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Lease-Adjusted Productivity Measurement

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March 2022

Working Paper 20220301

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Keywords: Leased capital, productivity estimation, firm-level productivity, productivity growth, mismeasurement
JEL Classification: E2, D24, O4

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1 Introduction

Productivity, especially critical to the US economy, remains an enduring challenge in its measurement. Numerous studies have estimated productivity utilizing factor inputs (e.g., capital and labor); however, most of these studies, due to previous lease accounting treatment, have failed to acknowledge the existence of leasing, despite the fact that lease contracts are extensively used in production-related activities. We document that leased capital accounts for more than 30% of the total productive assets used by US public firms, which is in line with the prior literature (see Eisfeldt and Rampini (2009), Rauh and Sufi (2012) and Rampini and Viswanathan (2013), among others).[1] Consistent with this view, recent lease accounting rule updates in ASC 842 require lessees to recognize most leases from off-balance-sheet activities back onto their balance sheets.[2] In this paper, we carefully adjust firm productivity measurement by lease, and show how omitting leased capital, as prior studies do, leads to an overestimation of the productivity level and growth. For US public firms, we find that the average overestimation for both productivity level and growth are around 60%. Explicitly considering leased capital affects productivity measurements through distinct channels, which we call the factor-share channel and the omitted-leased-capital channel. We then assess how each channel determines the difference of overestimations across firms.

Our starting point is that firms produce with not only owned physical capital, but also leased capital, while most prior studies in literature focus solely on the former when estimating productivity.[3] The productivity provides a broad gauge of firm-level performance, measuring the overall effectiveness with which capital and labor are used in a production process. Thus, ignoring leased capital should lead to a significant overestimation of productivity, in the sense that contributions to firm output from leased capital are mistakenly attributed to firm productivity.

To formally see this, we develop an analytical framework and assume that firms produce with total capital (i.e., owned capital augmented with leased capital) and labor using a standard Cobb-Douglas technology. By comparing our true adjusted measurements with the traditional unadjusted ones, we analytically show that there indeed exist overestimations of productivity level and growth when leased capital is neglected. Moreover, this overesti-

[1] Eisfeldt and Rampini (2009) provide a comprehensive review of this literature. See also Li and Tsou (2019) and Li and Xu (2020).

[2] Appendix B shows detailed institutional backgrounds related to this accounting rule change. The literature has yet to fully study its implications on macroeconomics and finance; that said, Li and Xu (2020) and Dou et al. (2021) explicitly consider leased capital as a source of productive asset and focus on capital misallocation.

[3] We use "owned capital" and "purchased capital," "leasing" and "renting" interchangeably in this paper.

mation can be decomposed into two distinct channels: one results from factor share mismeasurements (the factor-share channel), while the other results from a firm's leased capital ratio (the omitted-leased-capital channel). The factor-share channel comes from the empirical procedures on estimating production function parameters, as ignoring leased capital would underestimate the capital share and, consequently, overestimate the labor share. The omitted-leased-capital channel, on the other hand, comes from the proportion of firm output that is generated by leased capital but mistakenly attributed to firm productivity, under the same production parameters.

To empirically adjust for lease when estimating productivity, we first construct a measure of firms' leased capital. Guided by standard accounting practice and following Rauh and Sufi (2012) and Rampini and Viswanathan (2013), we capitalize rental expense to obtain a gauge of the amount of leased capital. We document that there is a significant heterogeneity in firms' leased capital ratios (defined as the ratio of leased capital to the total physical capital used in firm production). The leased capital ratio correlates with several firm characteristics: small, financially constrained, and young firms all tend to have higher leased capital ratios, implying that these firms rent more of their capital. Also, we observe that firms with high leased capital ratios have lower debt leverage but higher rental leverage, suggesting that leasing becomes a more crucial external financing channel than debt for these firms, consistent with the findings in Rampini and Viswanathan (2013).[4]

We then use the production function estimation techniques developed in prior studies to estimate productivity. Our benchmark is the commonly used control function approach initiated by Olley and Pakes (1996) with Ackerberg et al. (2015) corrections. We carefully adjust a firm's utilized capital and its investment expenditure by lease, and construct a panel of productivity level and growth for US public firms. Alternatively, we use the dynamic panel approach and fixed effect model for such estimates.

We find that appropriately adjusting for lease leads to an increase in the capital share and a decrease in the labor share. On average, both the level and growth of productivity drop substantially after adjusting for lease, meaning that they are overestimated - the log productivity level is significantly overestimated by over 60%, whereas the overestimation for log productivity growth is 0.4%, which is around 40% of the unadjusted growth measure. We decompose the overestimation into a factor-share channel and an omitted-leased-capital channel, and find that these two channels generate 37% and 16% overestimations of log productivity level, respectively, while they generate 0.24% and 0.08% in terms of growth

[4] Debt leverage is the ratio of the long-term debt (Compustat: DLTT) over the sum of leased capital and total assets (Compustat: AT). Rental leverage is the ratio of leased capital over the sum of leased capital and total assets (Compustat: AT). Leased-adjusted leverage is the sum of debt leverage and rental leverage.

overestimations, respectively. We then document that the overestimation from distinct channels are asymmetric across firms with different characteristics (i.e., leased capital ratio, size, financial-constraint level, and age). The large overestimation also exists at the industry level. Our results suggest that studies on productivity level (e.g., İmrohoroğlu and Tüzel (2014); Ang et al. (2020)) and on productivity growth (e.g., Basu and Fernald (2007); Solow (1957); Cette et al. (2016)) may need to explicitly adjust for lease in relevant cross-sectional or time-series results.

Specifically, we first sort firms into quintile portfolios according to their leased capital ratios, and document an asymmetric overestimation of productivity level across different portfolios: firms with higher leased capital ratios are subject to more overestimations in productivity level. We next sort firms by size, financial-constraint level, and age. We find that the overestimation only presents a mild positive relation with firm size, along with a mild negative relation with a firm's financial-constraint level. Also, when firm age increases, the overestimation first drops and then increases. However, and more importantly, after we decompose the overestimation of productivity level into two channels, we notice that the factor-share channel is larger for firms with lower leased capital ratios, for larger firms, for firms that are less financially constrained, and for older firms, while the second omitted-leased-capital channel is more important for firms in the opposite groups. These monotone relations are consistent with our intuition, since firms that are small, financially constrained, and young lease more, which makes the omitted-leased-capital channel more important for such firms. As for the overall overestimation for a given firm, it is determined by which of the two channels dominates.

With respect to the productivity growth, we also find that, on average, there is a large overestimation when leased capital is ignored. We again decompose such overestimation and find that the omitted-leased-capital channel monotonically increases with leased capital ratio and financial-constraint level, but decreases with firm size. The factor-share channel shows just the opposite for these three characteristics. Ultimately, the factor-share channel initially decreases and then increases with firm age, while the omitted-leased-capital channel presents the opposite pattern, i.e., these two channels show a U-shaped and an inverse U-shaped relation with firm age, respectively. Such monotone and U-shaped relations are mainly determined by the relative growth of owned capital and labor.

We provide robustness checks by using an alternative proxy of value-added, adopting different estimation strategies, as well as changing the treatment of missing key variables. The asymmetric patterns of the overestimation and the corresponding decompositions preserve, indicating the robustness of our benchmark results.

Related literature This paper relates to the strand of literature that seeks to understand why firms lease. Miller and Upton (1976), Myers et al. (1976), Smith Jr and Wakeman (1985), Lewis and Schallheim (1992), and Graham (2000) all show that taxes create incentives to lease. Sharpe and Nguyen (1995), Eisfeldt and Rampini (2009), and Chu (2020) all show that differential treatment for leases and debt during bankruptcy also induces leasing. Gavazza (2011) meanwhile argues that leasing is more attractive for volatile firms that must frequently rescale production, since the purchase and resale of assets incurs transaction costs. Finally, Li and Tsou (2019) argue that leasing allows firms to hedge asset price uncertainty associated with the resale of purchased assets.

This paper is connected to literature that estimates leased capital and brings lease adjustment to a firm's liability and asset side. Eisfeldt and Rampini (2009), Rauh and Sufi (2012) and Lim et al. (2017) provide the estimations of the amount of leased capital. On the liability side, Ang and Peterson (1984), Bayless and Diltz (1986), Marston and Harris (1988), Beattie et al. (2000), Yan (2006), and Schallheim et al. (2013) provide correlative evidence on whether lease and debt are substitutes or complements. Rampini and Viswanathan (2013) provide a strong case that leased capital cannot be ignored if one wants to understand the capital structure; specifically, they find that taking lease-leverage into account drastically reduces the fraction of firms with low leverage, and that the lease-adjusted leverage is of similar magnitude across firm size groups. On the asset side, Li and Xu (2020) adjust firms' marginal product of capital (MPK) by leased capital, and emphasize that leasing provides an additional channel of capital reallocation and would thus change the patterns of capital reallocation and capital misallocation. While our paper is closely related to Li and Xu (2020), we focus on both the productivity and productivity growth of individual firms (and correspondingly the measurements), rather than MPK.

Our paper is also related to the literature on productivity estimation. To retrieve productivity at the firm level, one must estimate the parameters of the production function. The most natural and common approach for doing so is to use the simple ordinary least squares (OLS) regression. However, OLS is subject to endogeneity problems, as factor inputs are typically choice variables of the firm, which depend on the unobserved productivity. Consequently, the estimates would be biased (Marschak and Andrews, 1944). Two traditional solutions to the endogeneity problem include the instrumental variables (IV) approach and the fixed effect (FE) model. While the IV approach hasn't been widely used in practice due to data availability, the FE model is more often used but could nevertheless generate strange capital coefficients (see, for instance, Griliches and Hausman (1986)).[5]

[5]The IV approach requires extra assumptions and information to justify the validity of potential instruments (which are the input or output prices). Also, for the FE model, the fixed effect assumption is strong

More recently, Olley and Pakes (1996) develop a dynamic investment model to tackle this endogeneity problem, known as the control function approach. They identify conditions under which a firm-level investment is strictly increasing in the firm's unobserved productivity shock. This strict monotonicity implies that we can invert the investment demand function, and thus control for the unobserved productivity shock by conditioning on a nonparametric representation of that inverse function. Levinsohn and Petrin (2003) extends Olley and Pakes (1996) by considering the fact that investment could be lumpy. Ackerberg et al. (2015) provide a clearer exposition of the potentially strong assumptions that one must make for this approach to work and propose variations that avoid several empirical problems with this approach.

