PHBS WORKING PAPER SERIES

The Technical Default Spread

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November 2021

Working Paper 20211101

Abstract

We study the quantitative impact of lender control rights on firm investment, asset prices, and the aggregate economy. We build a general equilibrium model with endogenous loan covenants, in which the breaching of a covenant (technical default) entails a switch in investment control rights from borrowers to lenders. Lenders optimally choose low-risk projects, thus mitigating borrowers' risk-taking incentives and reducing a firm's cost of equity. Such a mechanism mitigates the financial accelerator effect (Bernanke et al. (1999)), and generates a technical default spread that firms closer to technical default earn 4% lower average returns than those further away from it.

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1 Introduction

Traditional macro-finance models with financially-constrained firms (Bernanke and Gertler (1989), Bernanke et al. (1999), Kiyotaki and Moore (1997)) treat banks as passive bystanders without any involvement in their borrowers' repayment policies.[1] In these models, when a borrower is solvent, the bank recovers the value of the loan and obtains interest payments, and when the borrower defaults, then the bank recovers what is left of the borrower's assets.

In reality, banks exert significant effort to safeguard loan repayments. Using data on syndicated lending, Roberts and Sufi (2009a) document that around 97% of loans to large U.S. corporations contain at least one covenant. Using data from the U.S. shared credit registry, Chodorow-Reich and Falato (2021) show that around 6.5% of U.S. loans enter technical default by breaching a covenant in any given quarter, and that around 30% of U.S. loans were in technical default in the run-up to and during the financial crisis. When firms enter technical default, banks can freeze their credit lines, call back loans, and charge higher rates and fees. Importantly, technical default entails a switch in corporate control rights from borrowers to banks, whereby banks can exert influence and directly intervene in corporate investment decisions (Chava and Roberts (2008), Nini, Smith, and Sufi (2009)).[2] What is the quantitative impact of such widespread lender control rights on investment and asset prices?

In this paper, we study the impact of lender control rights on the aggregate economy and on the cross-section of equity returns. We develop a dynamic general equilibrium model based on the Bernanke et al. (1999) financial accelerator model, in which entrepreneurs borrow from lenders to finance their firms. We let the loan contract between the lender and the borrower endogenously specify a loan size, its interest rate, and, departing from the previous literature, a covenant threshold written on a signal of the firm's profits. We show that, when the signal is high and the entrepreneur is in control, the entrepreneur chooses relatively riskier projects due to her convex payoffs. Conversely, when the signal is low and the lender takes control of the firm, it chooses relatively safer projects to

[1] Quadrini (2011) and Brunnermeier, Eisenbach, and Sannikov (2012) offer comprehensive reviews of the vast literature on macroeconomic models with credit market frictions.

[2] For example, Chava and Roberts (2008) provide anecdotal evidence that lenders direct firms away from risky projects when firms enter technical default. Extreme examples of lender control rights are the so-called step-in rights in project finance, whereby projects can be entirely taken over by lenders (Madykov (2014)).

safeguard loan repayment. As a result, and in contrast to previous models, our model generates high investment risk and high returns in good times, but generates low risk and low returns in bad times. Importantly, we show that our model's ability to let loan covenants and technical default vary over the business cycle provides strong mitigation effects to the financial accelerator mechanism.

In the data, we construct a firm-level, time-varying measure of technical default probability in the spirit of Murfin (2012).[3] Consistent with our model, aggregate data shows that technical default is strongly counter-cyclical, and cross-sectional data shows that firms closer to technical default systematically display lower investment risk and lower expected returns than firms whose covenants are less binding. A calibrated version of our model allows us to jointly replicate these aggregate and cross-sectional patterns, as well as other macroeconomic and asset pricing moments. Our calibration exercise confirms that control rights changes in technical default are a quantitatively important determinant of investment and asset prices in the aggregate and in the cross-section.

Our analysis starts in an infinite-horizon economy populated by entrepreneurs, lenders, firms, and workers. Firms have unique access to productive capital and labor, and hire myopic entrepreneurs to make static investment and financing decisions. Entrepreneurs borrow resources from risk-neutral lenders and, outside of technical default, allocate resources between productive capital and a risk-free asset, as in Bernanke et al. (1999). Our main theoretical departure from Bernanke et al. (1999) is to introduce endogenous covenants in the loan contract and the transfer of control rights from entrepreneurs to lenders following covenant violations.

Motivated by recent empirical evidence, we make two assumptions with respect to the structure of loan covenants. First, we assume that covenants are written on firms' profits. This assumption is motivated by recent work documenting the vast prevalence of cash-flow based covenants (as opposed to stock-based covenants) in U.S. corporate lending (see, e.g., Lian and Ma (2021)).[4] Second, we assume that covenant violations trigger a switch of control rights to lenders who then allocate a firm's assets between risky capital and the risk-free asset by maximizing *their own* payoffs. This

[3] In what follows, we use the terms "distance to technical default," "technical default probability," and "covenant strictness" interchangeably.

[4] Popular cash-flow based covenants specify minimum levels for a firm's interest coverage ratios (EBITDA to interest expense), and fixed charge coverage ratios (EBITDA to fixed charges, the sum of interest expense, debt in current liabilities, and rent expense). Popular stock-based covenants specify maximum levels for firm leverage (debt to total assets).

assumption is supported by extensive empirical evidence on the active role that lenders play in shaping corporate policies when firms enter technical default (e.g., Chava and Roberts (2008), Nini et al. (2009, 2012), Falato and Liang (2016)).[5]

In our model, the allocation of investment control rights happens *after* the loan contract has been signed but *before* payoffs are realized, based on a signal of the risky investment's idiosyncratic profitability. At the beginning of each period, entrepreneurs meet with lenders and negotiate a one-period loan contract specifying the loan size, its interest rate, and a covenant on a *signal* of the risky investment's idiosyncratic profitability. Specifically, we assume that if the idiosyncratic signal falls below the covenant threshold, then investment control rights (i.e., the right to choose between risky and riskless assets) are assigned to the lender. If the idiosyncratic signal falls above the threshold, then investment control rights remain in the hands of the entrepreneur.

In the middle of the period, the firm's idiosyncratic profitability signal is realized and control rights are assigned to either the entrepreneur (if the signal is high) or the lender (if the signal is low). After investment decisions have been made, aggregate and idiosyncratic shocks are realized at the end of the period. If the value of the firm's assets is higher than the value of the loan, then the firm is solvent, the lender collects the value of the loan plus interest, and the entrepreneur collects the difference between the value of the firm's assets and the loan. If the value of the firm's assets is lower than the value of the loan, then the firm is in default, the lender collects a fraction of the value of the firm's assets, and the entrepreneur obtains nothing.

Our main theoretical result shows that, even if entrepreneurs are risk-averse, they always optimally choose to fully invest in the risky asset. On the other hand, lenders' payoffs induce full investment in the risk-free asset. As a result, our model predicts different levels of investment risk based on the endogenous allocation of control rights, with risky investment by the entrepreneur in control being associated with higher exposure to aggregate risk and higher expected stock returns. This mechanism yields the empirically-testable prediction that firms closer to and in technical default (and therefore more likely to experience a shift in control rights) should have *lower* investment risk

[5]Below, we argue that the assumption of full allocation of control rights to lenders can be relaxed by assuming that entrepreneurs and lenders bargain over a firm's asset allocation decision and also assuming that the lender's bargaining weight is a function of the firm's distance from technical default. In an extension of our model, we also consider the implications of costly asset reallocation between entrepreneurs and lenders.

and *lower* average stock returns than firms that are further away from technical default, a prediction that we test and confirm in the data.

We also prove that our general equilibrium model can achieve aggregation results, thus allowing us to study the macroeconomic implications of endogenous covenants and control rights transfers in a computationally-tractable, representative agent framework. As in the data, with the help of endogenous covenants and pro-cyclical variation in the average prospects of future profitability, our calibrated model generates counter-cyclical variation in technical default probability, a pattern rarely studied in the macroeconomic literature. Moreover, our model allows to match salient macroeconomic and asset pricing moments such as the level and the volatility of the risk-free rate, the equity premium, and the volatility and autocorrelation of consumption, output, and investment growth. Based on these calibration results, we go one step further to show that allowing for endogenous adjustments of covenants can mitigate the financial accelerator mechanism.

To demonstrate how loan covenants can slow down the financial accelerator, we compare the impulse response functions of key variables in two calibrated economies, one in which we allow the covenant threshold to change over the business cycle, and one in which the covenant threshold remains fixed. We show that in response to a financial shock that increases the entrepreneur's liquidation probability, flexible covenants allow the entrepreneur to take on relatively low leverage and promise the bank a relatively low interest payment while still satisfying the bank's break-even conditions. As a result, we find that in response to a one-standard-deviation aggregate financial shock, investment and capital price decrease by 30% less (on impact) in the economy with flexible covenant terms when compared to those in the fixed-covenant economy. In sum, allowing for lender control rights to vary over the business cycle mitigates the financial accelerator mechanism of Bernanke et al. (1999), which operates through a vicious cycle induced by a decline in entrepreneurial net worth, a decline in the price of capital, and an increase in external finance premia.

We then turn to the cross-section of firm investment and equity returns to test our mechanism. We use DealScan loan covenant data to construct a quarterly firm-level measure of distance to technical default. As in Murfin (2012), this covenant strictness measure is the probability that a firm will breach one of its covenant terms over the next quarter, and constitutes an ex-ante measure of future technical default. Different from ex-post measures of covenant violation, strictness is a continuous measure, which allows us to study the relationship between distance to technical default and expected returns through standard portfolio sorting techniques. Moreover, strictness captures the probability of breaching *any* covenant, irrespective of the covenant type. Using this measure, we study the empirical relationship between distance to technical default, investment conservatism (as measured by investment and acquisition growth; see Nini et al. (2012)), and future stock returns.

Consistent with our model, we document a positive relationship between strictness and investment conservatism, as well as a strong negative relationship between strictness and expected returns. We show that firms in the top quintile of the strictness distribution feature on average 3.6 lower investment growth rates and 3.8 lower acquisition growth rates than do firms in the fourth quintile of the strictness distribution. High strictness is also associated with low returns: a strategy that goes long on the high-strictness portfolio and short on the low-strictness portfolio earns an average *negative* excess return of 4.12% per year.

Our empirical tests show that the negative relationship between strictness and expected returns is non-monotonic, and driven by high-strictness firms. Relative to firms in the fourth quintile of the strictness distribution, firms in the top quintile earn 7.7% lower excess returns (unconditionally), and this relationship is partly explained by different exposure to the Fama and French (2015) and Hou et al. (2015) investment and profitability factors. We interpret these results as evidence that control rights reallocation leads to constrained investment choices, which reduce firms' exposure to aggregate investment opportunities and profitability. Additionally, Fama-MacBeth regressions of future returns on indicators for inclusion in strictness-based portfolios confirm that only high-strictness firms display lower excess returns when compared to low-strictness firms. Additional tests show that these results are robust to a number of robustness checks and empirical specifications, are not driven by financially-distressed firms (Campbell, Hilscher, and Szilagyi (2008), Bharath and Shumway (2008)), and hold ex-post in a regression discontinuity design (RDD) when firms actually breach their covenants.

Using the lens of our model, we interpret the non-monotonic, negative relationship between loan covenant strictness and expected returns as evidence of a shift in investment control rights for firms that are closest to technical default. Consistent with this intuition, we show that our calibrated model is able to generate a significant and sizable technical default spread. As in the data, high covenant strictness is associated with lower investment risk and lower average returns. Taken together, our empirical results and calibration support control rights reallocation as a quantitatively important determinant of a firm's investment, risk-taking incentives, and cost of capital, both in the aggregate and in the cross-section.

Our paper contributes to several streams of literature. Building on the seminal contributions of Bernanke and Gertler (1989), Bernanke et al. (1999), and Kiyotaki and Moore (1997), an established field of macro-finance studies the role of financial frictions in shaping firms' investment decisions and cost of equity. [6] Gomes et al. (2015) study the asset pricing implications of financial frictions in a model with costly state verification. Elenev et al. (2021) study a quantitative model of financial intermediation to evaluate the effects of macro prudential policy. Recent work in this area highlights the pervasive presence of cash-flow based covenants in lending contracts (Lian and Ma (2021)), studies the trade-off between covenants and interest rates (Green (2018)), and highlights the role of covenant in shaping corporate investment policies and transmitting monetary policy (Greenwald (2019), Xiang (2019), and Adler (2020)). Different from this literature, in our paper we explicitly account for the fact that lenders and borrowers have different payoffs and investment incentives when in control of the firm, and we build on this insight to study the aggregate and cross-sectional implications of lender control rights allocation on firms' investment risk and cost of capital. To the best of our knowledge, our paper is also the first to show that lender control rights can have a quantitatively large mitigation effect on the financial accelerator.

A long literature in corporate finance theory highlights loan covenants as a tool to mitigate agency problems between shareholders and debt-holders (Jensen and Meckling (1976), Myers (1977), Smith and Warner (1979)). Empirical work in this area shows that covenant violations can impact firm investment (Chava and Roberts (2008), Nini et al. (2009), Bradley and Roberts (2015)) and hiring poli-

[6] There is a vast literature on macro models with credit market frictions, but we do not attempt to summarize it here. A partial list includes Carlstrom and Fuerst (1997), Kiyotaki and Moore (2005), Krishnamurthy (2003), Kiyotaki and Moore (2008), Mendoza (2010), Gertler and Kiyotaki (2010), Gertler and Karadi (2011a), Jermann and Quadrini (2012a), He and Krishnamurthy (2012), He and Krishnamurthy (2013), Li (2013),He and Krishnamurthy (2014), Brunnermeier and Sannikov (2014) and Bianchi and Bigio (2014). Quadrini (2011) and Brunnermeier, Eisenbach, and Sannikov (2012) provide comprehensive reviews of this literature.

cies (Benmelech, Bergman, and Seru (2011), Falato and Liang (2016)), ultimately affecting firm risk (Gilje (2016), Ersahin, Irani, and Le (2021)), and firm value (Beneish and Press (1995), Harvey, Lins, and Roper (2004), Nini et al. (2012)). We complement this literature by showing that the allocation of control rights between borrowers and lenders has quantitatively important implications not only for individual firms but also for the aggregate economy. Moreover, we directly show that changes in firm risk-taking induced by technical default manifests themselves in the cost of equity capital, a result that we believe is new in this literature.

Our paper also contributes to the cross-sectional asset pricing literature by showing that the reallocation of control rights in technical default contributes to a significant variation in the cross-section of expected stock returns. While our paper is related to the literature on financial distress and expected stock returns, we show that the technical default spread is strongest for the set of firms that are *not* financially-distressed. In this sense, the data provide evidence that the reallocation of control rights serves as a distinct economic mechanism compared to those previously proposed in the literature for explaining the Campbell, Hilscher, and Szilagyi (2008) distress anomaly.[7] In this respect, it is particularly worth noting that our mechanism is economically different from the shareholder recovery in default mechanism proposed by Garlappi and Yan (2011). While their paper argues for a reallocation of risks from shareholders to creditors in financial distress, our theory hinges on a reduction of risky investment by creditors in control of the firm.

Our paper belongs to the literature on production-based asset pricing, for which Kogan and Papanikolaou (2012) provides an excellent survey. It also contributes to the broader literature linking investment to the cross-section of expected returns. Zhang (2005) provides an investment-based explanation for the value premium. Belo, Lin, and Yang (2019) study implications of equity financing frictions on the cross-section of stock returns. Ai, Li, Li, and Schlag (2020a) study the relation between asset collateralizability and stock returns. Our focus is to understand the quantitative impact of lender control rights on firm investment, asset prices, and the aggregate economy.

[7] A non-exhaustive list of such explanations includes shareholder recovery in default (Garlappi and Yan (2011)) and changes in equity beta for financially-distressed firms (George and Hwang (2010), Boualam, Gomes, and Ward (2019), and Chen, Hackbarth, and Strebulaev (2019)).

2 A General Equilibrium Model of Investment Control Rights

We present a discrete-time, infinite-horizon general equilibrium model featuring a representative household of entrepreneurs and workers, a continuum of firms, and a national lender. Our model extends the workhorse financial accelerator model (Bernanke et al. (1999)) to include loan covenants and creditor control rights.[8]

Time is infinite and discrete. In each period t, entrepreneurs sign one-period contracts with the national lender to make one-period investment decisions between a risk-free asset and risky capital. The lending contract specifies the size of the loan, its interest rate, and *a covenant threshold* such that if the firm's cash flows fall below the threshold, then the firm's investment control rights are assigned to the lender. Specifically, after the lending contract is signed, but before investment decisions are made, entrepreneurs and the lender observe a signal on the firm's future idiosyncratic productivity. If this signal is above its covenant threshold, then the entrepreneur maintains the firm's control rights and decides how to allocate hisher own funds and the bank's loan between the risk-free asset and risky capital. If the signal is below the covenant threshold, investment control rights are assigned to the lender.

2.1 Model Setup

Representative Household The representative household consists of a continuum of workers and a continuum of entrepreneurs. In each period t, workers inelastically supply labor L_t to firms and return wages W_t to the household. Entrepreneurs own and operate firms. As we describe below, entrepreneurs transfer a share Π_t of their wealth to the household for consumption and savings purposes.[9] We denote household consumption by C_t, its savings in the bank's deposits by D_t^H, and the bank's risk-free deposit rate by R_{t+1}^D. The representative household owns the national bank and it is entitled to the national bank's profits Π_t^B, but also absorbs the bank's losses in the event of bank

[8] A dynamic model of lender control rights is developed in Gete and Gourio (2015). Different from our paper, Gete and Gourio (2015) do not consider the aggregate implications of lender control rights for the financial accelerator or for asset prices in the cross-section.

