of investing more in period t is lower, and they have incentives to supply more capital goods to the rental market. Due to capital adjustment costs, the overall supply of capital goods gradually increases. As a result, wholesalers in both sectors will use more capital goods until each individual firm's marginal product of capital equals the capital goods price of r_t^K. At the same time, wholesalers also hire more labor since the production function is Cobb-Douglas. The output goods produced will increase, and the goods prices will tend to fall slightly due to nominal frictions. Therefore, we also observe fewer incumbent firms exit and more new potential firms enter.

Second, when we compare the magnitude of the two different responses, however, the differences are quite clear and large. From the dynamics in Figure 2 we can observe that for most variables demand shocks explain much of the dynamics. More precisely, we provide intuitive summary statistics in Figure 3: for each variable's responses, we compute the averages for the percent deviations across different horizons (years 2 to 10, years 2 to 5, and years 6 to 10). For each experiment and for a given variable, we then report the averages scaled by its counterpart from benchmark model and plot it with a bar. Negative numbers in the figure mean that the impact on that variable is decreasing relative to its steady state value.

For example, for the dynamics of real GDP in the figure, shocks to consumers can account for about 75% of increases in 10 years on average (the yellow bar), while shocks to producers can only account for about 25% (the blue bar).[26] In the first few years (2 to 5 years), demand-side shocks are even more important, and the share could be close to 90%. This pattern is also very similar for non-tradable output, total employment, and labor productivity. For consumption, the effects of demand-side shocks are even larger, accounting for almost all of the responses. However, this is not the case for investment, and the pattern is the opposite actually: supply shocks explain more than 83%, a little bit smaller on impact and larger in the longer run. The dynamics for firms' exit rates are somewhat in between, with demand-side shocks being more important for the first 5 years and firms' actions being more important for years after. Thus, overall, we see that supply-side shocks mainly affect investment and capital stock, and it matters relatively more in the longer run; demand-side shocks mainly affect consumers' consumption demand and goods prices, and matter more on impact and

[26]Also note that by design, the sum of the two bars should be very close to 100% but not exactly.

Figure 3: Shocks to Firms vs. Households Only: The Dynamics Over Time



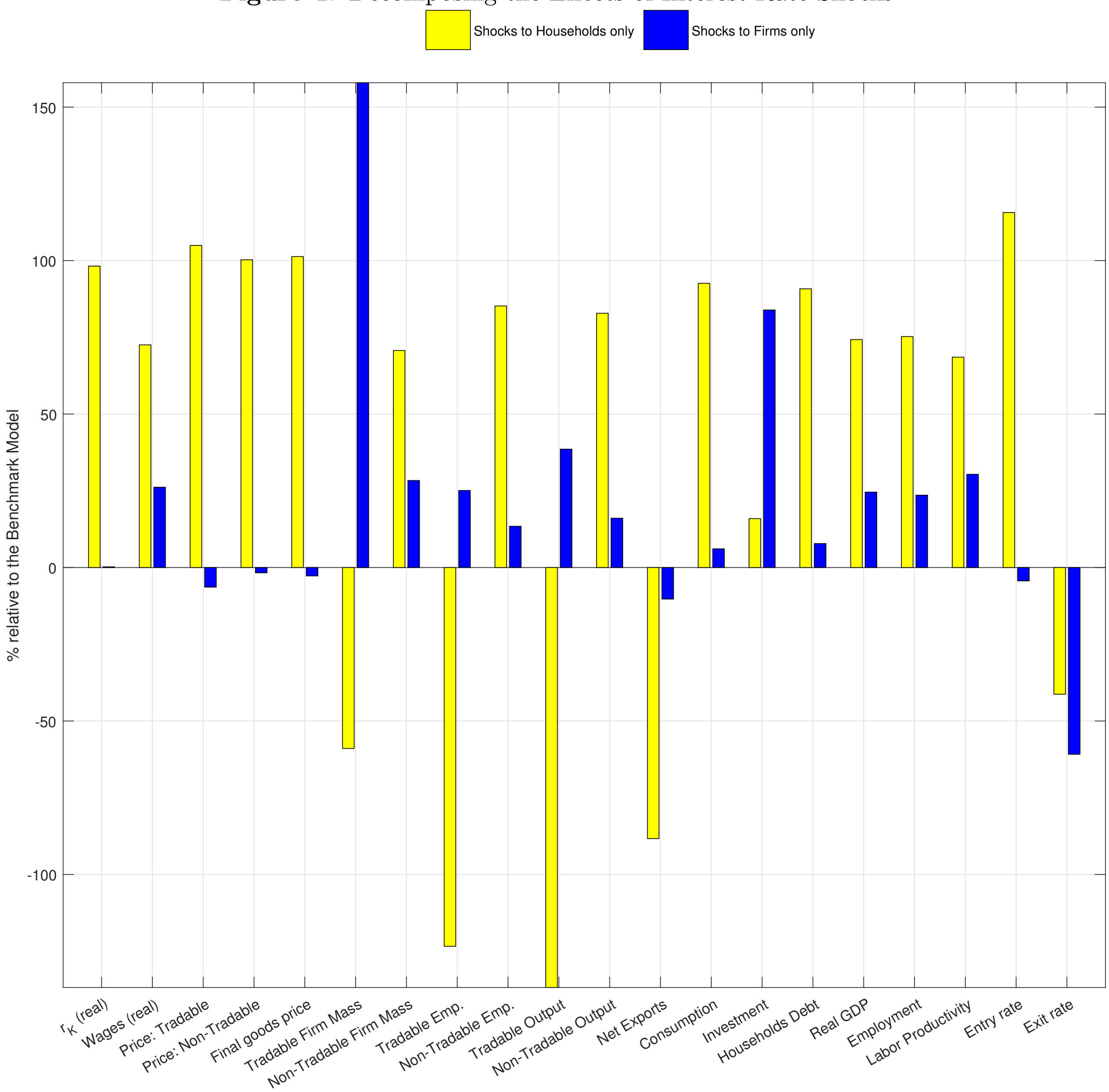
the effect is not so persistent. For other variables of the economy, Figure 4 provides a more complete picture, and includes prices for capital goods, prices for different sectors' output goods, employment and output in different sectors, and so on (for the horizon of years 2 to 10). Overall, in the first few years after credit market deregulation, we find that shocks to the household side explain a very large fraction of the responses for most of the variables.

Lastly, it is worth pointing out that several variables respond just opposite under the two types of shocks. This difference is mostly in the tradable goods sector. When there are favorable demand-side shocks and consumers increase their consumption demand for goods in both tradable and non-tradable sectors, then prices for output goods in both sectors increase. However, since there is some foreign demand for domestic tradable goods and the demand is elastic, price increases for tradable goods is limited. At the same time, producers overall have higher demand for input factors, and the input factor prices also increase (see Figure 4, for example). These two forces will make the firm's profitability (e.g., $\overline{\pi}_t^W$ in Section 3.2.2, the profit rate for a given level of z) actually decrease on impact. With supply-side shocks, capital goods firms' investment is increased significantly in the first few periods, thus capital goods price begins to drop after a few periods, and this is the main driving force that firms in both sectors start hiring more and producing more; for the tradable goods sector, even if the goods prices drop, the reduction in capital costs outweighs the output prices' drop. Thus, with supply-side shocks, tradable employment and output also increase, as in the non-tradable sector. This also mitigates the drop of tradable output in the benchmark model; without shocks to the firm side, the tradable output would recover much slower. In Figure 3, we see that tradable output moves in opposite directions under the two types of shocks (the sum of the two bars is, again, very close to 100% by design). The dynamics for net exports also follows closely with that for tradable output.

The effect on labor productivity is negative on impact when we have shocks to households, whereas it is positive when we have shocks to firms. Intuitively, in the case of demand-side shocks, since nominal consumption demand suddenly increases and but capital stock moves slowly, the responses in employment are large on impact, and thus real labor productivity is low. When investment and capital stock gradually increase over the first 10 years, labor productivity also increases over time.

Robust to Different Levels of Firm Debt. We also confirm our results and experiment with different dividend distribution ratios, such that firm debt is larger or smaller. Overall, we find that if firms have more debt in the steady state and along the transitions

Figure 4: Decomposing the Effects of Interest Rate Shocks



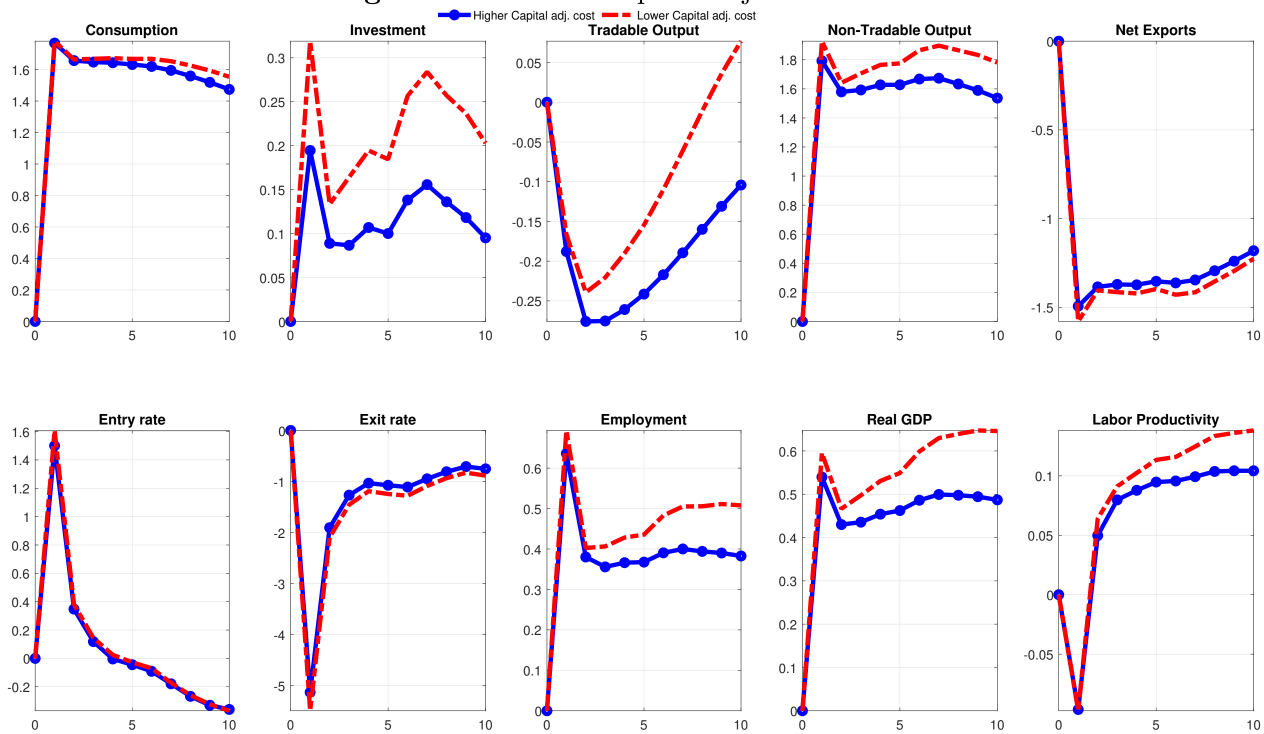
during the counterfactual analysis, then they could benefit relatively more from decreased interest rates; in turn, firms explain a larger proportion of the dynamics, especially in the shorter run. For example, we experiment with a larger firm debt (firm debt increased by about 50%, and the firm debt/total debt ratio is about 92% in the initial steady state), and we decompose the relative importance of consumers compared to that of firms as before. The numbers are about 72% vs. 26% for overall 10 years, 82% vs. 16% during the first 5 years, 64% vs. 34% for the next 5 years (we report our impulse results in Figure 23 in the Appendix). For comparison, the numbers in the benchmark model are about 74% vs. 25%, 84% vs. 14%, and 67% vs. 32%, respectively. When firms do not borrow (no retained earnings and no initial debt), the numbers are about 80% vs. 20%, 90% vs. 10%, and 72% vs. 28%, respectively (see Figures 24, 25, 26 in the Appendix for more details on impulse responses and decompositions). In short, we see that the main pattern in the benchmark model still holds quantitatively when firm debt is different.

5.2.1 Understanding the Persistent Impacts

Previously, we found that when we compare the relative importance of demand-side shocks and supply-side shocks, changes in firms' interest rates are important for the economy's dynamics in the relatively longer run. As we discussed before, capital stock in our economy gradually builds up when producers find that the cost of investing is effectively lower; over time, even if interest rates gradually revert to the initial steady state level, capital stock moves slowly. Thus, capital adjustment costs in our model may be important for understanding why the impacts of deregulation to be persistent.

In Figure 5, we compare responses with different capital adjustment cost parameter κ, 50% higher or 50% lower than the benchmark value. We observe that different degrees of adjustment costs will have a qualitatively similar pattern of responses; however, responses for investment are much larger with smaller adjustment costs. As a result, production and employment also have different responses. On impact, since demand-side shocks are important, differences in production and employment are very small; over time, however, the differences are larger and more evident. For example, the total employment response in the longer run is about 0.5% with a smaller κ and about 0.4% with a bigger κ. These suggest that investment and capital stock are important for persistent impacts. To further look at the role of capital adjustment cost when we have supply-side shocks, we only have shocks to

Figure 5: Different Capital Adjustment Costs



the firm side in Figure 6. It is clear that with smaller capital adjustment cost, investment, employment, and output in both sectors all increase more, and net exports drops more since domestic production expands relatively more and demands for more imported tradable goods. In Figure 7, we plot the average responses of the benchmark models (under each κ) that can be explained by supply-side shocks only. Similarly as before, we find that the supply-side is important for investment dynamics but not so much for consumption dynamics. With smaller capital adjustment costs, the relative importance of the supply-side in explaining the benchmark responses also increases, and this is more evident if we look at horizons from year 6 to 10. For example, in the panel for real GDP, the relative importance for supply-side shocks is about 13% from years 2 to 5 with a larger κ and that number is about 17% for a smaller κ; for years 6 to 10, the importance increases to about 28% for a larger κ, to more than 39% for a smaller κ. The pattern is also very similar for total employment (see Table 7 in the Appendix for details of other variables).

5.2.2 Further Inspecting the Role of Other Model Elements

Sticky Nominal Wages. One important nominal friction in the model is that nominal wages cannot be flexibly adjusted. Accordingly, we inspect the role of sticky wages for the

Figure 6: Supply Side Shocks Only: Persistent Impacts on Firms

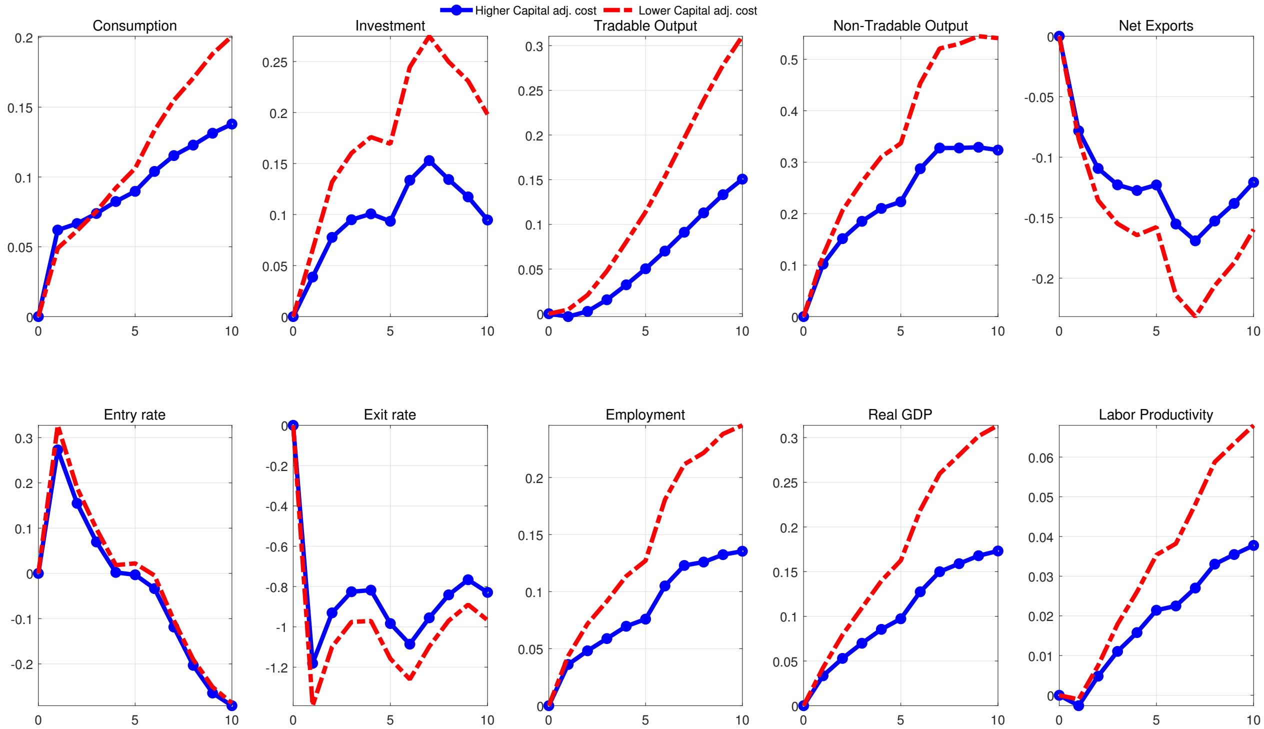
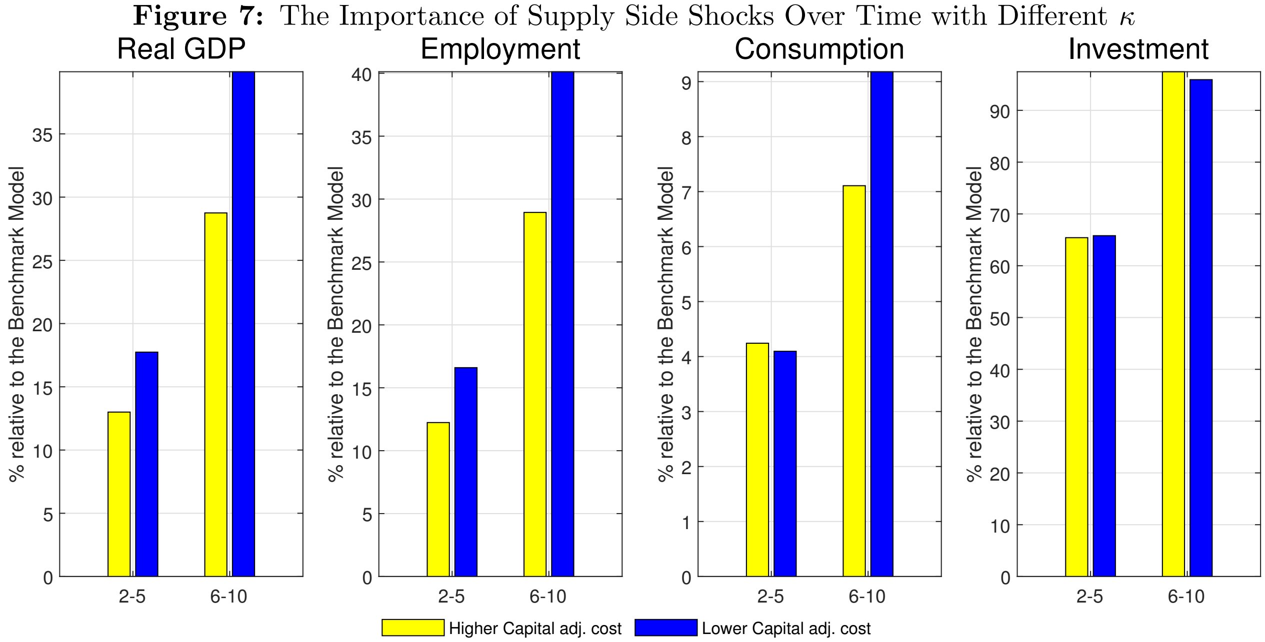


Figure 7: The Importance of Supply Side Shocks Over Time with Different κ



impacts of credit market deregulations. We change the value of θ_w, the Calvo parameter for not adjusting nominal wages within one period, to be 50% higher or 50% lower than the benchmark value. For each case, we also examine the cases where there are only shocks to households or to firms. The results are reported in Figure 8.

Intuitively, when nominal wages are assumed to be more sticky as compared to the benchmark case, wholesale firms both in the tradable and non-tradable goods sector will have higher profits following lowered interest rates and increased consumption demand. Therefore, these firms will hire more workers, and produce more, and more new firms are willing to enter. In the labor market, when wages are more sticky, households take any labor demand from firms as given (GHH preferences) and equilibrium employment increases more for the first few periods. For firms in the tradable goods sector, since wages are more sticky, the price increases for tradable output goods are relatively smaller (since for retailer firms, they have a constant markup ratio).

Overall, we observe a slightly larger effect for GDP and employment. After about 5 years, however, the differences due to sticky wages are no longer important, as we show in Figure 8. Turning to the relative importance of demand-side shocks vs. supply-side shocks when we have different degrees of wage stickiness, we report in Table 3 the decompositions as before. In columns (1) and (2), we report the percentage of responses that can be accounted by shocks to households or shocks to firms, respectively, for the case of more sticky wages. Similarly, columns (3) and (4) are for less sticky wages. We can still confirm our previous conclusion that demand-side shocks can explain about 75% of the responses in GDP, employment, and non-tradable outputs; shocks to firms are important for investment and more important in the longer run. Tradable output have opposite responses, and firm dynamics are affected by both shocks. This pattern is quite robust across different degrees of sticky wages.

Sticky Nominal Price for Output Goods. We also change the value of ϑ_P, the Calvo parameter for not adjusting nominal goods prices within one period, to be 50% higher or 50% lower than the benchmark value. The results are in Figure 9. As was the case for sticky wages, with more sticky prices in goods, increased consumption demand will increase employment and production; also as before, the differences between higher and lower nominal price rigidities die out in two periods after the shocks. For the relative importance of shocks to households only and to firms only, we find the pattern is still quite robust, as shown in Table 3 columns (5) to (8).

Consumers: Credit Constraints, Inter-temporal Substitution, and Labor Supply

Figure 8: The Role of Sticky Wages

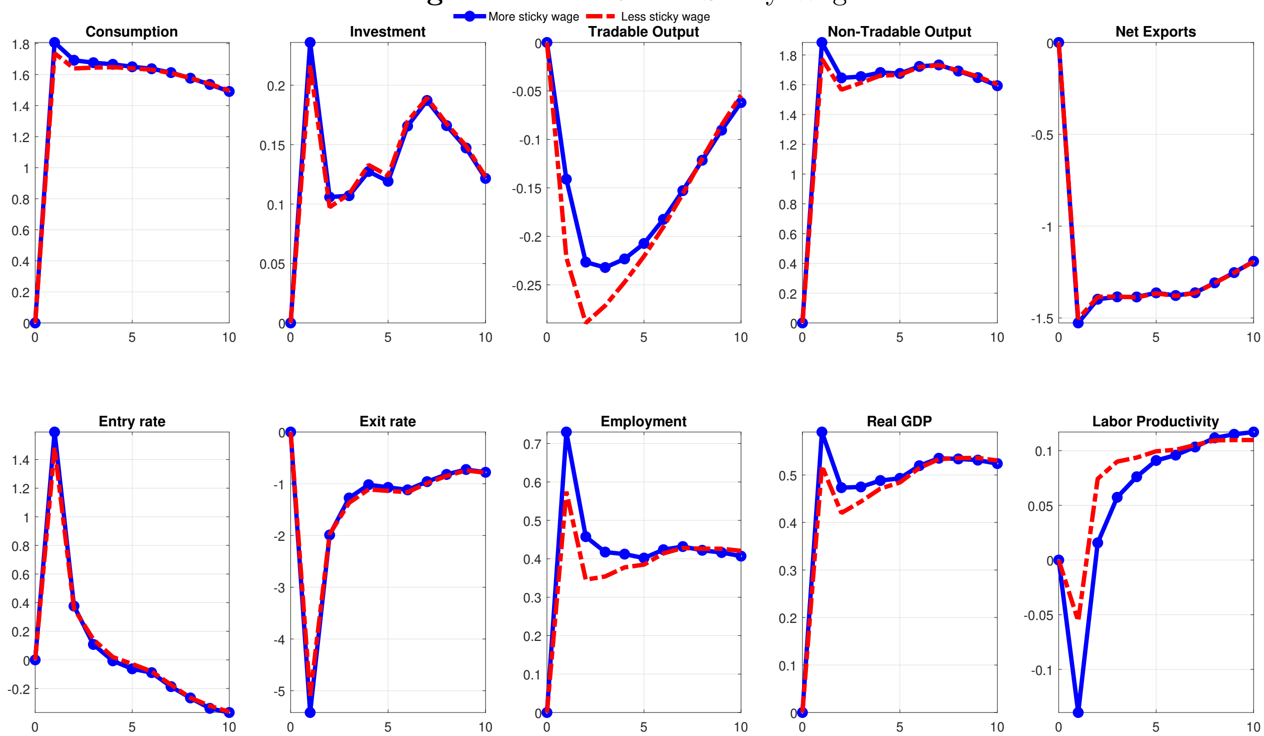
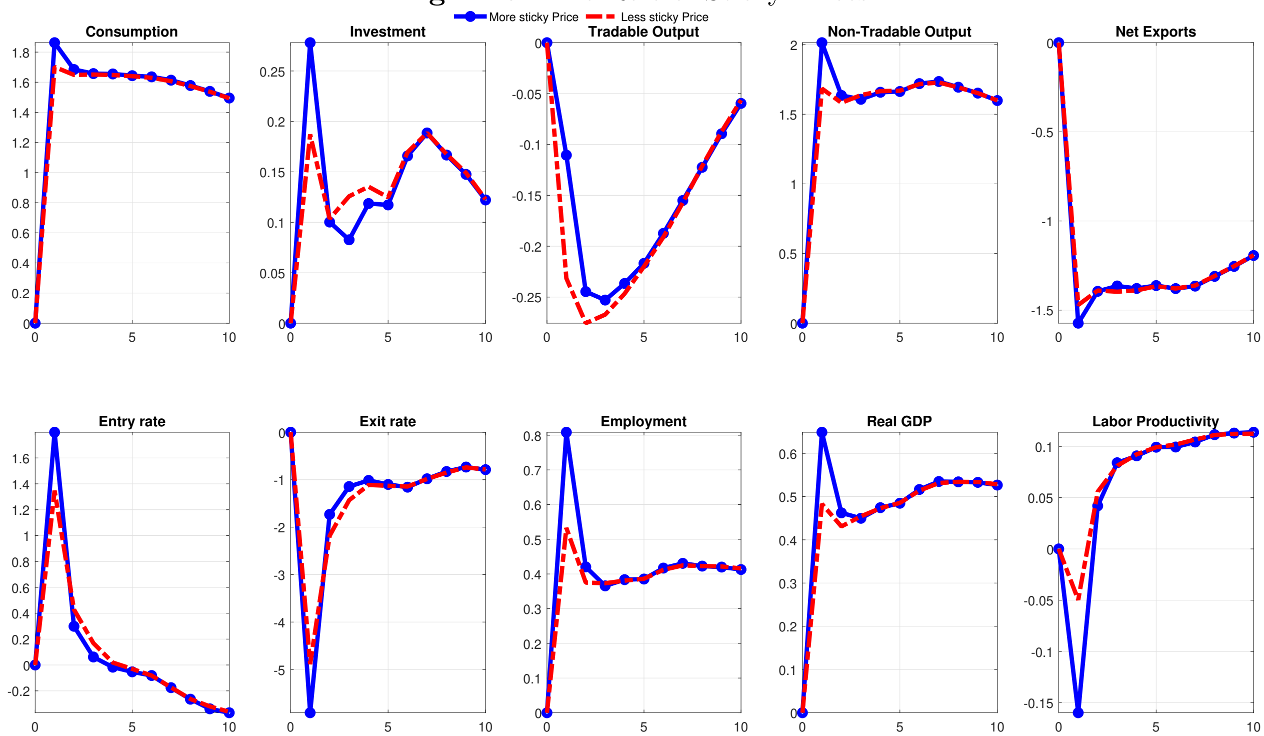


Table 3: Relative Importance of Demand Side Shocks vs. Supply Side Shocks (%): the Role of Sticky Wages and Sticky Prices

	More sticky wage		Less sticky wage		More sticky Price		Less sticky Price	
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
	HHs	Firms	HHs	Firms	HHs	Firms	HHs	Firms
GDP	74.6	24.2	74.1	24.8	74.7	24.1	74.0	24.9
Employment	76.0	22.8	74.8	24.0	76.0	22.8	74.7	24.1
Consumption	92.6	6.1	92.6	6.1	92.6	6.1	92.6	6.1
Investment	16.1	83.7	15.8	84.0	16.4	83.4	15.6	84.2
Tradable output	-141.4	43.3	-134.8	36.5	-139.9	41.8	-135.0	36.8
Non-Tradable output	82.9	16.0	82.8	16.1	83.0	15.9	82.8	16.1
Entry rates	115.2	-4.9	115.2	-4.0	112.5	-3.4	117.4	-7.9
Exit rates	-40.8	-60.9	-41.5	-60.6	-41.2	-60.1	-41.6	-60.4

Figure 9: The Role of Sticky Prices



Elasticity. Previous analysis shows that when changing interest rates, responses from household consumption demand is quantitatively important for most of the economic activity in the first few years. Thus, we next further inspect three model elements that are important determinants of consumption demand. First, as a large literature on the Heterogenous-Agents-New-Keynesian model (such as Kaplan, Moll, and Violante (2018) and Auclert (2019), Aguiar, Bils, and Boar (2020)) suggests that credit-constrained households may be important for business cycle analysis, we experiment and let the economy have more or fewer constrained households. We assume χ is 50% higher or 50% lower compared to benchmark model; that is, the percentage of households that are constrained close to 40% or 10%, which is a fairly wide range. We report our results in Table 6 in the Appendix. We find that, although more constrained households will have a slightly larger responses in GDP and employment (see Figure 27 in the Appendix for the responses), the basic pattern regarding the relative importance of different channels is still quite robust. We also study the role of labor supply elasticity υ_L (with different values of 5.0 and 1.25), and the parameter for inter-temporal substitution/risk aversion σ. We find that when labor supply is more elastic, supply-side shocks can account for more responses in employment and GDP across years 2 to 10; however, the differences with different υ_L overall are not large. However, when households have lower inter-temporal substitution incentives, the demand-side effects will be greatly reduced, and shocks to firms largely account for responses in GDP, employment, and consumption. Intuitively, for interest rate shocks on households to work, it is important that workers are able to adjust their consumption path over time.