Another advance of the literature extends the fixed effect models by using more generalized econometric procedures. Arellano and Bond (1991), Arellano and Bover (1995), Arellano and Honoré (2001), and Blundell and Bond (2000) perform such generalization by allowing productivity shocks to vary across time. This strategy allows us to quasi-difference the estimating equation and use lagged instruments, known as the system generalized method of moments (GMM) method. In this process, no assumptions on the unobservable being a scalar and strict monotonicity are required, both of which are essential for the inversions in control function approaches.[6]

In this paper, we rely on the control function approach pioneered by Olley and Pakes (1996) with Ackerberg et al. (2015) corrections in the main analysis, and use the dynamic panel approach and fixed effect models for our robustness checks.[7] What we do is to explicitly bring back leased capital and see how the corrected estimates under the same approach differ.

Our paper also contributes to the literature on the measurement of productivity growth (Solow, 1957; Jorgenson and Griliches, 1967; Basu et al., 2001). The closest papers in this literature study the rise in intangible capital. In particular, Corrado et al. (2009) study how including omitted intangibles in R& D capital could affect measures of both GDP growth and TFP growth. Crouzet and Eberly (2021) focus on organization capital for describing biases in measured TFP growth. Basu et al. (2013) study how unmeasured capital investments, which are complementary to information technology capital, affects TFP growth. Our work, however, starts from a completely different angle through emphasizing the omitted leased capital.

and is more likely to suffer measurement errors.

[6]This GMM estimation method can be poorly behaved in small samples.

[7]Given that pros and cons exist for using the control function approach, the dynamic panel approach, as well as the fixed effect models, and given that theory offers little guidance for choosing among them, we try all approaches in our paper and find that our results are robust across all estimation approaches.

The rest of the paper is organized as follows. We provide an analytical framework of the mismeasurement when leased capital is ignored in Section 2. In Section 3, we discuss the estimation procedures for firm-level productivity. We then summarize empirical facts on the importance of leased capital, on mismeasurements and corresponding decompositions, as well as provide robustness checks in Section 4. We conclude this paper in Section 5. Details on data construction, lease accounting rule change, estimation procedures, industry level results, and robustness tables are avaiable in Appendices A to E.

2 Analytical framework

In this section, we establish a simple framework to analytically show sources of mismeasurement and to motivate our empirical exercises.

We start by assuming the true production function of firm i is:

$$Y_{it} = A_{it} \left(K_{it}^o + K_{it}^l\right)^\alpha L_{it}^\beta \tag{1}$$

where Y_{it} is the total output of firm i, L_{it} is the labor employed, α is the capital share, and β is the labor share. Owned capital K_{it}^o and leased capital K_{it}^l are assumed to be perfect substitutes in production.[8] The firm i's productivity is A_{it}.

2.1 Productivity level estimates

Prior estimates of firm-level productivity in the literature largely ignore leased capital - most measures only utilize the owned tangible asset. Hence, following the prior estimation tradition, our "measured" log productivity of firm i, $\log A_{it}^M$, is:

$$\log A_{it}^M = \log Y_{it} - \widetilde{\alpha} \log K_{it}^o - \widetilde{\beta} \log L_{it}$$

where $\widetilde{\alpha}$ is the capital share and $\widetilde{\beta}$ is the labor share.

If we correct this estimation with leased capital, we have the lease-adjusted log productivity of firm i, $\log A_{it}$, as:

$$\log A_{it} = \log Y_{it} - \alpha \log \left(K_{it}^o + K_{it}^l\right) - \beta \log L_{it}$$

[8] The assumption that owned capital and leased capital are perfect substitutes is consistent with the previous literature, see Eisfeldt and Rampini (2009) and Rampini and Viswanathan (2013). The assumption can be verified in the data. It is also how Compustat records the data after the accounting rule changes.

The overestimation of firm-level productivity is hence equal to:

$$\log A_{it}^{M} - \log A_{it} = \alpha \log\left(K_{it}^{o} + K_{it}^{l}\right) - \widetilde{\alpha} \log K_{it}^{o} + \left(\beta - \widetilde{\beta}\right) \log L_{it}$$

We can further simplify it to allow for a decomposition:

$$\log A_{it}^{M} - \log A_{it} = \left[(\alpha - \widetilde{\alpha}) \log K_{it}^{o} + \left(\beta - \widetilde{\beta}\right) \log L_{it}\right] + \alpha \log\left(\frac{K_{it}^{o} + K_{it}^{l}}{K_{it}^{o}}\right) \tag{2}$$

where we denote $\left[(\alpha - \widetilde{\alpha}) \log K_{it}^{o} + \left(\beta - \widetilde{\beta}\right) \log L_{it}\right]$ as the first channel of the overestimation (i.e., the factor-share channel), and denote $\alpha \log\left(\frac{K_{it}^{o}+K_{it}^{l}}{K_{it}^{o}}\right)$ as the second channel of the overestimation (i.e., the omitted-leased-capital channel).[9] As leased capital is non-negative, we have $\alpha > \widetilde{\alpha}$ and $\widetilde{\beta} > \beta$.[10] Hence, ceteris paribus, a firm that has larger K_{it}^{o} and lower L_{it} will possess a larger factor-share channel, whereas a firm that uses more leased capital (relative to owned capital) will possess a larger omitted-leased-capital channel.

If we further assume that the production function is of constant return to scale, then the overestimation can be further simplified as:

$$\log A_{it}^{M} - \log A_{it} = (\alpha - \widetilde{\alpha}) \log\left(\frac{K_{it}^{o}}{L_{it}}\right) + \alpha \log\left(\frac{K_{it}^{o} + K_{it}^{l}}{K_{it}^{o}}\right) \tag{3}$$

From the first component of the above equation, we can see that apart from the factor share difference, this component depends upon the owned capital intensity $\frac{K_{it}^{o}}{L_{it}}$, which measures the owned capital per unit of labor.[11] This further confirms that the overestimation is jointly determined by the capital intensity (multiplied by factor difference) and the leased capital ratio, which are the two channels we analyze above. Within the same industry, the first component tends to be large for capital-intensive firms (i.e., firms with high $\frac{K_{it}^{o}}{L_{it}}$), whereas this component is likely to be small for labor-intensive firm (i.e., firms with low $\frac{K_{it}^{o}}{L_{it}}$). Because small and financially constrained firms tend to be labor-intensive firms, the

[9]The overestimation is also equal to:

$$\log A_{it}^{M} - \log A_{it} = \left[(\alpha - \widetilde{\alpha}) \log\left(K_{it}^{o} + K_{it}^{l}\right) + \left(\beta - \widetilde{\beta}\right) \log L_{it}\right] + \widetilde{\alpha} \log\left(\frac{K_{it}^{o} + K_{it}^{l}}{K_{it}^{o}}\right)$$

[10]Indeed, we show in Section 4 and Appendix E that such relations hold for different estimation techniques.

[11]Again, alternatively, we have:

$$\log A_{it}^{M} - \log A_{it} = (\alpha - \widetilde{\alpha}) \log\left(\frac{K_{it}^{o} + K_{it}^{l}}{L_{it}}\right) + \widetilde{\alpha} \log\left(\frac{K_{it}^{o} + K_{it}^{l}}{K_{it}^{o}}\right)$$

factor-share channel would play a smaller role in the total estimation error for these firms.

2.2 Productivity growth estimates

We next analyze the productivity growth and provide a decomposition. We denote firm i's total utilized capital amount at time t by K_{it}^U, which is the sum of owned capital and leased capital. Under the production function in Eq. (1), we can conduct a log linearization around the time-series average point and obtain:

$$\begin{aligned}\log Y_{it} &= \log A_{it} + \alpha\left[\log\left(\overline{K}_i^U\right) + \frac{\overline{K}_i^o}{\overline{K}_i^U}\left(\log K_{it}^o - \log \overline{K}_i^o\right) + \frac{\overline{K}_i^l}{\overline{K}_i^U}\left(\log K_{it}^l - \log \overline{K}_i^l\right)\right] \\ &\quad +\beta \log L_{it}\end{aligned}$$

where $\overline{K}_i^U$ denotes the time-series average of firm i's utilized capital amount, $\overline{K}_i^o$ denotes the time-series average of firm i's owned capital amount, and $\overline{K}_i^l$ is the time-series average of firm i's leased capital amount.

The log linearization implies that the firm-level productivity growth is:

$$\begin{aligned}\Delta a_{it} &= \log A_{it} - \log A_{it-1} \\ &= \left(\log Y_{it} - \log Y_{it-1}\right) - \alpha\left[\frac{\overline{K}_i^o}{\overline{K}_i^U}\left(\log K_{it}^o - \log K_{it-1}^o\right) + \frac{\overline{K}_i^l}{\overline{K}_i^U}\left(\log K_{it}^l - \log K_{it-1}^l\right)\right] \\ &\quad -\beta\left(\log L_{it} - \log L_{it-1}\right) \\ &= \Delta y_{it} - \alpha\left[\frac{\overline{K}_i^o}{\overline{K}_i^U}\Delta k_{it}^o + \frac{\overline{K}_i^l}{\overline{K}_i^U}\Delta k_{it}^l\right] - \beta \Delta l_{it}\end{aligned}$$

where we denote $(\log X_{it} - \log X_{it-1})$ using Δx_{it}, for which $X_{it} = Y_{it}, K_{it}^o, K_{it}^l$, and L_{it}.

However, if we ignore leased capital, as is the case in the prior literature, the measured productivity growth Δa_{it}^M is:

$$\Delta a_{it}^M = \Delta y_{it} - \widetilde{\alpha}\Delta k_{it}^o - \widetilde{\beta}\Delta l_{i,t} \tag{4}$$

Consequently, the mismeasurement (overestimation) of firm productivity growth is equal to the difference between Δa_{it}^M and Δa_{it}:

$$\Delta a_{i,t}^M - \Delta a_{i,t} = \left[\left(\alpha - \widetilde{\alpha}\right)\Delta k_{i,t}^o + \left(\beta - \widetilde{\beta}\right)\Delta l_{i,t}\right] - \alpha\frac{\overline{K}_i^l}{\overline{K}_i^U}\left(\Delta k_{i,t}^o - \Delta k_{i,t}^l\right) \tag{5}$$

where we denote $\left[(\alpha - \widetilde{\alpha})\,\Delta k^o_{i,t} + \left(\beta - \widetilde{\beta}\right)\Delta l_{i,t}\right]$ as the first channel of the mismeasurement in productivity growth, and denote $-\alpha\frac{\overline{K}^l_i}{\overline{K}^U_i}\left(\Delta k^o_{i,t} - \Delta k^l_{i,t}\right)$ as the second channel of the mismeasurement in productivity growth. The first channel represents the error resulting from the mismeasurements of capital and labor share, while the second channel stands for the error originating from the difference of the growth rate of leased capital versus that of owned capital. Additionally, the second channel can be written as: $-\alpha\left\{\Delta k^o_{i,t} - \left[\frac{\overline{K}^o_i}{\overline{K}^U_i}\Delta k^o_{i,t} + \frac{\overline{K}^l_i}{\overline{K}^U_i}\Delta k^l_{i,t}\right]\right\}$, which is the difference between owned capital growth and the weighted average of capital growth.