[9] As in Bernanke et al. (1999), this assumption prevents an entrepreneur's wealth from growing indefinitely and from growing out of the financial constraint.

default.[10] Given these assumptions, the household's budget constraint is

$$C_t + D_t^H = W_t L_t + R_t^D D_{t-1}^H + \Pi_t + \Pi_t^B. \tag{1}$$

We assume perfect consumption insurance within the household.[11] The household evaluates the utility of its consumption plans according to the Epstein and Zin (1989) recursive specification

$$U_t = \left\{ (1-\beta)\left(C_t\right)^{1-\frac{1}{\psi}} + \beta \left(E_t \left[U_{t+1}^{1-\gamma} \right] \right)^{\frac{1-\frac{1}{\psi}}{1-\gamma}} \right\}^{\frac{1}{1-\frac{1}{\psi}}}, \tag{2}$$

in which U_t denotes time-t utility, β denotes the household's time discount factor, ψ denotes its intertemporal elasticity of substitution, and γ denotes its relative risk aversion. Under these preferences, the household's stochastic discount factor (SDF) between time t and $t+1$ is

$$M_{t+1} \equiv \beta \left(\frac{C_{t+1}}{C_t} \right)^{-\frac{1}{\psi}} \left(\frac{U_{t+1}^{1-\gamma}}{E_t \left[U_{t+1}^{1-\gamma} \right]} \right)^{\frac{\frac{1}{\psi}-\gamma}{1-\gamma}}, \tag{3}$$

and no arbitrage ensures that

$$E_t \left[M_{t+1} \right] R_{t+1}^D = 1. \tag{4}$$

Firms All the firms in our economy are ex-ante identical and produce the same consumption good. Each firm is indexed by $i \in [0,1]$, and is operated by an entrepreneur to produce output $Y_{i,t}$ following a constant return to scale Cobb-Douglas production technology

$$Y_{i,t} = \bar{Z}_t \left(\exp(\omega_{i,t}) K_{i,t} \right)^{\alpha} \left(L_{i,t} \right)^{1-\alpha}, \tag{5}$$

in which $K_{i,t}$ and $L_{i,t}$ are firm-i's capital and labor inputs, respectively, $\bar{Z}_t$ is an aggregate productivity shock, and $\omega_{i,t}$ is a firm-specific shock that transforms one unit of capital into $\exp(\omega_{i,t})$ efficiency units

[10] This assumption is not restrictive and ensures that the bank's deposits are risk-free without introducing another asset in the economy.

[11] This assumption follows Gertler and Kiyotaki (2010), Gertler and Karadi (2011b) and many others. By making this assumption, we identify a heterogeneous role of entrepreneurs and workers within a representative agent framework.

of capital (Bernanke et al. (1999)).

Since labor is perfectly mobile, wages are identical across all firms in this economy, and firms choose labor $L_{i,t}$ to maximize

$$\max_{L_{i,t}} Y_{i,t} - W_t L_{i,t}. \tag{6}$$

The first-order condition of (6) allow us to show that the marginal product of capital is also constant across all firms and given by[12]

$$MPK_t \equiv \alpha \bar{Z}_t \left[\frac{(1-\alpha)\bar{Z}_t}{W_t} \right]^{\frac{1-\alpha}{\alpha}} = \alpha \bar{Z}_t K_t^{\alpha-1}. \tag{7}$$

The price Q_t of capital purchased by firms is determined by the first-order conditions of capital producers, which we detail in Appendix A.1.

We follow Frankel (1962) and Romer (1990) to assume that aggregate productivity is augmented by the aggregate stock of capital K_t,

$$\bar{Z}_t = Z_t K_t^{1-\alpha}, \tag{8}$$

in which $\ln Z_t$ follows an AR(1) process that we describe in Section 3. This assumption offers a parsimonious way to generate endogenous growth. From a technical standpoint, the assumption further simplifies the construction of the equilibrium as it implies that equilibrium quantities are homogeneous of degree one in the total capital stock, K_t, and that equilibrium prices do not depend on K_t. Combined with recursive preferences, this assumption also increases the volatility of the pricing kernel, as showed in the long-run risk literature (see, e.g., Bansal and Yaron (2004); Kung and Schmid (2015)).

[12]The first-order conditions of (6) give, for all i's,

$$\frac{\exp(\omega_{i,t})K_{i,t}}{L_{i,t}} = \left[\frac{W_t}{\bar{Z}_t(1-\alpha)} \right]^{1/\alpha}.$$

Re-arranging, integrating over i, and using the labor market clearing condition $\int L_{i,t} di = 1$, we get

$$K_t = \int \exp(\omega_{i,t}) K_{i,t} di = \left[\frac{W_t}{\bar{Z}_t(1-\alpha)} \right]^{1/\alpha} \int L_{i,t} di = \left[\frac{W_t}{\bar{Z}_t(1-\alpha)} \right]^{1/\alpha},$$

where K_t is the aggregate capital stock in the economy.

Idiosyncratic Productivity Our main departure from the Bernanke et al. (1999) setup is the inclusion of endogenous loan covenants written on a signal of the firm's future cash flows. To this end, we split the idiosyncratic shock $\omega_{i,t}$ into two components:

$$\omega_{i,t} = \omega^0_{i,t} + \omega^1_{i,t}, \tag{9}$$

in which $\omega^0_{i,t}$ and $\omega^1_{i,t}$ follow *i.i.d.* normal distributions with means $\mu_{0,t}$ and $\mu_{1,t}$, and standard deviations $\sigma_{0,t}$ and $\sigma_{1,t}$, respectively. We impose that $\mathbb{E}\left(\exp\left(\omega^0_{i,t}\right)\right) = \mathbb{E}\left(\exp\left(\omega^1_{i,t}\right)\right) = 1$ such that $\mu_{0,t} = -\frac{1}{2}\sigma^2_{0,t}$ and $\mu_1 = -\frac{1}{2}\sigma^2_{1,t}$. As a result, $\omega_{i,t}$ also follows a normal distribution, and $\mathbb{E}\left(\exp\left(\omega_{i,t}\right)\right) = 1$, meaning that the cross-sectional mean of the idiosyncratic shocks is 1. We further assume that the first component of the idiosyncratic shock, ω^0, is realized *after* the loan contract is signed, but *before* investment decisions are made, and that the exact realization of ω^0 can be perfectly observed only by the entrepreneurs and the lenders, but not by outside investors.[13] When entrepreneurs and lenders sign the debt contract, the debt contract specifies a threshold $\bar{\omega}^0_{i,t}$ such that if $\omega^0_{i,t} \geq \bar{\omega}^0_{i,t}$, then the entrepreneur keeps control rights to make investment decisions. If $\omega^0_{i,t} < \bar{\omega}^0_{i,t}$, then investment control rights switch from the entrepreneur to the lender.[14]

Entrepreneurs, Debt Contracts, and Investment Entrepreneur i enters period t with inherited wealth $N_{i,t}$, and borrows an amount $B_{i,t}$ from the national bank for one period. The national bank is risk-neutral, and sets up the loan contract to make zero profits ex-ante. Absent technical default, following a realization of the idiosyncratic productivity signal, the entrepreneur decides how to allocate this budget between productive capital for production in the next period and the bank's risk-free asset. Denoting by $A_{i,t}$ entrepreneur i's total assets and by $D^E_{i,t}$ the amount invested in the risk-free asset,

[13] This assumption is motivated by the fact that entrepreneurs and banks have an informational advantage relative to outside investors. Given this assumption, the covenant threshold $\bar{\omega}^0$ is contractible and explicitly written into the debt contract described.

[14] One can interpret this control rights allocation rule as resulting from investment Nash bargaining between lenders and borrowers, in which the bargaining power of the entrepreneur ϑ is a function of the distance between ω^0 and $\bar{\omega}^0$:

$$\vartheta\left(\omega^0, \bar{\omega}^0\right) = \begin{cases} 1 & \text{if } \omega^0 - \bar{\omega}^0 \geq 0, \\ 0 & \text{otherwise.} \end{cases}$$

the balance sheet identity of entrepreneur i is therefore

$$A_{i,t} = N_{i,t} + B_{i,t} = Q_t K_{i,t+1} + D^E_{i,t}. \tag{10}$$

We denote the fraction of the entrepreneur's assets invested in productive capital by $\theta_{i,t}$, and the fraction invested in bank deposits by $1 - \theta_{i,t}$, so that $\theta_{i,t} A_{i,t} = Q_t K_{i,t+1}$, and $(1 - \theta_{i,t}) A_{i,t} = D^E_{i,t}$. As in Bernanke et al. (1999) and Gertler and Kiyotaki (2010), we assume that the entrepreneur re-sells her entire stock of undepreciated productive capital after production. The total cash flow to entrepreneur i is $\exp(\omega_{i,t+1}) R^K_{t+1} Q_t K_{i,t+1}$, in which

$$R^K_{t+1} = \frac{MPK_{t+1} + Q_{t+1}(1-\delta)}{Q_t}. \tag{11}$$

We note once more that, since the marginal product of capital and its price are the same across all entrepreneurs, the return on capital is also the same across all entrepreneurs.

Time Line The timing of the model is as follows. In period t, entrepreneur i with net worth $N_{i,t}$ meets with the bank and negotiates a financial contract. The financial contract specifies the loan amount $B_{i,t}$, its interest rate $R^B_{i,t+1}$, and its covenant threshold $\bar{\omega}^0_{i,t+1}$. We rule out distressed lending from our framework by imposing that if the entrepreneur invests all available resources (including her own wealth) in the risk-free asset, she is always able to repay the loan. Formally, this implies that for all i, t, $R^B_{i,t+1} B_{i,t} < R^D_{i,t+1} A_{i,t}$. Empirically, we also rule out distressed lending by showing that our results are stronger for firms that are not in financial distress.

After signing the contract, entrepreneurs and lenders perfectly observe the idiosyncratic shock component $\omega^0_{i,t+1}$. If $\omega^0_{i,t+1} \geq \bar{\omega}^0_{i,t+1}$, the entrepreneur keeps the investment control rights. If $\omega^0_{i,t+1} < \bar{\omega}^0_{i,t+1}$, investment control rights are assigned to the lender. Moreover, consistent with a long empirical literature on loan terms' renegotiation in technical default (see, e.g., Roberts and Sufi (2009b) and Roberts and Sufi (2009a)), we allow the loan interest rate to be a function of whether the firm is in technical default. Formally, we let $R^B_{i,t+1}$ be such that

$$R_{i,t+1}^{B} = \begin{cases} R_{i,t+1}^{B,E} & \text{if } \omega_{i,t+1}^{0} - \bar{\omega}_{i,t+1}^{0} \geq 0, \\ R_{i,t+1}^{B,L} & \text{otherwise,} \end{cases} \tag{12}$$

in which the superscripts E and L denote the entrepreneur and the lender being in control, respectively. As we show below, $R_{i,t+1}^{B,E}$ reflects the credit risk of capital investment when the entrepreneur is in control, while $R_{i,t+1}^{B,L}$ carries no credit risk because the lender in control optimally chooses not to invest in risky capital. That is, $R_{i,t+1}^{B,L} = R_{t+1}^{D}$ for all $\omega_{i,t+1}^{0} < \bar{\omega}_{i,t+1}^{0}$.

At the beginning of $t+1$, the aggregate shock $\bar{Z}$ and the residual component of the idiosyncratic shock ω_i (i.e., ω_i^1) are realized, and entrepreneurs collect the payoffs from their portfolios. For given values of $\theta_{i,t}$ and $R_{i,t+1}^{B}$, and for a given realization of the aggregate state, entrepreneurs default on the loan if $\omega_{i,t+1}^{1}$ is below the endogenous default cutoff $\hat{\omega}_{i,t+1}^{1}$ as determined by

$$\left[\theta_{i,t} \exp\left(\omega_{i,t+1}^{0} + \hat{\omega}_{i,t+1}^{1}\right) R_{t+1}^{K} + (1-\theta_{i,t})\, R_{t+1}^{D}\right] A_{i,t} = R_{i,t+1}^{B} B_{i,t}. \tag{13}$$

2.2 Investment Decisions

In this section, we study the optimal investment choices θ^E for entrepreneurs and θ^L for lenders when entrepreneurs and lenders are in control (i.e., conditional on a realization of ω^0). We respectively denote by V^j and W^j, with $j \in \{E; L\}$, the payoffs of entrepreneurs and lenders when agent-j is in control (e.g., we let V^L denote the payoff of the entrepreneur when the lender is in control). In the next section, we derive the optimal loan contract by integrating the payoffs of the lender and the payoff of the entrepreneur across all possible realizations of ω^0.

Entrepreneurs are risk-averse. When in control, entrepreneurs choose their optimal portfolio mix between risky capital and risk-less deposits using the representative household's stochastic discount factor. We make the assumption that loans are issued to finance specific investments, which implies that loans and investments have the same (one-period) duration in our model. This assumption is consistent with our our empirical analysis: we find that in DealScan, loans issued to finance long-term projects such as project finance and real estate have an average duration of approximately 7 years. Conversely, loans issued to finance short-term investments such as working capital expenditure have an average duration of approximately 4 years. Importantly, this assumption allows us to represent entrepreneurs' problem as if entrepreneurs were myopic, and to greatly simplify the model solution.[15] Suppressing the dependency of V on its arguments, the entrepreneur's problem is to maximize her payoff in the non-default states, or

$$V^E = \max_{\theta^E_{i,t}} \mathbb{E}_t \left\{ M_{t+1} \int_{\hat{\omega}^1_{i,t+1}}^{\infty} \left\{ \left[\theta^E_{i,t} \exp\left(\omega_{i,t+1}\right) R^K_{t+1} + \left(1 - \theta^E_{i,t}\right) R^D_{t+1} \right] A_{i,t} - R^{B,E}_{i,t+1} B_{i,t} \right\} dF\left(\omega^1_{i,t+1}\right) \right\}. \tag{14}$$

Taking the entrepreneur's optimal decision for θ^E as given, the lender's payoff when the entrepreneur is in control is

$$\begin{aligned} W^E &= \mathbb{E}_t \left\{ M_{t+1} \left[\int_{-\infty}^{\hat{\omega}^1_{i,t+1}} (1-\zeta) \left[\theta^E_{i,t} \exp\left(\omega_{i,t+1}\right) R^K_{t+1} + \left(1 - \theta^E_{i,t}\right) R^D_{t+1} \right] A_{i,t} dF\left(\omega^1_{i,t+1}\right) \right.\right. \\ &\quad \left.\left. + \int_{\hat{\omega}^1_{i,t+1}}^{\infty} R^{B,E}_{i,t+1} B_{i,t} dF\left(\omega^1_{i,t+1}\right) \right] \right\}, \end{aligned} \tag{15}$$

in which $\zeta \in (0,1)$ is a deadweight loss in default (see, e.g., Elenev, Landvoigt, and Van Nieuwerburgh (2021)).

Since the household owns the national lender's equity, the lender also uses the household's stochastic discount factor to evaluate its future payoffs.[16] When in control of firm i, the national lender solves

$$\begin{aligned} W^L &= \max_{\theta^L_{i,t}} \mathbb{E}_t \left\{ M_{t+1} \left[\int_{-\infty}^{\hat{\omega}^1_{i,t+1}} (1-\zeta) \left[\theta^L_{i,t} \exp\left(\omega_{i,t+1}\right) R^K_{t+1} + \left(1 - \theta^L_{i,t}\right) R^D_{t+1} \right] A_{i,t} dF\left(\omega^1_{i,t+1}\right) \right.\right. \\ &\quad \left.\left. + \int_{\hat{\omega}^1_{i,t+1}}^{\infty} R^{B,L}_{i,t+1} B_{i,t} dF\left(\omega^1_{i,t+1}\right) \right] \right\}, \end{aligned} \tag{16}$$

[15] Without this assumption, the optimal financial contract would depend on the histories of realizations of the aggregate and idiosyncratic state variables, and the model would lose its tractability in a general equilibrium setting.

[16] Our results in terms of optimal investment policies are identical if we instead assume that the national lender is risk-neutral.

with associated payoff for the entrepreneur

$$V^L = \mathbb{E}_t \left\{ M_{t+1} \int_{\hat{\omega}^1_{i,t+1}}^{\infty} \left\{ \left[\theta^L_{i,t} \exp\left(\omega_{i,t+1}\right) R^K_{t+1} + \left(1 - \theta^L_{i,t}\right) R^D_{t+1} \right] A_{i,t} - R^{B,L}_{t+1} B_{i,t} \right\} dF\left(\omega^1_{i,t+1}\right) \right\}. \quad (17)$$

In the following proposition, we show that even if the entrepreneur is risk-averse, she always chooses *full investment* in the risky asset when in control. Intuitively, the entrepreneur's payoff function is convex in θ, such that—absent control rights reallocation—low realizations of ω^0 would always lead the entrepreneur to full investment in risk-free assets, and high realizations of ω^0 would always lead to full investment in risky capital. However, we show that the loan contract terms are such that the entrepreneur is always (weakly) better off assigning control rights to the lender instead of choosing to invest in the risk-free asset directly. This proposition allows us to greatly simplify the equilibrium characterization in the following section.