Producers: Entry Costs. With respect to producers' responses after credit deregulations, there are both incumbent firms' responses and also new firms' responses. As we discussed previously, changes in aggregate nominal demand due to demand-side shocks, as well as the investment and capital stock dynamics in the economy due to supply-side shocks, all affect the profitability for wholesaler firms. When profitability for incumbent firms increases, new potential firms are more likely enter and those incumbent firms who were on the margin of exiting absent credit deregulations will now choose not to exit. In Table 7 in the Appendix we further experiment with different values in entry cost parameters: ν, the linear coefficient for entry cost, and Θ, the elasticity of entry cost with respect to aggregate entry rate (also recall the model comparative statics in Figures 20 and 21). We find that the entry rates and exit rates that can be accounted for by different types of shocks change slightly, with supply-side shocks having slightly larger importance with lower costs; however, the overall differences on aggregate economic variables with different cost parameters are quite small, and our previous main pattern is still robust. Intuitively, this is mainly because in the short run after credit deregulations, new firms and exiting firms comprise only a small fraction of all active firms in the economy, and the majority of incumbent firms are not affected by these parameter changes.

6 Concluding Remarks

This paper explores, both theoretically and empirically, the effects of changes in credit conditions on the real economy. To the best of our knowledge, our paper is the first to integrate a quantitative, Heterogenous-Firm-New-Keynesian model with causal empirical evidence from a natural experiment for credit market deregulations. The model is suitable for studying the effects of changing interest rates in a small open economy, and we include rich features for quantitative exercises, including both tradable and non-tradable goods sectors, nominal frictions in both goods prices and in wages, and both endogenous firm entries and exits.[27] We find that our model can generate quite realistic responses for several key economic variables that are consistent with causal empirical evidence. Using our model as a laboratory, we can quantify the relative importance of demand-side effect vs. supply-side effect when there are changes in interest rates. Consumers' responses can account for about 80% of the total responses for most of the variables in the first 10 years after shocks, while firms can only explain about 20% of these responses. Producers' responses are more important for the relatively longer run, and the key variables on the supply-side, including investment as well as firm entry and exit, are all more responsive when interest rates faced by firms change. Overall, our empirical and quantitative analysis in this paper help us better understand the impacts and channels of credit market changes on the macroeconomy.

[27] As we deliberately keep our model relatively simple and standard, we acknowledge that several elements not currently in the model could potentially improve the model's fit for future studies. These elements could include time-to-build technology for firms (e.g., see Kydland and Prescott (1982)), firms may face additional credit market frictions such as collateral constraints (e.g., Kiyotaki and Moore (1997), Cooley and Quadrini (2001), and Midrigan and Xu (2014)), and firms may have endogenous saving and self financing, and habit formation for consumers (e.g., see Smets and Wouters (2003)), among others.

Online Appendix: Not Intended for Publication

A Data

In this section we investigate the effect of bank deregulation on economic activity by studying the relaxation of intrastate branching restrictions in the United States. Until the late 1970s most US states prohibited intrastate branching. Between mid 1970s and mid 1990s states lifted these restrictions in cohorts, such that by 1993 almost all states had deregulated their banking sector. The restrictions to banking were completely repealed with the passage of the Reigle-Neal Interstate Banking and Branching Efficiency Act of 1994.

A.1 Specification

We are interested in examining the dynamic impact of bank deregulation on various economic outcomes. To that end, we estimate the impulse response function of inequality to the deregulation of the banking sector by using the local projections estimator proposed by Jorda (2005) and extended by Teulings and Zubanov (2014) to correct for the bias that arises when controlling for state fixed effects.[28] The specification that we estimate is in equation (39).

A.2 Data

To assess the effect of bank deregulation on economic activity, we gather data on the timing of bank deregulation and state level economic characteristics. There are two bank reforms that occurred in the US between mid 1970s and mid 1990s: the interstate banking deregulation and the intrastate branching deregulation. In what follows, we provide some historical context for these reforms and highlight how we construct the bank deregulation indicators D_{it}^{intra} and D_{it}^{inter} used in estimating equation (39). We draw similar conclusions from the two reforms, so in estimating the parameters of the model we use the impulse response functions to the intrastate branching deregulation. This also isolates the mechanism we explore from consideration of geographic expansion that firms might have and that we abstract from.

After the United States Constitution prohibited the states from issuing fiat money and from taxing interstate trade, states used their regulatory authority over banks to generate a substantial part of their revenues (Sylla, Legler, and Wallis, 1987). Since states could not exert charter fees from banks incorporated in other states, they prohibited out-of-state banks from operating in their territories – hence the origin of the prohibition on *interstate banking*. In addition to excluding banks from other states, the legislatures often restricted intrastate bank expansion – prohibition on *intrastate branching*. The 1927 McFadden Act formalized the

[28] Teulings and Zubanov (2014) show that, when controlling for group fixed effects (state fixed effects in our application), the local projection impulse response function is subject to a downward bias for $k \geq 1$. The bias increases with the forecast horizon k. Their suggested solution is to include in the local projection equations dummy variables for the event of interest (bank deregulation in our application) occurring between t and $t+k$. This not only improves efficiency, but also takes away the bias in the local projection estimator. In our specification, the effect of these dummy variables in captured by the coefficients $\delta_{j,k}$.

authority of the states over the regulation of national banks branching activities within their borders (Kroszner and Strahan, 2014). After the passage of the McFadden Act, banks tried to circumvent state branching restrictions by building multibank holding companies, but The Douglas Amendment to the 1956 Bank Holding Company Act ended this practice. According to the Douglas Amendment, banking was a state right and, therefore, states had the option to exclude out-of-state banks or holding companies from buying or building a bank or branch in their state. All states exercised this option, effectively prohibiting interstate banking.

Interstate banking deregulation. In 1978 Maine took the first step towards interstate banking deregulation by passing a law that allowed entry by bank holding companies from any state that allowed entry by Maine banks. The next to follow were Alaska and New York, who passed similar laws in 1982. Other states followed gradually, so that by 1992 all states but Hawaii had lifted interstate banking restrictions. These state changes, however, did not permit banks to open branches across state lines (Rice and Strahan, 2010). The restrictions to interstate banking were completely repealed with the passage of the Reigle-Neal Interstate Banking and Branching Efficiency Act of 1994. Reigle-Neal made interstate banking a bank right, not a state right, so that banks or holding companies could now enter another state without permission. Table 4 compiles the interstate banking deregulation dates and illustrates how states deregulated in cohorts rather than at the same time to reform, which we exploit in our empirical analysis. Based on this table we construct a dummy indicator D_{it}^{inter} that we set equal to 1 if state i lifted its restrictions to interstate banking in year t and zero otherwise.

Intrastate branching deregulation. Although there was some deregulation of branching restrictions in the 1930s, most states continued to enforce intrastate branching restrictions well into the 1970s. Only 11 states allowed unrestricted statewide branching in 1970 and a total of 16 states prohibited branching entirely. Between 1970 and 1994, however, 38 states eased their restrictions on branching (Rice and Strahan, 2010). First, states permitted branching via mergers and acquisitions through the holding company structure. Second, states began allowing banks to open new branches anywhere within state borders. Table 5 compiles the dates at which states started lifting intrastate branching restrictions. These dates refer to the year in which a state allowed branching via mergers and acquisitions. As before, we construct a dummy indicator D_{it}^{intra} that we set equal to 1 if state i lifted its restrictions to intrastate branching in year t and zero otherwise. Since the states listed in the first line of Table 5 lifted these restrictions before 1970 and the exact date is not known with certainty, the dummy variable is not defined before 1970 for these states.[29]

Indicators of economic activity. We consider various indicators of economic activity. To assess the immediate effect of the bank deregulation on the banking sector we examine the impulse responses of branch density, interest rate on loans, quantity of loans to households and businesses. We obtain these from the Call Report data of institutions regulated by the Federal Deposit Insurance Corporation (FDIC).

To asses the effect of bank deregulation on the activity of firms we measure the response of firm entry and exit, wages and employment. We use data on the number of establishments

[29] This is one of the reasons we choose to estimate equations for each type of reform. The obvious alternative, which we implement as a robustness exercise, is to simultaneously control for both. However, in this case we lose all observations related to interstate banking deregulation for all the states which lifted intrastate branching restrictions prior to 1970.

and establishment entry and exit from the Business Dynamics Statistics database. We use wage and employment data from the Bureau of Economic Analysis (BEA).

Table 4: Timing of Interstate Banking Deregulation

Year	States	Total
1978	Maine	1
1982	Alaska, New York	2
1983	Connecticut, Massachusetts	2
1984	Kentucky, Rhode Island, Utah	3
1985	District of Columbia, Florida, Georgia, Idaho, Maryland, Nevada, North Carolina, Ohio, Tennessee, Virginia	10
1986	Arizona, Illinois, Indiana, Michigan, Minnesota, Missouri, New Jersey, Oregon, Pennsylvania, South Carolina	10
1987	Alabama, California, Louisiana, New Hampshire, Oklahoma, Texas, Washington, Wisconsin, Wyoming	9
1988	Colorado, Delaware, Mississippi, South Dakota, Vermont, West Virginia	6
1989	Arkansas, New Mexico	2
1990	Nebraska	1
1991	Iowa, North Dakota	2
1992	Kansas	1
1993	Montana	1
1999	Hawaii*	1

Source: Morgan, Rime, and Strahan (2004); *Park (2012)

B Solving for the Optimal Choices of the Agents

B.1 Solving the Household's Problem

We then consider the optimality conditions of the households. Let $\Gamma\lambda_t^S P_t$ denote the marginal utility of consumption for savers at time t. Then, we have:

$$\lambda_t^S P_t = \left[C_t^S - \phi\int_{j=0}^{1}\frac{N_{j,t}^{1+\upsilon}}{1+\upsilon}\mathrm{d}j\right]^{-\sigma} \tag{B.1}$$

$$\lambda_t^S - \beta(1+r_t^H)E_t\lambda_{t+1}^S = 0. \tag{B.2}$$

Table 5: Timing of Intrastate Branching Deregulation

Year	States	Total
< 1970	Alaska, Arizona, California, District of Columbia, Idaho, Maryland, Nevada, North Carolina, Rhode Island, South Carolina	10
1970	Vermont	1
1975	Maine	1
1976	New York	1
1977	New Jersey	1
1978	Virginia	1
1979	Ohio	1
1980	Connecticut	1
1981	Alabama, Utah	2
1982	Pennsylvania	1
1983	Georgia	1
1984	Massachusetts	1
1985	Nebraska, Oregon, Tennessee, Washington	4
1986	Hawaii, Mississippi	2
1987	Michigan, New Hampshire, North Dakota, West Virginia, Kansas	5
1988	Oklahoma, Florida, Illinois, Louisiana, Texas, Wyoming	6
1989	Indiana	1
1990	Kentucky, Missouri, Montana, Wisconsin	4
1991	Colorado, New Mexico	2
1993	Minnesota	1
1994	Arkansas	1
1999	Iowa	1

Source: Morgan, Rime, and Strahan (2004)

B.2 Wage Setting

Following the arguments of (Galí, 2015, Chapter 6), it is straightforward to show that firm i's demand for labor j will satisfy:

$$n_{i,j,t} = \left(\frac{w_{j,t}}{w_t}\right)^{-\epsilon_w} n_{i,t}$$

where w_t is an aggregate wage index:

$$w_t = \left(\int_0^1 w_{j,t}^{1-\epsilon_w} \mathrm{d}j \right)^{\frac{1}{1-\epsilon_w}}$$

Aggregating across intermediate goods firms, we therefore have:

$$N_{j,t} = \left(\frac{w_{j,t}}{w_t} \right)^{-\epsilon_w} N_t = w_{j,t}^{-\epsilon_w} \hat{w}_t$$

where $\hat{w}_t = w_t^{\epsilon_w} N_t$ is an aggregate variable, and

$$N_t = \int_{i \in I_N cup I_H} n_{i,t} di = \int_{i \in I_N \cup I_H} \left(\int_0^1 n_{i,j}^{\frac{\epsilon_w - 1}{\epsilon_w}} \mathrm{d}j \right)^{\frac{\epsilon_w}{\epsilon_w - 1}} di$$

Recall that a worker j who is free to choose their wage in period t sets $w_{j,t} \equiv w_{j,t}^\star$ to maximize (4):

$$\mathbb{E}_t \sum_{k=0}^{\infty} (\beta\theta_w)^k \left(\frac{w_{j,t}^\star N_{j,t+k}}{P_{t+k}} - \phi \frac{N_{j,t+k}^{1+\upsilon}}{1+\upsilon} \right),$$

Substituting in the labor demand function, this is:

$$\mathbb{E}_t \sum_{k=0}^{\infty} (\beta\theta_w)^k \left(\frac{w_{j,t}^{\star 1-\epsilon_w} \hat{w}_{t+k}}{P_{t+k}} - \phi \frac{\hat{w}_{t+k}^{1+\upsilon} w_{j,t}^{\star -\epsilon_w(1+\upsilon)}}{1+\upsilon} \right),$$

The first order condition for optimization is:

$$0 = \mathbb{E}_t \sum_{k=0}^{\infty} (\beta\theta_w)^k \left(\frac{(1-\epsilon_w) w_{j,t}^{\star -\epsilon_w} \hat{w}_{t+k}}{P_{t+k}} + \epsilon_w (1+\upsilon) \phi w_{j,t}^{\star -\epsilon_w(1+\upsilon)-1} \cdot \frac{\hat{w}_{t+k}^{1+\upsilon}}{1+\upsilon} \right)$$

which rearranges to:

$$0 = \mathbb{E}_t \sum_{k=0}^{\infty} (\beta\theta_w)^k \left(\frac{(1-\epsilon_w) w_{j,t}^{\star 1+\epsilon_w \upsilon} \hat{w}_{t+k}}{P_{t+k}} + \epsilon_w \phi \hat{w}_{t+k}^{1+\upsilon} \right)$$

which rearranges to:

$$w_{j,t}^{\star 1+\epsilon_w \upsilon} = \frac{\mathbb{E}_t \sum_{k=0}^{\infty} (\beta\theta_w)^k \left(\epsilon_w \phi \hat{w}_{t+k}^{1+\upsilon} \right)}{\mathbb{E}_t \sum_{k=0}^{\infty} (\beta\theta_w)^k \left((\epsilon_w - 1) P_{t+k}^{-1} \hat{w}_{t+k} \right)}$$

Now, let

$$\mathcal{A}_{w,t} = \mathbb{E}_t \sum_{k=0}^{\infty} (\beta\theta_w)^k \left(\epsilon_w \phi \hat{w}_{t+k}^{1+\upsilon} \right)$$

$$\mathcal{B}_{w,t} = \mathbb{E}_t \sum_{k=0}^{\infty} (\beta\theta_w)^k (\epsilon_w - 1) P_{t+k}^{-1} \hat{w}_{t+k}$$

These satisfy the recursive equations:

$$\mathcal{A}_{w,t} = \epsilon_w \phi \hat{w}_t^{1+\upsilon} + \beta\theta_w \mathbb{E}_t \mathcal{A}_{w,t+1}$$

$$\mathcal{B}_{w,t} = (\epsilon_w - 1) P_t^{-1} \hat{w}_t + \beta\theta_w \mathbb{E}_t \mathcal{B}_{w,t+1}$$

So we use these recursions to solve for $\mathcal{A}_{w,t}$ and $\mathcal{B}_{w,t}$, and let:

$$w_{j,t}^{\star} = \left(\frac{\mathcal{A}_{w,t}}{\mathcal{B}_{w,t}}\right)^{\frac{1}{1+\epsilon_w \upsilon}}$$

Thus, we have that, for both savers and spenders:

$$\mathcal{A}_{w,t} = \epsilon_w \phi \left(w_t^{\epsilon_w} N_t\right)^{1+\upsilon} + \beta\theta_w \mathbb{E}_t \mathcal{A}_{w,t+1} \tag{B.3}$$

$$\mathcal{B}_{w,t} = (\epsilon_w - 1) P_t^{-1} w_t^{\epsilon_w} N_t + \beta\theta_w \mathbb{E}_t \mathcal{B}_{w,t+1} \tag{B.4}$$

$$w_{j,t}^{\star} = \left(\frac{\mathcal{A}_{w,t}}{\mathcal{B}_{w,t}}\right)^{\frac{1}{1+\epsilon_w \upsilon}}$$

and we see that $w_{j,t}$ and N_t are the same for each j and the same for savers and spenders.

Following the arguments of (Galí, 2015, Chapter 6), the evolution of the aggregate wage w_t is given by:

$$w_t = \left(\theta_w w_{t-1}^{1-\epsilon_w} + (1-\theta_w) w_t^{\star 1-\epsilon_w}\right)^{\frac{1}{1-\epsilon_w}}$$

Substituting in the equation for $w_{j,t}^{\star}$ above, we can write:

$$w_t = \left(\theta_w w_{t-1}^{1-\epsilon_w} + (1-\theta_w)\left(\frac{\mathcal{A}_{w,t}}{\mathcal{B}_{w,t}}\right)^{\frac{1-\epsilon_w}{1+\epsilon_w \upsilon}}\right)^{\frac{1}{1-\epsilon_w}} \tag{B.5}$$

We showed above that the saver's marginal utility of consumption at time t is:

$$\lambda_t^S P_t \quad = \quad \left[C_t^S - \phi \int_{j=0}^{1} \frac{N_{j,t}^{1+\upsilon}}{1+\upsilon} \mathrm{d}j\right]^{-\sigma}$$

Using the labor demand curve above, this is:

$$\lambda_t^S P_t \quad = \quad \left[C_t^S - \frac{\phi N_t^{1+\upsilon}}{1+\upsilon} \int_{j=0}^{1} \left(\left(\frac{w_{j,t}}{w_t}\right)^{-\epsilon_w}\right)^{1+\upsilon} \mathrm{d}j\right]^{-\sigma}$$

which is

$$\lambda_t^S P_t = \left[C_t^S - \frac{\phi N_t^{1+\upsilon} \mathcal{D}_{w,t}^{-\epsilon_w(1+\upsilon)}}{1+\upsilon} \right]^{-\sigma} \tag{B.6}$$

where the wage dispersion term $\mathcal{D}_w$ satisfies:

$$\mathcal{D}_{w,t} = \frac{\left[\int_0^1 w_{j,t}^{-\epsilon_w(1+\upsilon)} dj\right]^{\frac{-1}{\epsilon_w(1+\upsilon)}}}{w_t}$$

The dispersion term evolves according to:

$$\mathcal{D}_{w,t} w_t = \left(\theta_w \left(\mathcal{D}_{w,t-1} w_{t-1}\right)^{-\epsilon_w(1+\upsilon)} + (1-\theta_w) \left(\frac{\mathcal{A}_{w,t}}{\mathcal{B}_{w,t}} \right)^{\frac{-\epsilon_w(1+\upsilon)}{1+\epsilon_w \upsilon}} \right)^{\frac{-1}{\epsilon_w(1+\upsilon)}} \tag{B.7}$$

B.3 Solving the problem of the capital goods producer

Substituting the capital goods production function into the Bellman equation for the capital goods producer, we obtain:

$$\begin{aligned} \mathcal{Q}_t K_t = (r_t^K + \delta_K)(K_t) + M_t E_t \left[\mathcal{Q}_{t+1}\right] K_{t+1} \\ - \left(K_{t+1} - (1-\delta_K) K_t + \kappa \left(\frac{K_{t+1}}{K_t} - 1 \right)^2 (K_t) \right) \end{aligned}$$

The first order condition for K_{t+1} is: (omitting the S subscript)

$$E_t \left[M_t \mathcal{Q}_{t+1}\right] - 1 - 2\kappa \left(\frac{K_{t+1}}{K_t} - 1 \right) = 0$$

The envelope condition is:

$$\mathcal{Q}_t = \delta_K + r_t^K + \left((1-\delta_K) - \kappa \left(\frac{K_{t+1}}{K_t} - 1 \right)^2 + 2\kappa \left(\frac{K_{t+1}}{K_t} - 1 \right) \frac{K_{t+1}}{K_t} \right)$$

which simplifies to:

$$\mathcal{Q}_t = \delta_K + r_t^K + \left(1 - \delta_K + \kappa \left(\left(\frac{K_{t+1}}{K_t} \right)^2 - 1 \right) \right)$$

Combining the first order condition and the envelope condition to eliminate $\mathcal{Q}$, we get:

$$E_t[M_{t+1}(\delta_K + r^K_{S,t+1})] + E_t\left[M_{t+1}\left(1-\delta_K+\kappa\left(\left(\frac{K_{S,t+2}}{K_{S,t+1}}\right)^2-1\right)\right)\right] = \left(1+2\kappa\left(\frac{K_{S,t+1}}{K_{S,t}}-1\right)\right) \tag{B.8}$$

B.4 Solving the Wholesaler's Problem

For the static profit of wholesale firms. $\pi^W(z;\mathbf{X}_t)$, we have

$$\begin{aligned}\pi^W(z;\mathbf{X}_t,S) &\equiv \max_{k,n}\left\{\left(\frac{P_{W,S,t}}{P_t}\right)y_i-(r^K_t+\delta_K)k-\frac{w_t n}{P_t}-c^F\right\}\\ &= \max_{k,n}\left\{\left(\frac{P_{W,S,t}}{P_t}\right)z_i^{1-\theta}(k_i^{1-\alpha}n_i^{\alpha})^{\theta}-(r^K_t+\delta_K)k-\frac{w_t n}{P}-c^F\right\}\end{aligned} \tag{B.9}$$

The first order conditions are:

$$(r^K_t+\delta_K)k = (1-\alpha)\theta\left(\frac{P_{W,S,t}}{P_t}\right)z_i^{1-\theta}\cdot(k_i^{1-\alpha}n_i^{\alpha})^{\theta}$$

$$\left(\frac{w_t n}{P_t}\right) = \alpha\theta\left(\frac{P_{W,S,t}}{P_t}\right)z_i^{1-\theta}\cdot(k_i^{1-\alpha}n_i^{\alpha})^{\theta}$$

Then

$$k^{1-\alpha}n^{\alpha} = \left(\frac{\alpha^{\alpha}(1-\alpha)^{1-\alpha}}{(r^K_t+\delta_K)^{1-\alpha}\left(\frac{w_t}{P_t}\right)^{\alpha}}\right)\theta\left(\frac{P_{W,S,t}}{P_t}\right)z_i^{1-\theta}\cdot(k_i^{1-\alpha}n_i^{\alpha})^{\theta}$$

and so

$$\left(k^{1-\alpha}n^{\alpha}\right)^{\theta} = \left(\frac{\alpha^{\alpha}(1-\alpha)^{1-\alpha}}{(r^K_t+\delta_K)^{1-\alpha}\left(\frac{w_t}{P_t}\right)^{\alpha}}\right)^{\frac{\theta}{1-\theta}}\left(\theta\left(\frac{P_{W,S,t}}{P_t}\right)\right)^{\frac{\theta}{1-\theta}}z_i^{\theta}$$

and so

$$\pi^W(z;\mathbf{X}_t,S) = (1-\theta)\left(\frac{\alpha^{\alpha}(1-\alpha)^{1-\alpha}}{(r^K_t+\delta_K)^{1-\alpha}\left(\frac{w_t}{P_t}\right)^{\alpha}}\right)^{\frac{\theta}{1-\theta}}\theta^{\frac{\theta}{1-\theta}}\left(\frac{P_{W,S,t}}{P_t}\right)^{\frac{1}{1-\theta}}z_i-c^F$$