3 Estimation procedure

In this section, we discuss the detailed procedure for estimating firm productivity. Our procedure addresses the endogeneity problem, which is generated by the correlation of input usage and the unobserved productivity.

Consistent with our prior specification in Eq. (1), we assume that the production function of each firm is a Cobb-Douglas form and allow the parameters of the production function to be industry-specific:

$$Y_{i,j,t} = A_{i,j,t}\left(K^o_{i,j,t} + K^l_{i,j,t}\right)^{\alpha_j} L^{\beta_j}_{i,j,t}, \tag{6}$$

where $A_{i,j,t}$ is the firm-specific productivity level at time t, $K^o_{i,j,t}$ corresponds to the mass of production units owned by the firm, and $K^l_{i,j,t}$ corresponds to the mass of production units leased by the firm.

We allow the industry-specific factor shares to vary at the industry level defined by Fama and French (1997) 30 industry classifications (FF30). The error in this regression is a function of $A_{i,j,t}$, and because $A_{i,j,t}$ is naturally correlated with both capital and labor, the error is correlated with the regressors. In our main analysis, we employ the two-step control function approach suggested by Olley and Pakes (1996) with Ackerberg et al. (2015) corrections to avoid endogeneity issues.

Given the industry-level estimates for $\widehat{\alpha}_j$ and $\widehat{\beta}_j$, we compute the estimated log productivity of firm i as follows:

$$\log \widehat{A}_{i,j,t} = \log Y_{i,j,t} - \widehat{\alpha}_j \cdot \log\left(K^o_{i,j,t} + K^l_{i,j,t}\right) - \widehat{\beta}_j \cdot \log L_{i,j,t}.$$

We allow for $\alpha_j + \beta_j \neq 1$, but our results also hold when we impose constant returns to scale in the estimation (i.e., $\alpha_j + \beta_j = 1$). We discuss our estimates in more detail in Appendix C.

As a comparison, the estimated log productivity of firm i when leased capital is ignored can be computed as:

$$\log \widehat{A}^{M}_{i,j,t} = \log Y_{i,j,t} - \widehat{\widehat{\alpha}}_j \cdot \log K^{o}_{i,j,t} - \widehat{\widehat{\beta}}_j \cdot \log L_{i,j,t}.$$

To confirm, this is the productivity estimate used in prior literatures, where leased capital is not considered.

We also consider alternative identification strategies - the ordinary least squares (OLS) estimation with fixed effects and the dynamic panel approach by Blundell and Bond (2000). We provide these results in our robustness check section.

4 Empirical analysis

In this section, we provide evidence that highlights the role of leasing as an important source of "unmeasured" capital in estimating firm-level productivity and productivity growth. We then discuss the distinct channels that generate mismeasurements and consider several robustness checks.

4.1 Data

Our sample consists of firms in Compustat, ranging from 1977 to 2017. The key variables for estimating firm-level productivity are firm-level owned capital, leased capital, value-added, and employment. We measure firm-level owned capital stock given by net plant, property, and equipment (PPENT), deflated following Hall (1990) using the data of investment deflators from the US National Income and Product Accounts (NIPA). We follow Rampini and Viswanathan (2013), Lim et al. (2017) and Li and Tsou (2019) to estimate the amount of leased capital. Because no data for total hours worked are available, we use the number of employees (EMP) to proxy for a firm's labor input. We calculate the value-added of firm i in industry j at time t using data on sales (SALE), operating income (OIBDP), rental expense (XRENT), and the number of employees (EMP). We then deflate this value-added by the aggregate gross domestic product (GDP) deflator from the US NIPA. Detailed information about the data is provided in Appendix A.

4.2 Importance of leased capital

To measure a firm's leasing activities, we use the leased capital ratio and the rental share. We define the leased capital ratio as leased capital divided by the sum of leased and owned capital. Alternatively, we define the rental share as the ratio between rental expense over the sum of capital expenditure and rental expense. In Table 1, we report the summary statistics of leased capital ratio and leverage in our sample.

Table 1: Summary Statistics

	Panel A	**Panel B**								
	Aggregate	**Size**			**WW index**			**Age**		
Variables	**Median**	**L**	**3**	**H**	**L**	**3**	**H**	**L**	**3**	**H**
LCR	0.388	0.530	0.391	0.301	0.298	0.381	0.531	0.422	0.400	0.346
Rental Share	0.281	0.387	0.280	0.220	0.219	0.274	0.394	0.271	0.293	0.268
Debt Leverage	0.130	0.048	0.142	0.173	0.166	0.138	0.062	0.110	0.123	0.153
Lease adj. Lev.	0.309	0.275	0.330	0.308	0.302	0.325	0.291	0.300	0.308	0.315

This table presents summary statistics for the variables related to leased capital in our sample. LCR is the leased capital ratio, calculated as the ratio of leased capital over the sum of leased capital and owned capital (PPENT). Leased capital is defined as 8 times rental expense (XRENT). Rental share is defined as the ratio between rental expense over the sum of capital expenditure (CAPX) plus rental expense. Debt leverage is the ratio of the long-term debt (DLTT) over the sum of leased capital and total assets (AT). Lease-adjusted leverage is the sum of debt leverage and rental leverage. On the right panel, we split the entire sample into subgroups according to their size and age. Size is defined by total assets. We use the WW index to measure the financial constrained level, according to Whited and Wu (2006). Age refers to the number of years since the firm first shows up in Compustat. We use "L" and "H" to denote firms with the lowest and highest sorting variable, respectively. We report time series averages of the cross-section median in the table. The sample is from 1977 to 2017 and excludes financial, utility, public administrative, and lessor industries from our analysis.

In Panel A of Table 1, we present two salient observations. First, the amount of leased capital is substantial - it accounts for 39% of overall productive assets, as measured by the leased capital ratio. The rental share gives a proportion of 28%. This empirical evidence is consistent with Eisfeldt and Rampini (2009) and Rampini and Viswanathan (2013), implying that leasing is a key source of production.[12] Second, the lease-adjusted leverage, defined as the sum of debt and rental leverage, is twice as large as debt leverage. It reveals that leasing is an essential source of external finance, which complements financial debt.

In Panel B, we sort firms into five quantiles based on their size, financial-constraint level, and age. We use book assets (AT) and the WW-index (according to Whited and Wu (2006)) to determine firm size and financial-constraint level, respectively. We calculate the number of years since the firm first shows up in Compustat as firm age. Looking across size groups,

[12] Similar findings are documented in Li and Tsou (2019) and Li and Xu (2020).

we note that the average leased capital ratio of small firms (0.53) is significantly higher than that of large firms (0.30). Also, we observe a large dispersion in the financial debt leverage, ranging from 0.05 to 0.17. The lease-adjusted leverage, however, is flat across size groups. Similar patterns can be found for both the financial-constraint and age groups. It is noteworthy that the dispersion of leased capital ratio and debt leverage in age groups is small, since firm age is a discrete variable and thus less dispersed intrinsically.

In summary, our findings in Table 1 recognize that leasing can be a more important source of productive asset for small, financially constrained, and young firms; and therefore, we should take leasing into account when estimating firm productivity. In the next section, we will present evidence to show that, leased capital is left "unmeasured," which leads to significant mismeasurements in firm productivity level and growth.

4.3 Implications for factor share estimations

Following our empirical strategies in Section 3, we calculate firm productivity level and growth both before and after we adjust for leased capital. We first compare the estimated factor shares.

Table 2: Factor Share Mismeasurements

	Capital Share α	**Labor Share** β
Unadj.		
Mean	0.250	0.709
Median	0.235	0.740
Adj.		
Mean	0.352	0.620
Median	0.332	0.648
Diff.		
Mean	0.101	−0.089
Median	0.098	−0.084

This table presents the factor share estimates at Fama and French (1997) 30 industry classification (FF30), before and after we adjust for lease. The sample is from 1977 to 2017 and excludes financial, utility, public administrative, and lessor industries from our analysis.

In Table 2, we summarize the sector-level estimates of capital and labor shares. Without adjusting for leased capital, the estimated sector-level capital share has a mean and median of 0.25 and 0.24, respectively, while the labor share has a mean and median of 0.71 and 0.74, respectively. When we explicitly consider leased capital, the capital share increases while the labor share decreases, consistent with our intuition. The capital share median increases by

0.10, whereas the labor share median decreases by 0.09. Though both estimates are within the range of standard values, the differences in factor shares are large. This finding indicates that the wrongly-estimated factor share is indeed important in generating mismeasurements. As we highlight in Section 2, this source is denoted as the factor-share channel.

4.4 Implications for productivity level and growth - aggregate level

In this section, we compare productivity level and growth at the aggregate level and report our results in Table 3.

We document substantial overestimation in both productivity level and growth. The average log productivity level is significantly overestimated by 60%, as shown in Row 3 of Panel A. In Panel B, we find that the overestimation in productivity growth has a magnitude of 0.37%. This accounts for over 60% of the (true) adjusted growth, or roughly 40% of the unadjusted growth.

The last two rows of Table 3 show the decomposition of the overestimation. With respect to productivity level captured in Panel A, we observe that the factor-share channel and omitted-leased-capital channel generate 37% and 16% overestimations of log productivity, respectively. In terms of growth captured in Panel B, the overestimations are 0.24% and 0.08% from these two channels, respectively.

Our findings in Table 3 suggest that the overestimations in productivity level and growth are indeed sizable. More importantly, the overestimation generated from ignoring leased capital can be captured by the mismeasured factor share and the omitted capital. While both these channels are significant, the factor-share channel seems larger than the omitted-leased-capital channel; nevertheless, the sources of these channels are from the "off-balance-sheet" leased capital being ignored. This again emphasizes the necessity to appropriately adjust lease for productivity measurements. In the next section, we examine how the overestimation is reflected in firms with different characteristics.

4.5 Implications for productivity level in the cross-section

In this section, we zoom in and study the mismeasurement of productivity levels in the cross-section.