Proposition 1. *The optimal investment choices for the entrepreneur and the lender in control are $\theta^E = 1$ and $\theta^L = 0$, respectively. Therefore, $R^{B,L} = R^D$.*

Proof. See Appendix A.2. □

2.3 Loan Contracts

We solve the optimal loan contract by integrating the value functions for entrepreneurs and lenders across all possible realizations of ω^0. The loan terms are chosen so as to maximize the value of the entrepreneur subject to the lender's ex-ante break-even condition. That is, $R^B_{i,t+1}, B_{i,t}, \bar{\omega}^0_{i,t+1}$ maximize entrepreneur i's ex-ante expected payoff,

$$\max_{R^L_{i,t+1}, B_{i,t}, \bar{\omega}^0_{i,t+1}} V\left(N_{i,t}, R^B_{i,t+1}, B_{i,t}, \bar{\omega}^0_{i,t+1}\right), \quad (18)$$

subject to lender's break-even condition

$$W\left(N_{i,t}, R^B_{i,t+1}, B_{i,t}, \bar{\omega}^0_{i,t+1}\right) = B_{i,t}, \quad (19)$$

in which

$$V\left(N_{i,t}, R^B_{i,t+1}, B_{i,t}, \bar{\omega}^0_{i,t+1}\right) = \int_{-\infty}^{\bar{\omega}^0_{i,t+1}} V^L\left(N_{i,t}, R^B_{i,t+1}, B_{i,t}, \bar{\omega}^0_{i,t+1}, \omega^0_{i,t+1}\right) dF\left(\omega^0_{i,t+1}\right), + \int_{\bar{\omega}^0_{i,t+1}}^{\infty} V^E\left(N_{i,t}, R^B_{i,t+1}, B_{i,t}, \bar{\omega}^0_{i,t+1}, \omega^0_{i,t+1}\right) dF\left(\omega^0_{i,t+1}\right), \tag{20}$$

and

$$W\left(N_{i,t}, R^B_{i,t+1}, B_{i,t}, \bar{\omega}^0_{i,t+1}\right) = \int_{-\infty}^{\bar{\omega}^0_{i,t+1}} W^L\left(N_{i,t}, R^B_{i,t+1}, B_{i,t}, \bar{\omega}^0_{i,t+1}, \omega^0_{i,t+1}\right) dF\left(\omega^0_{i,t+1}\right) + \int_{\bar{\omega}^0_{i,t+1}}^{\infty} W^E\left(N_{i,t}, R^B_{i,t+1}, B_{i,t}, \bar{\omega}^0_{i,t+1}, \omega^0_{i,t+1}\right) dF\left(\omega^0_{i,t+1}\right). \tag{21}$$

In the following proposition, we show that, when we rescale the problem by the amount of the entrepreneur's initial wealth, the optimal loan contract terms are identical across all entrepreneurs.

Proposition 2. *Define by $H_{i,t}$ the debt-to-net worth ratio $H_{i,t} \equiv B_{i,t}/N_{i,t}$. The optimal debt contract features the same $H_{i,t}$, $R^{B,E}_{i,t+1}$, and $\bar{\omega}^0_{i,t+1}$ across all the entrepreneurs.*

Proof. See Appendix A.3. □

The important result in Proposition 2 allows us to achieve aggregation in our model. By virtue of this aggregation result, we omit in the following sections the subscript i in $R^{B,E}_{t+1}$, H_t and $\bar{\omega}^0_{t+1}$. In the Appendix A.4, we provide a formal definition for the competitive equilibrium in our model.

2.4 Entrepreneurs' Wealth and Return on Equity

To close the model, we assume that a fraction λ of the undefaulted entrepreneurs are hit by a liquidity shock and need to liquidate all of their net worth to the household (Bernanke et al. (1999)). Defaulted entrepreneurs and those hit by the liquidity shock draw an initial wealth χN_t, with $\chi > 0$ at the beginning of period $t + 1$.[17]

We denote the wealth of the entrepreneur *before* the draw of new wealth by $\tilde{N}$. At the individual

[17] We assume that χ is small enough such that entrepreneurs do not have strategic default incentives.

level, $\tilde{N}$ evolves as

$$\tilde{N}_{i,t+1} = \begin{cases} R^D_{t+1} N_{i,t} & \text{if } \omega^0_{i,t+1} < \bar{\omega}^0_{i,t+1}, \\ \left[\exp\left(\omega_{i,t+1}\right) R^K_{t+1}\left(1+H_t\right) - R^{B,E}_{t+1} H_t\right] N_{i,t} & \text{if } \omega^0_{i,t+1} \geq \bar{\omega}^0_{i,t+1} \text{ and } \omega^1_{i,t+1} \geq \hat{\omega}^1_{i,t+1}, \\ 0 & \text{otherwise,} \end{cases} \tag{22}$$

which allows us to obtain the return on the entrepreneur's wealth as

$$\frac{\tilde{N}_{i,t+1}}{N_{i,t}} = \begin{cases} R^D_{t+1} & \text{if } \omega^0_{i,t+1} < \bar{\omega}^0_{i,t+1}, \\ \left[\exp\left(\omega_{i,t+1}\right) R^K_{t+1}\left(1+H_t\right) - R^{B,E}_{t+1} H_t\right] & \text{if } \omega^0_{i,t+1} \geq \bar{\omega}^0_{i,t+1} \text{ and } \omega^1_{i,t+1} \geq \hat{\omega}^1_{i,t+1}, \\ 0 & \text{otherwise.} \end{cases} \tag{23}$$

Equation (23) shows that a larger distance to technical default $\omega^0 - \bar{\omega}^0$ leads to investment in risky assets and higher returns (absent default). Finally, the aggregate law of motion for capital is

$$\begin{aligned} N_{t+1} &= N_t \lambda \left[\int_{\bar{\omega}^0_{t+1}}^{\infty} \int_{\hat{\omega}^1_{t+1}}^{\infty} \left\{\left[\exp\left(\omega^0_{t+1} + \omega^1_{t+1}\right) R^K_{t+1}\right]\left(1+H_t\right) - R^{B,E}_{t+1} H_t\right\} dF\left(\omega^1_{t+1}\right) dF\left(\omega^0_{t+1}\right) \right. \\ &\left. + \int_{-\infty}^{\bar{\omega}^0_{t+1}} R^D_{t+1} dF\left(\omega^0_{t+1}\right)\right] + \chi N_t \left[1 - \lambda\left(1 - \int_{\bar{\omega}^0_{t+1}}^{\infty} \int_{-\infty}^{\hat{\omega}^1_{t+1}} dF\left(\omega^1_{t+1}\right) dF\left(\omega^0_{t+1}\right)\right)\right], \end{aligned} \tag{24}$$

where the last element is the newly-realized wealth of entrepreneurs in default and of those hit by the liquidity shock.[18]

[18] This implies that the amount of wealth Π_t transferred to the household is

$$\Pi_t = N_{t-1}\left(1-\lambda\right)\left\{\int_{\bar{\omega}^0_t}^{\infty} \int_{\hat{\omega}^1_t}^{\infty} \left\{\exp\left(\omega^0_t + \omega^1_t\right) R^K_t \left(1 + H_{t-1}\right) - R^{B,E}_t H_{t-1}\right\} dF\left(\omega^1_t\right) dF\left(\omega^0_t\right) + \int_{-\infty}^{\bar{\omega}^0_t} R^D_{t+1} dF\left(\omega^0_t\right)\right\}.$$

Similarly, bank ex-post profits Π^B_t are the difference between the ex-post value of W and deposit debt repayments:

$$\begin{aligned} \Pi^B_t &= \int_{\bar{\omega}^0_t}^{\infty} \int_{\hat{\omega}^1_t}^{\infty} R^{B,E}_t B_{t-1} dF\left(\omega^1_t\right) dF\left(\omega^0_t\right) + \int_{-\infty}^{\bar{\omega}^0_t} R^D_t B_{t-1} dF\left(\omega^0_t\right) \\ &+ \int_{\bar{\omega}^0_t}^{\infty} \int_{-\infty}^{\hat{\omega}^1_t} \left[\left(1-\zeta\right) \exp\left(\omega_t\right) R^K_t A_{t-1}\right] dF\left(\omega^1_t\right) dF\left(\omega^0_t\right) - R^D_t B_{t-1}. \end{aligned}$$

3 Quantitative Model Results

In this section, we first specify the exogenous aggregate shocks in our economy. We then calibrate our model and evaluate its ability to replicate key moments of both macroeconomic quantities and asset prices at the aggregate level. Finally, we investigate the model's aggregate quantitative implications. We show that the model can generate counter-cyclical variation in the aggregate technical default probability consistent with the data, and that allowing for endogenous covenant terms to vary over the cycle mitigates the financial accelerator mechanism. In Section 4.3, we also show that the model can produce a sizable technical default spread in equity returns consistent with the data.

3.1 Specification of Aggregate Shocks

We consider two aggregate shocks in our benchmark model. First, we assume that the log of aggregate productivity $\ln Z_t$ follows the process

$$\ln Z_t - \ln\left(\bar{Z}\right) = \rho_z\left(\ln Z_{t-1} - \ln\left(\bar{Z}\right)\right) + \sigma_z \epsilon_{Z,t}, \tag{25}$$

in which $\bar{Z}$ is the steady state level of Z_t, and $\epsilon_{Z,t}$ represents white noise that follows an *i.i.d.* standard normal distribution. Second, we introduce the shocks to the volatility of ω^0 and ω^1 (i.e, σ_t^0 and σ_t^1). We interpret these shocks as uncertainty shocks following Christiano, Motto, and Rostagno (2014), who document that risk (uncertainty) shocks are the major drivers of business cycle fluctuations in the financial accelerator model. Specifically, we assume that the log of σ_t^0 and σ_t^1 follow the process

$$\ln \sigma_t^j - \ln\left(\bar{\sigma}^j\right) = \rho_s\left(\ln \sigma_{t-1}^j - \ln\left(\bar{\sigma}^j\right)\right) + \sigma_{sj}\epsilon_{s,t}, \qquad j = 0, 1, \tag{26}$$

in which σ_{s0} and σ_{s1} are the standard deviations of the uncertainty shocks to $\ln \sigma_t^0$ and $\ln \sigma_t^1$, respectively.[19]

We further assume that the innovations terms, $\epsilon_{Z,t}$ and $\epsilon_{s,t}$, are perfectly negatively correlated,

[19] For simplification, here we assume that the process of $\ln \sigma_t^0$ and $\ln \sigma_t^1$ have the same persistence parameter ρ_s, and share the same exogenous innovation $\epsilon_{s,t}$. More general specifications can be incorporated in this framework, including specifications for which we only allow for shocks to the total volatility of ω^0 and ω^1 and split the shocks proportionally between $\ln \sigma_t^0$ and $\ln \sigma_t^1$.

which implies the following structure for the innovation terms:

$$\begin{bmatrix} \epsilon_{Z,t} \\ \epsilon_{s,t} \end{bmatrix} \sim \text{Normal}\left(\begin{bmatrix} 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 & \rho_{Z,s} \\ \rho_{Z,s} & 1 \end{bmatrix} \right). \tag{27}$$

The perfect negative correlation between the innovation shocks indicates that a negative productivity shock is associated with a positive uncertainty shock, consistent with the fact that uncertainty spikes during in bad times (see Jurado, Ludvigson, and Ng (2015) and Bloom, Floetotto, Jaimovich, Saporta-Eksten, and Terry (2018)). These perfectly correlated shocks are a parsimonious way to generate a positive correlation between contemporaneous consumption and investment that is consistent with the data.[20]

3.2 Calibration

We calibrate the benchmark model at the quarterly frequency and present our calibrated parameters in Table 1. The first group of parameters in Table 1 are borrowed from the literature. Following the long-run risk literature (specifically, Bansal and Yaron (2004)), we set the relative-risk aversion parameter γ to 10 and the inter-temporal elasticity of substitution (IES) parameter ψ to 2. The capital share parameter α is set to 0.33, as in the real business cycle literature (Kydland and Prescott (1982)).

The parameters in the second group are determined by matching the first moments of macroeconomic quantities and prices. We set the time discount parameter $\beta = 0.997$ to generate a steady state real growth rate of 1.5% per year. The depreciation rate parameter δ is set to 0.025 such that the annual capital depreciation rate is 10%, consistent with the real business cycle literature. The steady state value of the entrepreneur's survival probability $\bar{\lambda}$ is set to 0.95, roughly targeting an average life of 10 years for U.S. firms in Compustat. Additionally, we let our parameters jointly target the following steady state outcomes: i) a steady state debt-to-net worth ratio of 0.5, implying an average non-financial corporate sector leverage ratio of 0.33, consistent with the data; ii) a steady state annual

[20]Note that uncertainty shocks by themselves are not enough to generate this positive correlation. Consider an alternative economy featuring only uncertainty shocks. Upon the uncertainty shock, output does not change since capital is pre-determined. On the other hand, from the aggregate resource constraint (see Equation (A.42) in the Appendix), it is clear that contemporaneous consumption and investment respond in opposite directions. As a result, the model generates a counter-factually negative correlation between consumption and investment growths over the cycle.

technical default probability of 24%, in line with the findings in Chodorow-Reich and Falato (2021) and Roberts and Sufi (2009a); and iii) a steady state annual loan spread of 2.6%, close to the average loan spread in the DealScan data. To obtain these steady state values, we set the steady state standard deviation of ω^0 (i.e., $\bar{\sigma}_0$) to 0.008, the steady state standard deviation of ω^1 (i.e., $\bar{\sigma}_1$) to 0.46, the steady state value of the TFP shock $\bar{Z}$ to 0.13, and the loss in default parameter ζ to 0.23, similar to Bernanke et al. (1999) and Gomes and Schmid (2016). The parameter χ that determines the transfer to entering entrepreneurs is set to 0.49, and it is implied by the law of motion of net worth at the steady state.

The parameters in the third group are determined by matching second moments in the data. The capital adjustment cost parameter ξ is set to 13 in order to match the volatility of the investment growth rate. The persistence of the TFP shock ρ_z and the uncertainty shock ρ_s are set to approximately match the auto-correlation of output growth, investment growth, and consumption growth, as well as the standard deviation of TFP shocks. Finally, shocks to σ_t^0 and σ_t^1 (i.e., σ_z, σ_{s0}, and σ_{s1}) are set to approximately match the volatility of consumption growth and output growth, as well as average technical default probability and average probability of actual default.

3.3 Aggregate Moments

Next, we study the performance of our benchmark calibration in matching aggregate quantities by solving and simulating our model at the quarterly frequency and then aggregating the model-generated data to compute annual moments. We show that our model is broadly consistent with the key empirical features of macroeconomic quantities and asset prices. Importantly, the model produces a reasonable set of aggregate moments in the financial market, including the levels of the risk-free rate, the corporate credit spread, and the probability of technical default as well as the volatility of the risk-free rate. Table 2 reports the model-simulated moments of macroeconomic quantities and asset returns and compares them to their counterparts in the data.

In terms of the aggregate moments of macroeconomic quantities, our calibration features a low volatility of consumption growth (3.12%) and a relatively high volatility of investment growth (7.34%). The model also produces correlations among consumption, investment, and output (as well as auto-correlations) that are reasonably close to the data. The investment-to-output ratio is 22%, close to the value of 17% in the data. In terms of asset pricing moments, the model can reproduce a low real interest rate (around 1.3%), a high equity premium (7.01%), and a loan spread consistent with the data. Moreover, it can generate an annual technical default probability (around 30%) consistent with the findings in Chodorow-Reich and Falato (2021), and a actual default probability (around 5%), which is in line with the findings in Altman and Suggitt (2000).

3.4 Impulse-Response Functions and Counter-Cyclical Technical Default

Next, we present our model's benchmark calibration impulse response functions, and we show that the model can generate counter-cyclical aggregate variation in the technical default probability (a result that we confirm in the empirical section).

Figure 1 plots the responses of quantities and prices to a negative productivity shock of a one-standard-deviation. As captured in Figure 1, we can make the following observations. First, in response to a negative productivity shock, capital prices and net worth-to-capital ratios drop as in standard macro models. Second, the entrepreneur's net worth decreases by more than the debt, such that the entrepreneur's debt-to-net worth ratio (i.e., leverage) increases. This increases the cost of external finance for the entrepreneur and depresses investment, thus lowering entrepreneurial net worth in subsequent periods and generating a vicious cycle in economic activity. In sum, our model successfully replicates the key mechanism of the financial accelerator of Bernanke et al. (1999). Third, due to the sharp decline in the return to capital (which negatively affects net worth) when the shock hits, the entrepreneur's actual default probability increases on impact. Moreover, the default probability further increases in subsequent periods due to higher leverage (and greater uncertainty). This increase in the actual default probability leads to an increase in loan spreads (measured as the ratio of the loan rate $R_t^{B,E}$ to the risk-free rate R_t^D).

Fourth, the model generates a counter-cyclical technical default probability consistent with the data, as TFP shocks are negatively correlated with uncertainty shocks, and also since a high uncertainty shock implies that firms on average draw worse signals ω^0 in bad states. On the other hand, we show that the endogenous covenant threshold chosen by the entrepreneur slightly decreases in bad times, thus allowing the entrepreneur to sustain a relative low level of leverage and interest rate. The uncertainty effect dominates, and the entrepreneur's overall technical default probability increases in bad times as shown in the bottom left panel of Figure 1. In quarterly simulated data, we show that the technical default probability and the output growth rate have a correlation coefficient of -0.18 (with a t-stat = -7.35).