Therefore, we may write:

$$\pi^W(z;\mathbf{X}_t,S) = \overline{\pi}^W_{S,t}z_i-c^F$$

where

$$\overline{\pi}^W_{S,t} = (1-\theta)\left(\frac{\alpha^{\alpha}(1-\alpha)^{1-\alpha}}{(r^K_t+\delta_K)^{1-\alpha}\left(\frac{w_t}{P_t}\right)^{\alpha}}\right)^{\frac{\theta}{1-\theta}}\theta^{\frac{\theta}{1-\theta}}\left(\frac{P_{W,S,t}}{P_t}\right)^{\frac{1}{1-\theta}} \tag{B.10}$$

The total revenue of wholesaler i is:

$$P_{W,S,t}y_i = P_t\left(\frac{1}{1-\theta}\right)\left(\pi^W(z;\mathbf{X}_t,S)+c_F\right) = \frac{P_t\overline{\pi}^W_{S,t}z_i}{1-\theta}$$

C Aggregate Output and Factor Demands

The total nominal wholesaler output in sector S can be written as

$$P_{W,S,t}Y_{W,S,t} = \int_{i\in I_S} P_{W,S,t}y_i di$$

Using our result above for $P_{W,S}y_i$, this is:

$$P_{W,S,t}Y_{W,S,t} = \frac{P_t\overline{\pi}^W_{S,t}M^W_{S,t}\overline{z}_{S,t}}{1-\theta} \tag{C.1}$$

Factor Demands: Adding up the factor demands in each sector, we obtain:

$$N^D_{S,t} = \alpha\theta\frac{P_{W,S,t}Y_{W,S,t}}{w_t}, \tag{C.2}$$

$$K^D_{S,t} = (1-\alpha)\,\theta\frac{P_{W,S,t}Y_{W,S,t}}{P_t(r^K_{S,t}+\delta_K)}. \tag{C.3}$$

Aggregate Profits : Adding up the profits for the wholesale firms in each sector and combining with the equation for aggregate output above, we obtain:

$$\pi^W_{S,t} = (1-\theta)P_{W,S,t}Y_{W,S,t} - c^F P_t M^W_{S,t}. \tag{C.4}$$

C.1 Implications of Market Clearing

Our results above for the final goods firm and for exports imply immediately that:

$$P_HY_H = \overline{X}P_H^{-\zeta} + \omega_H\gamma Y^dP\left(P_H^{\gamma}P_N^{1-\gamma}P^{-1}\right)^{\frac{1-\zeta}{\gamma}}$$

$$P_NY_N = (1-\gamma)PY^d$$

$$P = P_N^{1-\gamma}\left(\left[\omega_HP_H^{1-\zeta}+(1-\omega_H)\right]^{\frac{1}{1-\zeta}}\right)^{\gamma}$$

C.2 Further Aggregation

Above, we defined aggregate profits of all firms Π_t:

$$\Pi_t = \sum_{S\in\{N;H\}}\pi^W_{S,t} + \Pi^K_t + \Pi^R_t - \nu P_tM^e_{H,t}\left(\frac{M^e_{H,t}}{M^W_{H,t-1}}\right)^{\Theta} - \nu P_tM^e_{N,t}\left(\frac{M^e_{N,t}}{M^W_{N,t-1}}\right)^{\Theta}.$$

Using equations (C.4) and (7) to eliminate $\pi^W_{S,t}$ and Π^K_t from this, we obtain:

$$\Pi_t = \sum_{S\in\{N;H\}} (1-\theta)P_{W,S,t}Y_{W,S,t} - c^F P_t M^W_{S,t} - \nu P_t M^e_{S,t}\left(\frac{M^e_{S,t-1}}{M^W_{S,t-1}}\right)^\Theta + (r^K_t + \delta_K)(K_t) - P_t I_t + \Pi^R_t.$$

Using (C.3) and (29) to eliminate r^K_t from this, we obtain:

$$\Pi_t = \sum_{S\in\{N;H\}} ((1-\theta) + (1-\alpha)\theta)\, P_{W,S,t}Y_{W,S,t} - c^F P_t M^W_{S,t} - \nu P_t M^e_{S,t}\left(\frac{M^e_{S,t}}{M^W_{S,t-1}}\right)^\Theta - P_t I_t + \Pi^R_t.$$

Using (6) to eliminate I_t from this, we obtain

$$\begin{aligned}\Pi_t = \sum_{S\in\{N;H\}} & (1-\alpha\theta)\, P_{W,S,t}Y_{W,S,t} - c^F P_t M^W_{S,t} - \nu P_t M^e_{S,t}\left(\frac{M^e_{S,t}}{M^W_{S,t-1}}\right)^\Theta \\ & -P_t\left(K_{t+1} - (1-\delta_K)K_t + \kappa\left(\frac{K_{t+1}}{K_t} - 1\right)^2 K_t\right) + \Pi^R_t.\end{aligned}$$

Using (15) to eliminate Π^R_t, we obtain:

$$\begin{aligned}\Pi_t = \sum_{S\in\{N;H\}} & P_{S,t}Y_{S,t} - \alpha\theta P_{W,S,t}Y_{W,S,t} - c^F P_t M^W_{S,t} - \nu P_t M^e_{S,t}\left(\frac{M^e_{S,t}}{M^W_{S,t-1}}\right)^\Theta \\ & -P_t\left(K_{t+1} - (1-\delta_K)K_t + \kappa\left(\frac{K_{t+1}}{K_t} - 1\right)^2 K_t\right) - c^R P_t.\end{aligned}$$

Since final goods firms make zero profits, it holds that their revenue must equal their costs, so that:

$$P_t Y^d_t = \sum_{S\in\{N,H,F\}} P_{S,t} Y^d_{S,t}$$

Using the definition of net exports in (13) and the non-tradable market clearing condition $Y^d_{N,t} = Y_{N,t}$, this is:

$$P_t Y^d_t + NX_t = \sum_{S\in\{N,H\}} P_{S,t} Y_{S,t}.$$

Substituting this into the equation above and using the labor demand equation (C.2), we obtain:

$$\begin{aligned}\Pi_t = P_t Y^d_t + NX_t - w_t N_t - \sum_{S\in\{N;T\}} & \left(c^F P_t M^W_{S,t} + \nu P_t M^e_{S,t}\left(\frac{M^e_{S,t}}{M^W_{S,t-1}}\right)^\Theta\right) \\ & -P_t\left(K_{t+1} - (1-\delta_K)K_t + \kappa\left(\frac{K_{t+1}}{K_t} - 1\right)^2 K_t\right) - P_t c^R.\end{aligned}$$

Finally, using final goods market clearing, we obtain:

$$\Pi_t = P_t C_t + NX_t - w_t N_t \tag{C.5}$$

Combining the market clearing conditions for tradables and non-tradables with the final goods producers' first order conditions, we get:

$$\begin{aligned} P_{N,t}Y_{N,t} &= P_{N,t}Y^d_{N,t} = (1-\gamma)P_t Y^d_t, \\ P_{H,t}Y_{H,t} &= P_{H,t}Y^d_{H,t} + P_{H,t}Y^*_{H,t} = \overline{X}P^{1-\zeta}_{H,t} + \omega_H \gamma Y^d_t P_t \left(P^{\gamma}_{H,t} P^{1-\gamma}_{N,t} P^{-1}_t\right)^{\frac{1-\zeta}{\gamma}} \end{aligned}$$

Substituting in the expression for net exports, we get:

$$P_{H,t}Y_{H,t} = \gamma P_t Y^d_t + NX_t$$

D Characterizing the Equilibrium

We now list the equations from above that completely characterize the equilibrium. We repeat all these equations here, for convenience.

First, the saver's decision must satisfy their first order condition laid out in Section B.1 and (B.6)

$$\lambda^S_t P_t = \left[C^S_t - \frac{\phi N^{1+\upsilon}_t D^{-\epsilon_w(1+\upsilon)}_{W,t}}{1+\upsilon}\right]^{-\sigma} \tag{D.1}$$

$$\lambda^S_t - \beta(1+r^H_t)E_t\lambda^S_{t+1} = 0. \tag{D.2}$$

Household choices must satisfy their budget constraints (2) and (3) and they face the interest rates given by (5):

$$P_t C^L_t = w_t N_t, \tag{D.3}$$

$$B_t + (1-\chi)P_t C^S_t = \Pi_t + (1-\chi)w_t N_t + B_{t-1}\left(1 + r^H_{t-1}\right) \tag{D.4}$$

Savers earn the total profits of firms net of entry and capital costs (C.5) :

$$\Pi_t = P_t\left((1-\chi)C^S_t + \chi C^L_t\right) + NX_t - w_t N_t. \tag{D.5}$$

Wages evolve according to (B.5), with $\mathcal{A}_{w,t}$ and $\mathcal{B}_{w,t+1}$ evolving according to equations (B.3) and (B.4):

$$w_t = \left(\theta_w w^{1-\epsilon_w}_{t-1} + (1-\theta_w)\left(\frac{\mathcal{A}_{w,t}}{\mathcal{B}_{w,t}}\right)^{\frac{1-\epsilon_w}{1+\epsilon_w \upsilon}}\right)^{\frac{1}{1-\epsilon_w}} \tag{D.6}$$

$$\mathcal{A}_{w,t} = \epsilon_w \phi \left(w^{\epsilon_w}_t N_t\right)^{1+\upsilon} + \beta\theta_w \mathbb{E}_t \mathcal{A}_{w,t+1} \tag{D.7}$$

$$\mathcal{B}_{w,t} = (\epsilon_w - 1)P^{-1}_t w^{\epsilon_w}_t N_t + \beta\theta_w \mathbb{E}_t \mathcal{B}_{w,t+1} \tag{D.8}$$

Combining the market clearing conditions for tradables and non-tradables with the final goods producers' first order conditions, we get:

$$P_{N,t}Y_{N,t} = (1-\gamma)P_tY_t^d, \tag{D.9}$$
$$P_{H,t}Y_{H,t} = \gamma P_tY_t^d + NX_t \tag{D.10}$$

where we found above that the final goods price satisfies:

$$P_t = P_{N,t}^{1-\gamma}\left(\left[\omega_H P_{H,t}^{1-\zeta} + (1-\omega_H)\right]^{\frac{1}{1-\zeta}}\right)^{\gamma} \tag{D.11}$$

Net exports are given by:

$$NX_t = \overline{X}P_{H,t}^{1-\zeta} - (1-\omega_H)\gamma Y_t^d P_t\left(P_{N,t}^{1-\gamma}P_t^{-1}\right)^{\frac{1-\zeta}{\gamma}} \tag{D.12}$$

The market for final goods clears:

$$Y_t^d = (1-\chi)C_t^S + \chi C_t^L + \sum_{S\in\{N;T\}}\left(c^F M_{S,t}^W + \nu M_{S,t}^e\left(\frac{M_{S,t-1}^e}{M_{S,t-1}^W}\right)^{\Theta}\right) \tag{D.13}$$
$$+\left(K_{t+1} - (1-\delta_K)K_t + \kappa\left(\frac{K_{t+1}}{K_t} - 1\right)^2 K_t\right) + c^R. \tag{D.14}$$

Prices in each sector evolve according to (14):

$$\left(\frac{P_{S,t}}{P_{S,t-1}}\right)^{1-\eta} = \vartheta_P + (1-\vartheta_P)\left(\frac{\mathcal{A}_{P,S,t}}{\mathcal{B}_{P,S,t}P_{S,t-1}}\right)^{1-\eta}. \tag{D.15}$$

where

$$\mathcal{A}_{P,S,t} = \left(\frac{\eta}{\eta-1}\right)P_{S,t}^{\eta}P_t^{-1}Y_{S,t}P_{W,S,t} + \vartheta_P M_{t+1}\mathbb{E}_t\mathcal{A}_{P,,S,t+1} \tag{D.16}$$
$$\mathcal{B}_{P,S,t} = P_{S,t}^{\eta}P_t^{-1}Y_{S,t} + \vartheta_P M_{t+1}\mathbb{E}_t\mathcal{B}_{P,S,t+1}. \tag{D.17}$$

The dynamics of the total number of wholesalers in each sector and their productivity is given by Proposition 1:

$$j_{S,t} = \max\left[\left\{k \in \{0,1,...,t-1\} \middle| z^\star_{S,k} > z^\star_{S,t} \text{ or } z^\star_{S,k} \geq z^\star_{S,i},\ \forall i < k\right\} \cup \{0\}\right] \tag{D.18}$$

$$\overline{z}_{S,t} = \frac{\xi z^\star_{S,t}}{\xi - 1} + \frac{\rho_z^{t-j_{S,t}} M^W_{S,j_{S,t}}}{M^W_{S,t}}\left(\overline{z}_{S,j_{S,t}} - \frac{\xi \min\{z^\star_{S,t}; z^\star_{S,j_{S,t}}\}}{\xi - 1}\right) \tag{D.19}$$

$$M^W_{S,t} = \rho_z^{t-j_{S,t}} M^W_{S,j_{S,t}} \min\left\{1; \left(\frac{z^\star_{S,t}}{z^\star_{S,j_{S,t}}}\right)^{-\xi}\right\} + (z^\star_{S,t})^{-\xi} \sum_{i=1}^{t-j_{S,t}} \rho_z^{i-1}((1-\rho_z)M^W_{S,t-i} + M^e_{S,t-i+1}) \tag{D.20}$$

The total nominal output of wholesalers in each sector is given by equation (C.1), with $\overline{\pi}^W_{S,t}$ given by equation (B.10):

$$P_{W,S,t}Y_{W,S,t} = \frac{P_t \overline{\pi}^W_{S,t} M^W_{S,t} \overline{z}_{S,t}}{1-\theta} \tag{D.21}$$

$$\overline{\pi}^W_{S,t} = (1-\theta)\left(\frac{\alpha^\alpha (1-\alpha)^{1-\alpha}}{(r^K_t + \delta_K)^{1-\alpha}\left(\frac{w_t}{P_t}\right)^\alpha}\right)^{\frac{\theta}{1-\theta}} \theta^{\frac{\theta}{1-\theta}} \left(\frac{P_{W,S,t}}{P_t}\right)^{\frac{1}{1-\theta}}. \tag{D.22}$$

The total demands for factors of production in each sector is given by (C.2) and (C.3), with goods, labor and capital markets clearing (according to (29)):

$$N_t = \sum_S \alpha\theta \frac{P_{W,S,t}Y_{W,S,t}}{w_t}, \tag{D.23}$$

$$K_t = \sum_S (1-\alpha)\,\theta \frac{P_{W,S,t}Y_{W,S,t}}{P_t(r^K_{S,t} + \delta_K)}. \tag{D.24}$$

The free entry (34)and free exit (35) conditions need to be satisfied:

$$\nu \left(\frac{M^e_{S,t}}{M^W_{S,t-1}}\right)^\Theta = \textstyle\sum_{j=0}^\infty \mathcal{A}_{t,j}\left(\frac{z^{\star-(\xi-1)}_{t,j}}{\xi-1}\right) \tag{D.25}$$

$$z^\star_t = \frac{c_F - M_t(1-\rho_z)\sum_{j=0}^\infty \mathcal{A}_{t+1,j}\left(\frac{z^{\star-(\xi-1)}_{t+1,j}}{\xi-1}\right)}{(1-\rho_Z M_t)A_t} - \left(\frac{1}{(1-\rho_z M_t)A_t}\right)\textstyle\sum_{j=1}^\infty \mathcal{A}_{t,j}\left(z^\star_{t,j} - z^\star_{t+1,j-1}\right) \tag{D.26}$$

where

$$A_{S,t} = \frac{\overline{\pi}^W_{S,t}}{1 - \rho_z M_t} \tag{D.27}$$

$$z^\star_{S,t,j} = \max\{z^\star_{S,\tau}\}_{\tau=t}^{\tau=t+j} \tag{D.28}$$

$$\mathcal{A}_{t,j} = (1 - \rho_Z M_{t+j}) A_{t+j} \frac{\Pi_{k=0}^{k=j}(M_{t+k}\rho_z)}{M_{t+j}\rho_z} \tag{D.29}$$

Each firm's SDF is given by (8)

$$M_t = \frac{P_{t+1}}{P_t(1 + r_{F,t})}, \tag{D.30}$$

The quantity of capital produced satisfies (B.8):

$$E_t[M_{t+1}(\delta_K + r^K_{S,t+1})] + E_t\left[M_{t+1}\left(1 - \delta_K + \kappa\left(\left(\frac{K_{S,t+2}}{K_{S,t+1}}\right)^2 - 1\right)\right)\right]$$
$$= \left(1 + 2\kappa\left(\frac{K_{S,t+1}}{K_{S,t}} - 1\right)\right) \tag{D.31}$$

Total wholesale output in each sector is given by (16)

$$Y_{W,S,t} = Y_{S,t}\left(\mathcal{D}_{S,t}\right)^{-\eta} \tag{D.32}$$

Price dispersion in each sector and wage dispersion evolve according to (17) and (B.7):

$$\left(\frac{P_{S,t}\mathcal{D}_{S,t}}{P_{S,t-1}\mathcal{D}_{S,t-1}}\right)^{-\eta} = \vartheta_P + (1 - \vartheta_P)\left(\frac{\mathcal{A}_{P,S,t}}{\mathcal{B}_{P,S,t}P_{S,t-1}\mathcal{D}_{S,t-1}}\right)^{-\eta}. \tag{D.33}$$

$$\mathcal{D}_{w,t}w_t = \left(\theta_w\left(\mathcal{D}_{w,t-1}w_{t-1}\right)^{-\epsilon_w(1+\upsilon)} + (1 - \theta_w)\left(\frac{\mathcal{A}_{w,t}}{\mathcal{B}_{w,t}}\right)^{\frac{-\epsilon_w(1+\upsilon)}{1+\epsilon_w\upsilon}}\right)^{\frac{-1}{\epsilon_w(1+\upsilon)}} \tag{D.34}$$

These equations constitute a system of 47 equations with 47 endogenous variables. Since none of the equations are redundant, this system of equations therefore (more or less) pins down the equilibrium. The 47 endogenous variables are: λ^S_t, w_t, Y^d_t, P_t NX_t, B_t, Π_t, r^H_t, M_t, $\mathcal{A}_{w,t}$, $\mathcal{B}_{w,t}$, C^S_t, C^L_t, K_t, r^K_t, N_t, $\mathcal{D}_{w,t}$, and, for each S, $\mathcal{A}_{P,S,t}$, $\mathcal{B}_{P,S,t}$, $P_{S,t}$, $P_{W,S,t}$, $Y_{S,t}$, $M^W_{S,t}$, $\overline{z}_{S,t}$, $M^e_{S,t}$, $\hat{W}^S_t$, $j_{S,t}$, $z^\star_{S,t}$, $\overline{\pi}_{S,t}$, $A_{S,t}$, $Y_{W,S,t}$, $\mathcal{D}_{S,t}$.

D.1 Simplifying the dynamic equations

First, we simplify the equations on the household side. Substitute the budget constraint of the spender, and the equation defining aggregate profits Π_t into the equation for λ^S and into the saver's budget constraint, we obtain:

$$\lambda_t^S = P_t^{-1}\left[\frac{C_t - \frac{\chi w_t N_t}{P_t}}{1-\chi} - \frac{\phi \mathcal{D}_{w,t}^{-\epsilon_w(1+\upsilon)} N_t^{1+\upsilon}}{1+\upsilon}\right]^{-\sigma}$$

and

$$B_t - B_{t-1}(1+r_{t-1}^H) = NX_t$$

Combining equations (D.21), (D.22) and (D.32)and rearranging, we obtain that, for each $S \in \{H, N\}$:

$$Y_{S,t} = \aleph^\theta \left(\mathcal{D}_{S,t}\right)^\eta \left(\overline{\pi}_{S,t}^W\right)^\theta M_{S,t}^W \overline{z}_{S,t} \tag{D.35}$$

where

$$\aleph^\theta = \frac{\left(\overline{\pi}_{S,t}^W\right)^{1-\theta} P_t}{(1-\theta)P_{W,S,t}} = \left[\frac{\theta}{1-\theta}\left(\frac{\alpha^\alpha(1-\alpha)^{1-\alpha}}{(r_t^K+\delta_K)^{1-\alpha}\left(\frac{w_t}{P_t}\right)^\alpha}\right)\right]^\theta$$

So that

$$\aleph = \frac{\theta}{1-\theta}\left(\frac{\alpha^\alpha(1-\alpha)^{1-\alpha}}{(r_t^K+\delta_K)^{1-\alpha}\left(\frac{w_t}{P_t}\right)^\alpha}\right) \tag{D.36}$$

Substituting (D.21) into (D.16) and using (D.32), we have:

$$\mathcal{A}_{P,S,t} = \left(\frac{\eta}{\eta-1}\right) P_{S,t}^\eta \left(\frac{\left(\mathcal{D}_{S,t}\right)^\eta \overline{\pi}_{S,t}^W M_{S,t}^W \overline{z}_{S,t}}{1-\theta}\right) + \vartheta_P M_{t+1}\mathbb{E}_t \mathcal{A}_{P,,S,t+1}. \tag{D.37}$$

We can rearrange (D.12) to get:

$$Y_t^d P_t = \frac{\overline{X} P_{H,t}^{1-\zeta} - NX_t}{(1-\omega_H)\gamma\left(P_{N,t}^{1-\gamma}P_t^{-1}\right)^{\frac{1-\zeta}{\gamma}}} \tag{D.38}$$

Substituting this in to (D.9) and (D.10), we get:

$$Y_{N,t} = \frac{(1-\gamma)\left(\overline{X} P_{H,t}^{1-\zeta} - NX_t\right)}{(1-\omega_H)\gamma P_{N,t}\left(P_{N,t}^{1-\gamma}P_t^{-1}\right)^{\frac{1-\zeta}{\gamma}}} \tag{D.39}$$

$$Y_{H,t} = \frac{\gamma\left(\overline{X} P_{H,t}^{1-\zeta} - NX_t\right)}{(1-\omega_H)\gamma P_{H,t}\left(P_{N,t}^{1-\gamma}P_t^{-1}\right)^{\frac{1-\zeta}{\gamma}}} + \frac{NX_t}{P_{H,t}} \tag{D.40}$$

Substituting these into (D.35) and rearranging, we have:

$$\overline{\pi}^W_{N,t} = \left[\frac{(1-\gamma)\left(\overline{X}P_{H,t}^{1-\zeta} - NX_t\right)}{(1-\omega_H)\gamma P_{N,t}\left(P_{N,t}^{1-\gamma}P_t^{-1}\right)^{\frac{1-\zeta}{\gamma}}} \right]^{\frac{1}{\theta}} \left((\mathcal{D}_{N,t})^{\eta} \aleph^{\theta} M^W_{N,t}\overline{z}_{N,t}\right)^{\frac{-1}{\theta}}$$

$$\overline{\pi}^W_{H,t} = \left[\frac{\gamma\left(\overline{X}P_{H,t}^{1-\zeta} - NX_t\right)}{(1-\omega_H)\gamma P_{H,t}\left(P_{N,t}^{1-\gamma}P_t^{-1}\right)^{\frac{1-\zeta}{\gamma}}} + \frac{NX_t}{P_{H,t}} \right]^{\frac{1}{\theta}} \left((\mathcal{D}_{H,t})^{\eta} \aleph^{\theta} M^W_{H,t}\overline{z}_{H,t}\right)^{\frac{-1}{\theta}}$$

Combining these with $B_t - B_{t-1}(1+r^H_{t-1}) = NX_t$, we have:

$$\overline{\pi}^W_{N,t} = \left[\frac{(1-\gamma)\left(\overline{X}P_{H,t}^{1-\zeta} - (B_t - B_{t-1}(1+r^H_{t-1}))\right)}{(1-\omega_H)\gamma P_{N,t}\left(P_{N,t}^{1-\gamma}P_t^{-1}\right)^{\frac{1-\zeta}{\gamma}}} \right]^{\frac{1}{\theta}} \left((\mathcal{D}_{N,t})^{\eta} \aleph^{\theta} M^W_{N,t}\overline{z}_{N,t}\right)^{\frac{-1}{\theta}}$$

$$\overline{\pi}^W_{H,t} = \left[\frac{\gamma\left(\overline{X}P_{H,t}^{1-\zeta} - (B_t - B_{t-1}(1+r^H_{t-1}))\right)}{(1-\omega_H)\gamma P_{H,t}\left(P_{N,t}^{1-\gamma}P_t^{-1}\right)^{\frac{1-\zeta}{\gamma}}} + \frac{B_t - B_{t-1}(1+r^H_{t-1})}{P_{H,t}} \right]^{\frac{1}{\theta}} \left((\mathcal{D}_{H,t})^{\eta} \aleph^{\theta} M^W_{H,t}\overline{z}_{H,t}\right)^{\frac{-1}{\theta}}$$

We substitute (D.21) into the optimal capital demand condition (D.24) and rearrange to obtain:

$$r^K_t + \delta_K = \sum_S (1-\alpha)\,\theta \frac{\overline{\pi}^W_{S,t}M^W_{S,t}\overline{z}_{S,t}}{(1-\theta)K_t} = (1-\alpha)\,\theta \cdot \frac{\sum_S \overline{\pi}^W_{S,t}M^W_{S,t}\overline{z}_{S,t}}{(1-\theta)K_t}$$

Now, we substitute this in turn into the first order condition for capital accumulation to obtain:

$$E_t\left[M_{t+1}\left((1-\alpha)\,\theta \cdot \frac{\sum_S \overline{\pi}^W_{S,t+1}M^W_{S,t+1}\overline{z}_{S,t+1}}{(1-\theta)K_{t+1}} + 1 - \delta_K + \kappa\left(\left(\frac{K_{t+2}}{K_{t+1}}\right)^2 - 1\right)\right)\right] = \left(1 + 2\kappa\left(\frac{K_{t+1}}{K_t} - 1\right)\right) \quad \text{(D.41)}$$

We substitute (D.21) into (D.23) to get:

$$N_t = \sum_S \alpha\theta \frac{P_t\overline{\pi}^W_{S,t}M^W_{S,t}\overline{z}_{S,t}}{(1-\theta)w_t}$$