As we present in Section 4.2, firms use leased capital differently in the cross-section: small, financially constrained, and young firms lease more. Hence, we should expect that

Table 3: Productivity Level and Growth Mismeasurement

	Panel A		Panel B
	Aggregate		Aggregate
Variables	**Median**	**Variables**	**Median**
$\log A_i^M$	3.539	Δa_i^M (%)	1.004 %
$\log A_i$	2.908	Δa_i (%)	0.592 %
$\log A_i^M - \log A_i$	0.613	$\Delta a_i^M - \Delta a_i$ (%)	0.379 %
Channel 1	0.366	g(Channel 1) (%)	0.240 %
Channel 2	0.161	g(Channel 2) (%)	0.084 %

This table presents the time series averages of the cross-section median for main outcome variables at the aggregate level. $\log A_i^M$ and $\log A_i$ are estimates of productivity level, before and after adjusting for lease, respectively. Δa_i^M and Δa_i are estimates of productivity growth, before and after adjusting for lease, respectively. $\log A_i^M - \log A_i$ and $\Delta a_i^M - \Delta a_i$ are the corresponding mismeasurements of productivity level and growth. Channel 1 and Channel 2 refer to the factor-share channel and the omitted-leased-capital channel in productivity level, respectively. g(Channel 1) and g(Channel 2) refer to the factor-share channel and the omitted-leased-capital channel in productivity growth, respectively. The sample is from 1977 to 2017 and excludes financial, utility, public administrative, and lessor industries from the analysis.

the omitted-leased-capital channel differs asymmetrically across firms: this channel is more salient for small, financially constrained, and young firms. Moreover, we obtain $\alpha > \widetilde{\alpha}$ and $\widetilde{\beta} > \beta$ when we correct total utilized capital by leased capital.[13] Consequently, for small, financially constrained, and young firms, the factor-share channel should be smaller than that among large, unconstrained, and old firms, due to the fact that the former groups tend to have lower (owned) capital-to-labor ratios. As the total estimation error is a combination of the factor-share and omitted-leased-capital channels, its pattern depends on which channel dominates.

Panel A of Table 4 presents the results when we sort firms according to their leased capital ratios. Our results indicate a strong monotonic relationship between the overestimation of productivity level and leased capital ratio. Specifically, the overestimation monotonically increases with leased capital ratio. The average overestimation in the lowest leased capital ratio quintile is around 0.53, while that in the highest quintile reaches 0.77. Additionally, the factor-share channel decreases monotonically with leased capital ratio, indicating that factor-share mismeasurement plays a less important role in overall estimation errors for firms that lease capital intensively. However, the omitted-leased-capital channel monotonically increases with leased capital ratio, from 0.04 to 0.50.

In Panel B of Table 4, we investigate the relation between overestimation and size. The overall overestimation of productivity level slightly increases with firm size. Decomposing

[13] Such inequality holds under different estimation techniques.

the overestimation, we find that the factor-share channel increases monotonically with size, while the omitted-leased-capital channel does just the opposite.

We then examine how overestimations change with respect to a firm's financial-constraint level. A mild decreasing pattern of the overall overestimation with the financial-constraint level is found in Panel C. As we show in Panel C, we observe a mild decreasing pattern of the overall overestimation with the financial-constraint level.

The decomposition indicates that the two channels exhibit opposite patterns: the factor-share channel drops with a firm's financial-constraint level, whereas the omitted-leased-capital increases sharply (from 0.12 to 0.25).

Finally, we report the results of age portfolios in Panel D. In terms of the overall overestimation, we find that it initially drops with firm age, and then increases. The factor-share channel has a positive relation with firm age, and is particularly high for old firms. The omitted-leased-capital channel meanwhile is decreasing with firm age, from 0.18 to 0.14. These rather flat patterns are consistent with the low dispersion of leased capital ratios across age groups.

4.6 Implications for productivity growth in the cross-section

This section analyzes the mismeasurement of productivity growth in the cross-section and reports the results in Table 5. For firms sorted by leased capital ratio, we find that the growth mismeasurement is positive, indicating that it is an overestimation. The overestimation monotonically increases with leased capital ratio. With respect to the decomposition, we note that the factor-share channel decreases monotonically with leased capital ratio, and that the group with the highest leased capital ratio shows an overestimation close to 0. The omitted-leased-capital channel, however, increases monotonically with leased capital ratio - it is negative in Group 1 but then increases to almost 100% of the overall growth overestimation in Group 5.

Panel B of Table 5 presents the results of size portfolio. The relation between the overestimation and firm size follows a roughly increasing pattern. The factor-share channel increases monotonically with size, while the omitted-leased-capital channel decreases with firm size. As for the results sorted by financial-constraint level in Panel C, we discover exactly the opposite patterns of those in Panel B for the overall overestimation as well as for the channel

Table 4: Univariate Sorting and Productivity Level Mismeasurement

Variables	L	2	3	4	H
			Panel A: LCR		
Size	6.005	6.079	5.888	5.483	4.673
LCR	0.119	0.263	0.395	0.555	0.770
Age	13.293	15.122	14.659	12.829	11.512
$\log A_i^M$	3.439	3.492	3.532	3.569	3.656
$\log A_i$	2.905	2.924	2.924	2.911	2.879
$\log A_i^M - \log A_i$	0.533	0.561	0.607	0.665	0.774
Channel 1	0.466	0.415	0.376	0.333	0.257
Channel 2	0.041	0.107	0.185	0.297	0.493
			Panel B: Size		
Size	3.372	4.613	5.553	6.539	8.321
LCR	0.530	0.436	0.391	0.352	0.301
Age	10.622	10.988	12.171	15.366	24.646
$\log A_i^M$	3.444	3.497	3.537	3.567	3.658
$\log A_i$	2.826	2.882	2.917	2.939	2.991
$\log A_i^M - \log A_i$	0.590	0.595	0.610	0.628	0.651
Channel 1	0.291	0.342	0.370	0.407	0.450
Channel 2	0.247	0.188	0.163	0.141	0.118
			Panel C: WW Index		
Size	8.308	6.552	5.619	4.714	3.486
LCR	0.298	0.346	0.381	0.436	0.531
Age	26.793	16.415	12.732	11.317	10.683
$\log A_i^M$	3.648	3.564	3.532	3.492	3.434
$\log A_i$	2.985	2.943	2.917	2.875	2.810
$\log A_i^M - \log A_i$	0.640	0.618	0.608	0.597	0.596
Channel 1	0.444	0.402	0.371	0.346	0.295
Channel 2	0.116	0.136	0.160	0.188	0.246
			Panel D: Age		
Size	5.011	5.219	5.528	5.902	7.315
LCR	0.422	0.426	0.400	0.364	0.346
Age	5.085	9.463	14.244	22.220	38.293
$\log A_i^M$	3.560	3.534	3.526	3.507	3.562
$\log A_i$	2.933	2.898	2.901	2.902	2.905
$\log A_i^M - \log A_i$	0.616	0.614	0.608	0.590	0.639
Channel 1	0.355	0.356	0.358	0.361	0.415
Channel 2	0.179	0.181	0.166	0.147	0.141

This table shows the firm characteristics, average mismeasurements, and the decomposition of mismeasurements for productivity level when lease is ignored across five portfolios sorted on leased capital ratio, size, and age. LCR is the leased capital ratio, calculated as the ratio of leased capital over the sum of leased capital and owned capital. Size is measured by total assets. We use the WW index to measure the financial constrained level, according to Whited and Wu (2006). Age is measured by the number of years since the firm first shows up in Compustat. We use "L" and "H" to denote groups with the lowest and highest sorting variables, respectively. $\log A_i^M$ and $\log A_i$ are estimates of productivity level, before and after adjusting for lease, respectively. $\log A_i^M - \log A_i$ is the corresponding mismeasurement of productivity level. Channel 1 and Channel 2 refer to the factor-share channel and the omitted-leased-capital channel in productivity level, respectively. g(Channel 1) and g(Channel 2) refer to the factor-share channel and the omitted-leased-capital channel in productivity growth, respectively. We report time series averages of the cross-section median in the table. The sample is from 1977 to 2017 and excludes financial, utility, public administrative, and lessor industries from our analysis.

Table 5: Univariate Sorting and Productivity Growth Mismeasurement

Variables	L	2	3	4	H
			Panel A: LCR		
Δa_i^M (%)	0.519%	0.917%	0.920%	1.100%	1.697%
Δa_i (%)	0.369%	0.663%	0.576%	0.638%	0.764%
$\Delta a_i^M - \Delta a_i$ (%)	0.225%	0.279%	0.371%	0.453%	0.933%
g(Channel 1) (%)	0.338%	0.297%	0.281%	0.207%	0.014%
g(Channel 2) (%)	-0.131%	-0.006%	0.094%	0.312%	0.887%
			Panel B: Size		
Δa_i^M (%)	1.085%	0.859%	0.951%	0.977%	1.169%
Δa_i (%)	0.719%	0.502%	0.520%	0.451%	0.760%
$\Delta a_i^M - \Delta a_i$ (%)	0.264%	0.353%	0.402%	0.436%	0.389%
g(Channel 1) (%)	0.058%	0.223%	0.257%	0.282%	0.305%
g(Channel 2) (%)	0.173%	0.102%	0.084%	0.082%	0.033%
			Panel C: WW Index		
Δa_i^M (%)	1.141%	1.014%	0.991%	0.744%	1.193%
Δa_i (%)	0.741%	0.515%	0.521%	0.414%	0.859%
$\Delta a_i^M - \Delta a_i$ (%)	0.384%	0.410%	0.411%	0.369%	0.282%
g(Channel 1) (%)	0.287%	0.276%	0.266%	0.224%	0.053%
g(Channel 2) (%)	0.047%	0.070%	0.067%	0.117%	0.183%
			Panel D: Age		
Δa_i^M (%)	0.688%	1.053%	0.900%	1.066%	1.213%
Δa_i (%)	0.100%	0.657%	0.606%	0.726%	0.753%
$\Delta a_i^M - \Delta a_i$ (%)	0.496%	0.394%	0.339%	0.327%	0.395%
g(Channel 1) (%)	0.384%	0.217%	0.189%	0.198%	0.266%
g(Channel 2) (%)	0.064%	0.117%	0.104%	0.076%	0.092%

This table shows the average mismeasurements and the decomposition of mismeasurements for productivity growth when lease is ignored across five portfolios sorted on leased capital ratio, size, and age. LCR is the leased capital ratio, calculated as the ratio of leased capital over the sum of leased capital and owned capital. Size is measured by total assets. We use the WW index to measure the financial constrained level, according to Whited and Wu (2006). Age is measured by the number of years since the firm first shows up in Compustat. We use "L" and "H" to denote groups with the lowest and highest sorting variables, respectively. Δa_i^M and Δa_i are estimates of productivity growth, before and after adjusting for lease, respectively. $\Delta a_i^M - \Delta a_i$ is the corresponding mismeasurement of productivity level. Channel 1 and Channel 2 refer to the factor-share channel and the omitted-leased-capital channel in productivity level, respectively. g(Channel 1) and g(Channel 2) refer to the factor-share channel and the omitted-leased-capital channel in productivity growth, respectively. We report time series averages of the cross-section median in the table. The sample is from 1977 to 2017 and excludes financial, utility, public administrative, and lessor industries from our analysis.

decomposition.