3.5 Lender Control Rights and the Financial Accelerator

We conclude our quantitative analysis by showing that endogenous adjustments in loan covenants over the business cycle mitigate the aggregate effects of the financial accelerator.

As is well known in the literature of macroeconomic models with financial frictions, TFP shocks alone do not generate quantitatively high volatility in capital prices or in the entrepreneur's net worth. As the entrepreneur's net worth has low volatility, the amplification effects generated by the financial accelerator mechanism are small, making the setting inappropriate to asses possible mitigation effects due to lender control rights allocation.

To highlight the quantitative importance of this mitigation channel, we slightly modify our calibration exercise by introducing a shock to the entrepreneur's survival probability λ as in Ai, Li, and Yang (2020b).[21] By affecting entrepreneurs' net worth without directly affecting real production, this shock is a financial shock to the discount rate (as in Jermann and Quadrini (2012b) and Ai et al. (2020b)), and can generate a quantitatively sizable financial accelerator effect.

To technically maintain $\lambda \in (0, 1)$ in a parsimonious way, we set

$$\lambda_t = \frac{\exp(x_t)}{\exp(x_t) + \exp(-x_t)}, \tag{28}$$

in which x_t follows the process

$$x_t - x_{ss} = \rho_x (x_{t-1} - x_{ss}) + \sigma_x \epsilon_{x,t}, \tag{29}$$

with x_{ss} again denoting the steady-state value of x. Note that in this framework a negative innovation

[21]The literature considers alternative shocks for the same purpose of generating a large volatility in net worth. For example, a capital quality shock is introduced in Gertler and Kiyotaki (2010) and Elenev et al. (2021), and a shock to entrepreneurs' liquidation probability is introduced in Ai, Li, Li, and Schlag (2020a) and Ai et al. (2020b).

$\epsilon_{x,t}$ leads to a lower x_t and a lower survival probability λ_t. To maintain tractability, we further assume that the three shocks in the economy are such that $\rho_{Z,x} = 1$ and $\rho_{s,x} = -1$. Setting $\rho_{Z,x} = 1$ induces a positive correlation between TFP shocks and financial shocks, such that entrepreneurs' survival probability is low in low-productivity states.[22] Similar to our baseline calibration, $\rho_{s,x} = -1$ forces a negative correlation between financial shocks and uncertainty shocks, such that firms on average draw worse signals and idiosyncratic shocks in bad states.

In this new calibration, we keep all the parameters of Section 3.2 except for the parameters related to productivity shocks.[23] To assess the model's mitigation effects, in Figure 2 we consider the impulse response functions of quantities and prices to a negative financial shock of a one-standard-deviation. We compare two cases, a "flexible" case in which we allow covenant thresholds to change over the business cycle, and a "fixed" case in which we fix the covenant threshold $\bar{\omega}$ at its steady state.

The first result of Figure 2 shows that the baseline impulse-response patterns in this new calibration are similar to those documented in the previous section with only productivity shocks. When the entrepreneur's survival probability decreases, this directly decreases the entrepreneur's net worth and capital prices. At the same time, external finance becomes more expensive: both the debt-to-net worth ratio and the external finance premium increase, which decreases investment rates. High leverage ratios lead to high default probabilities, driving credit spreads up.

The main result of Figure 2 shows that allowing covenant thresholds to vary over the cycle has significant mitigation effects on the financial accelerator. In response to a negative financial shock to the entrepreneur's net worth, capital prices decrease, and the expected return on capital increases. The loan contract offers two options to the entrepreneur to take advantage of this higher expected return on capital while satisfying the bank's break-even condition (19). The first option is to increase the entrepreneur's leverage, as in the fixed-covenant case. The second option is to increase the states of the world for which the entrepreneur can invest in risky capital (with high expected returns) in-

[22]This assumption is necessary to generate a positive correlation between consumption and investment growth consistent with the data. If we excluded TFP shocks from the model, consumption and investment would feature opposite responses to financial shocks, thus generating a counter-factually negative correlation in their growth rates. Moreover, note that this assumption effectively implies that there is only one aggregate shock in the economy.

[23]Specifically, since we now have two shocks, we lower the persistence of productivity shocks to 0.985 and their volatility to 0.155, and set the persistence parameter of the financial shock ρ_x to 0.985, and also set the volatility of the parameter σ_x to 0.015 to match the volatilities and autocorrelations of consumption, investment, and output. The simulated moments of this model are quantitatively similar to those in the benchmark model and are broadly in line with our data.

stead of giving up control rights to the bank (which, by Proposition 1, always results in investment in the risk-free asset).

Quantitatively, the second effect dominates, and the optimal contract features a lower covenant threshold relative to the fixed-covenant case. As Figure 2 shows, this dynamic covenant adjustment leads to a lower increase in the technical default probability relative to the fixed-covenant case, which allows the entrepreneur in control to take on relatively low leverage and to promise a relatively low interest payment to the bank while still allowing the bank to break even in expectation. As a result of the lower increase in leverage and loan interest rates (relative to the fixed-covenant case), the entrepreneur's default probability increases by less, and the investment in the risky asset decreases by less as well.

Figure 2 shows that the mitigation effects from allowing covenant thresholds to vary over the business cycle are quantitatively large. For example, Figure 2 shows that investment rates and capital prices respectively drop by 1.35% and 0.1% on impact in the model with flexible covenants, and they respectively drop by 1.95% and 0.15% in the model with fixed covenants (implying a 30% lower drop in investment and capital prices in the model with flexible covenants). Debt-to-net worth ratios, external finance premia, actual default probabilities, and loan spreads are also significantly less sensitive to a negative shock in the model with flexible covenants. For example, the increases in net worth ratios and loan spreads in the model with flexible covenants are respectively only 31% and 40% of their counterparts in the model with fixed covenants.[24]

4 Empirical Analysis and Cross-Sectional Quantitative Results

In this section, we present our empirical analysis. We start by describing our main data sources and variables, and we then investigate the relationship between lender control rights and expected stock returns in the cross-section.

[24]Interestingly, the initial drop of net worth in the fixed covenant cases is bigger yet recovers faster, because the higher leverage and high expected return of capital in bad states allows for faster accumulation of net worth.

4.1 Data

Our data comes from three sources. Loan covenant data comes from Loan Pricing Corporation's (LPC) DealScan database, which provides extensive coverage on syndicated and bilateral private loans made by bank and non-bank (e.g., pension funds) lenders to U.S. borrowers since 1984 (see, e.g., Carey and Hrycray (1999) and Bradley and Roberts (2015)). The information contained in DealScan is sourced from listed firms' SEC filings or directly obtained from borrowers and lenders, and it includes pricing, amount, and maturity for individual loans (also known as facilities), as well as covenant terms for groups of loans included in the same lending contract (also known as packages). Since DealScan is only sparsely populated before 1996, and since our portfolio sorting exercise requires a sufficiently high number of observations in each portfolio, we drop pre-1996 loan observations from the data.

In Appendix B, we describe how we construct our quarterly measure of firm-level distance to technical default following Murfin (2012). To construct our strictness measure, we focus on the five most common covenant ratios found on DealScan, namely maximum debt to EBITDA, minimum interest coverage (EBITDA to interest expense), minimum fixed charge coverage (EBITDA to fixed charges, the sum of interest expense, debt in current liabilities, and rent expense), maximum leverage (long-term and current debt to total assets), and minimum current ratio (current assets to current liabilities). Collectively, these five covenants represent around 80% of all the DealScan covenant ratios.

A firm might have multiple loan packages outstanding at the same time, and each package might specify a different threshold for the same covenant ratio. Since our strictness measure captures the probability of breaching *any* covenant, we compute the most restrictive covenant for each of our five covenant ratios across all outstanding packages in any given quarter. For example, if in a given quarter a firm has five active packages and three of those packages include covenants restricting the firm's maximum debt-to-equity ratio, we use the smallest of these debt-to-equity covenants to compute strictness for that firm.

Quarterly financial data for the firms in our sample come from Compustat, and monthly firm returns come from CRSP. We use Compustat to compute the realized value of each of the financial ratios included in our strictness measure, as well as the variables that we use for portfolio double-sorting. Using the DealScan link table by Sudheer Chava and Michael Roberts (see Chava and Roberts (2008)), we obtain a firm-quarter panel that contains values for the most restrictive covenant for each of our five covenant types, as well as values for the annualized financial ratios associated with these covenants.[25] Following the methodology described in Section B.1, we use these financial ratios and covenants to construct our firm-quarter strictness measure.

Our sample starts with the first quarter of 1996 and ends with the last quarter of 2016. Following standard practice in the asset pricing literature, we drop firms in the financial and utilities industries. Figure 3 shows the time series properties of covenant strictness in our sample. Also, Figure 3 shows that average strictness across all firms in our sample is counter-cyclical, peaking at the beginning of 2001 and during the financial crisis. Finally, as documented by Griffin, Nini, and Smith (2018), Figure 3 also shows a general decreasing trend in covenant strictness since the early 2000's.

In the Appendix, we provide a validation test for our strictness measure by showing that lagged strictness positively predicts future covenant violations. Covenant violation data at the firm-quarter level come from Greg Nini, and represent an updated version of the covenant violation data in Nini, Smith, and Sufi (2012).[26] As documented in Chava and Roberts (2008), Nini et al. (2009), Nini et al. (2012), Falato and Liang (2016), Gilje (2016) and many other papers, covenant violation leads to the transfer of control rights to the lender. In this sense, our strictness measure effectively captures the ex ante probability of lender control rights in the next period.

Table 3 reports summary statistics for the main variables of the paper, including strictness, two measures of firm financial constraints (the Whited and Wu (2006) (WW) Index and the Hadlock and Pierce (2010) Size-Age (SA) Index), and two measures of firm default probability (failure probability as in Campbell, Hilscher, and Szilagyi (2008), and Expected Default Frequency (EDF) as in Bharath and Shumway (2008)). Table 3 also reports summary statistics (reported at the quarterly level) for the other variables that we use in the paper, such as the Book-to-Market (B/M) ratio, the Investment-

[25]As in Demerjian and Owens (2016), we annualized all flow variables (e.g., interest expense) when computing strictness by summing these variables over the current quarter and the past three quarters. If a covenant type is not present in any of the firm's active packages, we record a missing value for that covenant and compute strictness using the remaining covenants.

[26]We are grateful to Greg Nini for sharing the updated loan covenant violation data.

to-Capital (I/K) ratio, tangibility (the ratio of purchased capital (PPENTQ) to total assets (ATQ)), profitability (measured by Return on Equity (ROE) and Return on Assets (ROA)), leverage, and the Nini et al. (2012) investment and acquisition conservatism measures.

4.2 Lender Control Rights and Expected Stock Returns in the Cross-Section

In this section, we present the main empirical results on the relationship between loan covenant strictness and expected stock returns.

4.2.1 Portfolio Sorting

In our first empirical exercise, we form five strictness-based portfolios of firms in every quarter, and we compare the monthly excess returns across these portfolios in subsequent quarters.[27] Table 4 shows the characteristics of the five strictness-based portfolios in our sample. Specifically, Table 4 shows that high-strictness firms have on average higher failure probability, EDF, and book-to-market than low-strictness firms, and that high-strictness firms have lower profitability (as measured by their ROE), dividend payouts, and credit ratings than low-strictness firms. Our data show no evidence of a relationship between strictness and investment to capital ratios, and only weak evidence of a positive relationship between strictness and financial constraints as measured by the SA and WW indexes. Importantly, the data show that high-strictness firms have on average *higher leverage* than low-strictness firms, suggesting that reductions in leverage when firms enter technical default (due, for example, to loan call-backs by lenders) are unlikely driving our cross-sectional findings with respect to corporate risk-taking and equity returns.[28]

Table 4 shows that, unconditionally, firms in the high-strictness portfolio earn 40% of the average excess returns of firms in the low-strictness portfolio: the average excess return of firms in the high-strictness portfolio is 2.64%, while the average excess return of firms in the low-strictness portfolio is 6.76%. Moreover, the relationship between current strictness and future excess returns is non-monotonic: firms in the high-strictness portfolio earn only 25% of the excess returns earned by

[27] As in Fama and French (1992), we allow a two-quarter lag for information to be incorporated into stock returns.

[28] Our cross-sectional results hold when controlling for firm leverage in Fama and MacBeth (1973) regressions, further confirming this hypothesis.

firms in the fourth portfolio. In other words, Table 4 suggests that the negative relationship between strictness and returns is driven by high-strictness firms.

In Panel A of Table 5, we confirm these results by reporting excess returns and Newey and West (1987) t-statistics for the five strictness-based portfolios, for a strategy that goes long in the "High" portfolio and short in the fourth portfolio (High-4), for a strategy that goes long in the "High" portfolio and short in the "Low" portfolio (High-Low), and for a strategy that goes long in the fourth portfolio and short in the "Low" portfolio (4-Low). Four sets of results emerge from this panel. First, the High-Low portfolio earns a marginally significant negative excess return of 4.12% per year, confirming an overall negative relationship between strictness and expected returns.

Second, while a cursory look at the data seems to suggest a hump-shaped relationship between strictness and excess returns (the returns of the 4-Low portfolio are unconditionally positive and significant at the 10% level), this relationship becomes economically weaker when we control for standard risk factors, and is insignificant at conventional statistical levels when we control for the Fama and French (2015) factors. Third, the data show a significant drop in future excess returns for firms in the highest strictness quintile: the 4-High portfolio earns a statistically significant *negative* 7.72% annual return, which is only partially explained by differential exposure to the Fama and French (2015) factors.

Fourth, the difference in expected returns for firms in the fourth strictness portfolio and in the high-strictness portfolio is partly explained by differential exposure to the aggregate investment and profitability factors. We interpret these results as evidence that low-strictness firms have similar exposure to aggregate investment opportunities and profitability (Hou et al. (2015)), while firms close to technical default are more constrained in their investment choices. In the Appendix, we confirm this intuition by showing similar results for the Hou et al. (2015) q-factors.

In Panel B of Table 5, we also show that firms in the high-strictness portfolio exhibit statistically *higher* investment and acquisition conservatism than firms in the fourth portfolio. Firms in the high-strictness portfolio have around 3.6 times more conservative investment policies and around 3.8 more conservative acquisition policies than firms in the fourth portfolio. These results provide additional support for the intuition that technical default increases firms' conservativeness (Nini et al. (2012)), and also confirm a non-monotonic relationship between strictness and risk consistent with our theoretical model and with the empirical results of Panel A.

4.2.2 Robustness: Fama-MacBeth Regressions and Distressed Firms

In Table 6, we present estimates of Fama and MacBeth (1973) regressions of future monthly excess returns on strictness. We run monthly cross-sectional regressions of future excess return on strictness, and then average the estimates of these monthly cross-sectional regressions across all months in our sample. In Column (1), our Fama-MacBeth include firm size, book-to-market, reversal, leverage, and ROA as additional control variables.[29] In Columns (2) and (3), we respectively control for failure probability and EDF, so we may reduce concerns that our results might be driven by financial distress instead of technical default. In Column (4), we simultaneously control for failure probability and EDF. The results of Table 6 confirm the negative relationship between strictness and expected returns documented in Table 5, and show that this negative relationship is not driven by other firm-level variables potentially correlated with both strictness and returns.

In Appendix Tables B3 and B4, we conduct two additional sets of robustness checks on our main result. In Table B3, we confirm the second insight of Table 5—that the negative relationship between strictness and expected returns is mainly driven by firms close to technical default. To do so, we repeat the same exercise as in Table 6, but we replace our continuous measure of lagged strictness with indicators for whether a firm belongs to a different quintile of the strictness distribution. Table B3 provides strong support for our main insight. The estimates from Column (1) show that relative to a firm in the low-strictness portfolio (representing the baseline categorical variable), a firm in the high-strictness portfolio earns around 0.3% lower monthly returns, or 3.6% annual returns, on average. The baseline result of Column (1) is both economically and statistically robust when controlling for failure probability and EDF in Columns (2)-(4), confirming that the non-linear relationship between strictness-return relationship of high-strictness firms is not driven by financial distress.

In Table B4, we provide another test to confirm that the negative and non-monotonic relationship between strictness and returns is not driven by distressed firms. Since the financial distress puzzle

[29] In the Appendix, we also include the SA Index and the WW Index measured at the annual level to control for financial constraints.

is driven by firms with high default probability (see, e.g., Garlappi and Yan (2011)), in Table B4, we drop from the sample those firms above the 90^{th} percentile of the EDF distribution (Columns (1) and (2)) and those above the 90^{th} percentile of the failure probability distribution (Columns (3) and (4)). In Panel A, we compute the strictness-based portfolios using cutoffs from the unconditional strictness distribution in each quarter (i.e., including distressed firms). In Panel B, we first drop distressed firms and then construct strictness-based portfolios in the resulting sample. Both panels show that the results of Table B3 hold in the sub-sample of non-distressed firms, confirming that our main results are not driven by the distress anomaly.

4.2.3 Evidence from Regression Discontinuity around Covenant Violations

One possible concern is that our portfolio sorting results (which rely on an ex-ante measure of technical default, covenant strictness) may not be driven by changes in lenders' involvement in corporate policies if managers decrease the risk of their investments to avoid technical default. To address this concern, we provide evidence in Table 7 of a decrease in returns *after* covenants are violated using a regression discontinuity design (RDD) framework (e.g., Chava and Roberts (2008)).