We then substitute this into the wage Phillips curve equations:

$$\mathcal{A}_{w,t} = \epsilon_w \phi \left(w_t^{\epsilon_w - 1}\left(\sum_S \alpha\theta \frac{P_t\overline{\pi}^W_{S,t}M^W_{S,t}\overline{z}_{S,t}}{(1-\theta)}\right)\right)^{1+\upsilon} + \beta\theta_w \mathbb{E}_t \mathcal{A}_{w,t+1} \quad \text{(D.42)}$$

$$\mathcal{B}_{w,t} = (\epsilon_w - 1)P_t^{-1} w_t^{\epsilon_w - 1}\left(\sum_S \alpha\theta \frac{P_t \overline{\pi}_{S,t}^W M_{S,t}^W \overline{z}_{S,t}}{(1-\theta)}\right) + \beta\theta_w \mathbb{E}_t \mathcal{B}_{w,t+1} \tag{D.43}$$

We rearrange the non-tradable market clearing condition (D.9) to make Y_t^d the subject:

$$Y_t^d = \frac{P_{N,t} Y_{N,t}}{(1-\gamma)P_t} \tag{D.44}$$

We rearrange the final goods market clearing to make C_t the subject:

$$\begin{aligned} C_t = Y_t^d - \Bigg[\sum_{S\in\{N;H\}} &\left(c^F M_{S,t}^W + \nu M_{S,t}^e \left(\frac{M_{S,t}^e}{M_{S,t-1}^W}\right)^\Theta\right) \\ + &\left(K_{t+1} - (1-\delta_K)K_t + \kappa\left(\frac{K_{t+1}}{K_t} - 1\right)^2 K_t\right) + c^R \Bigg] \end{aligned}$$

Substitute the final goods market clearing condition in to the equation for λ to obtain λ. We should have the Euler equations always consistent with the path of interest rates:

$$r_t^H = \frac{\lambda_t^S}{\beta \lambda_{t+1}^S} - 1.$$

D.2 Solving for the Equilibrium

To solve for an equilibrium, we first guess a path for $\bar{\pi}_{H,t}$, $\bar{\pi}_{N,t}$, r_t^H, $\aleph_t$ and price dispersion in each sector $\mathcal{D}_{S,t}$. The remaining variables can be solved for using the following equations from above in the following order (we repeat the equations here for convenience.)

$$A_{S,t} = \frac{\overline{\pi}_{S,t}^W}{1 - \rho_z M_t} \tag{D.45}$$

$$z_{S,t,j}^\star = \max\{z_{S,\tau}^\star\}_{\tau=t}^{\tau=t+j} \tag{D.46}$$

$$\mathcal{A}_{t,j} = (1 - \rho_Z M_{t+j}) A_{t+j} \frac{\Pi_{k=0}^{k=j}(M_{t+k}\rho_z)}{M_{t+j}\rho_z} \tag{D.47}$$

$$\nu \left(\frac{M_{S,t}^e}{M_{S,t-1}^W}\right)^\Theta = \sum_{j=0}^\infty \mathcal{A}_{t,j}\left(\frac{z_{t,j}^{\star-(\xi-1)}}{\xi-1}\right) \tag{D.48}$$

$$z_t^\star = \frac{c_F - M_t(1-\rho_z)\sum_{j=0}^\infty \mathcal{A}_{t+1,j}\left(\frac{z_{t+1,j}^{\star-(\xi-1)}}{\xi-1}\right)}{(1-\rho_Z M_t)A_t} \tag{D.49}$$

$$- \left(\frac{1}{(1-\rho_z M_t)A_t}\right)\sum_{j=1}^\infty \mathcal{A}_{t,j}\left(z_{t,j}^\star - z_{t+1,j-1}^\star\right) \tag{D.50}$$

$$j_{S,t} = \max\left[\left\{k \in \{0,1,...,t-1\} \middle| z^\star_{S,k} > z^\star_{S,t} \text{ or } z^\star_{S,k} \geq z^\star_{S,i},\, \forall i < k\right\} \cup \{0\}\right] \tag{D.51}$$

$$\overline{z}_{S,t} = \frac{\xi z^\star_{S,t}}{\xi - 1} + \frac{\rho_z^{t-j_{S,t}} M^W_{S,j_{S,t}}}{M^W_{S,t}} \left(\overline{z}_{S,j_{S,t}} - \frac{\xi \min\{z^\star_{S,t}; z^\star_{S,j_{S,t}}\}}{\xi - 1}\right) \tag{D.52}$$

$$M^W_{S,t} = \rho_z^{t-j_{S,t}} M^W_{S,j_{S,t}} \min\left\{1; \left(\frac{z^\star_{S,t}}{z^\star_{S,j_{S,t}}}\right)^{-\xi}\right\} \tag{D.53}$$

$$+ (z^\star_{S,t})^{-\xi} \sum_{i=1}^{t-j_{S,t}} \rho_z^{i-1}((1-\rho_z) M^W_{S,t-i} + M^W_{S,t-i} \left(\frac{M^e_{S,t-i+1}}{M^W_{S,t-i}}\right)) \tag{D.54}$$

In each sector $S \in \{N; H\}$, output satisfies:

$$Y_{S,t} = \aleph_t^\theta \left(\mathcal{D}_{S,t}\right)^\eta \left(\overline{\pi}^W_{S,t}\right)^\theta M^W_{S,t} \overline{z}_{S,t}$$

In each sector, prices evolve according to the three Phillips curve equations:

$$\left(\frac{P_{S,t}}{P_{S,t-1}}\right)^{1-\eta} = \vartheta_P + (1-\vartheta_P)\left(\frac{\mathcal{A}_{P,S,t}}{\mathcal{B}_{P,S,t} P_{S,t-1}}\right)^{1-\eta}. \tag{D.55}$$

where

$$\mathcal{A}_{P,S,t} = \left(\frac{\eta}{\eta-1}\right) P^\eta_{S,t} \left(\frac{\left(\mathcal{D}_{S,t}\right)^\eta \overline{\pi}^W_{S,t} M^W_{S,t} \overline{z}_{S,t}}{1-\theta}\right) + \vartheta_P M_{t+1} \mathbb{E}_t \mathcal{A}_{P,,S,t+1}$$
$$\mathcal{B}_{P,S,t} = P^\eta_{S,t} P_t^{-1} Y_{S,t} + \vartheta_P M_{t+1} \mathbb{E}_t \mathcal{B}_{P,S,t+1}.$$

where the final goods price satisfies:

$$P_t = P^{1-\gamma}_{N,t} \left(\left[\omega_H P^{1-\zeta}_{H,t} + (1-\omega_H)\right]^{\frac{1}{1-\zeta}}\right)^\gamma$$

$$Y^d_t = \frac{P_{N,t} Y_{N,t}}{(1-\gamma) P_t}$$

K_t satisfies:

$$E_t\left[M_{t+1}\left((1-\alpha)\,\theta \cdot \frac{\sum_S \overline{\pi}^W_{S,t+1} M^W_{S,t+1} \overline{z}_{S,t+1}}{(1-\theta) K_{t+1}} + 1 - \delta_K + \kappa\left(\left(\frac{K_{t+2}}{K_{t+1}}\right)^2 - 1\right)\right)\right]$$
$$= \left(1 + 2\kappa\left(\frac{K_{t+1}}{K_t} - 1\right)\right)$$

$$r_t^K + \delta_K = (1-\alpha)\,\theta \cdot \frac{\sum_S \overline{\pi}^W_{S,t} M^W_{S,t} \overline{z}_{S,t}}{(1-\theta)K_t}$$

Wages evolve according to the three Phillips curve equations (which must be solved simultaneously):

$$\mathcal{A}_{w,t} = \epsilon_w \phi \left(w_t^{\epsilon_w - 1} \left(\sum_S \alpha\theta \frac{P_t \overline{\pi}^W_{S,t} M^W_{S,t} \overline{z}_{S,t}}{(1-\theta)} \right) \right)^{1+\upsilon} + \beta\theta_w \mathbb{E}_t \mathcal{A}_{w,t+1} \tag{D.56}$$

$$\mathcal{B}_{w,t} = (\epsilon_w - 1) P_t^{-1} w_t^{\epsilon_w - 1} \left(\sum_S \alpha\theta \frac{P_t \overline{\pi}^W_{S,t} M^W_{S,t} \overline{z}_{S,t}}{(1-\theta)} \right) + \beta\theta_w \mathbb{E}_t \mathcal{B}_{w,t+1} \tag{D.57}$$

$$w_t = \left(\theta_w w_{t-1}^{1-\epsilon_w} + (1-\theta_w) \left(\frac{\mathcal{A}_{w,t}}{\mathcal{B}_{w,t}} \right)^{\frac{1-\epsilon_w}{1+\epsilon_w \upsilon}} \right)^{\frac{1}{1-\epsilon_w}} \tag{D.58}$$

Wage dispersion evolves according to:

$$\mathcal{D}_{w,t} w_t = \left(\theta_w \left(\mathcal{D}_{w,t-1} w_{t-1} \right)^{-\epsilon_w(1+\upsilon)} + (1-\theta_w) \left(\frac{\mathcal{A}_{w,t}}{\mathcal{B}_{w,t}} \right)^{\frac{-\epsilon_w(1+\upsilon)}{1+\epsilon_w \upsilon}} \right)^{\frac{-1}{\epsilon_w(1+\upsilon)}}$$

Finally, we can solve for N_t, C_t and λ_t^S:

$$N_t = \sum_S \alpha\theta \frac{P_t \overline{\pi}^W_{S,t} M^W_{S,t} \overline{z}_{S,t}}{(1-\theta) w_t}$$

$$\begin{aligned} C_t = Y_t^d - \Bigg[& \sum_{S \in \{N;H\}} \left(c^F M^W_{S,t} + \nu M^e_{S,t} \left(\frac{M^e_{S,t}}{M^W_{S,t-1}} \right)^{\Theta} \right) \\ & + \left(K_{t+1} - (1-\delta_K) K_t + \kappa \left(\frac{K_{t+1}}{K_t} - 1 \right)^2 K_t \right) + c^R \Bigg] \end{aligned}$$

$$\lambda_t^S = P_t^{-1} \left[\frac{C_t - \frac{\chi w_t N_t}{P_t}}{1-\chi} - \frac{\phi \mathcal{D}_{w,t}^{-\epsilon_w(1+\upsilon)} N_t^{1+\upsilon}}{1+\upsilon} \right]^{-\sigma}$$

The path for λ_t^S should be consistent with Euler equations for given r_t^H:

$$r_t^H = \frac{\lambda_t^S}{\beta \lambda_{t+1}^S} - 1.$$

Then, with $(B_t - B_{t-1}(1 + r_{t-1}^H)) = NX_t$, the following three equations give us new guesses for the paths of M_t, $\aleph$, $\bar{\pi}_{H,t}$ and $\bar{\pi}_{S,t}$:

$$\overline{\pi}^W_{N,t} = \left[\frac{(1-\gamma)\left(\overline{X} P_{H,t}^{1-\zeta} - (B_t - B_{t-1}(1+r^H_{t-1}))\right)}{(1-\omega_H)\gamma P_{N,t}\left(P_{N,t}^{1-\gamma} P_t^{-1}\right)^{\frac{1-\zeta}{\gamma}}} \right]^{\frac{1}{\theta}} \left((\mathcal{D}_{N,t})^\eta \aleph^\theta M^W_{N,t} \overline{z}_{N,t}\right)^{\frac{-1}{\theta}}$$

$$\overline{\pi}^W_{H,t} = \left[\frac{\gamma\left(\overline{X} P_{H,t}^{1-\zeta} - (B_t - B_{t-1}(1+r^H_{t-1}))\right)}{(1-\omega_H)\gamma P_{H,t}\left(P_{N,t}^{1-\gamma} P_t^{-1}\right)^{\frac{1-\zeta}{\gamma}}} + \frac{B_t - B_{t-1}(1+r^H_{t-1})}{P_{H,t}} \right]^{\frac{1}{\theta}} \left((\mathcal{D}_{H,t})^\eta \aleph^\theta M^W_{H,t} \overline{z}_{H,t}\right)^{\frac{-1}{\theta}}$$

$$\aleph_t = \frac{\theta}{1-\theta}\left(\frac{\alpha^\alpha(1-\alpha)^{1-\alpha}}{(r^K_t+\delta_K)^{1-\alpha}\left(\frac{w_t}{P_t}\right)^\alpha}\right)$$

$$M_t = \frac{P_{t+1}}{P_t(1+r_{F,t})}$$

Price dispersion in each sector evolves according to:

$$\left(\frac{P_{S,t}\mathcal{D}_{S,t}}{P_{S,t-1}\mathcal{D}_{S,t-1}}\right)^{-\eta} = \vartheta_P + (1-\vartheta_P)\left(\frac{\mathcal{A}_{P,S,t}}{\mathcal{B}_{P,S,t}P_{S,t-1}\mathcal{D}_{S,t-1}}\right)^{-\eta}.$$

D.3 Some further rearrangements

We had that:

$$z^\star_{S,t,j} \quad = \max\{z^\star_{S,\tau}\}_{\tau=t}^{\tau=t+j} \tag{D.59}$$

Then, we may infer that:

$$z^\star_{S,t,j} = \max\{z^\star_{S,t}; z^\star_{S,t+1,j-1}\}$$

Then, since we had the following equation for $z^\star_{S,t}$:

$$z^\star_{S,t} \quad = \frac{c_F - M_t(1-\rho_z)\sum_{j=0}^\infty \mathcal{A}_{S,t+1,j}\left(\frac{z^{\star-(\xi-1)}_{t+1,j}}{\xi-1}\right)}{(1-\rho_Z M_t)A_{S,t}} - \left(\frac{1}{(1-\rho_z M_t)A_{S,t}}\right)\sum_{j=1}^\infty \mathcal{A}_{S,t,j}\left(z^\star_{S,t,j} - z^\star_{S,t+1,j-1}\right) \tag{D.60}$$

we may rewrite this as:

$$z^\star_{S,t} \quad = \frac{c_F - M_t(1-\rho_z)\sum_{j=0}^\infty \mathcal{A}_{S,t+1,j}\left(\frac{z^{\star-(\xi-1)}_{t+1,j}}{\xi-1}\right)}{(1-\rho_Z M_t)A_{S,t}} - \left(\frac{1}{(1-\rho_z M_t)A_{S,t}}\right)\sum_{j=1}^\infty \mathcal{A}_{S,t,j}\max\left\{z^\star_{S,t} - z^\star_{S,t+1,j-1}; 0\right\}$$

D.4 Steady State Equations

In the steady state, the Euler equation for λ, combined with the equation for M_t, imply that:

$$M = \beta$$

assuming that $r_t^F = r_t^H$.

It was shown in Appendix 3.2.2 that the equations for entry and exit become:

$$
\begin{aligned}
A_S^* &= \tfrac{1}{1-\rho_z M}\overline{\pi}_S^W \\
\nu\left(\frac{M_S^e}{M_S^W}\right)^\Theta &= A^*\left(\frac{z_S^{\star-(\xi-1)}}{\xi-1}\right) \\
z_S^\star &= \frac{c_F - M(1-\rho_z)\nu\left(\frac{M_S^e}{M_S^W}\right)^\Theta}{(1-\rho_Z M)A_S^*}
\end{aligned}
$$

Furthermore, the equations for firm dynamics become:

$$
\begin{aligned}
\overline{z}_{S,} &= \frac{\xi z_S^\star}{\xi-1} \\
M_{S,}^W &= \rho_z M^W + (z^\star)^{-\xi}((1-\rho_z)M_S^W + M_S^e)
\end{aligned}
\tag{D.61}
$$

The latter rearranges to:

$$
\frac{M_S^e}{M_S^W} = (z_S^{\star\xi} - 1)(1-\rho_z) \tag{D.62}
$$

Substituting this into the equations above, and using the first equation in this section to eliminate A_S^*, we get:

$$
\begin{aligned}
\nu(z_S^{\star\xi}-1)^\Theta(1-\rho_z)^\Theta &= \frac{\overline{\pi}_S^W}{1-\rho_z M}\left(\frac{z_S^{\star-(\xi-1)}}{\xi-1}\right) \\
z_S^\star &= \frac{c_F - M(1-\rho_z)\nu(z_S^{\star\xi}-1)^\Theta(1-\rho_z)^\Theta}{(1-\rho_Z M)(\xi-1)z_S^{\star(\xi-1)}\nu(z_S^{\star\xi}-1)^\Theta(1-\rho_z)^\Theta}
\end{aligned}
$$

and so

$$
\overline{\pi}_S^W = (1-\rho_Z M)(\xi-1)z_S^{\star(\xi-1)}\nu(z_S^{\star\xi}-1)^\Theta(1-\rho_z)^\Theta
$$

It is immediate from these equations that $z_S^\star$, and therefore $\overline{z}_S$ and $\overline{\pi}_S^W$ are functions of c_F, M, ρ_z, ξ, Θ and ν alone, and so must be the same for each S. Thus, we drop the S subscript from them.

Then, the output in each sector is proportional to the number of firms:

$$
Y_S = \aleph^\theta\left(\overline{\pi}^W\right)^\theta M_S^W \overline{z}
$$

which we can rewrite as:

$$
M_S^W = \frac{Y_S}{\aleph^\theta\left(\overline{\pi}^W\right)^\theta \overline{z}}
$$

Comparing the Phillips curve equations and the equations for the evolution of price and wage dispersion reveals that, in a steady state, the dispersion terms must satisfy:

$$\mathcal{D}_{P,S} = \mathcal{D}_w = 1$$

The Phillips curve equations in the steady state become:

$$1 = \vartheta_P + (1-\vartheta_P)\left(\frac{\mathcal{A}_{P,S}}{\mathcal{B}_{P,S}P_S}\right)^{1-\eta}.$$

where

$$\mathcal{A}_{P,S} = (1-\vartheta_P M)^{-1}\left(\frac{\eta}{\eta-1}\right)P_S^{\eta}\left(\frac{\overline{\pi}_S^W M_S^W \overline{z}}{1-\theta}\right)$$
$$\mathcal{B}_{P,S} = (1-\vartheta_P M)^{-1}P_S^{\eta}P^{-1}Y_S = (1-\vartheta_P M)^{-1}P_S^{\eta}P^{-1}\aleph^{\theta}\left(\overline{\pi}^W\right)^{\theta}M_S^W\overline{z}.$$

Combining these three equations, we have:

$$1 = \frac{\mathcal{A}_{P,S}}{\mathcal{B}_{P,S}P_S} = \left(\frac{\eta}{\eta-1}\right)PP_S^{-1}\left(\frac{\left(\overline{\pi}^W\right)^{1-\theta}\aleph^{-\theta}}{1-\theta}\right)$$

Since all terms in this equation are invariant across sector apart from P_S, it follows that P_S is the same for each sector S, so I will refer to it as $\hat{P}$. Then, the equation above becomes:

$$\hat{P} = \left(\frac{\eta}{\eta-1}\right)P\left(\frac{\left(\overline{\pi}^W\right)^{1-\theta}\aleph^{-\theta}}{1-\theta}\right)$$

and the final goods price P satisfies:

$$P = \hat{P}^{1-\gamma}\left(\left[\omega_H\hat{P}^{1-\zeta} + (1-\omega_H)\right]^{\frac{1}{1-\zeta}}\right)^{\gamma}$$

Substituting this into the equation we derived from the Phillips curve above, we have:

$$\hat{P}^{\gamma} = \left(\frac{\eta}{\eta-1}\right)\left(\left[\omega_H\hat{P}^{1-\zeta} + (1-\omega_H)\right]^{\frac{1}{1-\zeta}}\right)^{\gamma}\left(\frac{\left(\overline{\pi}^W\right)^{1-\theta}\aleph^{-\theta}}{1-\theta}\right)$$

The remaining equations from the previous section, in the steady state, become:

$$PY^d = \frac{P_N Y_N}{1-\gamma}$$

$$M\left(\frac{(1-\alpha)\theta)\overline{\pi}^W\overline{z}\sum_S M_S^W}{(1-\theta)K} + 1 - \delta_K\right) = 1$$

$$r^K + \delta_K = \frac{(1-\alpha)\theta)\overline{\pi}^W\overline{z}\sum_S M_S^W}{(1-\theta)K} = M^{-1} - 1 + \delta_K$$

$$\mathcal{A}_w = (1-\beta\theta_w)^{-1}\epsilon_w\phi\left(\frac{w^{\epsilon_w-1}P\alpha\theta\overline{\pi z}\sum_S M_S^W}{1-\theta}\right)^{1+\upsilon}$$

$$\mathcal{B}_w = (1-\beta\theta_w)^{-1}(\epsilon_w-1)P^{-1}\left(\frac{w^{\epsilon_w-1}P\alpha\theta\overline{\pi z}\sum_S M_S^W}{1-\theta}\right)$$

$$w = \left(\frac{\mathcal{A}_w}{\mathcal{B}_w}\right)^{\frac{1}{1+\epsilon_w\upsilon}}$$

The latter simplifies to:

$$w^{1+\epsilon_w\upsilon} = \left(\frac{\epsilon_w\phi}{\epsilon_w-1}\right)\left(\frac{w^{\epsilon_w-1}\alpha\theta\overline{\pi z}\sum_S M_S^W}{1-\theta}\right)^{\upsilon}P^{1+\upsilon}$$

which rearranges to give:

$$w^{1+\upsilon} = \left(\frac{\epsilon_w\phi}{\epsilon_w-1}\right)\left(\frac{\alpha\theta\overline{\pi z}\sum_S M_S^W}{1-\theta}\right)^{\upsilon}P^{1+\upsilon}$$

which rearranges to:

$$\frac{\alpha\theta\overline{\pi z}\sum_S M_S^W}{1-\theta} = \left(\frac{\epsilon_w-1}{\epsilon_w\phi}\right)^{\frac{1}{\upsilon}}\left(\frac{w}{P}\right)^{\frac{1+\upsilon}{\upsilon}}$$

In the steady state, our equation for N_t becomes:

$$N = \sum_S \alpha\theta\frac{P\overline{\pi}^W M_S^W\overline{z}}{(1-\theta)w}$$

Substituting this in to the equations for $r^K+\delta$ above, we get:

$$r^K+\delta = \left(\frac{1-\alpha}{\alpha}\right)\frac{wN}{PK} = M^{-1}-1+\delta$$

which rearranges to:

$$K = \frac{(1-\alpha)\frac{w}{P}N}{\alpha(r^K+\delta)}$$

Then, we have the remaining equations:

$$N = \frac{\alpha\theta\overline{\pi}^W\overline{z}P\sum_S M_S^W}{(1-\theta)w} = \left(\frac{\epsilon_w-1}{\epsilon_w\phi}\right)^{\frac{1}{\upsilon}}\left(\frac{w}{P}\right)^{\frac{1}{\upsilon}}$$

which rearranges to the following two equations:

$$\sum_S M_S^W = \frac{(1-\theta)\frac{w}{P}N}{\alpha\theta\overline{\pi}^W\overline{z}}$$

and

$$\left(\overline{\pi}^W\right)^\theta \overline{z} \sum_S M_S^W = \left(\frac{1-\theta}{\alpha\theta\left(\overline{\pi}^W\right)^{1-\theta}}\right)\left(\frac{\epsilon_w - 1}{\epsilon_w \phi}\right)^{\frac{1}{\upsilon}}\left(\frac{w}{P}\right)^{\frac{1}{\upsilon}+1}$$

We also have:

$$C = Y^d - \Bigg[\left(c_F + \nu(z^{\star\xi} - 1)^\Theta (1-\rho_z)^\Theta\right)\left(\sum_{S\in\{N;T\}} M_S^W\right) + \delta_K K + c^R\Bigg]$$

$$\lambda^S = \left[\frac{C - \frac{\chi w N}{P}}{1-\chi} - \frac{\phi N^{1+\upsilon}}{1+\upsilon}\right]^{-\sigma}$$

$$r^H = \frac{1}{\beta} - 1$$

In the steady state, we have $B = \overline{B}$, which will be calibrated so that debt/output in ss is consistent with data.