In Panel D, we examine the relation between the overestimation and age. Our results show that the growth overestimation follows an overall negative relation with firm age, although the oldest group possesses an overestimation similar to that of the second-youngest group. Moreover, the factor-share channel presents a U-shaped relation with firm age, while the omitted-leased-capital channel shows the inverse.

4.7 Adjustment at the industry level

Apart from the cross-sectional results at the firm and portfolio level, it is important that we look at the industry level as well. In Table 6, we show the adjustment to industries that have the ten highest leased capital ratios.

Table 6: TFP and TFP growth at industry level

FF30	LCR	$\frac{K^o_{it}}{L_{it}}$	$\log A^M_i - \log A_i$	Channel 1	Channel 2	$\Delta a^M_i - \Delta a_i$ %	g(Channel 1) %	g(Channel 2) %
22	0.694	2.709	0.725	0.343	0.375	0.446%	0.154%	0.295%
27	0.667	2.872	0.626	0.363	0.261	0.471%	0.391%	0.083%
7	0.654	2.475	0.505	0.133	0.368	0.709%	0.078%	0.712%
26	0.560	3.003	0.709	0.399	0.271	0.534%	0.275%	0.218%
23	0.450	3.133	0.720	0.472	0.204	0.591%	0.472%	0.065%
28	0.437	3.002	0.665	0.448	0.192	0.370%	0.301%	0.078%
5	0.406	3.340	0.717	0.535	0.150	0.345%	0.097%	0.220%
6	0.387	3.253	0.724	0.477	0.201	0.489%	0.287%	0.208%
8	0.381	3.439	0.979	0.702	0.214	0.672%	0.651%	0.015%
14	0.348	3.169	0.351	0.238	0.101	0.185%	0.117%	0.010%

This table shows the average mismeasurements and the decomposition of mismeasurements for productivity level when lease is ignored for the 10 industries with the highest leased capital ratios. LCR is the leased capital ratio, calculated as the ratio of leased capital over the sum of leased capital and owned capital. $\frac{K^o_{it}}{L_{it}}$ is the capital intensity. $\log A^M_i - \log A_i$ is the mismeasurement of productivity level. $\Delta a^M_i - \Delta a_i$ is the mismeasurement of productivity growth. Channel 1 and Channel 2 refer to the factor-share channel and the omitted-leased-capital channel in productivity level, respectively. g(Channel 1) and g(Channel 2) refer to the factor-share channel and the omitted-leased-capital channel in productivity growth, respectively. We report time series averages of the cross-section median in the table. The sample is from 1977 to 2017 and excludes financial, utility, public administrative, and lessor industries from our analysis.

We observe that operating lease is intensive in services-producing industries. For the industries in Table 6, the overestimation of productivity level ranges from 35% to 98%, and the difference between unadjusted and adjusted productivity growth is within the range of 0.19% to 0.71%. More importantly, for productivity level, the omitted-leased-capital channel in most of the top 10 industries is larger than the median magnitude. A similar relation holds for growth mismeasurements. Meanwhile, as the leased capital ratio decreases, the (owned) capital intensity increases, and the fraction of omitted-leased-capital channel in overall mismeasurements decreases for both productivity level and growth. Nevertheless, our results indicate that obvious overestimations exist in both the level and growth of productivity at the industry level.

4.8 Robustness

To finish our analysis, we consider several extensions in this section. First, we examine the robustness of our results in Tables 4 and 5 to the measurement of value-added. We follow David and Venkateswaran (2019) and estimate it as a constant fraction of revenues using a share of intermediates of 0.5. We present our results in Table E.2 and find that using this alternative measure of value-added has a negligible effect on the patterns found in our benchmark results, though the overall magnitude becomes larger.

To address the endogeneity concern in the production function estimation, we also consider alternative identification strategies. We first use ordinary least squares (OLS) estimation with fixed effects on the regression, an identifying strategy in which we assume that within an industry, $A_{i,j,t}$ varies only at the firm-level, but varies across industries and over time under industry and year fixed effects. The second identification strategy is based on the assumption that $A_{i,j,t}$ follows an autoregressive process. This strategy allows us to quasi-difference our estimating equation and use lagged instruments, as in Blundell and Bond (2000). The results are reported in Tables E.3 and E.4, respectively. We find that while the mismeasurement magnitudes become smaller, both methods produce nearly identical patterns of mismeasurement decomposition as those in the benchmark method of Olley and Pakes (1996) and Ackerberg et al. (2015).

Finally, we check to see whether our results are robust to different treatments of missing rental expense in our analysis. Here, instead of dropping firms with missing rental expense, we set them to zero. As can be seen in Table E.5, our results are largely similar when we treat firms with missing rental expenditure as zero rental expenditure instead of dropping them in our analysis.

5 Conclusion

In this paper, we explicitly adjust firm-level productivity measurements by the unmeasured but important leased capital. Through comparing the prior unadjusted productivity level and growth with the adjusted estimates, we find sizable overestimation of those prior measures.

We also provide a decomposition of the overestimation both analytically and empirically. Our results demonstrate that ignoring leased capital generates overestimation in productivity level and growth from two channels: one is from the wrongly measured factor share, which we denote as the factor-share channel; while the other originates from the wrongly measured utilized capital, which we denote as the omitted-leased-capital channel. The magnitudes of both channels are large and asymmetric for firms with different characteristics (i.e., leased capital ratio, size, financial-constraint level, and age), as well as for industries with different leased capital ratios. Our analysis provides an essential caveat to the new leases standard: lease-induced capital should be properly factored in, as they exert an asymmetric impact on productivity measurements for firms in the cross-section. In turn, such "unmeasured" capital and our findings should have important implications for studies related to productivity level and growth in the cross-section.

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Appendix for Online Publication

A: Data construction

Our sample, which we obtain from Compustat, consists of firms with positive rental expenditure data and non-missing SIC codes. The sample period ranges from 1977 to 2017. We focus on firms trading on NYSE, AMEX, and NASDAQ, except utility firms that have four-digit standard industrial classification (SIC) codes between 4900 and 4999, finance firms that have SIC codes between 6000 and 6999 (finance, insurance, trusts, and real estate sectors), and public administrative firms that have SIC codes between 9000 and 9999. We also explicitly exclude industries that serve as lessors (i.e., industries with SIC 3 digits of 735, 751 and SIC 4 digits: 7377).

Below, we present the key variables used to estimate firm-level productivity. Value-added, (Y_{ijt}), in our benchmark case follows İmrohoroğlu and Tüzel (2014) and is computed as Sales (Compustat item, SALE) - Materials, deflated by the GDP price deflator from US NIPA. Materials is measured as Total expenses minus Labor expenses. Total expenses is approximated as the difference between firm Sales and firm operating profit adjusted for lease. The adjusted operating profit is calculated as Operating Income Before Depreciation and Amortization + Rental expense, similar to Rauh and Sufi (2012), corresponding to [OIBDP+XRENT] in Compustat. Labor expenses is calculated by multiplying the number of employees from Compustat (EMP) by average wages from the Social Security Administration. The stock of labor (L_{ijt}) is measured by the number of employees from Compustat (EMP). These steps lead to our definition of value-added that is proxied by the sum of Operating Income Before Depreciation and Amortization, rental expenses, and labor expenses. The alternative calculation of value-added follows David and Venkateswaran (2019) and is estimated as as 0.5 times Sales.

The owned capital stock $K^o_{i,j,t}$ is given by net plant, property, and equipment (PPENT), deflated following Hall (1990) using the data of investment deflators from the US NIPA. The leased capital stock is given $K^l_{i,j,t}$ is estimated by 8 times rental expenses, as in Rampini and Viswanathan (2013), Lim et al. (2017), Li and Tsou (2019), and as well as common industry practice. Finally, capital expenditure is measured by CAPX, while the lease-adjusted capital expenditure is the sum of rental expense and CAPX.

SHAKE SHACK INC.
CONSOLIDATED BALANCE SHEETS
(in thousands, except share and per share amounts)

		December 25 2019		December 26 2018
ASSETS				
Current assets:				
Cash and cash equivalents	$	37,099	$	24,750
Marketable securities		36,508		62,113
Accounts receivable		9,970		10,523
Inventories		2,221		1,749
Prepaid expenses and other current assets		1,877		1,984
Total current assets		87,675		101,119
Property and equipment, net		314,862		261,854
Operating lease assets		274,426		—
Deferred income taxes, net		279,817		242,533
Other assets		11,488		5,026
TOTAL ASSETS	$	968,268	$	610,532
LIABILITIES AND STOCKHOLDERS' EQUITY				
Current liabilities:				
Accounts payable	$	14,300	$	12,467
Accrued expenses		24,140		22,799
Accrued wages and related liabilities		11,451		10,652
Operating lease liabilities, current		30,002		—
Other current liabilities		19,499		14,030
Total current liabilities		99,392		59,948

Figure B.1: This figure shows excerpts from Shake Shack's balance sheet in its 2019 financial statement. It includes the asset side and the liability side.

B: Accounting rule

In February 2016, the Financial Accounting Standards Board (FASB) issued updated accounting standards for lease (ASU 2016-02, Topic 842). Effective from 2019, firms are required to recognize lease assets and lease liabilities from off-balance-sheet activities on their balance sheets, which increases the transparency and comparability among organizations. Firms must also disclose key information about leasing transactions. The exact adoption rule differs across firms. For public firms and certain other entities, ASC 842 is effective for annual periods beginning after December 15, 2018. For private firms, the new lease accounting adoption was set to be effective for reporting periods beginning subsequent to December 15, 2019.

After adopting the new accounting rule, firms now report "Lease right-of-use asset" on the asset side, and both short-term and long-term lease liabilities on the liability side. These items were absent before the adoption of the new operating lease accounting rule. Additionally, firms are required to report the estimates of their operating leases, including the value, average regaining life, and discount rate, as well as disclose the possibility of renewing or extending existing leases. Figure B.1 shows the example from Shake Shack's financial statement in 2019. We can see that the new rule has a major impact on both Shake Shack's asset and liability side.

ASC 842 has proved a major change in accounting and FASB has issued several accounting standard updates and amendments to it since the publishment in 2016. In response to COVID-19, FASB has proposed the deferral of the new lease accounting standard effective date for certain entities such as private entities, including private not-for-profit entities.

In a joint effort, the International Accounting Standards Board (IASB) also released IFRS 16 on new lease standards, requiring nearly all leases to be reported on lessees' balance sheets as assets and liabilities in 2016, effective for annual periods beginning on or after January 1, 2019.

C: Detail on estimation procedure

C.1. Olley and Pakes (1996) and Ackerberg et al. (2015)

Olley and Pakes (1996) and Ackerberg et al. (2015) both belong to the control function literature, which use a two-step process based on the use of a control function for the productivity process. The main difference is at what stage production parameters are estimated. The procedure in Ackerberg et al. (2015) is a refinement of Olley and Pakes (1996). We denote the former as ACF and the latter as OP. Below we review both OP and ACF methods and highlight the benefits of using ACF.