In Table 7, we study how future returns are affected by violations of the most common covenant type in DealScan, maximum Debt-to-EBITDA. In the first two columns of Table 7, we report estimates of the specification

$$\text{Ex. Ret.}_{i,t+1} = a + b_1 \times \text{Violation}_{it} + b_2 \times \text{Distance}_{it} + b_3 \times \text{Violation}_{it} \times \text{Distance}_{it} + b_4 \times X_{it} + e_{it+1}, \tag{30}$$

in which $\text{Ex. Ret.}_{i,t+1}$ is firm i's future quarterly excess return, Violation_{it} is an indicator equal to one if firm i is in violation of its most restrictive Debt-to-EBITDA covenant in quarter t, Distance_{it} is the difference between firm i's Debt-to-EBITDA value and its most restrictive Debt-to-EBITDA covenant, and X_{it} is a matrix of time-varying controls, including size, market-to-book, book leverage, and ROA.[30] The coefficient of interest in (30) is b_1, the average difference in conditional excess returns

[30] In line with our portfolio sorting exercise, we allow for a two-quarter lag between our balance sheet measures and future returns. Our results are not sensitive to alternative lag choices.

for firms breaching their Debt-to-EBITDA covenants, relative to firms not breaching these covenants.

The first two columns of Table 7 show that breaching a covenant is associated with a 31 to 44 basis point reduction in future excess returns, depending on the specification. These results are economically and statistically similar when we include time-varying controls and when we allow for higher-order functional dependencies between returns and distance to violations in Columns (2) and (3), and hold in narrow bandwidths around covenant breaches (see Figure 4 and the Appendix). Overall, the evidence in Table 7, Figure 4, and the Appendix supports our control rights reallocation hypothesis, and confirms strictness as a forward-looking measure of technical default.

4.2.4 Additional Robustness

In the Appendix, we perform a number of additional robustness tests on the results of the previous sections. First, we show that the results of Table 5 are robust to a different empirical specification for the process governing the (log) growth of firm-level financial ratios used to compute strictness.[31] Second, we include the WW Index and the SA Index (measured at the annual level) to our Fama-MacBeth specifications from Table 6 to show that our results are not driven by financial constraints. Third, we repeat the exercise from Tables 6, B3, and B4 using pooled OLS regressions instead of Fama-MacBeth regressions, for which we keep the firm-month panel structure of the data instead of computing averages of monthly cross-sectional regressions. These pooled OLS results are both statistically and economically similar to those from Tables 6 and B3-B4, and provide additional support for the presence of a non-linear negative relationship between loan covenant strictness and expected returns.

4.3 Quantitative Results: Covenant Strictness and the Cross-Section of Equity Returns

We conclude by evaluating the model's ability to replicate the cross-sectional return patterns documented in the data. To do so, we first simulate the model for 800 quarters, thus obtaining a time series of aggregate quantities and prices.[32] Using these aggregate simulate data, we then draw id-

[31] Specifically, we replace the process specified in Equation (B.1) with an AR(1) process for the evolution of the natural logarithm of firms' financial ratios.

[32] The aggregation properties of our model allow us to simulate a time series first and then obtain a cross-section.

iosyncratic signals and shocks to their distribution to simulate a cross section of 5,000 firms in each period. Finally, we obtain firms' simulated returns in each period according to Equation (23).

Due to the aggregation properties of the model (see Proposition (2)), the covenant threshold $\bar{\omega}_t^0$ is the same for all entrepreneurs at the beginning of each period t before the idiosyncratic shock $\omega_{i,t}^0$ is realized. As mentioned in the model section, to bring our model closer to the data, we assume that the exact realization of the idiosyncratic shock $\omega_{i,t}^0$ can only be perfectly observed by the entrepreneur and the lender, but not by investors. Specifically, if the entrepreneur is not in technical default, investors can only observe a noisy signal of $\omega_{i,t}^0$, given by

$$\hat{\eta}_{i,t} = \omega_{i,t}^0 + \eta_{i,t}, \tag{31}$$

in which η is an $i.i.d$ white noise with a mean of 0 and a standard deviation of 1. If the firm enters technical default, however, investors can perfectly observe the idiosyncratic shock realization $\omega_{i,t}^0$.[33] Under these assumptions, we define strictness in the model as

$$\text{Strictness}_{i,t}^M = \begin{cases} \frac{\exp\left(\bar{\omega}_{t+1}^0 - \hat{\eta}_{i,t+1}\right)}{\exp\left(\max_{j\in I}\left\{\bar{\omega}_{t+1}^0 - \hat{\eta}_{j,t+1}\right\}\right)} & \text{if } \omega_{i,t+1}^0 \geq \bar{\omega}_{t+1}^0, \\ 1 & \text{if } \omega_{i,t+1}^0 < \bar{\omega}_{t+1}^0, \end{cases} \tag{32}$$

and we use this measure to conduct cross-sectional portfolio sorting using simulated data.[34]

We present the results of our cross-sectional replication in Table 8. In the model, lender control rights are such that firms with high asset strictness invest in less-risky assets and therefore have a significantly lower average return than those with low strictness. Quantitatively, our control rights allocation mechanism allows us to produce a sizable technical default spread of around 3%, in line with the magnitude we observe in the data.

[33]This assumption is consistent with the fact that firms publicly disclose loan covenant violations in their SEC filings, for example.

[34]Using the functional form in (32), and in particular normalizing by the largest value of $\bar{\omega}_{t+1}^0 - \hat{\eta}_{i,t+1}$ in the set of firms I, we can confirm that strictness is monotonically increasing in $\bar{\omega}_{t+1}^0 - \hat{\eta}_{i,t+1}$, and that its value is less than or equal to 1 when $\omega_{i,t+1}^0 \geq \bar{\omega}_{t+1}^0$.

5 Conclusion

We build a dynamic general equilibrium model in which endogenous loan covenants allocate investment control rights between borrowers and creditors, and we study its quantitative implications for the macroeconomy and also for investment risk and asset prices in the cross-section. In our model, when borrowers' expected cash flows fall below the covenant threshold specified by the contract, lenders take control and optimally invest in low-risk projects to safeguard loan repayment, thus reducing the firm's cost of equity.

A calibrated version of our model can match the counter-cyclicality of aggregate technical default that we observe in the data, the aggregate moments of key macroeconomic and asset pricing variables, as well as the technical default spread that we observe in the cross-section of equity returns. Using this model, we confirm that firms that are closer to breaching a covenant earn on average 4% lower returns than firms whose covenants are less binding. We also show that, by allowing entrepreneurs to lower the cost of bank borrowing in bad times, the flexible allocation of control rights over the business cycle has significant mitigation effects on the financial accelerator. Overall, our results show that lender control rights are a quantitatively important determinant of a firm's investment, risk-taking incentives, and cost of capital, both in the aggregate and in the cross-section.

Figure 1

Impulse Response Functions

This figure plots the percentage deviation from the steady state of the key variables in our baseline model with negatively correlated productivity shocks and uncertainty shocks. The parameters used in the calibration are listed in Table 1.

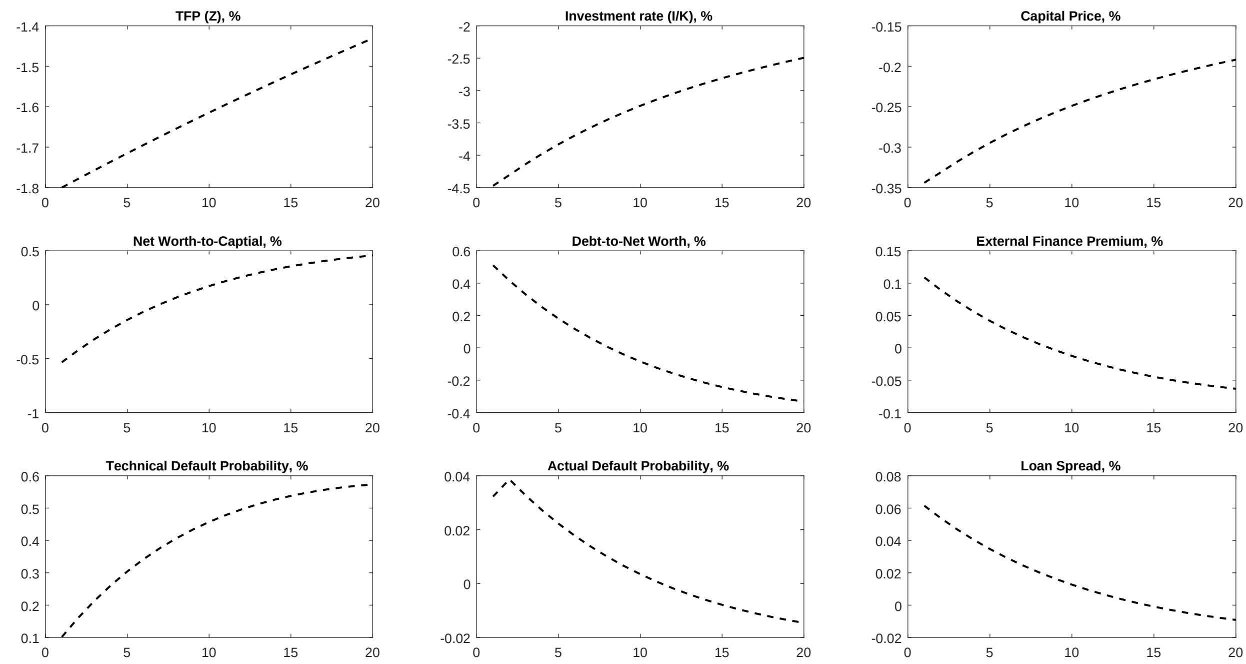

Figure 2

Lender Control Rights and the Mitigation of the Financial Accelerator

This figure plots the percentage deviation from the steady state of the key variables in the calibration from Section 3.5 with financial shocks and uncertainty shocks. The parameters used are listed in Table 1 and in Section 3.5. The dashed lines refer to the case with flexible covenant terms while the solid lines refer to the case with fixed covenant terms.

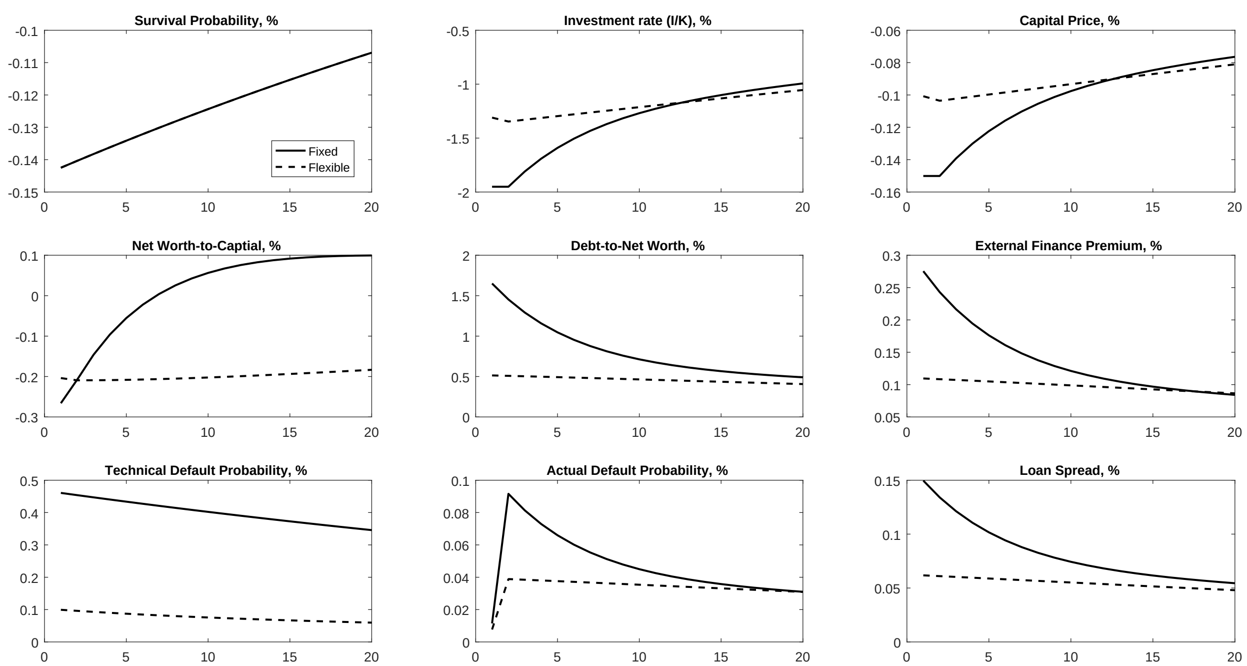

Figure 3

Covenant Strictness: Time Series

This figure provides a graphical illustration of the time series and business cycle properties of the average covenant strictness in our sample (equally-weighted aross all firms). Strictness is defined as in Section B.1. The sample starts with the first quarter of 1996 and ends with the last quarter of 2017.

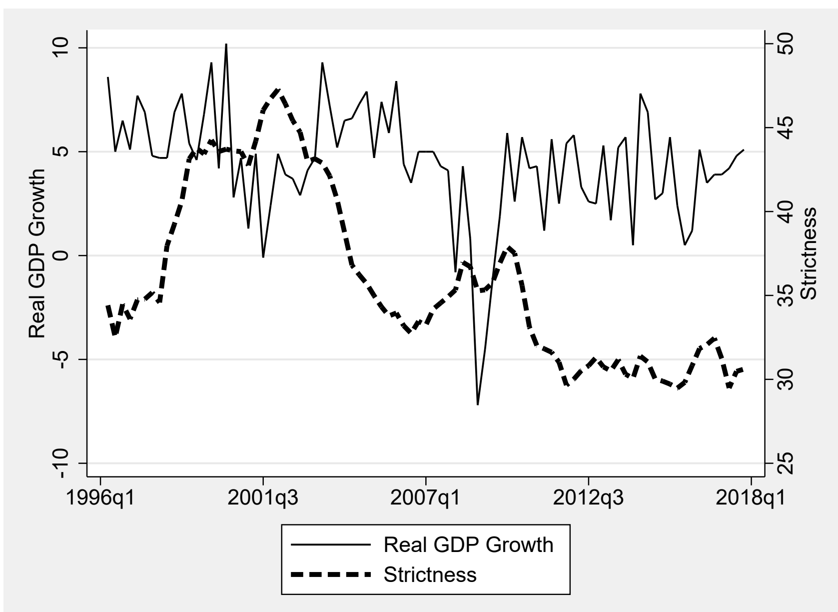

Figure 4

Covenant Violations and Stock Returns

This figure plots average annualized excess returns against a firm's distance from the Debt-to-EBITDA covenant threshold (the most common covenant in DealScan database). Observations to the right of the zero vertical line correspond to covenant violations. Each circle represents average excess returns within 28 equally-spaced distance bins around the threshold. The solid lines are fitted values from local polynomial regressions on either side of the threshold, and the dashed lines are 95% confidence intervals for these estimates (using bootstrapped standard errors). The sample starts with the first quarter of 1996 and ends with the last quarter of 2017.

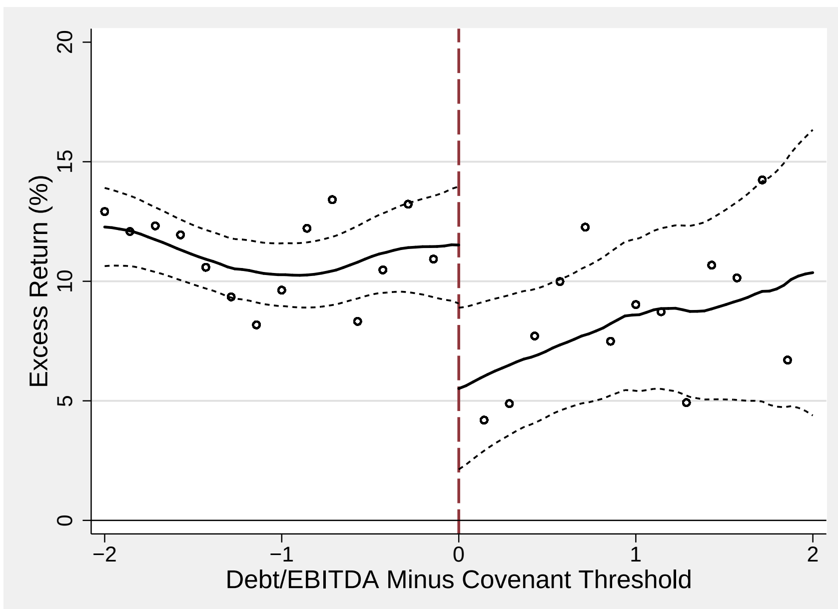

Table 1

Calibrated Parameter Values

This table lists the parameter values used to solve and simulate the model. We calibrate the model at the quarterly frequency using data moments of the U.S. economy from 1930 to 2016.