And our equations for $\overline{\pi}_H$, $\overline{\pi}_N$ become:

$$\overline{\pi}^W = \left[\frac{(1-\gamma)\left(\overline{X}\hat{P}^{1-\zeta} + Br^H\right)}{(1-\omega_H)\gamma\hat{P}\left(\hat{P}^{1-\gamma}P^{-1}\right)^{\frac{1-\zeta}{\gamma}}}\right]^{\frac{1}{\theta}}\left(\aleph^\theta M_N^W \overline{z}\right)^{\frac{-1}{\theta}}$$

$$\overline{\pi}^W = \left[\frac{\gamma\left(\overline{X}\hat{P}^{1-\zeta} + Br^H\right)}{(1-\omega_H)\gamma\hat{P}\left(\hat{P}^{1-\gamma}P^{-1}\right)^{\frac{1-\zeta}{\gamma}}} - \frac{Br^H}{\hat{P}}\right]^{\frac{1}{\theta}}\left(\aleph^\theta M_H^W \overline{z}\right)^{\frac{-1}{\theta}}$$

Recall that we showed above that:

$$Y_S = \aleph^\theta \left(\overline{\pi}^W\right)^\theta M_S^W \overline{z}$$

Comparing that to these two equations, we see that:

$$Y_H = \frac{\gamma\left(\overline{X}\hat{P}^{1-\zeta} + Br^H\right)}{(1-\omega_H)\gamma\hat{P}\left(\hat{P}^{1-\gamma}P^{-1}\right)^{\frac{1-\zeta}{\gamma}}} - \frac{Br^H}{\hat{P}}$$

and

$$Y_N = \frac{(1-\gamma)\left(\overline{X}\hat{P}^{1-\zeta} + Br^H\right)}{(1-\omega_H)\gamma\hat{P}\left(\hat{P}^{1-\gamma}P^{-1}\right)^{\frac{1-\zeta}{\gamma}}}$$

and, if we define $Y = \sum_S Y_S$, then we have:

$$Y = \aleph^{\theta}\overline{z}\left(\overline{\pi}^W\right)^{\theta}\sum_S M_S^W = \frac{\left(\overline{X}\hat{P}^{1-\zeta} + Br^H\right)}{(1-\omega_H)\gamma\hat{P}\left(\hat{P}^{1-\gamma}P^{-1}\right)^{\frac{1-\zeta}{\gamma}}} - \frac{Br^H}{\hat{P}}$$

Combining this with the equation for $\frac{w}{P}$ above, we have:

$$\aleph^{\theta}\left(\frac{1-\theta}{\alpha\theta\left(\overline{\pi}^W\right)^{1-\theta}}\right)\left(\frac{\epsilon_w - 1}{\epsilon_w\phi}\right)^{\frac{1}{\upsilon}}\left(\frac{w}{P}\right)^{\frac{1}{\upsilon}+1} = \frac{\left(\overline{X}\hat{P}^{1-\zeta} + Br^H\right)}{(1-\omega_H)\gamma\hat{P}\left(\hat{P}^{1-\gamma}P^{-1}\right)^{\frac{1-\zeta}{\gamma}}} - \frac{Br^H}{\hat{P}} = Y$$

We rewrite this as:

$$\frac{w}{P} = \hat{\aleph}\cdot\aleph^{\frac{-\theta\upsilon}{1+\upsilon}}\cdot Y^{\frac{\upsilon}{1+\upsilon}}$$

where

$$\hat{\aleph} = \left[\left(\frac{1-\theta}{\alpha\theta\left(\overline{\pi}^W\right)^{1-\theta}}\right)^{-1}\left(\frac{\epsilon_w - 1}{\epsilon_w\phi}\right)^{\frac{-1}{\upsilon}}\right]^{\frac{\upsilon}{1+\upsilon}}$$

Recall that the final goods price P satisfies:

$$P = \hat{P}^{1-\gamma}\left(\left[\omega_H\hat{P}^{1-\zeta} + (1-\omega_H)\right]^{\frac{1}{1-\zeta}}\right)^{\gamma}$$

Substituting this in to the expression for Y, we have:

$$Y = \frac{\left(\overline{X}\hat{P}^{1-\zeta} + Br^H\right)\left[\omega_H\hat{P}^{1-\zeta} + (1-\omega_H)\right]}{(1-\omega_H)\gamma\hat{P}} - \frac{Br^H}{\hat{P}}$$

Finally, in the steady state, our equation for $\aleph$ becomes:

$$\aleph = \frac{\theta}{1-\theta}\left(\frac{\alpha^{\alpha}(1-\alpha)^{1-\alpha}}{(r^K+\delta_K)^{1-\alpha}\left(\frac{w}{P}\right)^{\alpha}}\right)$$

which rearranges to:

$$\frac{w}{P} = \left[\frac{\theta}{1-\theta}\left(\frac{\alpha^{\alpha}(1-\alpha)^{1-\alpha}}{(r^K+\delta_K)^{1-\alpha}\aleph}\right)\right]^{\frac{1}{\alpha}}$$

Combining this with the equation we derived for $\frac{w}{P}$ immediately above, we obtain:

$$\hat{\aleph} \cdot \aleph^{\frac{-\theta \upsilon}{1+\upsilon}} \cdot Y^{\frac{\upsilon}{1+\upsilon}} = \left[\frac{\theta}{1-\theta} \left(\frac{\alpha^{\alpha}(1-\alpha)^{1-\alpha}}{(r^K + \delta_K)^{1-\alpha} \aleph} \right) \right]^{\frac{1}{\alpha}}$$

which rearranges to:

$$\aleph^{\frac{1+\upsilon-\alpha\theta\upsilon}{1+\upsilon}} = \hat{\aleph}^{-\alpha} Y^{\frac{-\alpha\upsilon}{1+\upsilon}} \left[\frac{\theta}{1-\theta} \left(\frac{\alpha^{\alpha}(1-\alpha)^{1-\alpha}}{(r^K + \delta_K)^{1-\alpha}} \right) \right]$$

Or,

$$\aleph = \hat{\hat{\aleph}} Y^{\frac{-\alpha\upsilon}{1+\upsilon-\alpha\theta\upsilon}}$$

where

$$\hat{\hat{\aleph}} = \left[\left(\frac{1-\theta}{\alpha\theta \left(\overline{\pi}^W\right)^{1-\theta}} \right)^{-1} \left(\frac{\epsilon_w - 1}{\epsilon_w \phi} \right)^{\frac{-1}{\upsilon}} \right]^{\frac{-\alpha\upsilon}{1+\upsilon-\alpha\theta\upsilon}} \cdot \left[\frac{\theta}{1-\theta} \left(\frac{\alpha^{\alpha}(1-\alpha)^{1-\alpha}}{(r^K + \delta_K)^{1-\alpha}} \right) \right]^{\frac{1+\upsilon}{1+\upsilon-\alpha\theta\upsilon}}$$

Substituting this into the Phillips-like equation for $\hat{P}$ that we derived above, we have:

$$\hat{P}^{\gamma} = \left(\frac{\eta}{\eta - 1} \right) \left(\left[\omega_H \hat{P}^{1-\zeta} + (1-\omega_H) \right]^{\frac{1}{1-\zeta}} \right)^{\gamma} \left(\frac{\left(\overline{\pi}^W\right)^{1-\theta} \hat{\hat{\aleph}}^{-\theta} Y^{\frac{\alpha\theta\upsilon}{1+\upsilon-\alpha\theta\upsilon}}}{1-\theta} \right)$$

D.5 Solving for the Steady State

The equations from the previous section can be solved in the following order to solve for the steady state, almost in closed form. We repeat the equations here for convenience.

$$M = \beta$$

(Assuming that $r_t^F = r_t^H$)

$$r^H = \frac{1}{\beta} - 1$$

$$B = \overline{B} \tag{D.63}$$

$$z^{\star} = \frac{c_F - M(1-\rho_z)\nu(z^{\star\xi} - 1)^{\Theta}(1-\rho_z)^{\Theta}}{(1-\rho_Z M)(\xi - 1) z^{\star(\xi-1)} \nu (z^{\star\xi} - 1)^{\Theta} (1-\rho_z)^{\Theta}}$$

$$\overline{\pi}^W = (1-\rho_Z M)(\xi - 1) z^{\star(\xi-1)} \nu (z^{\star\xi} - 1)^{\Theta} (1-\rho_z)^{\Theta}$$

$$\overline{z} = \frac{\xi z^{\star}}{\xi - 1}$$

$$A^{*} = \tfrac{1}{1-\rho_z M}\overline{\pi}^{W}$$

$$r^{K} = M^{-1} - 1$$

$$\hat{\hat{\aleph}} = \left[\left(\frac{1-\theta}{\alpha\theta \left(\overline{\pi}^{W}\right)^{1-\theta}} \right)^{-1} \left(\frac{\epsilon_w - 1}{\epsilon_w \phi} \right)^{\frac{-1}{\upsilon}} \right]^{\frac{-\alpha\upsilon}{1+\upsilon-\alpha\theta\upsilon}} \cdot \left[\frac{\theta}{1-\theta} \left(\frac{\alpha^{\alpha}(1-\alpha)^{1-\alpha}}{(r^{K}+\delta_K)^{1-\alpha}} \right) \right]^{\frac{1+\upsilon}{1+\upsilon-\alpha\theta\upsilon}}$$

Then, the following two equations can be solved simultaneously for Y and $\hat{P}$:

$$Y = \frac{\left(\overline{X}\hat{P}^{1-\zeta} + Br^{H}\right)\left[\omega_H \hat{P}^{1-\zeta} + (1-\omega_H)\right]}{(1-\omega_H)\gamma\hat{P}} - \frac{Br^{H}}{\hat{P}}$$

$$\hat{P}^{\gamma} = \left(\frac{\eta}{\eta-1}\right)\left(\left[\omega_H \hat{P}^{1-\zeta} + (1-\omega_H)\right]^{\frac{1}{1-\zeta}}\right)^{\gamma}\left(\frac{\left(\overline{\pi}^{W}\right)^{1-\theta}\hat{\hat{\aleph}}^{-\theta} Y^{\frac{\alpha\theta\upsilon}{1+\upsilon-\alpha\theta\upsilon}}}{1-\theta}\right)$$

Then, we have:

$$P_H = P_N = \hat{P}$$

$$P = \hat{P}^{1-\gamma}\left(\left[\omega_H \hat{P}^{1-\zeta} + (1-\omega_H)\right]^{\frac{1}{1-\zeta}}\right)^{\gamma}$$

$$\aleph = \hat{\hat{\aleph}} Y^{\frac{-\alpha\upsilon}{1+\upsilon-\alpha\theta\upsilon}}$$

$$w = P\left[\frac{\theta}{1-\theta}\left(\frac{\alpha^{\alpha}(1-\alpha)^{1-\alpha}}{(r^{K}+\delta_K)^{1-\alpha}\aleph}\right)\right]^{\frac{1}{\alpha}}$$

$$Y_H = \frac{\gamma\left(\overline{X}\hat{P}^{1-\zeta} + Br^{H}\right)}{(1-\omega_H)\gamma\hat{P}\left(\hat{P}^{1-\gamma}P^{-1}\right)^{\frac{1-\zeta}{\gamma}}} - \frac{Br^{H}}{\hat{P}}$$

and

$$Y_N = \frac{(1-\gamma)\left(\overline{X}\hat{P}^{1-\zeta} + Br^{H}\right)}{(1-\omega_H)\gamma\hat{P}\left(\hat{P}^{1-\gamma}P^{-1}\right)^{\frac{1-\zeta}{\gamma}}}$$

$$Y^d = \frac{P^{-1} P_N Y_N}{1-\gamma}$$

For each $S \in \{H; N\}$:

$$M_S^W = \frac{Y_S}{\aleph^\theta \left(\overline{\pi}^W\right)^\theta \overline{z}}$$

$$N = \left(\frac{\epsilon_w - 1}{\epsilon_w \phi}\right)^{\frac{1}{\upsilon}} \left(\frac{w}{P}\right)^{\frac{1}{\upsilon}}$$

$$K = \frac{(1-\alpha)\frac{wN}{P}}{\alpha(r^K + \delta)}$$

For each S:

$$M_S^e = M_S^W (z_S^{\star\xi} - 1)(1 - \rho_z) \tag{D.64}$$

$$\mathcal{A}_{P,S} = (1 - \vartheta_P M)^{-1} \left(\frac{\eta}{\eta - 1}\right) \hat{P}^\eta \left(\frac{\overline{\pi}^W M_S^W \overline{z}}{1-\theta}\right)$$
$$\mathcal{B}_{P,S} = (1 - \vartheta_P M)^{-1} \hat{P}^\eta P^{-1} Y_S.$$

$$\mathcal{A}_w = (1 - \beta\theta_w)^{-1} \epsilon_w \phi \left(\frac{w^{\epsilon_w - 1} P \alpha \theta \overline{\pi} \overline{z} \sum_S M_S^W}{1-\theta}\right)^{1+\upsilon}$$

$$\mathcal{B}_w = (1 - \beta\theta_w)^{-1} (\epsilon_w - 1) P^{-1} \left(\frac{w^{\epsilon_w - 1} P \alpha \theta \overline{\pi} \overline{z} \sum_S M_S^W}{1-\theta}\right)$$

$$C = Y^d - \Bigg[\left(c_F + \nu (z^{\star\xi} - 1)^\Theta (1 - \rho_z)^\Theta\right) \left(\sum_{S \in \{N;T\}} M_S^W\right) + \delta_K K + c^R \Bigg]$$

$$\lambda^S = \left[\frac{C - \frac{\chi w N}{P}}{1-\chi} - \frac{\phi N^{1+\upsilon}}{1+\upsilon}\right]^{-\sigma}$$

$$\mathcal{D}_{P,S} = \mathcal{D}_w = 1$$

E Proofs of Technical Propositions

E.1 Proof of Proposition 1

Recall that, for the dynamics of the measures of firms, we have:

$$\begin{aligned}
&\forall \hat{z} \geq z^\star(\mathbf{X}_{t+1}), M^F_{t+1} \int saI_{\{z' \geq z^\star(\mathbf{X}_{t+1})\}} I_{\{z' \leq \hat{z}\}} \mu_{t+1}(z') dz' \\
= \; & M^F_t \rho_z \int I_{\{z_t \geq z^\star(\mathbf{X}_t)\}} I_{\{z_t \leq \hat{z}\}} I_{\{z_t \geq z^\star(\mathbf{X}_{t+1})\}} \mu_t(z_t) dz_t \\
& + \left[M^F_t (1-\rho_z) + M^e_{t+1} \right] \int I_{\{z' \leq \hat{z}\}} I_{\{z' \geq z^\star(\mathbf{X}_{t+1})\}} dG_z(z')
\end{aligned} \tag{E.1}$$

$$M^F_{t+1} = M^F_t \rho_z \int I_{\{z_t \geq z^\star(\mathbf{X}_t)\}} I_{\{z_t \geq z^\star(\mathbf{X}_{t+1})\}} \mu_t(z_t) dz_t + \left[M^F_{t+1} (1-\rho_z) + M^e_{t+1} \right] \times \int I_{\{z' \geq z^\star(\mathbf{X}_{t+1})\}} dG_z(z') \tag{E.2}$$

$$M^{Exit}_{t+1} = M^F_t \rho_z \int I_{\{z_t \geq z^\star(\mathbf{X}_t)\}} I_{\{z_t \leq z^\star(\mathbf{X}_{t+1})\}} \mu_t(z_t) dz_t + \left[M^F_t (1-\rho_z) + M^e_{t+1} \right] \times \int I_{\{z' \leq z^\star(\mathbf{X}_{t+1})\}} dG_z(z') \tag{E.3}$$

To simplify these expressions, let $H_t(z)$ be the cdf of continuing firms associated with μ_t. That is,

$$H_t(y) \equiv \frac{\int_{z^\star_t}^{y} \mu_t(z) dz}{\int_{z^\star_t}^{\infty} \mu_t(z) dz}$$

Then, from the expressions above, we can say that H_t and M^W_t evolve according to:

$$M^W_{t+1}(1 - H_{t+1}(z)) = \begin{cases} \rho_z M^W_t (1 - H_t(z)) + ((1-\rho_z) M^W_t + M^e_{t+1})(1 - G(z)) & \text{if } z \geq z^\star_{t+1} \\ M^W_{t+1} & \text{if } z \leq z^\star_{t+1} \end{cases} \tag{E.4}$$

Here, there is no $z \geq z^\star_t$ in these expressions because $H_t(z^\star_t) = 0$, so all the necessary information is already contained in H_t.

At $z^\star_{t+1}$, both cases must give the same term on the left hand side, and so:

$$M^W_{t+1} = \rho_z M^W_t (1 - H_t(z^\star_{t+1})) + ((1-\rho_z) M^W_t + M^e_{t+1}))(1 - G(z^\star_{t+1})) \tag{E.5}$$

Finally,

$$M^{Exit}_t = M^W_t \rho_z H_t(z^\star_{t+1}) + ((1-\rho_z) M^W_t + M^e_{t+1}) G(z^\star_{t+1}) \tag{E.6}$$

Differentiating equation (E.4) with respect to z, we have:

$$M_{t+1}^W \mu_{t+1}(z) = \begin{cases} \rho_z M_t^W \mu_t(z) + ((1-\rho_z)M_t^W + M_{t+1}^e)g(z) & \text{if } z > z_{t+1}^\star \\ 0 & \text{if } z \leq z_{t+1}^\star \end{cases} \tag{E.7}$$

These equations fully describe the evolution of the firm distribution given M_t^e and $z_t^\star$, for each t.

We may guess and verify that, for any z, the solution has the following form:

$$M_t^W \mu_t(z) = \begin{cases} g(z) \sum_{i=1}^{k+1} \rho_z^{i-1}((1-\rho_z)M_{t-i}^W + M_{t-i+1}^e) & \text{if } z > z_t^\star \\ 0 & \text{if } z \leq z_t^\star \end{cases} \tag{E.8}$$

where k is the number of *consecutive* periods for which $z_t^\star < z$.

That is, $\max\{z_{t-i}^\star\}_{i=0}^{i=k} < z$, but $z_{t-k}^\star > z$. If $z_t^\star < z$ for all t, then $k = \infty$.

It can be proved by induction that this is the solution. Assume it holds for some t, then plug it into equation (E.7) and we see that it also holds for $t+1$.

Recall that, for the dynamics of the measures of firms, we have:

$$\begin{aligned} & \forall \hat{z} \geq z^\star(\mathbf{X}_{t+1}), M_{t+1}^F \int I_{\{z' \geq z^\star(\mathbf{X}_{t+1})\}} I_{\{z' \leq \hat{z}\}} \mu_{t+1}(z')dz' \\ = \;& M_t^F \rho_z \int I_{\{z_t \geq z^\star(\mathbf{X}_t)\}} I_{\{z_t \leq \hat{z}\}} I_{\{z_t \geq z^\star(\mathbf{X}_{t+1})\}} \mu_t(z_t)dz_t \\ & + \left[M_t^F(1-\rho_z) + M_{t+1}^e\right] \int I_{\{z' \leq \hat{z}\}} I_{\{z' \geq z^\star(\mathbf{X}_{t+1})\}} dG_z(z') \end{aligned} \tag{E.9}$$

$$\begin{aligned} M_{t+1}^F &= M_t^F \rho_z \int I_{\{z_t \geq z^\star(\mathbf{X}_t)\}} I_{\{z_t \geq z^\star(\mathbf{X}_{t+1})\}} \mu_t(z_t)dz_t \\ & \quad + \left[M_{t+1}^F(1-\rho_z) + M_{t+1}^e\right] \times \int I_{\{z' \geq z^\star(\mathbf{X}_{t+1})\}} dG_z(z') \end{aligned} \tag{E.10}$$

$$\begin{aligned} M_{t+1}^{Exit} &= M_t^F \rho_z \int I_{\{z_t \geq z^\star(\mathbf{X}_t)\}} I_{\{z_t \leq z^\star(\mathbf{X}_{t+1})\}} \mu_t(z_t)dz_t \\ & \quad + \left[M_t^F(1-\rho_z) + M_{t+1}^e\right] \times \int I_{\{z' \leq z^\star(\mathbf{X}_{t+1})\}} dG_z(z') \end{aligned} \tag{E.11}$$

To simplify these expressions, let $H_t(z)$ be the cdf of continuing firms associated with μ_t. That is,

$$H_t(y) \equiv \frac{\int_{z_t^\star}^{y} \mu_t(z)dz}{\int_{z_t^\star}^{\infty} \mu_t(z)dz}$$

Then, from the expressions above, we can say that H_t and M_t^W evolve according to:

$$M_{t+1}^W(1 - H_{t+1}(z))$$
$$= \begin{cases} \rho_z M_t^W(1 - H_t(z)) + ((1-\rho_z)M_t^W + M_{t+1}^e)(1 - G(z)) & \text{if } z \geq z_{t+1}^\star \\ M_{t+1}^W & \text{if } z \leq z_{t+1}^\star \end{cases} \tag{E.12}$$

Here, there is no $z \geq z_t^\star$ in these expressions because $H_t(z_t^\star) = 0$, so all the necessary information is already contained in H_t.

At $z_{t+1}^\star$, both cases must give the same term on the left hand side, and so:

$$M_{t+1}^W = \rho_z M_t^W(1 - H_t(z_{t+1}^\star)) + ((1-\rho_z)M_t^W + M_{t+1}^e))(1 - G(z_{t+1}^\star)) \tag{E.13}$$

Finally,

$$M_t^{Exit} = M_t^W \rho_z H_t(z_{t+1}^\star) + ((1-\rho_z)M_t^W + M_{t+1}^e)G(z_{t+1}^\star) \tag{E.14}$$

Differentiating equation (E.12) with respect to z, we have:

$$M_{t+1}^W \mu_{t+1}(z) = \begin{cases} \rho_z M_t^W \mu_t(z) + ((1-\rho_z)M_t^W + M_{t+1}^e)g(z) & \text{if } z > z_{t+1}^\star \\ 0 & \text{if } z \leq z_{t+1}^\star \end{cases} \tag{E.15}$$

These equations fully describe the evolution of the firm distribution given M_t^e and $z_t^\star$, for each t.

Now suppose that for some $k \geq 0$, that $z_t^\star \geq \max\{z_{t-i}^\star\}_{i=0}^k$. Then, iterating equation (E.7) forward, we have that

$$M_t^W \mu_t(z)$$
$$= \begin{cases} \rho_z^{k+1} M_{t-k-1}^W \mu_{t-k-1}(z) + g(z)\sum_{i=1}^{k+1} \rho_z^{i-1}((1-\rho_z)M_{t-i}^W + M_{t-i+1}^e) & \text{if } z > z_t^\star \\ 0 & \text{if } z \leq z_t^\star \end{cases} \tag{E.16}$$

More generally, we may guess and verify that, for any z, the solution has the following form:

$$M_t^W \mu_t(z) = \begin{cases} g(z)\sum_{i=1}^{k+1} \rho_z^{i-1}((1-\rho_z)M_{t-i}^W + M_{t-i+1}^e) & \text{if } z > z_t^\star \\ 0 & \text{if } z \leq z_t^\star \end{cases} \tag{E.17}$$

where k is the number of *consecutive* periods for which $z_t^\star < z$.

That is, $\max\{z_{t-i}^\star\}_{i=0}^{i=k} < z$, but $z_{t-k-1}^\star > z$. If $z_t^\star < z$ for all t, then $k = \infty$.

It can be proved by induction that this is the solution. Assume it holds for some t, then plug it into equation (E.7) and we see that it also holds for $t+1$.

Using equations (E.16) and (E.8) we now describe the evolution of $\overline{z}$ and M_t^W. First note that under Assumption 1, it is easy to show that

$$\int_{z>z^\star} z g(z) dz = \left(\frac{\xi}{\xi - 1}\right)(z^\star)^{1-\xi}$$

and that

$$\int_{z>z^\star} g(z)dz = (z^\star)^{-\xi}$$

We now prove the Proposition by considering two separate cases: first, the case in which $z^\star_{j_t} \leq z^\star_t$ and, second, the case in which $z^\star_{j_t} > z^\star_t$, with j_t defined in both cases according to equation (33).

Case 1: Suppose that $z^\star_{j_t} \leq z^\star_t$. Then, by the definition of j_t in equation (33), it must be that $z^\star_t \geq z^\star_{j_t} \geq z^\star_i$, for all $i < j_t$. Furthermore, it must be that there exists no $k > j_t$, with $k < t$ that satisfies $z^\star_k > z^\star_t$, since otherwise it would be the case that $j_t = k$. In combination, these inequalities imply that it must be the case that for all $i < t$, $z^\star_t \geq z^\star_i$. Then, integrating equation (E.8) and setting $k = \infty$ we get:

$$M_t^W \int_{z>z^\star} z\mu(z)dz = \left(\sum_{i=1}^{\infty} \rho_z^{i-1}((1-\rho_z)M_{t-i}^W + M_{t-i+1}^e)\right) \int_{z>z^\star} zg(z)dz \tag{E.18}$$

That is,

$$M_t^W \overline{z}_t = \left(\sum_{i=1}^{\infty} \rho_z^{i-1}((1-\rho_z)M_{t-i}^W + M_{t-i+1}^e)\right) \left(\frac{\xi}{\xi-1}\right) (z_t^\star)^{1-\xi}. \tag{E.19}$$

This expression is problematic in that M_t^W is not defined for $t < 0$. However, we can make this expression meaningful by assuming that at $t = 0$ the economy is in a steady state, so that $M_t^W = M_0^W$, $M_t^e = M_0^e$ and $z^\star_t = z^\star_0$ for all $t < 0$.

Similarly, we have that with $z^\star_t \geq z^\star_{t-i}$ for all $i \geq 0$,

$$M_t^W \int_{z>z^\star} \mu(z)dz = \left(\sum_{i=1}^{\infty} \rho_z^{i-1}((1-\rho_z)M_{t-i}^W + M_{t-i+1}^e)\right) \int_{z>z^\star} g(z)dz. \tag{E.20}$$

That is,

$$M_t^W = \left(\sum_{i=1}^{\infty} \rho_z^{i-1}((1-\rho_z)M_{t-i}^W + M_{t-i+1}^e)\right) (z_t^\star)^{-\xi}. \tag{E.21}$$

Comparing equations (E.19) and (E.21), we see that, in this case,

$$\overline{z}_t = \left(\frac{\xi}{\xi-1}\right) z_t^\star. \tag{E.22}$$

Equally, since by assumption $z^\star_{j_t} \geq z^\star_i$, for all $i < j_t$, we can also use equation (E.8), setting $k = \infty$, to conclude that:

$$\overline{z}_{j_t} = \left(\frac{\xi}{\xi-1}\right) z_{j_t}^\star \tag{E.23}$$

$$M_{j_t}^W = \left(\sum_{i=1}^{\infty} \rho_z^{i-1}((1-\rho_z)M_{j_t-i}^W + M_{j_t-i+1}^e)\right) (z_{j_t}^\star)^{-\xi} \tag{E.24}$$

Now, combining equations (E.22) and (E.23), we see immediately that

$$\overline{z}_t = \frac{\xi z_t^\star}{\xi - 1} = \frac{\xi z_t^\star}{\xi - 1} + \frac{\rho_z^{t-j_t} M_{j_t}^W}{M_t^W} \left(\overline{z}_{j_t} - \frac{\xi z_{j_t}^\star}{\xi - 1} \right) \tag{E.25}$$

Likewise, combining equations (E.21) and (E.24), we get:

$$M_t^W = \rho_z^{t-j_t} M_{j_t}^W \left(\frac{z_t^\star}{z_{j_t}^\star} \right)^{-\xi} + (z_t^\star)^{-\xi} \sum_{i=1}^{t-j_t} \rho_z^{i-1} ((1-\rho_z) M_{t-i}^W + M_{t-i+1}^e) \tag{E.26}$$

Since $z_t^\star \geq z_{j_t}^\star$, equations (E.25) and (E.26) correspond to the results of Proposition 1 in this case.

Case 2: Now consider instead the alternative case, where $z_{j_t}^\star > z_t^\star$. By the definition of j_t in equation (33), it must then be the case that $z_i^\star \leq z_t^\star$ for all i satisfying $j_t < i \leq t$. Then, setting $k = t - j_t - 1$ it follows that $z_t^\star \geq \max\{z_{t-i}^\star\}_{i=0}^k$. Then, integrating equation (E.16) with $k = t - j_t - 1$ we have that:

$$M_t^W \int_{z > z_t^\star} z \mu_t(z) dz = \rho_z^{t-j_t} M_{j_t}^W \int_{z > z_t^\star} z \mu_{j_t}(z) dz + \int_{z > z_t^\star} z g(z) dz \sum_{i=1}^{t-j_t} \rho_z^{i-1} ((1-\rho_z) M_{t-i}^W + M_{t-i+1}^e)$$

Since in this case it holds that $\int_{z > z_t^\star} z \mu_{j_t}(z) dz = \int_{z > z_{j_t}^\star} z \mu_{j_t}(z) dz = \overline{z}_{j_t}$, we may simplify this to:

$$M_t^W \overline{z}_t = \rho_z^{t-j_t} M_{j_t}^W \overline{z}_{j_t} + \left(\frac{\xi}{\xi - 1} \right) (z_t^\star)^{1-\xi} \sum_{i=1}^{t-j_t} \rho_z^{i-1} ((1-\rho_z) M_{t-i}^W + M_{t-i+1}^e) \tag{E.27}$$

Similarly, in this case we may also integrate (E.16) with $k = t - j_t - 1$ to get that

$$M_t^W \int_{z > z_t^\star} \mu_t(z) dz = \rho_z^{t-j_t} M_{j_t}^W \int_{z > z_t^\star} \mu_{j_t}(z) dz + \int_{z > z_t^\star} g(z) dz \sum_{i=1}^{t-j_t} \rho_z^{i-1} ((1-\rho_z) M_{t-i}^W + M_{t-i+1}^e).$$

This simplifies to:

$$M_t^W = \rho_z^{t-j_t} M_{j_t}^W + (z_t^\star)^{-\xi} \sum_{i=1}^{t-j_t} \rho_z^{i-1} ((1-\rho_z) M_{t-i}^W + M_{t-i+1}^e). \tag{E.28}$$

Dividing equation (E.27) by equation (E.28) implies that

$$\overline{z}_t = \frac{\xi z_t^\star}{\xi - 1} + \frac{\rho_z^{t-j_t} M_{j_t}^W}{M_t^W} \left(\overline{z}_{j_t} - \frac{\xi z_t^\star}{\xi - 1} \right) \tag{E.29}$$

Since, by assumption, $z_{j_t}^\star > z_t^\star$, equations (E.28) and (E.29) correspond to the results of Proposition 1 in this case.