Olley and Pakes (1996) In OP approach, the first step regression reads:

$$\log Y_{i,j,t} = \beta_j \log L_{i,j,t} + \phi_{i,j,t} + \epsilon_{i,j,t}$$

where we approximate $\phi_{i,j,t}$ with a n^{th} order polynomial series in capital and investment. This first stage equation is estimated with OLS, which results in a consistent estimate for $\widehat{\beta}_j$; this $\widehat{\beta}_j$ is free from the simultaneity problem. In the second stage, we estimate the survival probabilities $\widehat{P}_{\text{survival},t}$ using a probit model on investment and capital, as well as on their squares and cross products (i.e., polynomial expression) of last period. This survival probability estimation is used to control for selection bias. We then fit:

$$\log Y_{i,j,t+1} - \widehat{\beta}_j \log L_{i,j,t+1} = \alpha_j \log K_{i,j,t+1} + \rho \log A_{i,j,t} + \tau \widehat{P}_{\text{survival},t} + \epsilon_{i,j,t+1}$$

so that we can obtain the consistent estimate of $\widehat{\alpha}_j$. With these two parameter estimates, we can then compute firm-level productivity as:

$$\log \widehat{A}_{i,j,t} = \log Y_{i,j,t} - \widehat{\alpha}_j \cdot \log K_{i,j,t} - \widehat{\beta}_j \cdot \log L_{i,j,t}$$

Ackerberg et al. (2015) In ACF correction, we don't estimate any production parameters in the first stage. Instead, all parameters are estimated in the second stage. It is hence robust to labor adjustment costs and does not require us to adjust for sample selection. In the first stage, we conduct the OLS regression:

$$\log Y_{i,j,t} = \Phi_{i,j,t} + \epsilon_{i,j,t}$$

where we approximate $\Phi_{i,j,t}$ with a n^{th} order polynomial series in capital, labor and investment. This first stage estimation results in an estimate for $\Phi_{i,j,t}$, which we will use to infer productivity and innovations in productivity $\xi_{i,j,t}(\alpha_j; \beta_j)$. We use this step to detect and net out unanticipated productivity shocks and measurement errors in firm level $Y_{i,j,t}$. In the second stage, we use the following moment conditions to estimate the production parameters:

$$\mathbf{E}\left(\xi_{i,j,t}(\alpha_j; \beta_j) \times [\log K_{i,j,t}, \log L_{i,j,t-1}]'\right) = 0$$

which is followed by firm-level productivity:

$$\log \widehat{A}_{i,j,t} = \log Y_{i,j,t} - \widehat{\alpha}_j \cdot \log (K_{i,j,t}) - \widehat{\beta}_j \cdot \log L_{i,j,t}$$

For our estimation of lease-adjusted measure, capital refers to the total amount of owned and leased capital, while investment is the expenditure on both types of capital, defined as the sum of capital expenditure and rental expense. For the estimation without lease adjustment, capital refers to the owned capital, while investment is the only capital expenditure.[14]

[14] We allow for $\alpha_j + \beta_j \neq 1$. But if the estimates lead to $\alpha_j + \beta_j > 1$, we impose constant returns to scale in the estimation, i.e., $\alpha_j + \beta_j = 1$. We employ this procedure for different approaches.

C.2. Dynamic Panel Approach

The estimation of the production function with the dynamic panel data approach follows Blundell and Bond (2000). We can write the firm-level production function as follows:

$$\begin{aligned} \log Y_{i,j,t} &= z_{i,j} + w_t + \alpha_j \log\left(K^o_{i,j,t} + K^l_{i,j,t}\right) + \beta_j \log L_{i,j,t} + v_{i,j,t} + u_{i,j,t} \\ v_{i,j,t} &= \rho v_{i,j,t-1} + e_{i,j,t}, \end{aligned} \tag{C1}$$

where $z_{i,j}$, w_t and $v_{i,j,t}$ indicate a firm fixed effect, a time-specific intercept, and a possibly autoregressive productivity shock, respectively. We denote the residuals from the regression by $u_{i,j,t}$ and $e_{i,j,t}$, which we assume to be white noise processes. ρ is the persistence of the productivity process. The model has the following dynamic representation:

$$\begin{aligned} \Delta \log Y_{i,j,t} &= \rho \Delta \log Y_{i,j,t-1} + \alpha_j \Delta \log\left(K^o_{i,j,t} + K^l_{i,j,t}\right) - \rho\alpha_j \Delta \log\left(K^o_{i,j,t-1} + K^l_{i,j,t-1}\right) \\ &\quad + \beta_j \Delta \log L_{i,j,t} - \rho\beta_j \Delta \log L_{i,j,t-1} + \left(\Delta w_{j,t} - \rho \Delta w_{j,t-1}\right) + \Delta \kappa_{i,j,t}, \end{aligned} \tag{C2}$$

where $\kappa_{i,j,t} = e_{i,j,t} + u_{i,j,t} - \rho u_{i,j,t-1}$. We let $x_{i,j,t} = \left\{\log\left(K^o_{i,j,t} + K^l_{i,j,t}\right), \log\left(L_{i,j,t}\right), \log\left(Y_{i,j,t}\right)\right\}$. Assuming that $E\left[x_{i,j,t-l} e_{i,t}\right] = E\left[x_{i,j,t-l} u_{i,t}\right] = 0$ for $l > 0$, we obtain the following moment conditions:

$$\begin{aligned} E\left[x_{i,j,t-l} \Delta \kappa_{i,j,t}\right] &= 0 \text{ for } l \geq 3 \\ E\left[\Delta x_{i,j,t-l} \Delta \kappa_{i,j,t}\right] &= 0 \text{ for } l \geq 3 \end{aligned}$$

that are used to conduct a consistent GMM estimation of Eq. (C2). Given the estimates $\widehat{\alpha}_j$ and $\widehat{\beta}_j$, the log productivity of firm i is computed as:

$$\log \widehat{A}_{i,j,t} = \log Y_{i,j,t} - \widehat{\alpha}_j \cdot \log\left(K^o_{i,j,t} + K^l_{i,j,t}\right) - \widehat{\beta}_j \cdot \log L_{i,j,t} \tag{C3}$$

We then conduct the same procedures without leased capital $K^l_{i,j,t}$, and get the following estimated log productivity of firm i when leased capital is ignored:

$$\log \widehat{A}^M_{i,j,t} = \log Y_{i,j,t} - \widehat{\widehat{\alpha}}_j \cdot \log\left(K^o_{i,j,t}\right) - \widehat{\widehat{\beta}}_j \cdot \log L_{i,j,t}$$

C.3. OLS with Fixed Effects

An alternative way to estimate the production function avoiding endogeneity issues is to use the ordinary least squares (OLS) estimation with fixed effects on the regression:

$$\log Y_{i,j,t} = v_j + z_{i,j} + w_{j,t} + \alpha_j \log \left(K^o_{i,j,t} + K^l_{i,j,t}\right) + \beta_j \log L_{i,j,t} + u_{i,j,t} \tag{C4}$$

The parameters v_j, $z_{i,j}$, and $w_{j,t}$ indicate an industry dummy, a firm fixed effect, and an industry-specific time dummy, respectively. That is, the identifying assumption in our specification with fixed effects is that within an industry, firm-level productivity varies only at the firm level, but varies across industries and over time under industry and year fixed effects. The residual from the regression is denoted by $u_{i,j,t}$. We allow the industry-specific factor shares to vary at Fama and French (1997) 30 industry classification (FF30) level.

Given our point estimates of $\widehat{\alpha}_j$ and $\widehat{\beta}_j$, we can use Eq. (C3) to estimate $\log \widehat{A}_{i,j,t}$. We then conduct the same procedures without leased capital $K^l_{i,j,t}$, and obtain the estimated $\log \widehat{A}^M_{i,j,t}$.

D: Additional results at the industry level

In Table D.1, we report the results of mismeasurements for all industries according to Fama and French (1997) 30 industry classifications (FF30). Firms in some industries access capital intensively through leasing, such as the retail industry and the apparel industry. There are comparatively significant cross-industry variations in leased capital ratios, ranging from 0.80 for the retail industry to 0.06 for the coal industry. Our results indicate that it is important that we adjust lease for productivity measurements across all industries, and that channels from factor share mismeasurments and omitted capital affect differently across industries.

E: Detail on robustness

This section contains tables for robustness checks in Section 4.8. Table E.2 presents our results when we use the value-added definition in David and Venkateswaran (2019). Table E.3 shows the results when we use the ordinary least squares (OLS) estimation with fixed effects for productivity. Table E.4 reports our results under the dynamic panel approach, following Blundell and Bond (2000). Table E.5 shows the results when we treat missing rental expense as zero. All these tables suggest that the corresponding patterns of mismeasurements

Table D.1: Industry-Level Productivity Level and Growth Mismeasurement

FF30	Industry Name	LCR	$\frac{K^o_{it}}{L_{it}}$	$\log A_i^M - \log A_i$	Channel 1	Channel 2	$\Delta a_i^M - \Delta a_i$ %	g(Channel 1) %	g(Channel 2) %
22	Personal and Business Services	0.694	2.709	0.725	0.343	0.375	0.446%	0.154%	0.295%
27	Retail	0.667	2.872	0.626	0.363	0.261	0.471%	0.391%	0.083%
7	Apparel	0.654	2.475	0.505	0.133	0.368	0.709%	0.078%	0.712%
26	Wholesale	0.560	3.003	0.709	0.399	0.271	0.534%	0.275%	0.218%
23	Business Equipment	0.450	3.133	0.720	0.472	0.204	0.591%	0.472%	0.065%
28	Restaurants, Hotels, Motels	0.437	3.002	0.665	0.448	0.192	0.370%	0.301%	0.078%
5	Printing and Publishing	0.406	3.340	0.717	0.535	0.150	0.345%	0.097%	0.220%
6	Consumer Goods	0.387	3.253	0.724	0.477	0.201	0.489%	0.287%	0.208%
8	Healthcare, Medical Equipment, & Pharmaceutical Products	0.381	3.439	0.979	0.702	0.214	0.672%	0.651%	0.015%
14	Electrical Equipment	0.348	3.169	0.351	0.238	0.101	0.185%	0.117%	0.010%
4	Recreation	0.344	3.714	0.751	0.514	0.183	0.523%	0.367%	0.072%
13	Fabricated Products and Machinery	0.331	3.371	0.438	0.324	0.102	0.285%	0.175%	0.086%
25	Transportation	0.322	4.601	0.740	0.509	0.171	0.473%	0.319%	0.176%
30	Everything Else	0.318	3.537	0.507	0.384	0.104	0.306%	0.186%	0.127%
11	Construction and Construction Materials	0.309	3.381	0.419	0.294	0.086	0.280%	0.170%	0.067%
16	Aircraft, Ships, and Railroad Equipment	0.297	3.413	0.205	0.155	0.041	0.129%	0.145%	-0.015%
10	Textiles	0.288	3.408	0.570	0.469	0.085	0.325%	0.325%	0.062%
1	Food Products	0.270	3.715	0.360	0.237	0.107	0.180%	0.133%	0.040%
15	Automobiles and Trucks	0.249	3.430	0.366	0.246	0.093	0.292%	0.224%	0.049%
21	Communication	0.214	4.886	0.489	0.377	0.092	0.325%	0.126%	0.194%
9	Chemicals	0.214	4.531	0.520	0.402	0.098	0.318%	0.248%	0.034%
3	Tobacco Products	0.212	3.846	0.569	0.379	0.182	0.452%	0.212%	0.331%
2	Beer Liquor	0.209	4.334	0.492	0.404	0.082	0.404%	0.249%	0.144%
24	Business Supplies and Shipping Containers	0.207	4.038	0.300	0.221	0.058	0.224%	0.104%	0.059%
18	Coal	0.176	5.193	0.234	0.163	0.063	0.205%	0.189%	0.248%
12	Steel Works Etc	0.162	4.250	0.385	0.301	0.059	0.281%	0.141%	0.036%
17	Precious Metals, Non-Metallic, & Industrial Metal Mining	0.078	5.368	0.238	0.175	0.041	0.163%	0.077%	0.019%
19	Petroleum and Natural Gas	0.072	6.683	0.375	0.304	0.048	0.264%	0.267%	-0.036%