Parameters	Symbol	Value
Capital Share	α	0.33
Relative Risk Aversion	γ	10
IES	ψ	2
Time Discount Rate	β	0.997
Depreciation Rate	δ	0.025
Loss in Default	ζ	0.367
Transfer to Entering Entrepreneurs	χ	0.275
Entrepreneur Survival Probability	$\bar{\lambda}$	0.95
Steady State Standard Deviation of ω^0	$\bar{\sigma}_0$	0.012
Steady State Standard Deviation of ω^1	$\bar{\sigma}_1$	0.378
Steady State TFP Shock	$\bar{Z}$	0.155
Capital Adjustment Cost	ξ	13
Persistence of TFP Shock	ρ_z	0.988
Vol. of TFP Shock	σ_z	0.018
Persistence of σ Shock	ρ_s	0.985
Vol. of σ_0 Shock	σ_{s0}	0.025
Vol. of σ_1 Shock	σ_{s1}	0.0015
Persistence of Financial Shock	ρ_x	0.985
Vol. of Financial Shock	σ_x	0.015

Table 2

Key Aggregate Moments under the Benchmark Parametrization

This table reports a set of key moments generated under the benchmark parameters reported in Table 1. Empirical moments are computed using U.S. annual data from 1930 to 2016. Volatility, correlations, and first-order autocorrelations are denoted as $\sigma(\cdot)$, $corr(\cdot,\cdot)$, and $AC1(\cdot)$, respectively. The average annual loan spread is calculated using syndicated loan data in DealScan. The annual technical default probability come from **?**. The loan default rate data comes from Altman and Suggitt (2000). R^D is the risk-free rate, R^M is the market return (the return on entrepreneurs' aggregate net worth). $R^{B,E}$ is the loan rate charged by the bank when the entrepreneur is in control. For moments in the model, we first simulate the model at the quarterly frequency based on parameters calibrated in Table 1, and then time-aggregate the quarterly observations to the annual frequency.

Moments	Data (St. Dev.)	Model
$\sigma(\Delta y)$	3.05(0.60)	3.83
$\sigma(\Delta c)$	2.53(0.56)	3.12
$\sigma(\Delta i)$	10.3(2.36)	7.34
$corr(\Delta c, \Delta i)$	0.40(0.28)	0.81
$corr(\Delta c, \Delta y)$	0.82(0.07)	0.97
$AC1(\Delta c)$	0.56(0.17)	0.67
$AC1(\Delta i)$	0.18(0.09)	0.22
$AC1(\Delta y)$	0.49(0.15)	0.49
$E(I_t/Y_t)$	0.17(0.01)	0.21
Risk-Free Rate $E[R_D]$	1.1(0.16)	1.36
Volatility of Real Interest Rate $\sigma(R_D)$	0.97(0.31)	1.04
Market Return $E[R_M - R_D]$	5.71(2.25)	7.01
Loan Spread $E\left(R^{B,E} - R^f\right)$	2.6	2.13
Technical Default Probability	$24 \sim 30$	33.31
Actual Default Probability	3.59	5.56

Table 3

Summary Statistics, 1996-2016

This table presents summary statistics for the main variables in the paper over the period 1996-2016. The Whited-Wu (WW) Index and the the Size-and-Age (SA) Index are constructed as in Whited and Wu (2006) and Hadlock and Pierce (2010), respectively. Strictness is constructed as described as in Section B.1. Failure probability (Pr(Failure)) and expected default frequency are constructed following Campbell et al. (2008) and Bharath and Shumway (2008), respectively. B/M is the book-to-market ratio (the book value of a firm's equity (SEQQ) divided by the market value of the firm's outstanding shares). Investment rate (I/K) is investment (CAPXQ) over property, plant, and equipment (PPENTQ). Tangibility is the ratio of purchased capital (PPENTQ) to total assets (ATQ). Rating is a discrete score based on the firm's credit rating, Return on Equity (ROE) is net income divided by firm's book value of equity, and Return on Assets (ROA) is the ratio of operating income before depreciation (OIBDPQ) over total assets (ATQ). Book leverage is the sum of long-term liabilities (DLTTQ) and current liabilities (DLCQ) divided by total assets (ATQ). Size is the natural log of a firm's market capitalization. Leverage ratio is the sum of long-term liabilities (DLTTQ) and current liabilities (DLCQ) divided by stockholders' Equity (SEQQ). Reversal is the firm's one-month lagged return. Δ CAPX/Asset is the year-on-year difference in capital expenditures (CAPXQ) scaled by average assets over the same period, and Δ ACQU/Asset is the year-on-year difference in cash acquisitions (CHEQ) scaled by average assets over the same period; these two variables are constructed as in Nini et al. (2012). The WW Index and the SA Index are measured for each year. Size and Reversal are measured for each month. All the remaining variables are measured for each quarter.

	Mean	SD	p25	p50	p75	Observations
WW Index	-0.33	0.09	-0.40	-0.33	-0.26	23,874
SA Index	-3.73	0.64	-4.32	-3.66	-3.28	24,796
Strictness (pp)	35.09	36.52	1.12	20.09	67.68	83,025
Pr(Failure) (pp)	0.29	1.27	0.02	0.03	0.08	82,727
EDF (pp)	4.54	14.63	0.00	0.00	0.12	81,290
B/M	0.73	0.61	0.35	0.57	0.90	82,721
I/K	0.05	0.09	0.01	0.03	0.06	66,483
Tangibility	0.32	0.25	0.12	0.25	0.48	82,363
Rating	1.71	1.84	0.00	1.00	3.00	83,025
ROE (pp)	2.38	7.38	0.62	2.47	4.44	82,976
ROA (pp)	0.90	2.24	0.24	1.03	1.90	83,007
Leverage Ratio	1.28	2.28	0.32	0.67	1.28	82,553
Size	6.70	1.89	5.42	6.79	8.04	247,168
Reversal	0.99	13.04	-5.75	0.74	7.18	248,012
Δ CAPX/Asset	-0.10	1.34	-0.35	-0.01	0.26	77,964
Δ ACQU/Asset	-0.21	4.49	0.00	0.00	0.00	74,534

Table 4

Characteristics of Strictness-Based Portfolios

This table reports time-series averages of the cross-sectional averages of firm characteristics in five portfolios sorted by loan covenant strictness. The sample period starts in January 1996 and ends in December 2016, and the sample excludes financial and utility industries. All the variables are computed as in Table 3.

	Low	2	3	4	High
Strictness (pp)	0.13	5.10	23.36	56.41	92.75
Excess Return (pp)	6.76	8.40	6.90	10.36	2.64
Pr(Failure) (pp)	0.07	0.08	0.16	0.30	0.35
EDF (pp)	0.27	0.49	1.23	1.88	3.94
B/M	0.38	0.43	0.45	0.51	0.57
Size	10.19	10.07	9.86	9.63	9.74
I/K	0.05	0.05	0.04	0.04	0.05
ROE (pp)	6.17	4.96	4.80	3.70	1.77
Dividend Payout	0.80	0.72	0.64	0.53	0.48
Rating Score	4.06	3.64	3.34	2.97	2.87
Book Leverage	0.20	0.27	0.32	0.35	0.42
Leverage Ratio	0.85	0.98	1.32	1.69	2.20
SA Index	-4.23	-4.18	-4.10	-4.05	-4.04
WW Index	-0.46	-0.44	-0.42	-0.41	-0.41
Δ CAPX/Asset	-0.08	-0.03	-0.03	-0.03	-0.11
Δ ACQU/Asset	-0.18	-0.10	-0.19	-0.09	-0.34
Average Number of Firms	161.36	175.81	175.62	175.57	174.76

Table 5

Portfolios Sorted on Strictness

This table reports value weighted excess return and two investment conservatism measures for strictness-sorted portfolios. The sample starts in January 1996 and ends in December 2016, and it excludes financial and utility industries. In Panel A, we compute the annualized average monthly value-weighted excess returns for strictness-sorted portfolios, and their alphas and betas from the Fama and French (2015) five factor model. Portfolios are rebalanced at the end of each quarter. A firm's monthly returns are annualized and expressed in percentage terms. The t-statistics are estimated following Newey and West (1987). In Panel B, we repeat the same portfolio sorting procedures replacing excess returns and alphas with the Nini et al. (2012) investment conservatism measures Δ CAPX/Asset and Δ ACQU/Asset, as described in Table 3. The results of this table suggest non-monotonic negative relationships between strictness and excess returns and between strictness and investment conservatism, all driven by high-strictness firms.

Panel A: Excess Returns for Strictness-sorted Portfolios

	Low	2	3	4	High	High-4	High-Low	4-Low
Excess Return (pp)	6.76*	8.40**	6.90*	10.36**	2.64	-7.72**	-4.12	3.60*
t-stat.	1.90	2.27	1.83	2.59	0.49	-2.32	-1.52	1.88
α^{FF5}	-2.76*	-2.03	-3.06	-0.79	-6.56***	-5.77*	-3.80	1.97
t-stat.	-1.84	-1.12	-1.45	-0.42	-2.68	-1.97	-1.64	1.19
β^{MKT}	1.06***	1.03***	1.08***	1.09***	1.18***	0.10	0.12*	0.02
t-stat.	30.64	27.18	29.53	21.09	24.88	1.58	1.88	0.56
β^{SMB}	0.09	0.24***	0.19***	0.30***	0.37***	0.07	0.28***	0.21***
t-stat.	1.70	3.55	2.75	4.31	6.37	0.85	3.09	3.53
β^{HML}	0.05	0.02	0.12*	0.17	0.21**	0.04	0.17**	0.13
t-stat.	0.58	0.18	1.69	1.31	2.18	0.37	2.03	1.41
β^{RMW}	0.29***	0.45***	0.32***	0.39***	-0.09	-0.48***	-0.37***	0.10
t-stat.	4.86	4.65	4.42	4.74	-0.68	-3.82	-2.75	1.38
β^{CMA}	0.06	0.13	-0.02	0.13	-0.25	-0.37***	-0.30*	0.07
t-stat.	0.80	1.28	-0.17	1.23	-1.57	-2.64	-1.87	0.76

Panel B: Investment Conservatism Measures for Strictness-sorted Portfolios

	Low	2	3	4	High	High-4	High-Low	4-Low
Δ CAPX/Asset	-0.08*	-0.03	-0.03	-0.03	-0.11*	-0.08*	-0.03	0.04*
t-stat.	-1.78	-1.10	-1.00	-0.77	-1.80	-1.94	-0.66	1.66
Δ ACQU/Asset	-0.18***	-0.10	-0.19***	-0.09	-0.34***	-0.25**	-0.17	0.08
t-stat.	-2.99	-1.09	-2.96	-1.12	-3.15	-2.36	-1.58	0.97

Table 6

Fama-MacBeth Regressions on Strictness

This table presents the results of our Fama-MacBeth analysis of the link between excess returns and strictness. The table reports average coefficients of monthly cross-sectional regressions of monthly excess returns on lagged strictness and other control variables (as in Fama and French (1992), we allow for a six-month lag between independent variables and excess returns). Column (1) is our baseline specification. Columns (2)-(4) augment this baseline specification to control for failure probability and EDF. All variables are computed as in Table 3. The sample starts in January 1996 and ends in December 2016.

	Dependent Variable: Monthly Excess Returns			
	(1)	(2)	(3)	(4)
Strictness	-0.357***	-0.327***	-0.364***	-0.330***
	(0.12)	(0.12)	(0.12)	(0.12)
Size	-0.088*	-0.100**	-0.066	-0.079*
	(0.05)	(0.05)	(0.05)	(0.05)
Log B/M	0.141	0.136	0.081	0.075
	(0.13)	(0.12)	(0.12)	(0.11)
Reversal	-0.016**	-0.016**	-0.016**	-0.017**
	(0.01)	(0.01)	(0.01)	(0.01)
Book Leverage	-0.112	-0.081	-0.415	-0.407
	(0.48)	(0.47)	(0.46)	(0.46)
ROA	5.082	3.217	5.139	3.376
	(3.87)	(3.53)	(3.65)	(3.42)
Pr(Failure)		-80.929**		-91.997***
		(31.79)		(28.05)
EDF			0.192	2.272
			(2.48)	(2.41)
R-Squared	0.041	0.047	0.049	0.054
Observations	219,331	218,952	214,750	214,699

Note: Newey-West standard errors in parentheses. ***, **, and * respectively denote statistical significance at the 1%, 5%, and 10% levels.

Table 7

RDD Regressions: Covenant Violations and Future Returns

This table presents estimates of the RDD specification (30) using maximum Debt-to-EBITDA violations. Violation_{it} is a dummy variable equal to 1 if firm i's' Debt-to-EBITDA ratio exceeds its most restrictive covenant threshold in quarter t. Distance_{it} is the difference between firm i's actual Debt-to-EBITDA ratio and its most restrictive covenant value. The sample starts in January 1996 and ends in December 2016.

	Dependent Variable: Excess Returns		
	(1)	(2)	(3)
Violation	-0.443***	-0.309***	-0.272*
	(0.11)	(0.11)	(0.15)
Distance	0.134***	0.109***	-0.017
	(0.03)	(0.03)	(0.07)
Violation × Distance	-0.224***	-0.180***	0.083
	(0.04)	(0.04)	(0.13)
Size		-0.021	-0.024
		(0.02)	(0.02)
Log B/M		0.068	0.076
		(0.05)	(0.05)
Book Leverage		-0.475**	-0.309
		(0.21)	(0.23)
ROA		4.407***	3.981**
		(1.60)	(1.62)
High Order Polynomials	No	No	Yes
Year-Quarter FE	Yes	Yes	Yes
R-Squared	0.214	0.220	0.220
Observations	67,591	64,451	64,451

Note: Standard errors (in parentheses) are clustered at the firm level. ***, **, and * respectively denote statistical significance at the 1%, 5%, and 10% levels.

Table 8

Cross-Section Firm Characteristics and Expected Return

This table shows model simulated moments and their counterparts for portfolios sorted on strictness measure. The sample period is from January 1996 to December 2016. Panel A reports the statistics computed in the data. Panel B reports the statistic computed from the simulated data.

Panel A: Data

Firm Characteristics	Low	2	3	4	High
Strictness (pp)	0.13	5.10	23.36	56.41	92.75
Excess Return (pp)	6.76	8.40	6.90	10.36	2.64
Pr(Failure) (pp)	0.08	0.08	0.20	0.33	0.36
Size	10.19	10.07	9.86	9.63	9.74

Panel B: Model

Firm Characteristics	Low	2	3	4	High
Excess Return (pp)	7.58	7.56	7.52	7.51	4.55

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Online Appendix for "The Technical Default Spread"

A Appendix to Model Section

A.1 The Capital Goods Producer

The capital goods producer uses I_t amount of consumption goods to produce new capital using the technology

$$\Phi\left(\frac{I_t}{K_t}\right) K_t. \tag{A.1}$$

Following Jermann (1998), we let

$$\Phi\left(\frac{I_t}{K_t}\right) = \left[\frac{a_1}{1-\frac{1}{\xi}}\left(\frac{I_t}{K_t}\right)^{1-\frac{1}{\xi}} + a_2\right], \tag{A.2}$$

in which a_1 and a_2 are set such that there are no adjustment costs in our model's deterministic steady state, and ξ represents the elasticity of new capital investments relative to the existing stock of capital. The law of motion of aggregate capital is therefore

$$K_{t+1} = \Phi\left(\frac{I_t}{K_t}\right) K_t + (1-\delta)K_t, \tag{A.3}$$

and taking the capital price Q_t as given, the capital producer's optimal choice of investment I_t implies

$$Q_t = \left[\Phi'\left(\frac{I_t}{K_t}\right)\right]^{-1}. \tag{A.4}$$

A.2 Proof of Proposition 1

We start by showing that when a lender is in control, $\theta^L = 0$. Applying Leibniz's rule to (16) and taking the derivative of (16) with respect to $\theta^L_{i,t}$,

$$\begin{aligned}
\frac{\partial W^L\left(\theta^L_{i,t}\right)}{\partial \theta^L_{i,t}} &= -\mathbb{E}_t\left\{M_{t+1}R^{B,L}_{i,t+1}B_{i,t}\frac{\partial \hat{\omega}^1_{i,t+1}}{\partial \theta^L_{i,t}}f_1(\hat{\omega}^1_{i,t+1})\right\} \\
&+\mathbb{E}_t\left\{M_{t+1}(1-\zeta)\left[\theta^L_{i,t}\exp\left(\omega^0_{i,t+1}+\hat{\omega}^1_{i,t+1}\right)R^K_{t+1}+\left(1-\theta^L_{i,t}\right)R^D_{t+1}\right]A_{i,t}f_1(\hat{\omega}^1_{i,t+1})\frac{\partial \hat{\omega}^1_{i,t+1}}{\partial \theta^L_{i,t}}\right\} \\
&+\mathbb{E}_t\left\{M_{t+1}\int_{-\infty}^{\hat{\omega}^1_{i,t+1}}\left\{(1-\zeta)\left[\exp\left(\omega^0_{i,t+1}+\omega^1_{i,t+1}\right)R^K_{t+1}-R^D_{t+1}\right]A_{i,t}\right\}dF\left(\omega^1_{i,t+1}\right)\right\}, \quad \text{(A.5)} \\
&= -\mathbb{E}_t\left\{M_{t+1}\zeta R^{B,L}_{i,t+1}B_{i,t}f_1(\hat{\omega}^1_{i,t+1})\frac{\partial \hat{\omega}^1_{i,t+1}}{\partial \theta^L_{i,t}}\right\} \\
&+\mathbb{E}_t\left\{M_{t+1}\int_{-\infty}^{\hat{\omega}^1_{i,t+1}}\left\{(1-\zeta)\left[\exp\left(\omega^0_{i,t+1}+\omega^1_{i,t+1}\right)R^K_{t+1}-R^D_{t+1}\right]A_{i,t}\right\}dF\left(\omega^1_{i,t+1}\right)\right\}, \quad \text{(A.6)}
\end{aligned}$$

in which the second equality follows from Equation (13), and also in which $f_1(\cdot)$ is the probability density function of ω^1. From Equation (13), we have

$$\frac{\partial \hat{\omega}^1_{i,t+1}}{\partial \theta^L_{i,t}} = \frac{R^D_{t+1}A_{i,t}-R^{B,L}_{i,t+1}B_{i,t}}{\theta^L_{i,t}\left(R^{B,L}_{i,t+1}B_{i,t}-\left(1-\theta^L_{i,t}\right)R^D_{t+1}A_{i,t}\right)}. \tag{A.7}$$