The intuition for the proof of Proposition 1 is as follows. Let j_t denote the latest period $k < t$ such that $z_k^\star > z_t^\star$ (the definition of j_t has to be slightly more complicated than this in practice, to account for the cases where no such period k exists). The dynamics of wholesaler productivity imply that, in order to know the distribution of wholesaler productivities at time t, it suffices to know only the distribution at time j_t, along with $z_t^\star$ and the number of new entrants in periods $j_t + 1, ...t$. This is because each wholesaler either retains the same productivity z as it had in the previous period, or draws a new productivity according to the Pareto distribution g, with all new entrants drawing a productivity according to g. Then, since $z_{j_t}^\star > z_t^\star$, it follows that all wholesalers who were active in period j_t who have not drawn a new productivity will still be active in period t. Now, the measure $\mu(z_t)$ of wholesalers with a given z_t will be the sum of the measure of wholesalers who have retained the same z since period j_t and the measure of wholesalers who either entered or drew a new productivity between j_t and t. Wholesalers who draw a new productivity or entered between these periods will be active in period t if and only if their current productivity exceeds $z_t^\star$ (since $z_k^\star \leq z_t^\star$ for all periods $j_t < k \leq t$). Therefore, the distribution of z_t across wholesalers who drew a new z between j_t and t will be the Pareto distribution g truncated at $z_t^\star$. Combining this with the measure of wholesalers who have not changed productivity since j_t gives the distribution of z_t across all wholesalers. Integrating this to get $\overline{z}_t$ and M_t^F then gives the results of Proposition 1.

E.2 Proof of Proposition 2

To begin with, we substitute $\pi^W(z, \mathbf{X}_t) = z\overline{\pi}_t^W - c_F$ into the wholesaler Bellman equation to get:

$$\begin{aligned} W(z; \mathbf{X}_t, S) &= z\overline{\pi}_{S,t}^W - c_F + M_t\rho_z \max\{W(z; \mathbf{X}_{t+1}, S); 0\} \\ &\quad + M_t(1-\rho_z)E_{G_z}[\max\{W(z'; \mathbf{X}_{t+1}, S); 0\}] \end{aligned} \tag{E.30}$$

It turns out to be much more convenient to define $\hat{W}_S(z, \mathbf{X}_t)$ as:

$$\hat{W}_S(z, \mathbf{X}_t) = \max\{W(z; \mathbf{X}_t, S); 0\}$$

Then, we can write the free entry and free exit conditions as:

$$\nu\left(\frac{M_{S,t}^e}{M_{S,t-1}^W}\right)^\Theta = E_{G_z}\hat{W}_S(z, \mathbf{X}_t) \tag{E.31}$$

$$z_{S,t}^\star = \inf\{z \geq 1 : \hat{W}_S(z, \mathbf{X}_t) > 0\} \tag{E.32}$$

To prove the Proposition, we first show the following Lemma giving a closed form solution for $\hat{W}$-

Lemma 1. *Suppose that $z_{S,t}^\star > 1$ always. The maximized firm's value function $\hat{W}_S(z, \mathbf{X}_t)$*

satisfies:

$$\hat{W}_S(z, \mathbf{X}_t) = \sum_{j=0}^{\infty}(1 - M_{t+j}\rho_z)A_{S,t+j} \max\left\{z - z^\star_{S,t,j}; 0\right\} \frac{\Pi_{k=0}^{k=j}(M_{t+k}\rho_z)}{M_{t+j}\rho_z} \tag{E.33}$$

where

$$A_{S,t} = \frac{\overline{\pi}^W_{S,t}}{1 - \rho_z M_t} \tag{E.34}$$

$$z^\star_{S,t,j} = \max\{z^\star_{S,\tau}\}_{\tau=t}^{\tau=t+j} \tag{E.35}$$

In the steady state, these equations become:

$$\hat{W}_S(z) \equiv A^* \max\{z - z^\star_S; 0\} \tag{E.36}$$

$$A^*_S = \frac{1}{1 - \rho_z M}\overline{\pi}^W_S z \tag{E.37}$$

Proof. The proof below frequently omits the S subscript, but the proof is unchanged if this is included.

Using the definition of $\hat{W}$, the intermediate goods firm Bellman equation implies that:

$$\hat{W}_S(z, \mathbf{X}_t) =$$
$$\max\left\{0; \overline{\pi}^W_{S,t}z - c_F + M_t[\rho_z \hat{W}_S(z; \mathbf{X}_{t+1}) + (1 - \rho_z)E_{G_z}\hat{W}_S(z'; \mathbf{X}_{t+1})]\right\}$$

We can use the free entry condition to simplify the recursive equation for $\hat{W}_S(z, \mathbf{X}_t)$ further:

$$\hat{W}_S(z, \mathbf{X}_t) =$$
$$\max\left\{0; \overline{\pi}^W_{S,t}z - c_F + M_t[\rho_z \hat{W}_S(z; \mathbf{X}_{t+1}) + (1 - \rho_z)\nu\left(\frac{M^e_{S,t+1}}{M^W_{S,t}}\right)^\Theta]\right\} \tag{E.38}$$

We first solve for the intermediate goods firm's maximized value function $\hat{W}_S(\mathbf{X}_t, z)$ in the steady state.

First note that free exit means that $\hat{W}_S(z^\star) = 0$ in the steady state ($z^\star > 1$ must hold in the steady state since otherwise no firms would exit). Since the continuing firm's profits $\pi^W_t(z)$ are linear in z, we may guess (and will verify) that for the continuing firm the maximized value function should also be linear in z in the steady state. Since $\hat{W}_S(z^\star) = 0$, this implies that:

$$\hat{W}_S(z) \equiv A^* \max\{z - z^\star; 0\},$$

for some constant A^*.

Substituting this into the Bellman equation and assuming a steady state, we get:

$$\begin{aligned}\hat{W}_S(z) &= z\overline{\pi}_S^W z - c_F + M\rho_z A^* \max\{z - z^\star; 0\} \\ &+ M(1-\rho_z)A^* \int_{\{z' \geq z^\star(\mathbf{X}_{t+1}, S)\}} (z' - z^\star) dG,\end{aligned}$$

This is consistent with our guess that $\hat{W}_S(z) = A^* \max\{z - z^\star; 0\}$ provided that $\frac{\partial \hat{W}_S(z)}{\partial z} = A^*$ for $z > z^\star$, which holds as long as:

$$A^* = \overline{\pi}_S^W z + M\rho_z A^*$$

i.e.

$$A^* = \frac{1}{1 - \rho_z M}\overline{\pi}_S^W z$$

Now we prove the dynamic solution for $\hat{W}_S(z, \mathbf{X}_t)$ by induction. The dynamic solution for $\hat{W}_S(z, \mathbf{X}_t)$ equivalent to the steady state solution just derived, in steady state, so the dynamic solution is definitely correct once we reach steady state. Let

$$S_t(z) = \overline{\pi}_{S,t}^W z - c_F + M_t(1-\rho_z)\nu \left(\frac{M_{S,t+1}^e}{M_{S,t}^W}\right)^\Theta$$

Now, suppose the dynamic solution holds for $t+1$, then the Bellman equation for W implies that:

$$\begin{aligned}&\hat{W}_S(z, \mathbf{X}_t) \\ &\quad = \max\Bigg\{0; S_t(z) + M_t\rho_z(1 - M_{t+1}\rho_z)A_{t+1}\max\{z - z_{t+1}^\star; 0\} \\ &\qquad\qquad + M_t\rho_z \sum_{j=1}^{\infty}(1 - M_{t+1+j}\rho_z)A_{t+1+j}\max\{z - z_{t+1,j}^\star; 0\}\Pi_{\tau=t+1}^{\tau=t+j}(M_\tau\rho_Z)\Bigg\}\end{aligned}$$

which is:

$$\hat{W}_S(z, \mathbf{X}_t) = 1(z > z_t^\star)\left(S_t(z) + \sum_{j=0}^{\infty}(1 - M_{t+1+j}\rho_z)A_{t+1+j}\max\{z - z_{t+1,j}^\star; 0\}\Pi_{\tau=t}^{\tau=t+j}(M_\tau\rho_Z)\right)$$

and, since $\hat{W}_S(z^\star, \mathbf{X}_t) = 0$, it must also be the case that:

$$0 = S_t(z_t^\star) + \sum_{j=0}^{\infty}(1 - M_{t+1+j}\rho_z)A_{t+1+j}\max\{z_t^\star - z_{t+1,j}^\star; 0\}\Pi_{\tau=t}^{\tau=t+j}(M_\tau\rho_Z)$$

Combining these, we have:

$$\hat{W}_S(z, \mathbf{X}_t) = \mathbb{1}(z > z_t^\star)\left(S_t(z) - S_t(z_t^\star)\right) + 1(z > z_t^\star)\times$$
$$\left(\sum_{j=0}^{\infty}(1 - M_{t+1+j}\rho_z)A_{t+1+j}\left(\max\{z - z_{t+1,j}^\star; 0\} - \max\{z_t^\star - z_{t+1,j}^\star; 0\}\right)\Pi_{\tau=t}^{\tau=t+j}(M_\tau \rho_Z)\right),$$

which is

$$\hat{W}_S(z, \mathbf{X}_t) = A_t(1 - M_t\rho_z)\max\{z - z_t^\star; 0\} + \sum_{j=0}^{\infty}$$
$$\left(A_{t+1+j}(1 - M_{t+1+j}\rho_z)\mathbb{1}(z > z_t^\star)\left(\max\{z - z_{t+1,j}^\star; 0\} - \max\{z_t^\star - z_{t+1,j}^\star; 0\}\right)\Pi_{\tau=t}^{\tau=t+j}(M_\tau \rho_Z)\right),$$

which is

$$\hat{W}_S(z, \mathbf{X}_t) = A_t(1 - M_t\rho_z)\max\{z - z_t^\star; 0\} + \sum_{j=1}^{\infty}$$
$$\left(A_{t+j}(1 - M_{t+j}\rho_z)\mathbb{1}(z > z_t^\star)\left(\max\{z - z_{t+1,j-1}^\star; 0\} - \max\{z_t^\star - z_{t+1,j-1}^\star; 0\}\right)\Pi_{\tau=t}^{\tau=t+j-1}(M_\tau \rho_Z)\right),$$

which is

$$\begin{aligned}\hat{W}_S(z, \mathbf{X}_t) = {} & A_t(1 - M_t\rho_z)\max\{z - z_t^\star; 0\} \\ & + \sum_{j=1}^{\infty} A_{t+j}(1 - M_{t+j}\rho_z)\left(\Pi_{\tau=t}^{\tau=t+j-1}(M_\tau \rho_Z)\right) \\ & \times \left[\mathbb{1}(z_{t+1,j-1}^\star > z_t^\star)\mathbb{1}(z > z_{t+1,j-1}^\star)(z - z_{t+1,j-1}^\star) + \mathbb{1}(z_t^\star > z_{t+1,j-1}^\star)\mathbb{1}(z > z_t^\star)(z - z_t^\star)\right],\end{aligned}$$

which is

$$\begin{aligned}\hat{W}_S(z, \mathbf{X}_t) = {} & A_t(1 - M_t\rho_z)\max\{z - z_t^\star; 0\} \\ & + \sum_{j=1}^{\infty} A_{t+j}(1 - M_{t+j}\rho_z)\max\left\{z - \max\{z_t^\star; z_{t+1,j-1}^\star\}; 0\right\}\Pi_{\tau=t}^{\tau=t+j-1}(M_\tau \rho_Z)\end{aligned}$$

which is

$$\begin{aligned}\hat{W}_S(z, \mathbf{X}_t) = {} & A_t(1 - M_t\rho_z)\max\{z - z_t^\star; 0\} \\ & + \sum_{j=1}^{\infty} A_{t+j}(1 - M_{t+j}\rho_z)\max\left\{z - z_{t,j}^\star; 0\right\}\Pi_{\tau=t}^{\tau=t+j-1}(M_\tau \rho_Z)\end{aligned}$$

which is

$$\hat{W}_S(z, \mathbf{X}_t) = \sum_{j=0}^{\infty} A_{t+j}(1 - M_{t+j}\rho_z)\max\left\{z - z_{t,j}^\star; 0\right\}\frac{\Pi_{k=0}^{k=j}(M_{t+k}\rho_z)}{M_{t+j}\rho_z}$$

and so the induction is proved.

It is straightforward to show that in the steady state (38) becomes:

$$\mathcal{A}_{t,j} = (1 - M\rho_z)M^j \rho_z^j A^*$$

and so

$$\sum_{j=0}^{\infty} \mathcal{A}_{t,j} = A^*$$

Then, it is immediate that (34) and (35) become:

$$\begin{aligned} \nu \left(\frac{M^e}{M^W} \right)^{\Theta} &= A^* \left(\frac{z^{\star -(\xi-1)}}{\xi - 1} \right) \\ z^\star &= \frac{c_F - M(1-\rho_z)\nu \left(\frac{M^e}{M^W} \right)^{\Theta}}{(1-\rho_Z M)A^*} \end{aligned}$$

□

Now, we use the Lemma to prove Proposition 2. Recall the Proposition: *$z_t^\star$ and M_t^E satisfy:*

$$\nu \left(\frac{M_{S,t}^e}{M_{S,t-1}^W} \right)^{\Theta} = \sum_{j=0}^{\infty} \mathcal{A}_{t,j} \left(\frac{z_{t,j}^{\star -(\xi-1)}}{\xi - 1} \right) \tag{E.39}$$

and

$$z_t^\star = \frac{c_F - M_t(1-\rho_z)\nu \left(\frac{M_{S,t}^e}{M_{S,t-1}^W} \right)^{\Theta}}{(1-\rho_Z M_t)A_t} - \left(\frac{1}{(1-\rho_z M_t)A_t} \right) \sum_{j=1}^{\infty} \mathcal{A}_{t,j} \left(z_{t,j}^\star - z_{t+1,j-1}^\star \right) \tag{E.40}$$

where

$$\mathcal{A}_{t,j} = (1-\rho_Z M_{t+j})A_{t+j} \frac{\Pi_{k=0}^{k=j}(M_{t+k}\rho_z)}{M_{t+j}\rho_z} \tag{E.41}$$

Proof. First we derive (E.39). Recall that, when z is Pareto, $\mathbb{E}\left[\max\{z - z^\star; 0\}\right] = \frac{z^{\star -(\xi-1)}}{\xi-1}$, which can be verified by integration.

Using this to take the expectation of the equation for $\hat{W}(z, \mathbf{X}_t)$ in (E.33) above, we have:

$$\nu \left(\frac{M_{S,t}^e}{M_{S,t-1}^W} \right)^{\Theta} = \mathbb{E}\left[\hat{W}(z, \mathbf{X}_t)\right] = \sum_{j=0}^{\infty} (1-\rho_Z M_{t+j})A_{t+j} \left(\frac{z_{t,j}^{\star -(\xi-1)}}{\xi - 1} \right) \frac{\Pi_{k=0}^{k=j}(M_{t+k}\rho_z)}{M_{t+j}\rho_z}$$

Using the definition of $\mathcal{A}$ in (E.41), we obtain (E.39).

Now, we derive (E.40). We use the Bellman equation:

$$0 = \hat{W}(z_t^\star, \mathbf{X}_t) = 0 = S_t(z^\star) + M_t\rho_z \hat{W}(z_t^\star, \mathbf{X}_{t+1})$$

where, as above,

$$S_t(z) = z\overline{\pi}^W_{S,t} - c_F + M_t(1-\rho_z)\nu\left(\frac{M^e_{S,t+1}}{M^W_{S,t}}\right)^{\Theta}$$

Then, since $(1 - M_t\rho_z)A_t = S'_t(z)$, it follows that:

$$(1 - M_t\rho_z)A_t = \overline{\pi}^W_{S,t}$$

Then, $z^\star_t$ satisfies:

$$\begin{aligned}(1 - M_t\rho_z)A_t z^\star_t = c_F - M_t(1-\rho_z)\nu\left(\frac{M^e_{S,t+1}}{M^W_{S,t}}\right)^{\Theta} \\ - \sum_{j=0}^{\infty}(1 - M_{t+1+j}\rho_z)A_{t+1+j}\max\left\{z^\star_t - z^\star_{t+1,j}; 0\right\}\frac{M_t\Pi_{k=0}^{k=j}(M_{t+1+k}\rho_z)}{M_{t+1+j}}\end{aligned}$$

which is:

$$\begin{aligned}(1 - M_t\rho_z)A_t z^\star_t = c_F - M_t(1-\rho_z)\nu\left(\frac{M^e_{S,t+1}}{M^W_{S,t}}\right)^{\Theta} \\ - \sum_{j=0}^{\infty}A_{t+1+j}(1 - M_{t+1+j}\rho_z)\left(\max\left\{z^\star_t; z^\star_{t+1,j}\right\} - z^\star_{t+1,j}\right)\frac{M_t\Pi_{k=0}^{k=j}(M_{t+1+k}\rho_z)}{M_{t+1+j}}\end{aligned}$$

Or

$$\begin{aligned}(1 - M_t\rho_z)A_t z^\star_t = c_F - M_t(1-\rho_z)\nu\left(\frac{M^e_{S,t+1}}{M^W_{S,t}}\right)^{\Theta} \\ - \sum_{j=0}^{\infty}(1 - M_{t+1+j}\rho_z)A_{t+1+j}\left(z^\star_{H,j+1} - z^\star_{t+1,j}\right)\frac{M_t\Pi_{k=0}^{k=j}(M_{t+1+k}\rho_z)}{M_{t+1+j}}\end{aligned}$$

Using the definition of $\mathcal{A}$ in (E.41), this is:

$$z^\star_t = \frac{c_F - M_t(1-\rho_z)\nu\left(\frac{M^e_{S,t+1}}{M^W_{S,t}}\right)^{\Theta}}{(1-\rho_Z M_t)A_t} - \left(\frac{M_t\rho_z}{(1-\rho_z M_t)A_t}\right)\sum_{j=0}^{\infty}\mathcal{A}_{t+1,j}\left(z^\star_{H,j+1} - z^\star_{t+1,j}\right)$$

Now, note that

$$\mathcal{A}_{H,j+1} = \mathcal{A}_{t+1,j}M_t\rho_z$$

So we can write the equation for $z^\star_t$ as:

$$z^\star_t = \frac{c_F - M_t(1-\rho_z)\nu\left(\frac{M^e_{S,t+1}}{M^W_{S,t}}\right)^{\Theta}}{(1-\rho_Z M_t)A_t} - \left(\frac{1}{(1-\rho_z M_t)A_t}\right)\sum_{j=0}^{\infty}\mathcal{A}_{H,j+1}\left(z^\star_{H,j+1} - z^\star_{t+1,j}\right)$$

which is the same as (E.40). □

More intuitions for Proposition 2. The left hand side of equation (34) is the cost of entry at time t. The right hand side is the value of entry, which is proportional to a weighted average of future $\frac{z^{\star xi-1}}{\xi-1}$, because, since the distribution of new z draws is Pareto, it is straightforward to show that $E[z|z>z^\star]\text{Prob}(z>z^\star)=\frac{z^{\star xi-1}}{\xi-1}$. The term $\mathcal{A}_{t,j}$ in equation (34) is simply a present discounted sum of future $\overline{\pi}^F_{S,t}$, which determines wholesaler profits.

The optimal value of $z^\star_t$ is given by equation (35). The first term on the right hand side of (35) is $\frac{c_F}{(1-\rho_z M_t)A_t}\equiv\frac{c_F}{\overline{\pi}^F_{S,t}}$. This first term is the cutoff value of z needed to break even in the present period, since the wholesaler's period profits are $\overline{\pi}^F_{S,t}z-c_F$. The remaining terms on the right hand side of (35) reflect the option value to a wholesaler of remaining in operation in the hope of receiving a better draw of z in future. As such, these terms depend on a similar weighted sum of future $z^\star$ terms to the right hand side of equation (34).

It is straightforward to show that in the steady state (38) becomes:

$$\mathcal{A}_{t,j}=(1-M\rho_z)M^j\rho_z^jA^*$$

and so

$$\sum_{j=0}^{\infty}\mathcal{A}_{t,j}=A^*$$

Then, it is immediate that (37), (34) and (35) become:

$$\begin{aligned}
A^*_S &= \tfrac{1}{1-\rho_z M}\overline{\pi}^W_S\\
\nu\left(\frac{M^e}{M_F}\right)^{\Theta} &= A^*\left(\frac{z^{\star-(\xi-1)}}{\xi-1}\right)\\
z^\star &= \frac{c_F-M(1-\rho_z)\nu\left(\frac{M^e}{M_F}\right)^{\Theta}}{(1-\rho_Z M)A^*}
\end{aligned}$$

F Appendix for Quantitative Exercises

F.1 Distribution of Firms

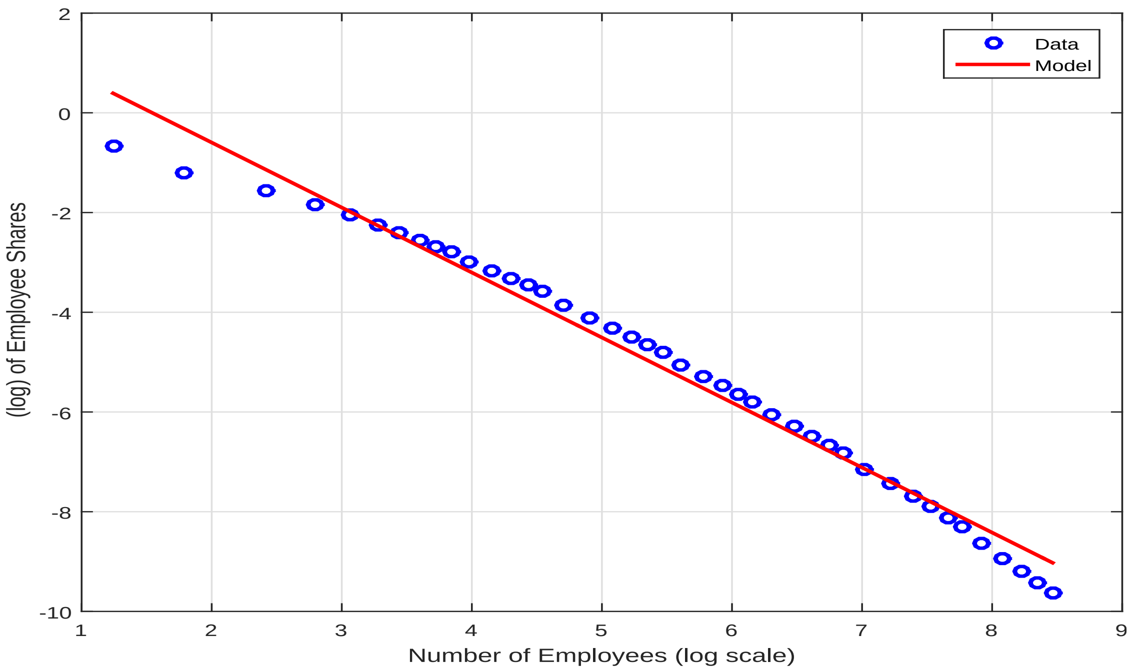


Figure 10: Distribution of establishments by employment: model vs. data.

F.2 Responses

Simple AR(1) Shocks to Interest Rates: Different Persistence. We also consider when the AR(1) shocks to interest rates have different persistence parameters. We consider a one-period, transitory to interest rates (in period 2, the first period after steady state), and the magnitude of shock is the same as the previous AR(1) case. The impulse responses are compared in Figure 12 in the Appendix. Not surprisingly we find that transitory shocks will have smaller responses in consumption, employment and GDP on impact, and also the responses are much less persistent. For investment, the initial responses are more or less similar but it quickly reverts in the case of transitory shocks, as the capital stock in the first two periods build up and firms have to dis-invest when shocks are gone. In turn, the figure shows that investment is much smoother with persistent shocks, and so is employment since capital and labor are complements for production.

Shocks to Aggregate Productivity. We then study the model's responses with an aggregate productivity shock. This is a quite standard exercise in the literature for business cycles, and we can further understand how the model works. Specifically, we do it as follows: in Equation 18, we slightly modify the wholesalers' production function to $y_i = (A_t z_i)^{1-\theta}(k_i^{1-\alpha} n_i^{\alpha})^{\theta}$, where $A_t = 1$ in steady states. Similarly, we assume there is an "MIT" shock to A_t in the beginning of period 1 so that A_1 is increased by 1%; over time, the shocks decay with a persistence parameter of 0.95. The results are now collected in Figure 13 in the Appendix. Like in a standard New Keynesian Model with productivity shocks, the nominal prices for the two types of goods both decline on impact and gradually return back, simply because with increased productivity, firms can produce more with the same amount of inputs. Since prices are with nominal frictions, firms still find it profitable to hire more labor and capital inputs and produce more. Thus, we see factor prices are all driven up, employment and output in the two sectors are also both increasing, net exports also increase in the first few period. Since the persistence of productivity shock is relatively high, labor income and profits increases are persistent and households increase their consumption in the first few periods. With GHH preferences in labor supply, households will supply more labor even if their wealth also increases, and thus the equilibrium employment overall increases. At the same time, households save more and accumulate more assets. The entry rate increases on impact and the exit rate decreases. Real GDP increases for about 1%, and real, labor productivity increases for about 0.35% on impact (using back of envelop calculation, $1-\theta$ is about 0.15, and $(A_t)^{1-\theta}$ would increase for about 0.3%).