This table shows the average mismeasurements and the decomposition of mismeasurements for productivity level when lease is ignored for each industry defined as Fama-French 30 (Fama and French, 1997). LCR is the leased capital ratio, calculated as the ratio of leased capital over the sum of leased capital and owned capital. $\frac{K^o_{it}}{L_{it}}$ is the capital intensity. $\log A_i^M - \log A_i$ is the mismeasurement of productivity level. $\Delta a_i^M - \Delta a_i$ is the mismeasurement of productivity growth. Channel 1 and Channel 2 refer to the factor-share channel and the omitted-leased-capital channel in productivity level, respectively. g(Channel 1) and g(Channel 2) refer to the factor-share channel and the omitted-leased-capital channel in productivity growth, respectively. We report time series averages of the cross-section median in the table. The sample is from 1977 to 2017 and excludes financial, utility, public administrative, and lessor industries from our analysis.

and decomposition are nearly identical to those found in our benchmark case.

Table E.2: Univariate Sorting Results (Alternative Value-Added)

Variables	L	2	3	4	H
Panel A: LCR					
$\log A_i^M$	3.590	3.730	3.800	3.852	3.985
$\log A_i$	2.834	2.954	2.991	3.008	2.983
$\log A_i^M - \log A_i$	0.765	0.783	0.822	0.871	1.008
Channel 1	0.681	0.603	0.554	0.499	0.380
Channel 2	0.053	0.135	0.231	0.348	0.592
Δa_i^M %	0.665%	0.876%	1.119%	1.365%	2.111%
Δa_i %	0.497%	0.666%	0.788%	0.784%	0.765%
$\Delta a_i^M - \Delta a_i$ %	0.262%	0.354%	0.446%	0.569%	1.123%
g(Channel 1) %	0.396%	0.362%	0.330%	0.231%	-0.045%
g(Channel 2) %	-0.154%	-0.009%	0.111%	0.404%	1.105%
Panel B: Size					
$\log A_i^M$	3.664	3.728	3.773	3.801	3.971
$\log A_i$	2.837	2.917	2.960	2.976	3.097
$\log A_i^M - \log A_i$	0.809	0.819	0.831	0.850	0.888
Channel 1	0.437	0.513	0.549	0.588	0.643
Channel 2	0.301	0.234	0.202	0.176	0.149
Δa_i^M %	1.297%	1.159%	1.117%	1.215%	1.130%
Δa_i %	0.856%	0.685%	0.642%	0.641%	0.691%
$\Delta a_i^M - \Delta a_i$ %	0.282%	0.418%	0.471%	0.547%	0.507%
g(Channel 1) %	-0.005%	0.216%	0.287%	0.326%	0.386%
g(Channel 2) %	0.228%	0.121%	0.106%	0.091%	0.048%
Panel C: WW Index					
$\log A_i^M$	3.945	3.799	3.768	3.734	3.672
$\log A_i$	3.079	2.989	2.958	2.911	2.833
$\log A_i^M - \log A_i$	0.874	0.842	0.830	0.822	0.818
Channel 1	0.637	0.581	0.551	0.515	0.443
Channel 2	0.148	0.174	0.199	0.235	0.302
Δa_i^M %	1.028%	1.272%	1.246%	1.203%	1.348%
Δa_i %	0.625%	0.688%	0.683%	0.784%	0.863%
$\Delta a_i^M - \Delta a_i$ %	0.496%	0.521%	0.502%	0.410%	0.314%
g(Channel 1) %	0.366%	0.313%	0.313%	0.209%	-0.007%
g(Channel 2) %	0.061%	0.083%	0.086%	0.151%	0.236%
Panel D: Age					
$\log A_i^M$	3.788	3.784	3.774	3.754	3.845
$\log A_i$	2.954	2.947	2.953	2.958	2.990
$\log A_i^M - \log A_i$	0.846	0.841	0.829	0.806	0.871
Channel 1	0.532	0.528	0.530	0.533	0.595
Channel 2	0.223	0.227	0.210	0.188	0.176
Δa_i^M %	1.496%	1.368%	1.091%	1.078%	1.032%
Δa_i %	0.805%	0.842%	0.654%	0.659%	0.592%
$\Delta a_i^M - \Delta a_i$ %	0.593%	0.517%	0.415%	0.420%	0.483%
g(Channel 1) %	0.414%	0.267%	0.224%	0.238%	0.300%
g(Channel 2) %	0.069%	0.123%	0.147%	0.089%	0.096%

This table shows the firm characteristics, average mismeasurements, and the decomposition of mismeasurements for productivity level and growth when lease is ignored across five portfolios sorted on leased capital ratio, size, and age. LCR is the leased capital ratio, calculated as the ratio of leased capital over the sum of leased capital and owned capital. Size is measured by total assets. We use the WW index to measure the financial constrained level, according to Whited and Wu (2006). Age is measured by the number of years since the firm first shows up in Compustat. We use "L" and "H" to denote groups with the lowest and highest sorting variables, respectively. $\log A_i^M$ and $\log A_i$ are estimates of productivity level, before and after adjusting for lease, respectively. $\log A_i^M - \log A_i$ is the corresponding mismeasurement of productivity level. Δa_i^M and Δa_i are estimates of productivity growth, before and after adjusting for lease, respectively. $\Delta a_i^M - \Delta a_i$ is the corresponding mismeasurement of productivity level. Channel 1 and Channel 2 refer to the factor-share channel and the omitted-leased-capital channel in productivity level, respectively. g(Channel 1) and g(Channel 2) refer to the factor-share channel and the omitted-leased-capital channel in productivity growth, respectively. We report time series averages of the cross section median in the table. The sample is from 1977 to 2017 and excludes financial, utility, public administrative, and lessor industries from our analysis.

Table E.3: Univariate Sorting Results (Fixed Effect Model)

Variables	L	2	3	4	H
			Panel A: LCR		
$\log A_i^M$	3.752	3.789	3.807	3.802	3.815
$\log A_i$	3.346	3.365	3.356	3.327	3.266
$\log A_i^M - \log A_i$	0.411	0.428	0.449	0.471	0.541
Channel 1	0.367	0.326	0.293	0.254	0.184
Channel 2	0.028	0.070	0.118	0.190	0.337
Δa_i^M %	1.020%	1.324%	1.298%	1.442%	1.961%
Δa_i %	0.792%	1.075%	1.053%	1.106%	1.249%
$\Delta a_i^M - \Delta a_i$ %	0.218%	0.256%	0.302%	0.377%	0.673%
g(Channel 1) %	0.296%	0.256%	0.227%	0.177%	0.027%
g(Channel 2) %	-0.091%	-0.004%	0.061%	0.227%	0.612%
			Panel B: Size		
$\log A_i^M$	3.582	3.705	3.793	3.884	4.030
$\log A_i$	3.150	3.256	3.330	3.406	3.535
$\log A_i^M - \log A_i$	0.421	0.434	0.453	0.472	0.516
Channel 1	0.193	0.245	0.277	0.317	0.378
Channel 2	0.168	0.129	0.110	0.096	0.080
Δa_i^M %	1.234%	1.384%	1.328%	1.416%	1.535%
Δa_i %	0.984%	0.996%	1.004%	0.997%	1.163%
$\Delta a_i^M - \Delta a_i$ %	0.244%	0.312%	0.337%	0.355%	0.323%
g(Channel 1) %	0.048%	0.188%	0.224%	0.245%	0.259%
g(Channel 2) %	0.121%	0.064%	0.059%	0.050%	0.028%
			Panel C: WW Index		
$\log A_i^M$	4.023	3.878	3.791	3.704	3.577
$\log A_i$	3.530	3.403	3.333	3.255	3.143
$\log A_i^M - \log A_i$	0.512	0.469	0.452	0.438	0.423
Channel 1	0.376	0.316	0.280	0.249	0.197
Channel 2	0.079	0.093	0.108	0.129	0.167
Δa_i^M %	1.490%	1.360%	1.478%	1.281%	1.258%
Δa_i %	1.128%	0.987%	1.154%	0.952%	1.046%
$\Delta a_i^M - \Delta a_i$ %	0.308%	0.341%	0.350%	0.322%	0.258%
g(Channel 1) %	0.240%	0.229%	0.241%	0.194%	0.052%
g(Channel 2) %	0.034%	0.050%	0.047%	0.084%	0.128%
			Panel D: Age		
$\log A_i^M$	3.784	3.762	3.775	3.784	3.879
$\log A_i$	3.328	3.302	3.318	3.322	3.393
$\log A_i^M - \log A_i$	0.446	0.452	0.456	0.454	0.494
Channel 1	0.261	0.270	0.277	0.284	0.341
Channel 2	0.122	0.124	0.113	0.100	0.094
Δa_i^M %	1.345%	1.397%	1.364%	1.359%	1.412%
Δa_i %	0.814%	0.983%	1.079%	1.114%	1.091%
$\Delta a_i^M - \Delta a_i$ %	0.467%	0.378%	0.311%	0.271%	0.277%
g(Channel 1) %	0.367%	0.225%	0.179%	0.164%	0.188%
g(Channel 2) %	0.039%	0.069%	0.080%	0.045%	0.058%

This table shows the firm characteristics, average mismeasurements, and the decomposition of mismeasurements for productivity level and growth when lease is ignored, for five portfolios sorted on leased capital ratio, size, and age. LCR is the leased capital ratio, calculated as the ratio of leased capital over the sum of leased capital and owned capital. Size is measured by total assets. We use the WW index to measure the financial constrained level, according to Whited and Wu (2006). Age is measured by the number of years since the firm first shows up in Compustat. We use "L" and "H" to denote groups with the lowest and highest sorting variables, respectively. $\log A_i^M$ and $\log A_i$ are estimates of productivity level, before and after adjusting for lease, respectively. $\log A_i^M - \log A_i$ is the corresponding mismeasurement of productivity level. Δa_i^M and Δa_i are estimates of productivity growth, before and after adjusting for lease, respectively. $\Delta a_i^M - \Delta a_i$ is the corresponding mismeasurement of productivity level. Channel 1 and Channel 2 refer to the factor-share channel and the omitted-leased-capital channel in productivity level, respectively. g(Channel 1) and g(Channel 2) refer to the factor-share channel and the omitted-leased-capital channel in productivity growth, respectively. We report time series averages of the cross-section median in the table. The sample is from 1977 to 2017 and excludes financial, utility, public administrative, and lessor industries from our analysis.