Plugging (A.7) into (A.6), and noting that, for $\omega^1_{i,t+1} < \hat{\omega}^1_{i,t+1}$,

$$\begin{aligned}
\left[\exp\left(\omega^0_{i,t+1}+\omega^1_{i,t+1}\right)R^K_{t+1}-R^D_{t+1}\right]A_{i,t} &< \left[\exp\left(\omega^0_{i,t+1}+\hat{\omega}^1_{i,t+1}\right)R^K_{t+1}-R^D_{t+1}\right]A_{i,t}, \quad \text{(A.8)} \\
&= \frac{R^{B,L}_{i,t+1}B_{i,t}-R^D_{t+1}A_{i,t}}{\theta^L_{i,t}}, \quad \text{(A.9)}
\end{aligned}$$

we obtain

$$\frac{\partial W^L\left(\theta^L_{i,t}\right)}{\partial \theta^L_{i,t}} \leq -\mathbb{E}_t\left\{M_{t+1}\zeta R^{B,L}_{i,t+1}B_{i,t}f(\hat{\omega}^1_{i,t+1})\frac{\partial\hat{\omega}^1_{i,t+1}}{\partial\theta^L_{i,t}}\right\}+(1-\zeta)\mathbb{E}_t\left\{M_{t+1}\frac{R^{B,L}_{i,t+1}B_{i,t}-R^D_{t+1}A_{i,t}}{\theta^L_{i,t}}F\left(\hat{\omega}^1_{i,t+1}\right)\right\}, \quad (A.10)$$

$$= \frac{R^{B,L}_{i,t+1}B_{i,t}-R^D_{t+1}A_{i,t}}{\theta^L_{i,t}}\mathbb{E}_t\left\{M_{t+1}\left[\underbrace{\frac{\zeta R^{B,L}_{i,t+1}B_{i,t}f(\hat{\omega}^1_{i,t+1})}{\left(R^{B,L}_{i,t+1}B_{i,t}-(1-\theta^L_{i,t})R^D_{t+1}A_{i,t}\right)}}_{>0}+\underbrace{(1-\zeta)F\left(\hat{\omega}^1_{i,t+1}\right)}_{>0}\right]\right\}. \quad (A.11)$$

Given our maintained assumption that in all contracts $R^B_{i,t+1}B_{i,t} < R^D_{i,t+1}A_{i,t}$, and given the positive correlation between M_{t+1} and the default cutoff value $\hat{\omega}^1_{i,t+1}$ (both M_{t+1} and $\hat{\omega}^1_{i,t+1}$ are negatively correlated to the aggregate states), it follows that $\partial W^L\left(\theta^L_{i,t}\right)/\partial\theta^L_{i,t} < 0$, and therefore $\theta^L = 0$. Since the lender invests all of the firm's resources in the risk-free asset, it follows that $R^{B,L}_{i,t+1} = R^D_{t+1}$.

Next, we show that when the entrepreneur is in control, then $\theta^E = 1$. In contrast to the case in which the lender is in control, the sign of

$$\frac{\partial V^E\left(\theta^E_{i,t}\right)}{\partial\theta^E_{i,t}} = \mathbb{E}_t\left\{M_{t+1}\int_{\hat{\omega}^1_{i,t+1}}^{\infty}\left\{\left[\exp\left(\omega^0_{i,t+1}+\omega^1_{i,t+1}\right)R^K_{t+1}-R^D_{t+1}\right]A_{i,t}\right\}dF\left(\omega^1_{i,t+1}\right)\right\} \quad (A.12)$$

is ambiguous. This ambiguity arises because increasing $\theta^E_{i,t}$ leads to higher expected payoffs on the upside but it also increases the default cutoff value and the entrepreneur's default probability.[A.1] However, since

$$\frac{\partial^2 V^E\left(\theta^E_{i,t}\right)}{\partial\theta^{E,2}_{i,t}} = \mathbb{E}_t\left\{M_{t+1}\frac{\left(R^{B,E}_{i,t+1}B_{i,t}-R^D_{t+1}A_{i,t}\right)^2}{\left(\theta^E_{i,t}\right)^2\left(R^{B,E}_{i,t+1}B_{i,t}-(1-\theta^E_{i,t})R^D_{t+1}A_{i,t}\right)}f_1(\hat{\omega}^1_{i,t+1})\right\}\geq 0, \quad (A.13)$$

$V^E\left(\theta^E_{i,t}\right)$ is convex in $\theta^E_{i,t}$, implying corner solutions for θ^E. In Lemma A.1, we prove the important intermediate result that there exists a *unique* $\tilde{\omega}^0$ such that for all realizations of ω^0 above $\tilde{\omega}^0$, the entrepreneur chooses $\theta^E = 1$, and for all realizations of ω^0 below $\tilde{\omega}^0$, the entrepreneur chooses $\theta^E = 0$.

Lemma A.1. *There exists an unique $\tilde{\omega}^0_{i,t+1}$ such that for any $\omega^0_{i,t+1} \geq \tilde{\omega}^0_{i,t+1}$, $\theta^E_{i,t} = 1$, and for any $\omega^0_{i,t+1} < \tilde{\omega}^0_{i,t+1}$, $\theta^E_{i,t} = 0$.*

[A.1] This is clear from (A.7). When $R^{B,E}_{i,t+1}B_{i,t}/< R^D_{t+1}A_{i,t}$, then $\partial\hat{\omega}^1_{i,t+1}/\partial\theta_{i,t} > 0$.

Proof. Fix $R^B_{i,t+1}$ and $B_{i,t}$. If $\theta^E_{i,t} = 0$, since $A_{i,t}R^D_{t+1} > R^{B,E}_{i,t+1}B_{i,t}$, the default cutoff $\hat{\omega}^1_{i,t+1}$ in (13) is not defined, and the value of the entrepreneur can be written as

$$V^E\left(\theta^E_{i,t} = 0\right) = \mathbb{E}_t\left\{M_{t+1}\left(R^D_{t+1}A_{i,t} - R^{B,E}_{i,t+1}B_{i,t}\right)\right\}, \tag{A.14}$$

which is independent on the realization of $\omega^0_{i,t+1}$. On the other hand, if $\theta^E_{i,t} = 1$, the value of entrepreneur can be written as

$$V^E\left(\theta^E_{i,t} = 1\right) = \mathbb{E}_t\left\{M_{t+1}\int_{\hat{\omega}^1_{i,t+1}}^{\infty}\left\{\exp\left(\omega_{i,t+1}\right)R^K_{t+1}A_{i,t} - R^{B,E}_{i,t+1}B_{i,t}\right\}dF\left(\omega^1_{i,t+1}\right)\right\}. \tag{A.15}$$

Taking the derivative of (A.15) with respect to $\omega^0_{i,t+1}$ gives

$$\frac{\partial V^E\left(\theta^E_{i,t} = 1\right)}{\partial \omega^0_{i,t+1}} = \mathbb{E}_t\left\{M_{t+1}\int_{\hat{\omega}^1_{i,t+1}}^{\infty}\exp\left(\omega_{i,t+1}\right)R^K_{t+1}A_{i,t}dF\left(\omega^1_{i,t+1}\right)\right\} > 0, \tag{A.16}$$

which means that the value of the entrepreneur in control is monotonically increasing in the realization of $\omega^0_{i,t+1}$. Moreover, consider the limiting case $\omega^0_{i,t+1} \to -\infty$. From (A.7), we have

$$\lim_{\omega^0_{i,t+1}\to-\infty}\hat{\omega}^1_{i,t+1} = \lim_{\omega^0_{i,t+1}\to-\infty}\ln\left(\frac{R^{B,E}_{i,t+1}B_{i,t}}{\exp\left(\omega^0_{i,t+1}\right)R^K_{t+1}A_{i,t}}\right) \to \infty. \tag{A.17}$$

Therefore, $V^E\left(\theta^E_{i,t} = 1\right) \to 0$ when $\omega^0_{i,t+1} \to -\infty$. In the the limiting case in which $\omega^0_{i,t+1} \to \infty$,

$$\lim_{\omega^0_{i,t+1}\to\infty}\hat{\omega}^1_{i,t+1} = \lim_{\omega^0_{i,t+1}\to\infty}\ln\left(\frac{R^{B,E}_{i,t+1}B_{i,t}}{\exp\left(\omega^0_{i,t+1}\right)R^K_{t+1}A_{i,t}}\right) \to -\infty, \tag{A.18}$$

and $V^E\left(\theta^E_{i,t} = 1\right) \to \infty$. Since $V^E\left(\theta^E_{i,t} = 1\right)$ is monotonically increasing from 0 to ∞ within the domain of $\omega^0_{i,t+1}$ while $V^E\left(\theta^E_{i,t} = 0\right)$ is strictly positive and independent of $\omega^0_{i,t+1}$, there exists a unique crossing point $\tilde{\omega}^0_{i,t+1}$, such that $V^E\left(\theta^E_{i,t} = 1\right) = V^E\left(\theta^E_{i,t} = 0\right)$. For any $\omega^0_{i,t+1} > \tilde{\omega}^0_{i,t+1}$, $V^E\left(\theta^E_{i,t} = 1\right) > V^E\left(\theta^E_{i,t} = 0\right)$, and it is optimal for the entrepreneur to choose $\theta^E_{i,t} = 1$. For any $\omega^0_{i,t+1} < \tilde{\omega}^0_{i,t+1}$, $V^E\left(\theta^E_{i,t} = 1\right) < V^E\left(\theta^E_{i,t} = 0\right)$, and it is the optimal for the entrepreneur to choose $\theta^E_{i,t} = 0$. □

Using this intermediate result, we can show that the entrepreneur in control *always* chooses $\theta^E_{i,t} = 1$. We do so by showing that the optimal contract $\Theta \equiv \{R^B, B, \bar{\omega}^0\}$ has to be such that $\bar{\omega}^0_{i,t+1} \geq \tilde{\omega}^0_{i,t+1}(\Theta)$. We proceed by contradiction. Suppose that the optimal contract Θ is such that $\bar{\omega}^0_{i,t+1} < \tilde{\omega}^0_{i,t+1}(\Theta)$. By Lemma A.1, for any realization of $\omega^0_{i,t+1}$ such that $\bar{\omega}^0_{i,t+1} < \omega^0_{i,t+1} < \tilde{\omega}^0_{i,t+1}(\Theta)$, the entrepreneur chooses $\theta^E_{i,t} = 0$ and obtains payoff $V^E\left(\theta^E_{i,t} = 0\right) = \mathbb{E}_t\left\{M_{t+1}\left(R^D_{t+1}A_{i,t} - R^{B,E}_{i,t+1}B_{i,t}\right)\right\}$. However, if the entrepreneur were to give control rights to the lender, he/she would get $V^L\left(\theta^L_{i,t} = 0\right) = \mathbb{E}_t\left\{M_{t+1}\left(R^D_{t+1}A_{i,t} - R^D_{t+1}B_{i,t}\right)\right\}$. Since $R^{B,E}_{i,t+1}$ carries compensation for credit risk, it must be that $R^{B,E}_{i,t+1} > R^D_{t+1}$, by which $V^E\left(\theta^E_{i,t} = 0\right) < V^L\left(\theta^L_{i,t} = 0\right)$, a contradiction with the optimality of Θ.

A.3 Proof of Proposition 2

To prove our statement, we show that our maximization problem can be expressed in terms of the entrepreneur's debt-to-net worth ratio $H_{i,t} = B_{i,t}/N_{i,t}$. Since $N_{i,t}$ is a state variable, the contract terms that maximize (18)-(19) also solve

$$\max_{R^B_{i,t+1}, B_{i,t}, \bar{\omega}^0_{i,t+1}} \frac{V\left(N_{i,t}, R^B_{i,t+1}, B_{i,t}, \bar{\omega}^0_{i,t+1}\right)}{N_{i,t}}, \tag{A.19}$$

subject to

$$\frac{W\left(N_{i,t}, R^B_{i,t+1}, B_{i,t}, \bar{\omega}^0_{i,t+1}\right)}{N_{i,t}} = \frac{B_{i,t}}{N_{i,t}}. \tag{A.20}$$

Next, we express the default cutoff $\hat{\omega}^1_{i,t+1}$ and the value functions of the entrepreneur and the lender in terms of $H_{i,t}$. Starting from $\hat{\omega}^1_{i,t+1}$, we note that

$$\begin{aligned}
\hat{\omega}^1_{i,t+1} &= \ln\left\{\frac{R^B_{i,t+1}B_{i,t} - (1-\theta_{i,t})\,R^D_{t+1}A_{i,t}}{\theta_{i,t}\exp\left(\omega^0_{i,t+1}\right)R^K_{t+1}A_{i,t}}\right\}, \\
&= \ln\left\{\frac{R^B_{i,t+1}H_{i,t} - (1-\theta_{i,t})\,R^D_{t+1}\,(1+H_{i,t})}{\theta_{i,t}\exp\left(\omega^0_{i,t+1}\right)R^K_{t+1}\,(1+H_{i,t})}\right\}.
\end{aligned} \tag{A.21}$$

Moreover, we define $\bar{V}\left(R^B_{i,t+1}, H_{i,t}, \bar{\omega}^0_{i,t+1}\right) \equiv V\left(N_{i,t}, R^B_{i,t+1}, B_{i,t}, \bar{\omega}^0_{i,t+1}\right)/N_{i,t}$, and note that

$$\begin{aligned}
\bar{V}\left(R^L_{i,t+1}, H_{i,t}, \bar{\omega}^0_{i,t}\right) &= \int_{\bar{\omega}^0_{i,t+1}}^{\infty} \frac{V^E\left(N_{i,t}, R^B_{i,t+1}, B_{i,t}, \bar{\omega}^0_{i,t+1}, \omega^0_{i,t+1}\right)}{N_{i,t}} dF\left(\omega^0_{i,t+1}\right) \\
&\quad + \int_{-\infty}^{\bar{\omega}^0_{i,t+1}} \frac{V^L\left(N_{i,t}, R^B_{i,t+1}, B_{i,t}, \bar{\omega}^0_{i,t+1}, \omega^0_{i,t+1}\right)}{N_{i,t}} dF\left(\omega^0_{i,t+1}\right), \qquad \text{(A.22)} \\
&= \int_{\bar{\omega}^0_{i,t+1}}^{\infty} \bar{V}^E\left(R^B_{i,t+1}, H_{i,t}, \bar{\omega}^0_{i,t+1}, \omega^0_{i,t+1}\right) dF\left(\omega^0_{i,t+1}\right) \\
&\quad + \int_{-\infty}^{\bar{\omega}^0_{i,t+1}} \bar{V}^L\left(R^B_{i,t+1}, H_{i,t}, \bar{\omega}^0_{i,t+1}, \omega^0_{i,t+1}\right) dF\left(\omega^0_{i,t+1}\right), \qquad \text{(A.23)}
\end{aligned}$$

in which (suppressing functional dependencies)

$$\begin{aligned}
\bar{V}^E &= \frac{V^E\left(N_{i,t}, R^B_{i,t+1}, B_{i,t}, \bar{\omega}^0_{i,t+1}, \omega^0_{i,t+1}\right)}{N_{i,t}}, \qquad \text{(A.24)} \\
&= \mathbb{E}_t\left\{\int_{\hat{\omega}^1_{i,t+1}}^{\infty} \left\{\left[\theta^E_{i,t}\exp\left(\omega_{i,t+1}\right)R^K_{t+1} + \left(1-\theta^E_{i,t}\right)R^D_{t+1}\right]\frac{A_{i,t}}{N_{i,t}} - R^{B,E}_{i,t+1}\frac{B_{i,t}}{N_{i,t}}\right\} dF\left(\omega^1_{i,t+1}\right)\right\}, \qquad \text{(A.25)} \\
&= \mathbb{E}_t\left\{\int_{\hat{\omega}^1_{i,t+1}}^{\infty} \left\{\left[\theta^E_{i,t}\exp\left(\omega_{i,t+1}\right)R^K_{t+1} + \left(1-\theta^E_{i,t}\right)R^D_{t+1}\right]\left(1+H_{i,t}\right) - R^{B,E}_{i,t+1}H_{i,t}\right\} dF\left(\omega^1_{i,t+1}\right)\right\}, \qquad \text{(A.26)}
\end{aligned}$$

and

$$\begin{aligned}
\bar{V}^L &= \frac{V^L\left(N_{i,t}, R^B_{i,t+1}, B_{i,t}, \bar{\omega}^0_{i,t+1}, \omega^0_{i,t+1}\right)}{N_{i,t}}, \qquad \text{(A.27)} \\
&= \mathbb{E}_t\left\{\int_{\hat{\omega}^1_{i,t+1}}^{\infty} \left\{\left[\theta^L_{i,t}\exp\left(\omega_{i,t+1}\right)R^K_{t+1} + \left(1-\theta^L_{i,t}\right)R^D_{t+1}\right]\frac{A_{i,t}}{N_{i,t}} - R^{B,L}_{i,t+1}\frac{B_{i,t}}{N_{i,t}}\right\} dF\left(\omega^1_{i,t+1}\right)\right\}, \qquad \text{(A.28)} \\
&= \mathbb{E}_t\left\{\int_{\hat{\omega}^1_{i,t+1}}^{\infty} \left\{\left[\theta^L_{i,t}\exp\left(\omega_{i,t+1}\right)R^K_{t+1} + \left(1-\theta^L_{i,t}\right)R^D_{t+1}\right]\left(1+H_{i,t}\right) - R^{B,L}_{i,t+1}H_{i,t}\right\} dF\left(\omega^1_{i,t+1}\right)\right\}. \qquad \text{(A.29)}
\end{aligned}$$