Figure 11: Model Responses to AR(1) shocks in interest rates

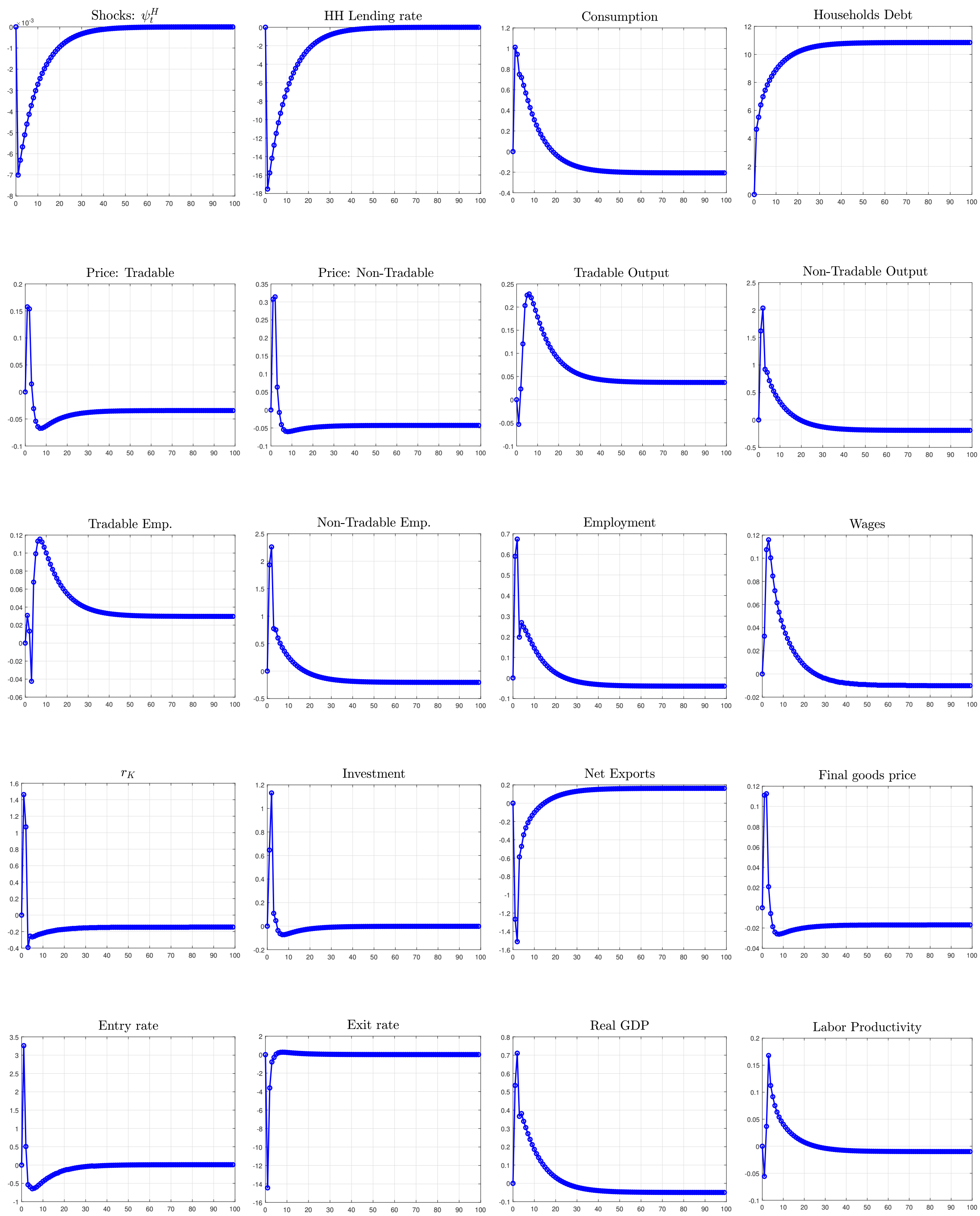


Figure 12: Model Responses to AR(1) shocks in interest rates: different persistence

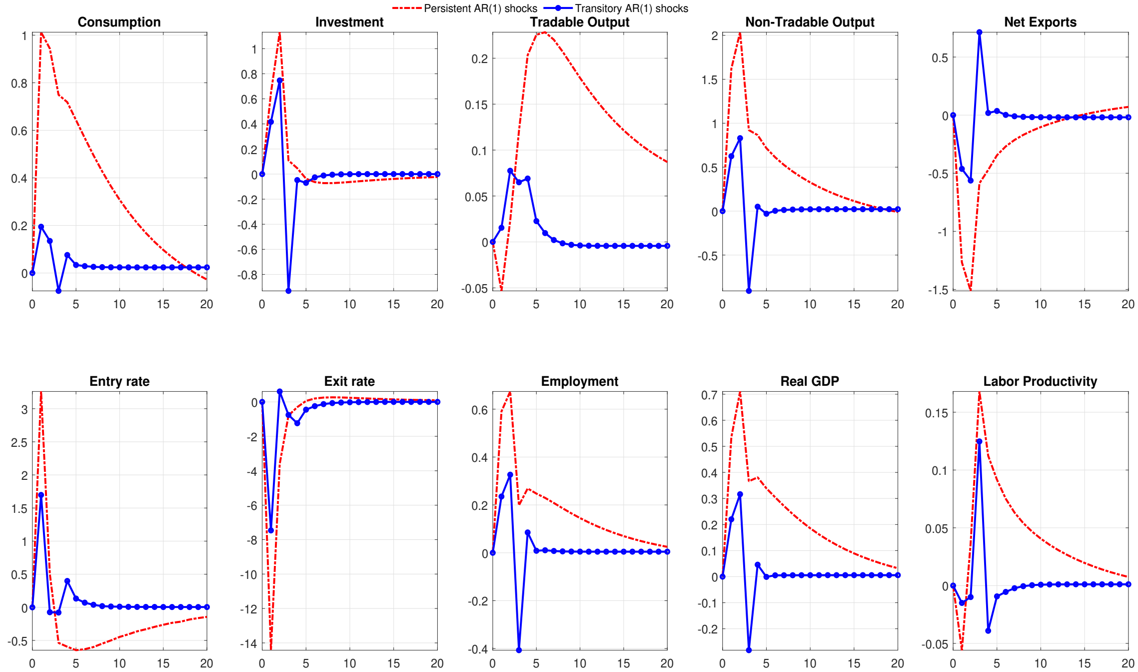


Figure 13: Model Responses to Aggregate Productivity AR(1) shocks

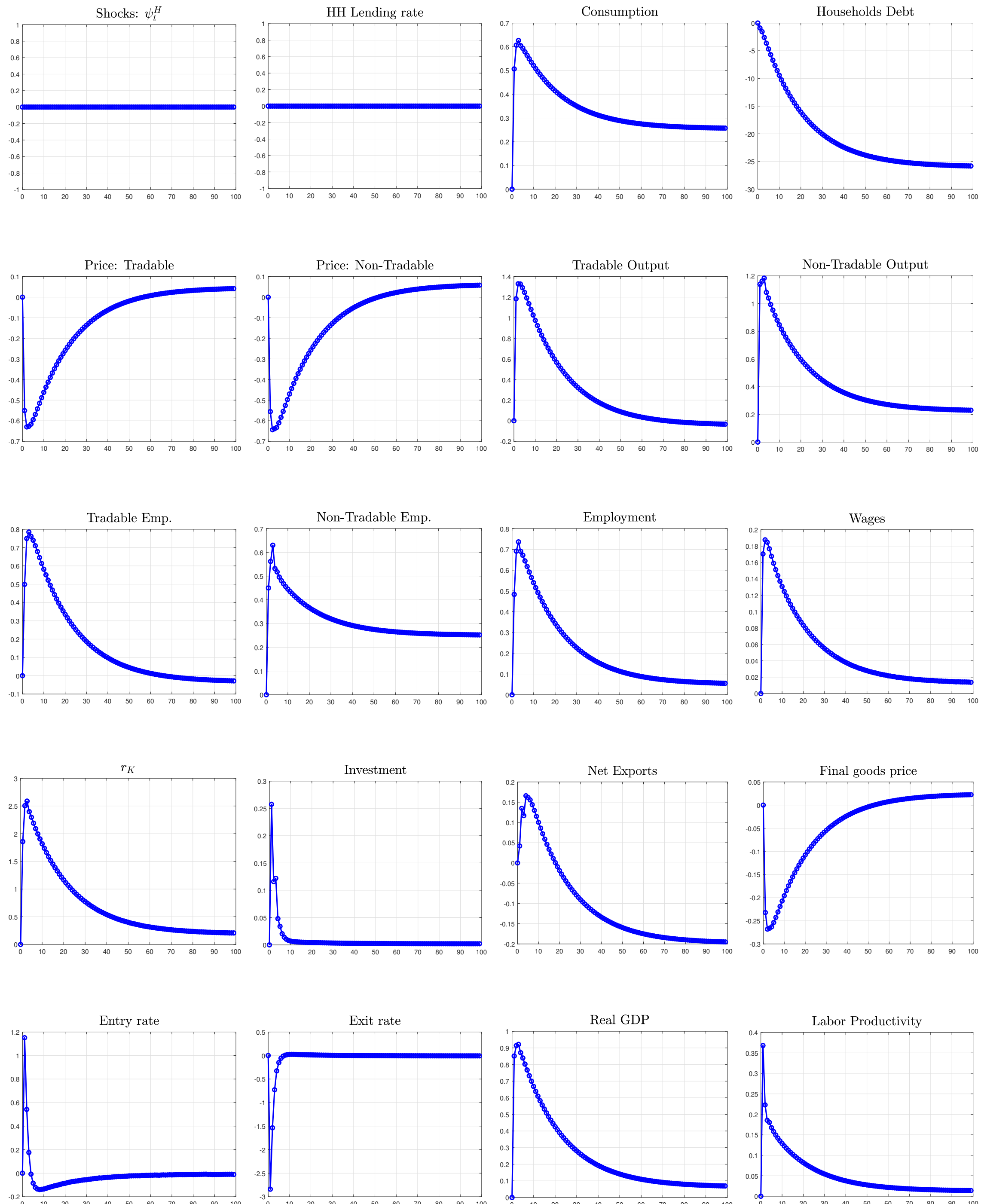


Figure 14: Model responses with different values in η, Demand Elas. for intermediate goods

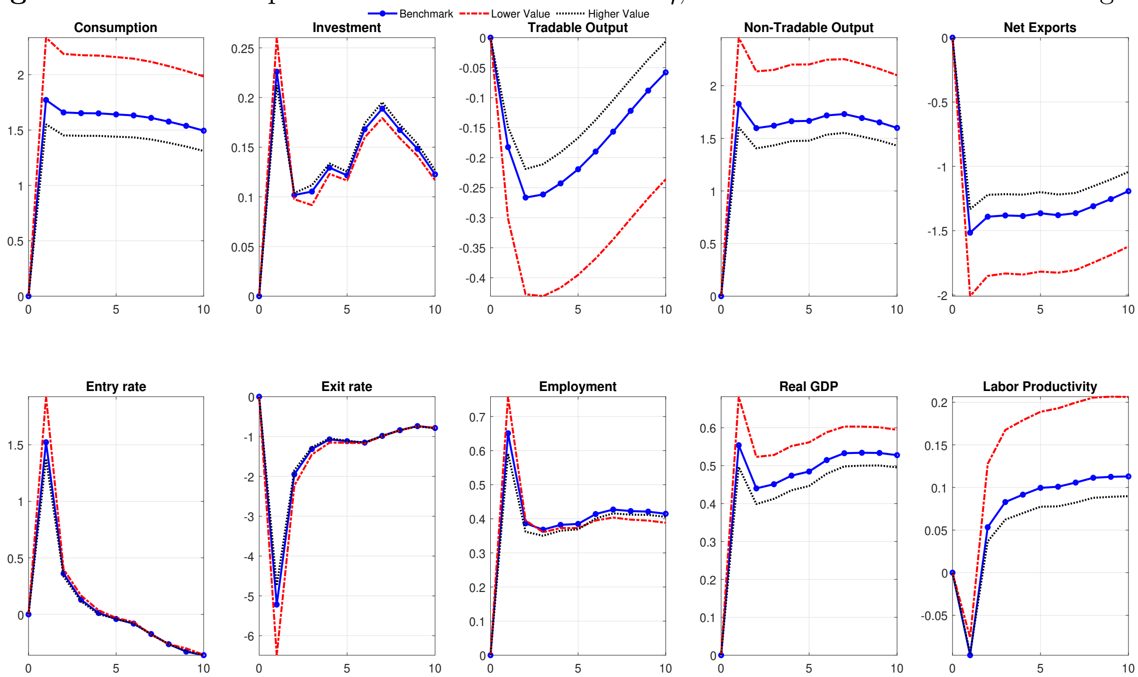


Figure 15: Model responses with different values in Labor supply Elasticity, υ_L

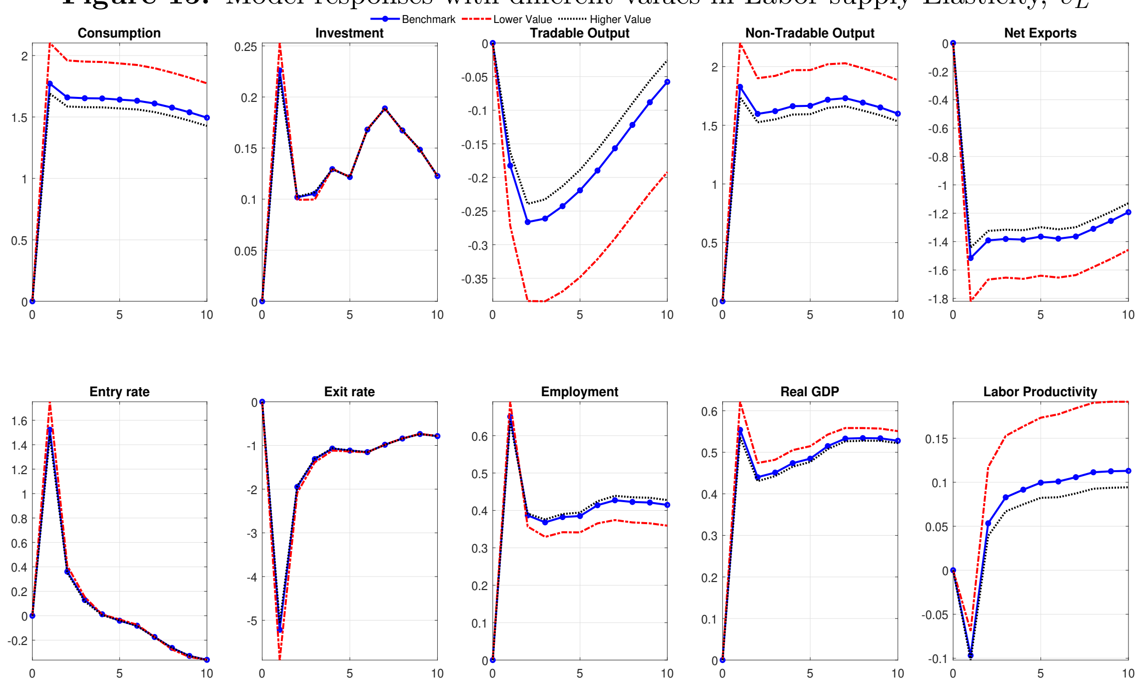


Figure 16: Model responses with different values in ζ (Elasticity between Home and Foreign tradable goods)

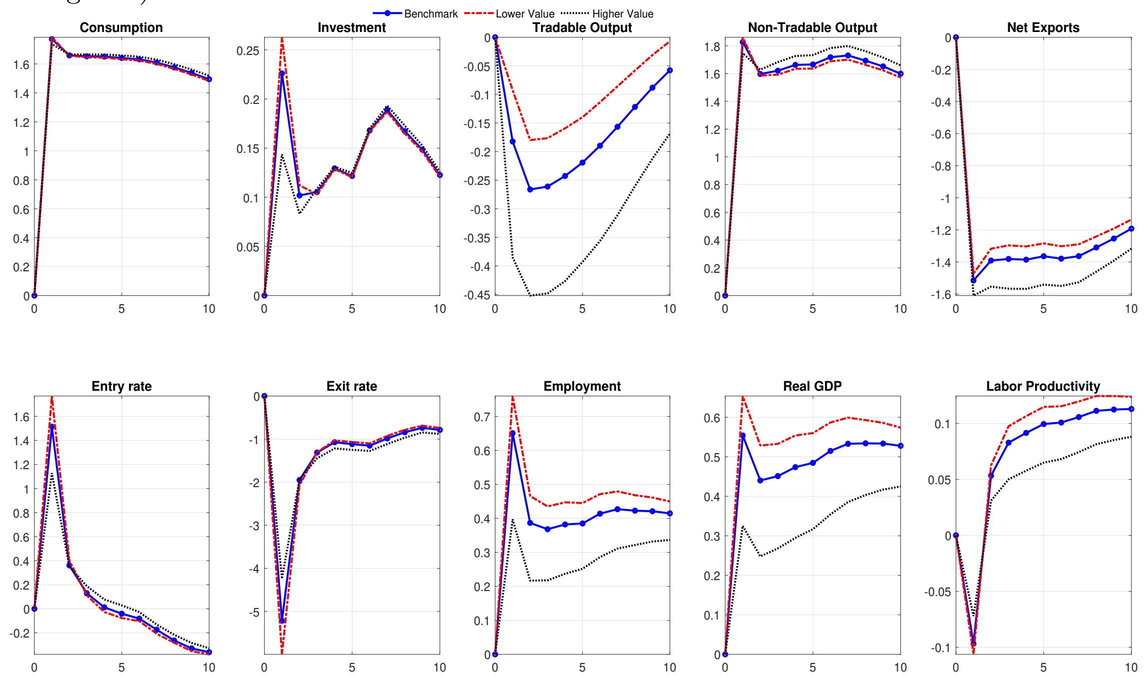


Figure 17: Model responses with different values in Capital adj. costs κ

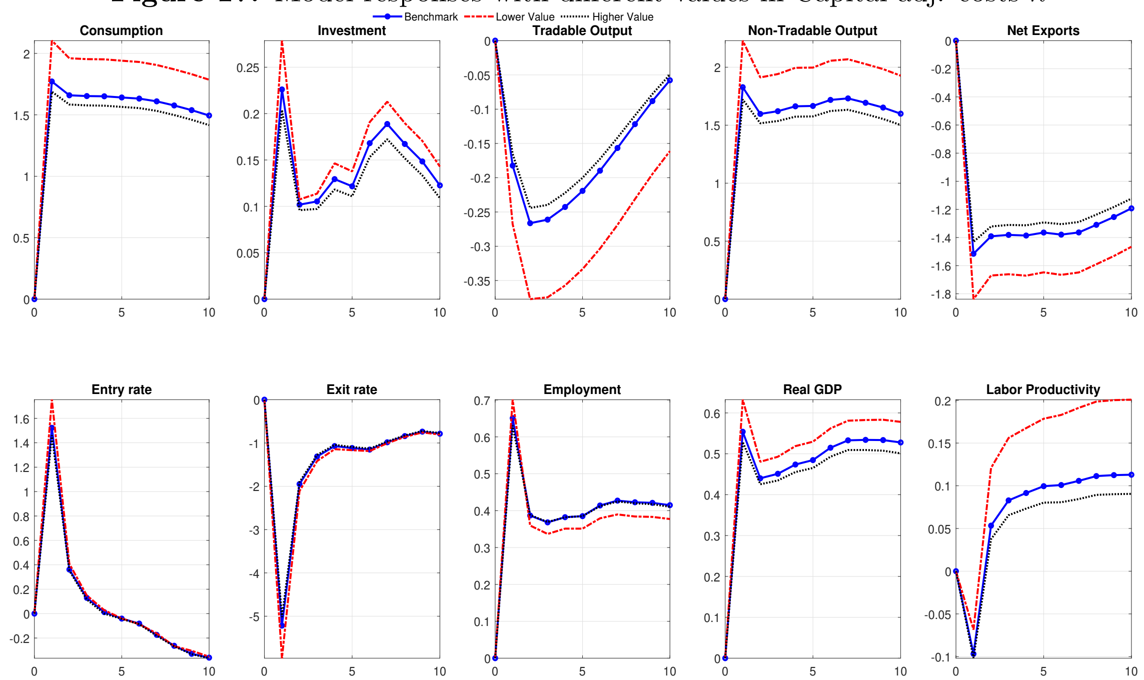


Figure 18: Model responses with different values in Flow operating costs c^F

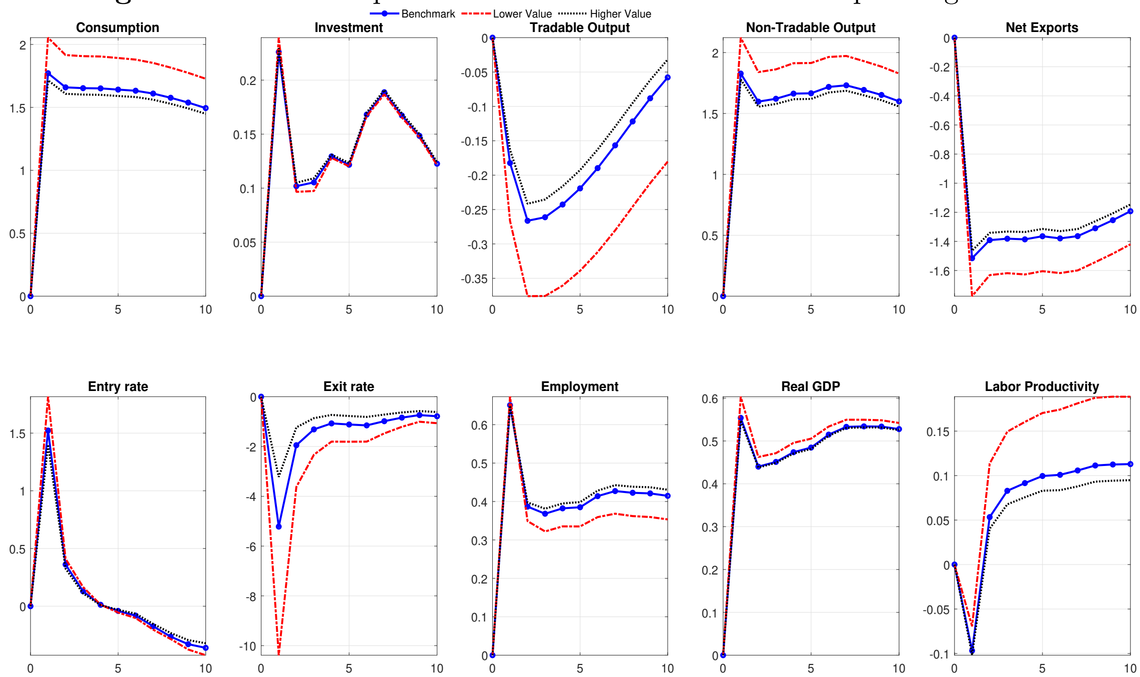


Figure 19: Model responses with different values in Persistence for firm-level productivity, ρ_z