Table E.4: Univariate Sorting Results (Dynamic Panel Approach)

Variables	L	2	3	4	H
Panel A: Leased Cap. Ratio					
$\log A_i^M$	3.915	3.954	3.952	3.932	3.889
$\log A_i$	3.452	3.465	3.449	3.406	3.317
$\log A_i^M - \log A_i$	0.452	0.459	0.472	0.494	0.549
Channel 1	0.412	0.373	0.340	0.297	0.222
Channel 2	0.024	0.058	0.097	0.159	0.298
Δa_i^M %	1.497%	1.772%	1.800%	1.956%	2.353%
Δa_i %	1.134%	1.420%	1.385%	1.436%	1.565%
$\Delta a_i^M - \Delta a_i$ %	0.320%	0.347%	0.398%	0.471%	0.732%
g(Channel 1) %	0.378%	0.349%	0.323%	0.267%	0.080%
g(Channel 2) %	-0.085%	-0.008%	0.059%	0.208%	0.597%
Panel B: Size					
$\log A_i^M$	3.572	3.780	3.943	4.105	4.365
$\log A_i$	3.146	3.313	3.436	3.558	3.721
$\log A_i^M - \log A_i$	0.400	0.447	0.481	0.520	0.594
Channel 1	0.206	0.275	0.324	0.381	0.473
Channel 2	0.152	0.117	0.099	0.086	0.073
Δa_i^M %	1.601%	1.936%	1.878%	1.883%	1.899%
Δa_i %	1.285%	1.363%	1.399%	1.338%	1.406%
$\Delta a_i^M - \Delta a_i$ %	0.304%	0.418%	0.456%	0.465%	0.402%
g(Channel 1) %	0.098%	0.279%	0.333%	0.349%	0.335%
g(Channel 2) %	0.115%	0.060%	0.055%	0.050%	0.026%
Panel C: WW Index					
$\log A_i^M$	4.357	4.091	3.938	3.786	3.575
$\log A_i$	3.723	3.555	3.437	3.313	3.144
$\log A_i^M - \log A_i$	0.587	0.511	0.479	0.451	0.409
Channel 1	0.466	0.379	0.327	0.281	0.213
Channel 2	0.072	0.084	0.097	0.117	0.152
Δa_i^M %	1.788%	1.869%	2.071%	1.878%	1.753%
Δa_i %	1.340%	1.353%	1.558%	1.304%	1.344%
$\Delta a_i^M - \Delta a_i$ %	0.382%	0.451%	0.470%	0.438%	0.321%
g(Channel 1) %	0.308%	0.331%	0.354%	0.288%	0.099%
g(Channel 2) %	0.031%	0.047%	0.044%	0.074%	0.123%
Panel D: Age					
$\log A_i^M$	3.870	3.873	3.907	3.951	4.102
$\log A_i$	3.386	3.380	3.402	3.433	3.509
$\log A_i^M - \log A_i$	0.464	0.472	0.479	0.482	0.527
Channel 1	0.299	0.311	0.321	0.341	0.404
Channel 2	0.113	0.114	0.104	0.091	0.086
Δa_i^M %	2.177%	2.102%	1.905%	1.686%	1.650%
Δa_i %	1.419%	1.498%	1.414%	1.329%	1.243%
$\Delta a_i^M - \Delta a_i$ %	0.647%	0.521%	0.405%	0.349%	0.319%
g(Channel 1) %	0.558%	0.361%	0.260%	0.239%	0.219%
g(Channel 2) %	0.039%	0.067%	0.075%	0.045%	0.052%

This table shows the firm characteristics, average mismeasurements, and the decomposition of mismeasurements for productivity level and growth when lease is ignored across five portfolios sorted on leased capital ratio, size, and age. LCR is the leased capital ratio, calculated as the ratio of leased capital over the sum of leased capital and owned capital. Size is measured by total assets. We use the WW index to measure the financial constrained level, according to Whited and Wu (2006). Age is measured by the number of years since the firm first shows up in Compustat. We use "L" and "H" to denote groups with the lowest and highest sorting variables, respectively. $\log A_i^M$ and $\log A_i$ are estimates of productivity level, before and after adjusting for lease, respectively. $\log A_i^M - \log A_i$ is the corresponding mismeasurement of productivity level. Δa_i^M and Δa_i are estimates of productivity growth, before and after adjusting for lease, respectively. $\Delta a_i^M - \Delta a_i$ is the corresponding mismeasurement of productivity level. Channel 1 and Channel 2 refer to the factor-share channel and the omitted-leased-capital channel in productivity level, respectively. g(Channel 1) and g(Channel 2) refer to the factor-share channel and the omitted-leased-capital channel in productivity growth, respectively. We report time series averages of the cross-section median in the table. The sample is from 1977 to 2017 and excludes financial, utility, public administrative, and lessor industries from our analysis.

Table E.5: Univariate Sorting Results (Alternative Treatment of Missing Rental Expense)

Variables	L	2	3	4	H
			Panel A: LCR		
$\log A_i^M$	3.440	3.485	3.523	3.561	3.650
$\log A_i$	3.025	2.999	3.000	2.979	2.936
$\log A_i^M - \log A_i$	0.422	0.493	0.529	0.591	0.705
Channel 1	0.392	0.382	0.345	0.307	0.231
Channel 2	0.010	0.082	0.150	0.253	0.441
Δa_i^M %	0.595%	0.820%	0.957%	1.149%	1.689%
Δa_i %	0.515%	0.646%	0.665%	0.730%	0.828%
$\Delta a_i^M - \Delta a_i$ %	0.126%	0.205%	0.271%	0.405%	0.812%
g(Channel 1 %	0.190%	0.248%	0.220%	0.162%	-0.004%
g(Channel 2 %	-0.138%	-0.061%	0.029%	0.258%	0.754%
			Panel B: Size		
$\log A_i^M$	3.420	3.485	3.534	3.567	3.670
$\log A_i$	2.878	2.945	2.991	3.028	3.101
$\log A_i^M - \log A_i$	0.516	0.521	0.530	0.532	0.544
Channel 1	0.264	0.304	0.328	0.356	0.388
Channel 2	0.189	0.151	0.133	0.114	0.099
Δa_i^M %	1.045%	0.996%	0.811%	1.026%	1.210%
Δa_i %	0.764%	0.589%	0.521%	0.605%	0.829%
$\Delta a_i^M - \Delta a_i$ %	0.183%	0.264%	0.280%	0.311%	0.309%
g(Channel 1) %	0.016%	0.136%	0.174%	0.202%	0.232%
g(Channel 2) %	0.183%	0.101%	0.077%	0.071%	0.041%
			Panel C: WW Index		
$\log A_i^M$	3.659	3.564	3.527	3.483	3.410
$\log A_i$	3.098	3.038	2.991	2.939	2.860
$\log A_i^M - \log A_i$	0.538	0.519	0.526	0.523	0.523
Channel 1	0.383	0.352	0.332	0.307	0.268
Channel 2	0.097	0.109	0.127	0.154	0.192
Δa_i^M %	1.166%	0.994%	1.012%	0.853%	1.104%
Δa_i %	0.801%	0.604%	0.647%	0.566%	0.781%
$\Delta a_i^M - \Delta a_i$ %	0.297%	0.292%	0.295%	0.260%	0.208%
g(Channel 1) %	0.218%	0.195%	0.190%	0.143%	0.011%
g(Channel 2) %	0.049%	0.064%	0.072%	0.109%	0.198%
			Panel D: Age		
$\log A_i^M$	3.550	3.524	3.518	3.511	3.565
$\log A_i$	3.002	2.971	2.976	2.985	3.007
$\log A_i^M - \log A_i$	0.540	0.536	0.529	0.507	0.532
Channel 1	0.320	0.321	0.321	0.322	0.356
Channel 2	0.144	0.145	0.133	0.116	0.117
Δa_i^M %	0.651%	0.964%	0.922%	1.111%	1.136%
Δa_i %	0.185%	0.594%	0.725%	0.790%	0.793%
$\Delta a_i^M - \Delta a_i$ %	0.341%	0.307%	0.251%	0.251%	0.304%
g(Channel 1) %	0.261%	0.168%	0.143%	0.142%	0.198%
g(Channel 2) %	0.052%	0.102%	0.104%	0.071%	0.079%

This table shows the firm characteristics, average mismeasurements, and the decomposition of mismeasurements for productivity level and growth when lease is ignored across five portfolios sorted on leased capital ratio, size, and age. LCR is the leased capital ratio, calculated as the ratio of leased capital over the sum of leased capital and owned capital. Size is measured by total assets. We use the WW index to measure the financial constrained level, according to Whited and Wu (2006). Age is measured by the number of years since the firm first shows up in Compustat. We use "L" and "H" to denote groups with the lowest and highest sorting variables, respectively. $\log A_i^M$ and $\log A_i$ are estimates of productivity level, before and after adjusting for lease, respectively. $\log A_i^M - \log A_i$ is the corresponding mismeasurement of productivity level. Δa_i^M and Δa_i are estimates of productivity growth, before and after adjusting for lease, respectively. $\Delta a_i^M - \Delta a_i$ is the corresponding mismeasurement of productivity level. Channel 1 and Channel 2 refer to the factor-share channel and the omitted-leased-capital channel in productivity level, respectively. g(Channel 1) and g(Channel 2) refer to the factor-share channel and the omitted-leased-capital channel in productivity growth, respectively. We report time series averages of the cross-section median in the table. The sample is from 1977 to 2017 and excludes financial, utility, public administrative, and lessor industries from our analysis.