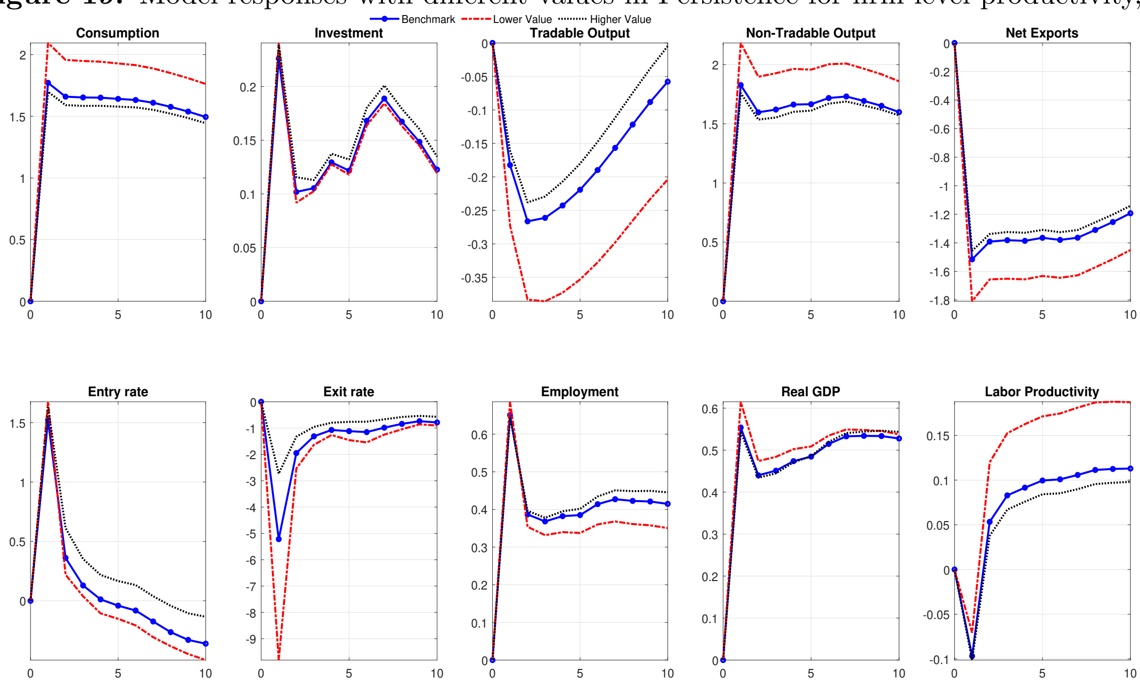


Figure 20: Model responses with different values in Entry cost parameter ν

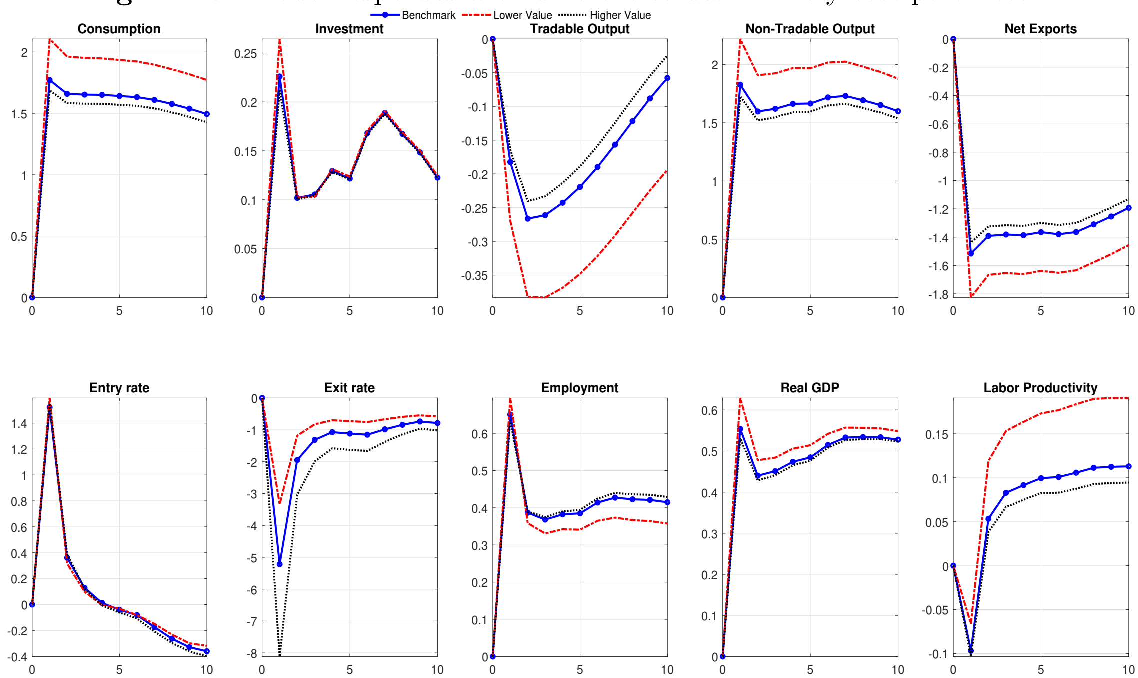


Figure 21: Model responses with different values in Elasticity of entry costs, Θ

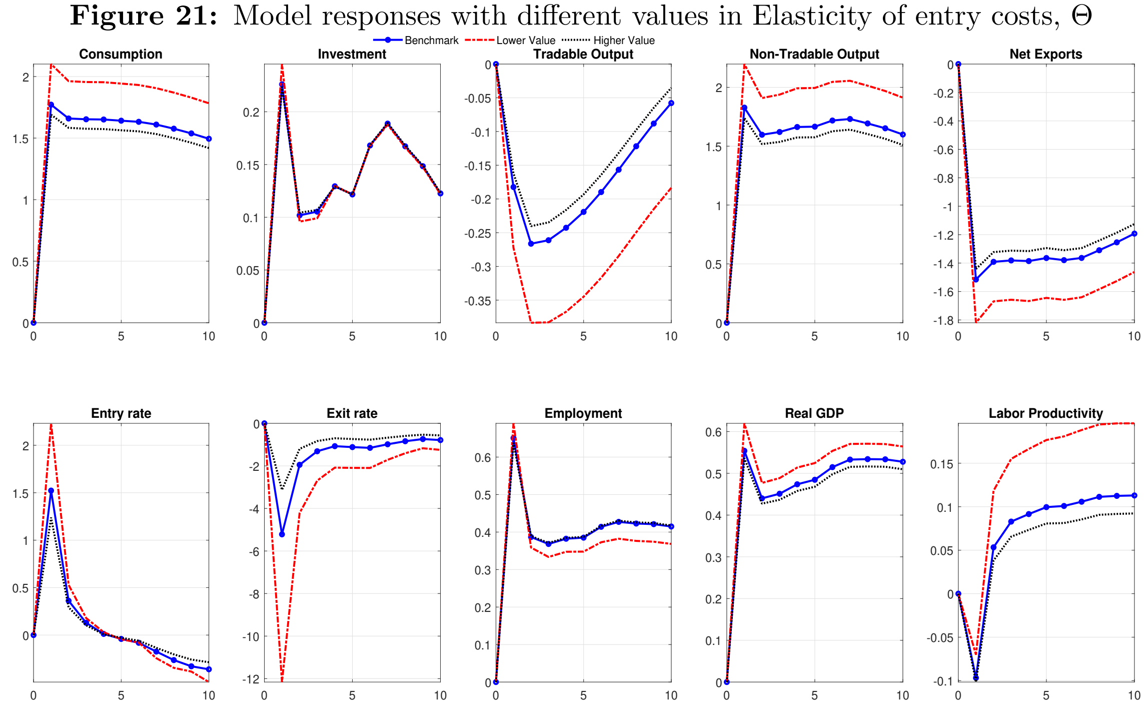
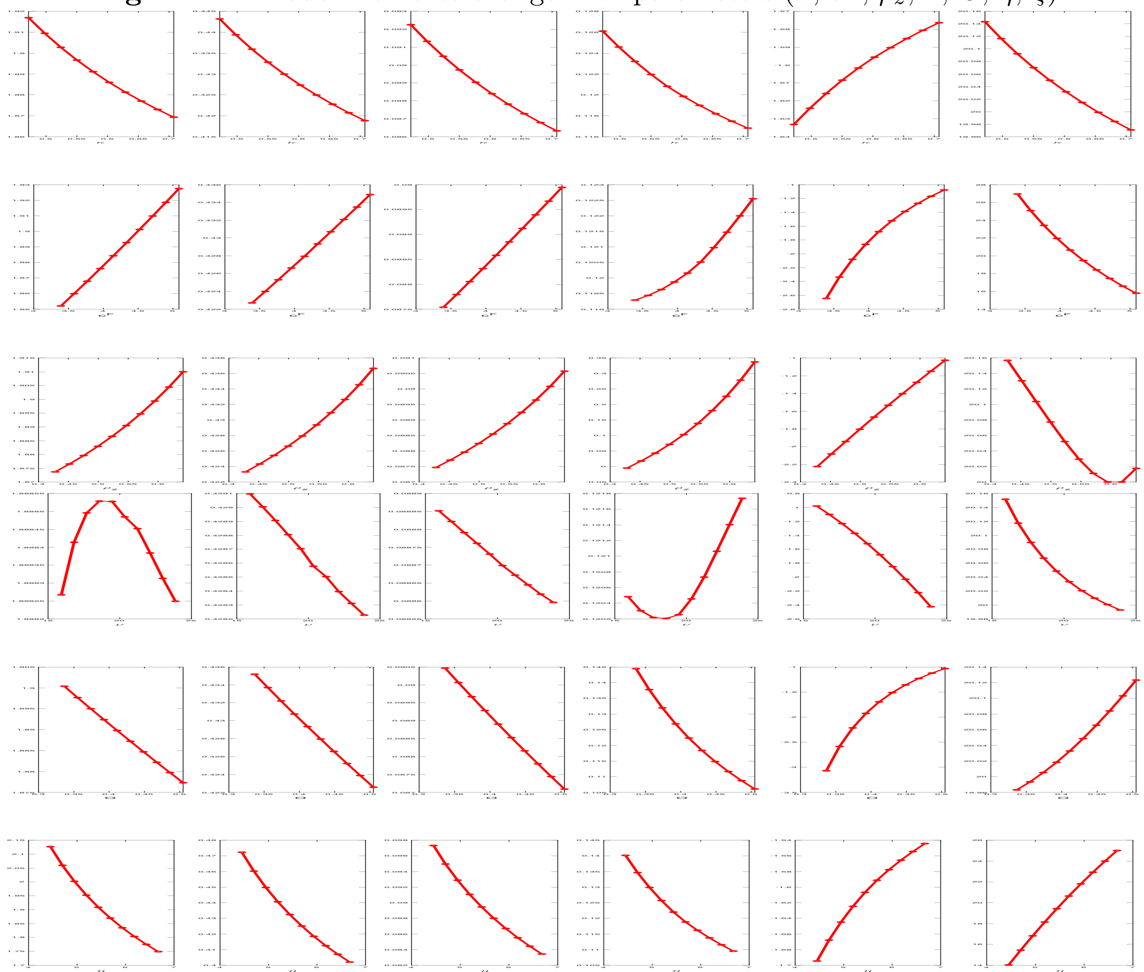


Figure 22: Model moments change with parameters (κ, c^F, ρ_z, ν, Θ, η, ζ)



NOTE: This figure plots the model moment changes as one of the model parameter changes. In the first row, we change κ as in the x-axis, and plot the model's responses for 6 variables in the y-axis (from left to right, Real GDP, Total Employment, Real Wages, Entry rates, Exit rates, Household Debt). The model's responses are computed as the percent deviations from corresponding steady state values averaged from periods 2 to periods 10. Rows from 1 to 7 are for κ, c^F, ρ_z, ν, Θ, η, ζ, respectively.

F.3 Counterfactuals

Figure 23: Larger Firm Debt

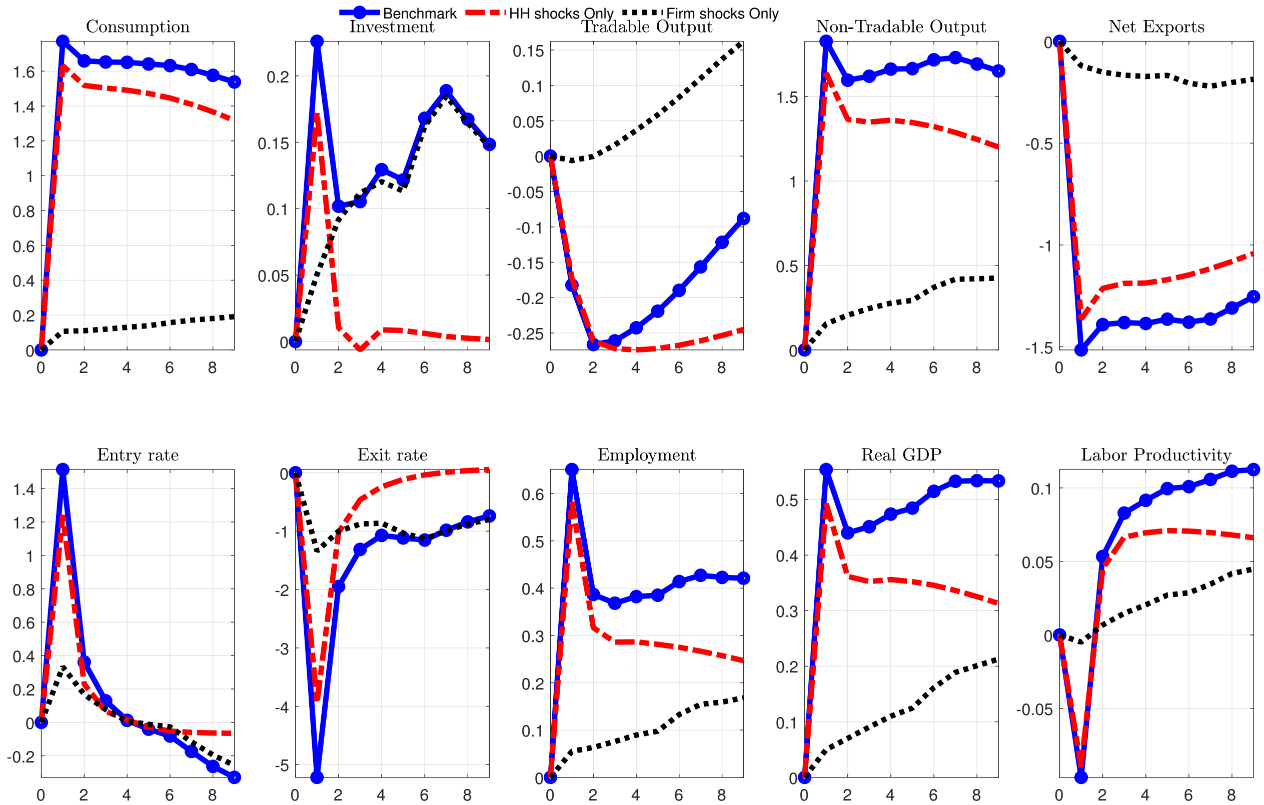


Figure 24: No Firm Debt

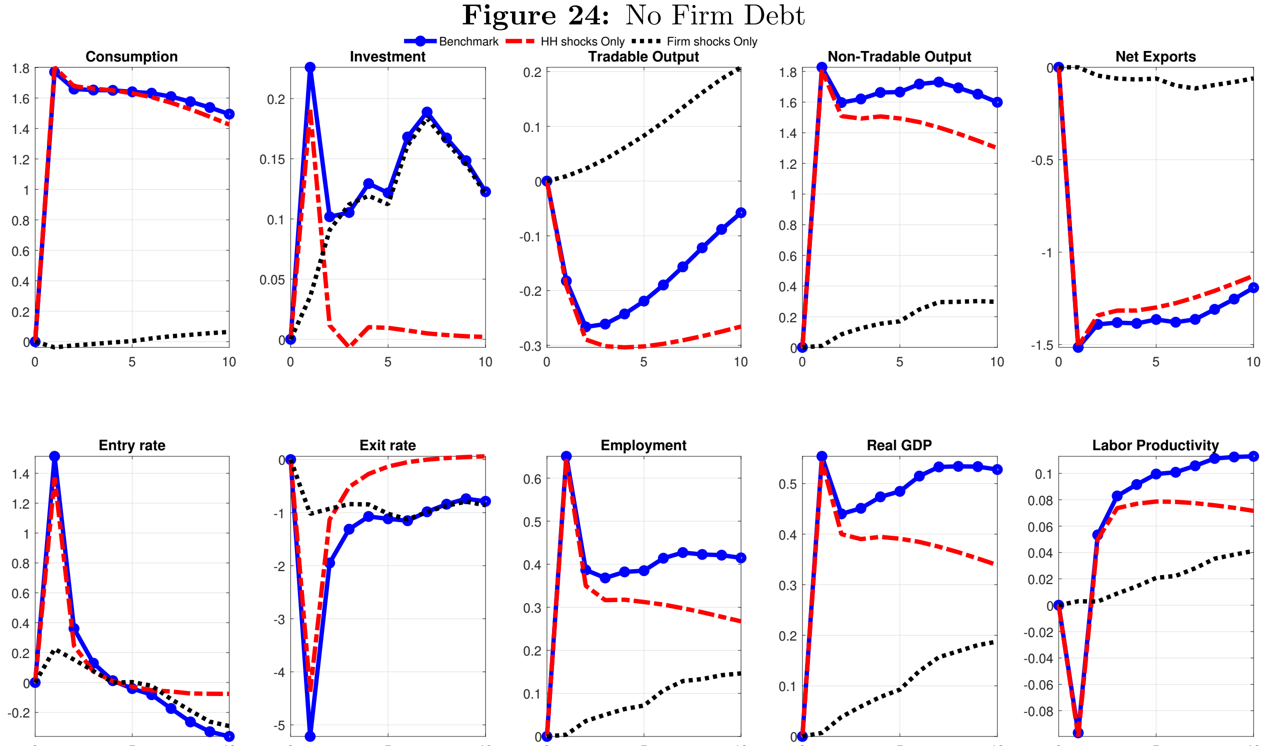


Figure 25: No Firm Debt: the Dynamics of Importance Over Time

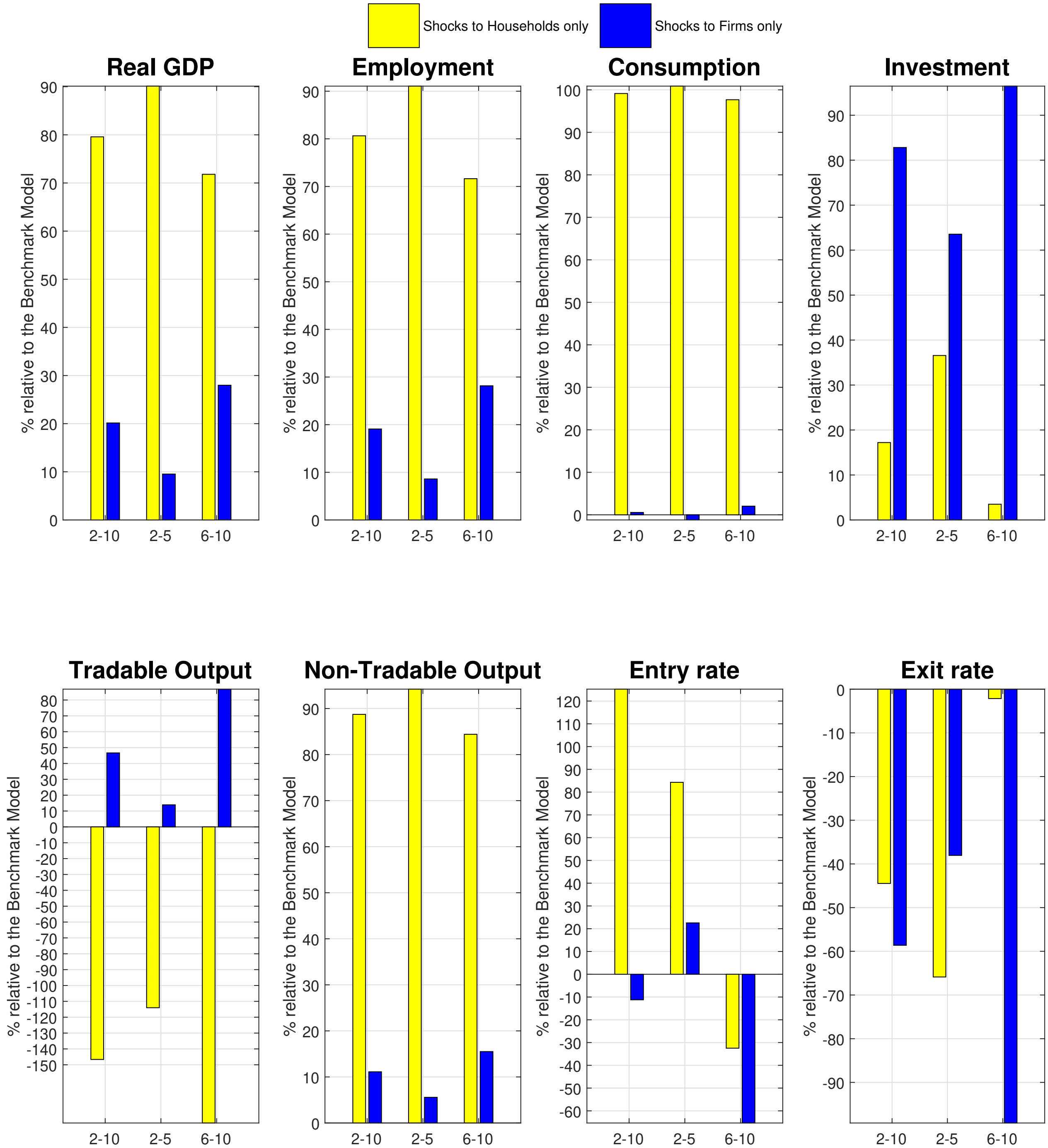


Figure 26: No Firm Debt: the Relative Importance

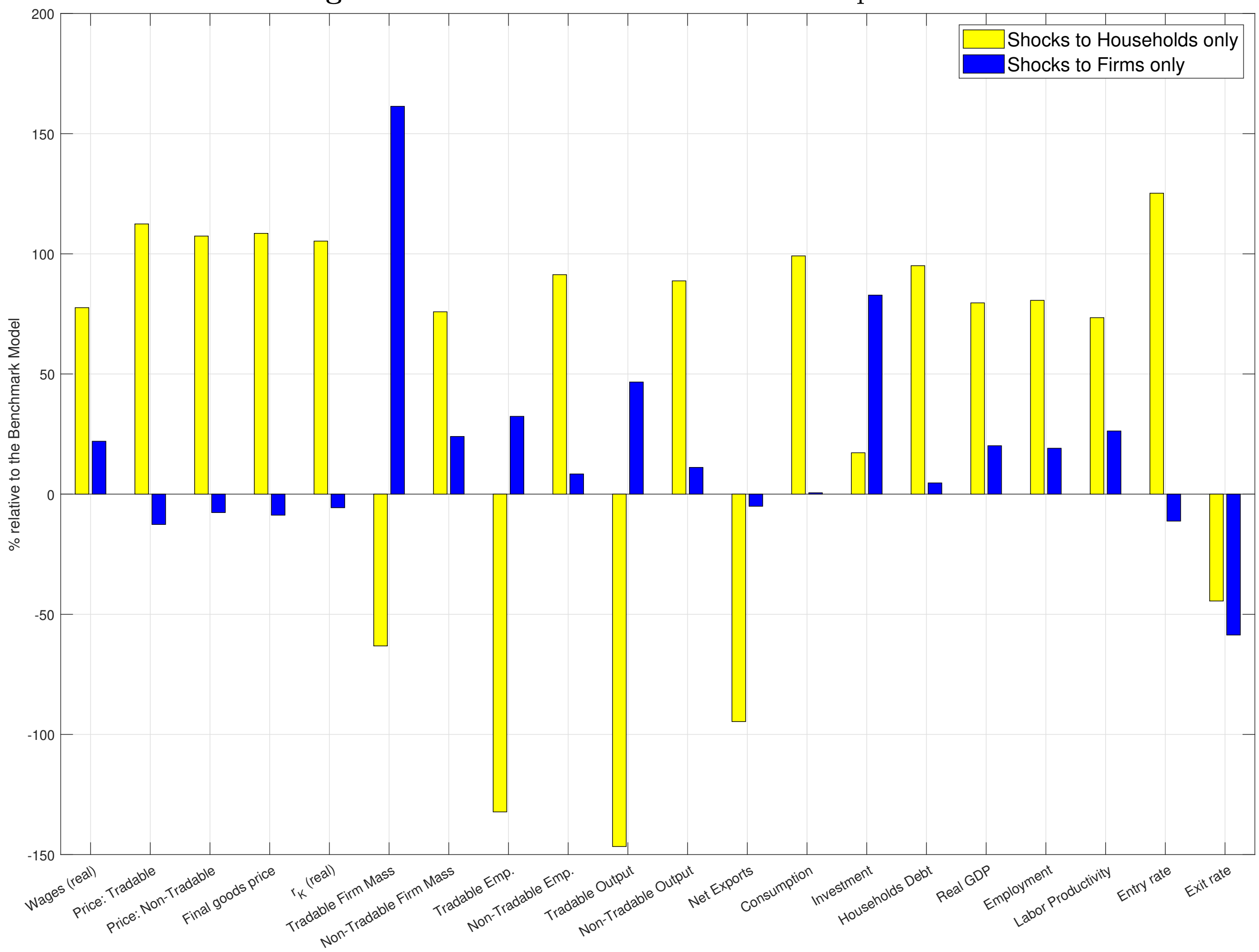


Figure 27: The role of credit constraints for consumers

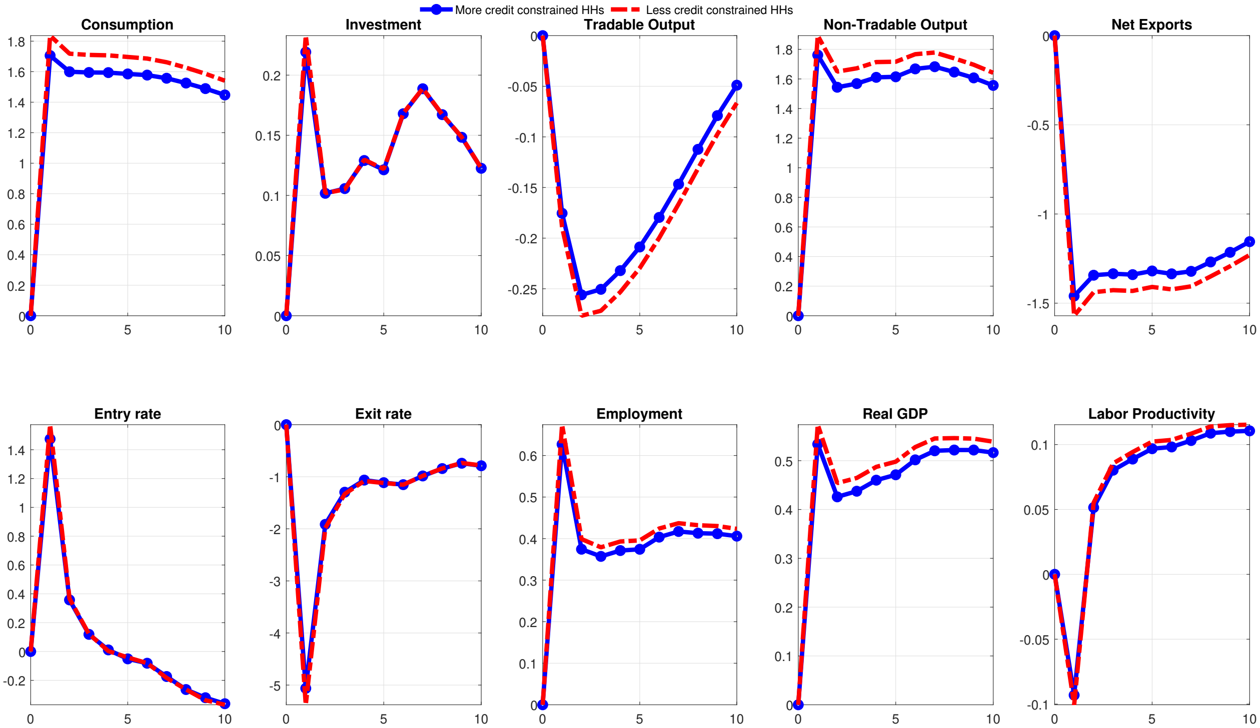


Table 6: Relative importance of demand side shocks vs. supply side shocks (%): changes in household side parameters

	More Constrained HHs		Less Constrained HHs		More Elastic Labor Supply		Less Elastic Labor Supply		More Risk Aversion		Less Risk Aversion	
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)
	HHs	Firms	HHs	Firms	HHs	Firms	HHs	Firms	HHs	Firms	HHs	Firms
GDP	73.6	25.2	74.9	24.0	73.1	25.6	78.1	20.9	60.2	38.1	79.1	19.9
Employment	74.6	24.2	75.9	23.0	74.1	24.6	79.0	20.0	61.4	36.9	80.0	19.0
Consumption	92.4	6.2	92.8	6.0	92.0	6.6	94.5	4.4	86.6	11.1	94.3	4.7
Investment	15.4	84.4	16.4	83.4	15.4	84.4	17.8	82.1	9.1	90.7	19.9	80.0
Tradable output	-138.9	40.7	-135.0	36.6	-145.7	47.7	-118.1	19.4	-204.2	109.4	-125.8	27.1
Non-Tradable output	82.4	16.5	83.3	15.6	81.8	17.0	86.1	13.0	71.5	26.6	86.4	12.7
Entry rates	119.6	-4.5	116.0	-4.4	115.9	-4.8	115.9	-4.6	126.5	-9.4	114.5	-3.1
Exit rates	-40.3	-61.7	-42.1	-59.9	-39.8	-62.1	-46.2	-55.7	-26.5	-74.9	-48.1	-53.7

Table 7: Relative importance of demand side shocks vs. supply side shocks (%): changes in Firm side parameters

	More Capital adj. cost		Less Capital adj. cost		Higher Entry cost ν		Lower Entry cost ν		Higher Entry cost Θ		Lower Entry cost Θ	
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)
	HHs	Firms	HHs	Firms	HHs	Firms	HHs	Firms	HHs	Firms	HHs	Firms
GDP	76.9	21.9	67.9	30.9	74.1	24.7	74.8	24.1	74.9	23.9	57.4	20.9
Employment	77.8	21.0	69.1	29.7	75.1	23.7	75.7	23.1	75.9	22.9	56.2	19.3
Consumption	92.9	5.8	91.7	6.9	92.7	6.0	92.5	6.2	92.6	6.0	82.0	4.1
Investment	16.0	83.9	15.9	83.9	14.7	85.1	18.7	81.1	16.3	83.5	11.3	81.2
Tradable output	-124.5	26.1	-199.0	101.6	-137.2	39.0	-135.0	36.8	-131.0	32.7	-193.3	51.3
Non-Tradable output	84.4	14.5	78.7	20.2	82.8	16.1	83.0	15.9	83.2	15.7	71.5	14.1
Entry rates	124.9	-11.8	102.5	8.1	136.4	-24.0	71.1	37.2	93.3	16.1	92.1	-41.6
Exit rates	-42.1	-60.0	-38.9	-62.9	-45.0	-56.8	-38.0	-63.4	-40.6	-61.4	-32.3	-55.6

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