Similarly, we let $\bar{W}\left(R^B_{i,t+1}, H_{i,t}, \bar{\omega}^0_{i,t+1}\right) \equiv W\left(N_{i,t}, R^B_{i,t+1}, B_{i,t}, \bar{\omega}^0_{i,t+1}\right)/N_{i,t}$. Then,

$$\begin{aligned}
\bar{W}\left(R^B_{i,t+1}, H_{i,t}, \bar{\omega}^0_{i,t+1}\right) &= \int_{\bar{\omega}^0_{i,t+1}}^{\infty} \bar{W}^E\left(R^B_{i,t+1}, H_{i,t}, \bar{\omega}^0_{i,t}, \omega^0_{i,t+1}\right) dF\left(\omega^0_{i,t+1}\right) \\
&\quad + \int_{-\infty}^{\bar{\omega}^0_{i,t}} \bar{W}^L\left(R^B_{i,t+1}, H_{i,t}, \bar{\omega}^0_{i,t+1}, \omega^0_{i,t+1}\right) dF\left(\omega^0_{i,t+1}\right), \qquad \text{(A.30)}
\end{aligned}$$

in which

$$
\begin{aligned}
\bar{W}^E &= \frac{W^E\left(N_{i,t}, R^B_{i,t+1}, B_{i,t}, \bar{\omega}^0_{i,t+1}, \omega^0_{i,t+1}\right)}{N_{i,t}}, && \text{(A.31)}\\
&= \mathbb{E}_t\left\{M_{t+1}\int_{\hat{\omega}^1_{i,t+1}}^{\infty} R^{B,E}_{i,t+1}\frac{B_{i,t}}{N_{i,t}}dF\left(\omega^1_{i,t+1}\right)\right\}\\
&\quad +\mathbb{E}_t\left\{M_{t+1}\int_{-\infty}^{\hat{\omega}^1_{i,t+1}}(1-\zeta)\left[\theta^E_{i,t}\exp\left(\omega_{i,t+1}\right)R^K_{t+1}+\left(1-\theta^E_{i,t}\right)R^D_{t+1}\right]\frac{A_{i,t}}{N_{i,t}}dF\left(\omega^1_{i,t+1}\right)\right\}, && \text{(A.32)}\\
&= \mathbb{E}_t\left\{M_{t+1}\int_{\hat{\omega}^1_{i,t+1}}^{\infty} R^{B,E}_{i,t+1}H_{i,t}dF\left(\omega^1_{i,t+1}\right)\right\}\\
&\quad +\mathbb{E}_t\left\{M_{t+1}\int_{-\infty}^{\hat{\omega}^1_{i,t+1}}(1-\zeta)\left[\theta^E_{i,t}\exp\left(\omega_{i,t+1}\right)R^K_{t+1}+\left(1-\theta^E_{i,t}\right)R^D_{t+1}\right](1+H_{i,t})dF\left(\omega^1_{i,t+1}\right)\right\}, && \text{(A.33)}
\end{aligned}
$$

and

$$
\begin{aligned}
\bar{W}^L &= \frac{W^L\left(N_{i,t}, R^B_{i,t+1}, B_{i,t}, \bar{\omega}^0_{i,t+1}, \omega^0_{i,t+1}\right)}{N_{i,t}}, && \text{(A.34)}\\
&= \mathbb{E}_t\left\{M_{t+1}\int_{\hat{\omega}^1_{i,t+1}}^{\infty} R^{B,L}_{i,t+1}\frac{B_{i,t}}{N_{i,t}}dF\left(\omega^1_{i,t+1}\right)\right\}\\
&\quad +\mathbb{E}_t\left\{M_{t+1}\int_{-\infty}^{\hat{\omega}^1_{i,t+1}}(1-\zeta)\left[\theta^L_{i,t}\exp\left(\omega_{i,t+1}\right)R^K_{t+1}+\left(1-\theta^L_{i,t}\right)R^D_{t+1}\right]\frac{A_{i,t}}{N_{i,t}}dF\left(\omega^1_{i,t+1}\right)\right\}, && \text{(A.35)}\\
&= \mathbb{E}_t\left\{M_{t+1}\int_{\hat{\omega}^1_{i,t+1}}^{\infty} R^{B,L}_{i,t+1}H_{i,t}dF\left(\omega^1_{i,t+1}\right)\right\}\\
&\quad +\mathbb{E}_t\left\{M_{t+1}\int_{-\infty}^{\hat{\omega}^1_{i,t+1}}(1-\zeta)\left[\theta^L_{i,t}\exp\left(\omega_{i,t+1}\right)R^K_{t+1}+\left(1-\theta^L_{i,t}\right)R^D_{t+1}\right](1+H_{i,t})dF\left(\omega^1_{i,t+1}\right)\right\} && \text{(A.36)}
\end{aligned}
$$

Finding the optimal contract terms now boils down to maximizing entrepreneur i's expected value by choosing $R^B_{i,t+1}, H_{i,t}, \bar{\omega}^0_{i,t+1}$

$$\max_{R^B_{i,t+1}, H_{i,t}, \bar{\omega}^0_{i,t+1}} \bar{V}\left(R^B_{i,t+1}, H_{i,t}, \bar{\omega}^0_{i,t}\right) \tag{A.37}$$

subject to the lender's ex-ante breakeven condition

$$\bar{W}\left(R^B_{i,t+1}, H_{i,t}, \bar{\omega}^0_{i,t}\right) = H_{i,t} \tag{A.38}$$

The problem (A.37)-(A.38) does not depend on any entrepreneur-specific state variable, implying that the solution to the problem is the same for all entrepreneurs.

A.4 Competitive Equilibrium

A competitive equilibrium is a set of quantities for the household $\left\{C_t, D_t^H, L_t\right\}_{t=0}^{\infty}$, quantities for entrepreneurs $\left\{N_{i,t}, K_{i,t}, B_{i,t}, D_{i,t}^E, \bar{\omega}_{i,t}^0\right\}_{t=0}^{\infty}$, quantities for the national bank $\{B_t\}_{t=0}^{\infty}$, quantities for the capital goods producer $\{I_t, K_t\}_{t=0}^{\infty}$ and prices $\left\{R_t^D, R_{i,t}^B, Q_t, R_t^K\right\}_{t=0}^{\infty}$, such that given prices, these quantities solve households', banks', capital goods producers' and entrepreneurs' maximization problems, firms maximize their profits, and the market clear. The market clearing conditions are

$$K_t = \int K_{i,t} di, \tag{A.39}$$

$$L_t = \int L_{i,t} di = 1, \tag{A.40}$$

$$B_t = H_t N_t = D_t^H + D_t^E, \tag{A.41}$$

and

$$Y_t = C_t + I_t + \zeta N_t \int_{\bar{\omega}_{t-1}^0}^{\infty} \int_{-\infty}^{\hat{\omega}_t^1} \exp\left(\omega_t^0 + \omega_t^1\right) R_t^K \left(1 + H_{t-1}\right) dF\left(\omega_t^1\right) dF\left(\omega_t^0\right). \tag{A.42}$$

A.5 Finding the Optimal Contract Terms

We use the results from Propositions 1 and 2 to find the optimal contract terms in our problem. These contract terms are solutions to the three first-order conditions (A.51), (A.54), and (A.60) below.

We start by using the results of Proposition 1 to show that the ex post value functions of the entrepreneur can be rewritten as

$$\begin{aligned} V^E &= \mathbb{E}_t \left\{ M_{t+1} \int_{\hat{\omega}_{i,t+1}^1}^{\infty} \left\{ \left[\exp\left(\omega_{i,t+1}^0 + \omega_{i,t+1}^1\right) R_{t+1}^K \right] A_{i,t} - R_{i,t+1}^{B,E} B_{i,t} \right\} dF\left(\omega_{i,t+1}^1\right) \right\}, && \text{(A.43)} \\ V^L &= \mathbb{E}_t \left[M_{t+1} \left(R_{t+1}^D A_{i,t} - R_{i,t+1}^{B,L} B_{i,t} \right) \right], && \text{(A.44)} \end{aligned}$$

and the ex-post value functions of lender can be rewritten as

$$
\begin{aligned}
W^E &= \mathbb{E}_t\left\{M_{t+1}\int_{\hat{\omega}^1_{i,t+1}}^{\infty} R^{B,E}_{i,t+1}B_{i,t}dF\left(\omega^1_{i,t+1}\right)\right\} \\
&+\mathbb{E}_t\left\{M_{t+1}\int_{-\infty}^{\hat{\omega}^1_{i,t+1}}\left[(1-\zeta)\left[\exp\left(\omega^0_{i,t+1}+\omega^1_{i,t+1}\right)R^K_{t+1}\right]A_{i,t}\right]dF\left(\omega^1_{i,t+1}\right)\right\}, \qquad \text{(A.45)} \\
W^L &= \mathbb{E}_t\left(M_{t+1}R^{B,L}_{i,t+1}B_{i,t}\right). \qquad \text{(A.46)}
\end{aligned}
$$

Using (A.43)-(A.46), we obtain the ex-ante value function of the entrepreneur as

$$
\begin{aligned}
V &= \mathbb{E}_t\left\{M_{t+1}\int_{\bar{\omega}^0_{i,t+1}}^{\infty}\int_{\hat{\omega}^1_{i,t+1}}^{\infty}\left\{\left[\exp\left(\omega_{i,t+1}\right)R^K_{t+1}\right]A_{i,t}-R^{B,E}_{i,t+1}B_{i,t}\right\}dF\left(\omega^1_{i,t+1}\right)dF\left(\omega^0_{i,t+1}\right)\right\} \\
&+\mathbb{E}_t\left\{M_{t+1}\int_{-\infty}^{\bar{\omega}^0_{i,t+1}}\left(R^D_{t+1}A_{i,t}-R^{B,L}_{i,t+1}B_{i,t}\right)dF\left(\omega^0_{i,t+1}\right)\right\}, \qquad \text{(A.47)}
\end{aligned}
$$

and the ex-ante value function of the lender as

$$
\begin{aligned}
W &= \mathbb{E}_t\left\{M_{t+1}\int_{\bar{\omega}^0_{i,t+1}}^{\infty}\int_{\hat{\omega}^1_{i,t+1}}^{\infty}R^{B,E}_{i,t+1}B_{i,t}dF\left(\omega^1_{i,t+1}\right)dF\left(\omega^0_{i,t+1}\right)\right\} \\
&+\mathbb{E}_t\left\{M_{t+1}\int_{\bar{\omega}^0_{i,t+1}}^{\infty}\int_{-\infty}^{\hat{\omega}^1_{i,t+1}}\left[(1-\zeta)\left[\exp\left(\omega_{i,t+1}\right)R^K_{t+1}\right]A_{i,t}\right]dF\left(\omega^1_{i,t+1}\right)dF\left(\omega^0_{i,t+1}\right)\right\} \\
&+\mathbb{E}_t\left\{M_{t+1}\int_{-\infty}^{\bar{\omega}^0_{i,t+1}}R^{B,L}_{i,t+1}B_{i,t}dF\left(\omega^0_{i,t+1}\right)\right\}. \qquad \text{(A.48)}
\end{aligned}
$$

Since $R^{B,L}_{i,t+1} = R^D_{t+1}$ by Proposition 1, the choice variables in this problem are $R^{B,E}_{i,t+1}, \bar{\omega}^0_{i,t+1}$ and $H_{i,t}$. As shown in Proposition 2, these contract terms are the same across all entrepreneurs, and we therefore drop the subscript i in what follows. To simplify notation, we also define two auxiliary functions

$$
G\left(\omega^0_{t+1}, H_t, R^B_{t+1}\right) \equiv \int_{\hat{\omega}^1_{t+1}}^{\infty}\left[\exp\left(\omega^0_{t+1}+\omega^1_{t+1}\right)R^K_{t+1}\left(1+H_t\right)-R^{B,E}_{t+1}H_t\right]dF\left(\omega^1_{t+1}\right), \qquad \text{(A.49)}
$$

and

$$
T\left(\omega^0_{t+1}, H_t, R^B_{t+1}\right) \equiv \int_{\hat{\omega}^1_{t+1}}^{\infty}R^{B,E}_{t+1}H_t dF\left(\omega^1_{i,t+1}\right)+\int_{-\infty}^{\hat{\omega}^1_{t+1}}(1-\zeta)\exp\left(\omega_{t+1}\right)R^K_{t+1}\left(1+H_t\right)dF\left(\omega^1_{t+1}\right). \text{(A.50)}
$$

Denoting by ψ the Lagrangian multiplier of the lender's breakeven condition, we derive the first-order condition with respect to H_t as

$$\mathbb{E}_t \left\{ M_{t+1} \left(\int_{\bar{\omega}_{t+1}^0}^{\infty} \frac{\partial G}{\partial H_t} dF\left(\omega_{t+1}^0\right) \right) \right\} = \psi_t \mathbb{E}_t \left(1 - M_{t+1} \int_{\bar{\omega}_{t+1}^0}^{\infty} \frac{\partial T}{\partial H_t} dF\left(\omega_{t+1}^0\right) - M_{t+1} \int_{-\infty}^{\bar{\omega}_{t+1}^0} R_{t+1}^D dF\left(\omega_{t+1}^0\right) \right), \tag{A.51}$$

in which

$$\frac{\partial G}{\partial H_t} = \int_{\hat{\omega}_{t+1}^1}^{\infty} \left[\exp\left(\omega_{t+1}^0 + \omega_{t+1}^1\right) R_{t+1}^K - R_{t+1}^{B,E} \right] dF\left(\omega_{t+1}^1\right), \tag{A.52}$$

and (using) together with the expression of $\hat{\omega}_{t+1}^1$ and $\partial\hat{\omega}_{t+1}^1/\partial H_t$, we have

$$\frac{\partial T}{\partial H_t} = \int_{\hat{\omega}_{t+1}^1}^{\infty} R_{t+1}^{B,E} dF\left(\omega_{t+1}^1\right) + \int_{-\infty}^{\hat{\omega}_{t+1}^1} (1-\zeta) \exp\left(\omega_{t+1}^0 + \omega_{t+1}^1\right) R_{t+1}^K dF\left(\omega_{t+1}^1\right) - \frac{\zeta R_{t+1}^{B,E}}{1+H_t} f_1\left(\hat{\omega}_{t+1}^1\right). \tag{A.53}$$

Similarly, the first-order condition with respect to $R_{t+1}^{B,E}$ is

$$\mathbb{E}_t \left\{ M_{t+1} \left(\int_{\bar{\omega}_{t+1}^0}^{\infty} \frac{\partial G}{\partial R_{t+1}^{B,E}} dF\left(\omega_{t+1}^0\right) \right) \right\} = -\psi_t \mathbb{E}_t \left(M_{t+1} \int_{\bar{\omega}_{t+1}^0}^{\infty} \frac{\partial T}{\partial R_{t+1}^{B,E}} dF\left(\omega_{t+1}^0\right) \right), \tag{A.54}$$

in which

$$\frac{\partial G}{\partial R_{t+1}^{B,E}} = -\int_{\hat{\omega}_{t+1}^1}^{\infty} H_t dF\left(\omega_{t+1}^1\right), \tag{A.55}$$

$$\frac{\partial T}{\partial R_{t+1}^{B,E}} = \int_{\hat{\omega}_{t+1}^1}^{\infty} H_t dF\left(\omega_{t+1}^1\right) - \zeta H_t f_1\left(\hat{\omega}_{t+1}^1\right). \tag{A.56}$$

Finally, the first-order condition with respect to $\bar{\omega}_{t+1}^0$ is s

$$\begin{aligned} &\mathbb{E}_t \left\{ M_{t+1} \left(\int_{\bar{\omega}_{t+1}^0}^{\infty} \frac{\partial G}{\partial \bar{\omega}_{t+1}^0} dF\left(\omega_{t+1}^0\right) - G\left(\bar{\omega}_{t+1}^0, H_t, R_{t+1}^B\right) f_0\left(\bar{\omega}_{t+1}^0\right) + R_{t+1}^D f_0\left(\bar{\omega}_{t+1}^0\right) \right) \right\} \\ &= -\psi_t \mathbb{E}_t \left[M_{t+1} \left(\int_{\bar{\omega}_{t+1}^0}^{\infty} \frac{\partial T}{\partial \bar{\omega}_{t+1}^0} dF\left(\omega_{t+1}^0\right) - T\left(\bar{\omega}_{t+1}^0, H_t, R_{t+1}^B\right) f_0\left(\bar{\omega}_{t+1}^0\right) + R_{t+1}^D H_t f_0\left(\bar{\omega}_{t+1}^0\right) \right) \right], \end{aligned} \tag{A.57}$$

in which

$$\frac{\partial G}{\partial \bar{\omega}^0_{t+1}} = 0, \tag{A.58}$$

$$\frac{\partial T}{\partial \bar{\omega}^0_{t+1}} = 0. \tag{A.59}$$

Since $f_0\left(\bar{\omega}^0_{t+1}\right) > 0$ and known at time t, we can divide both sides of (A.57) by $f_0\left(\bar{\omega}^0_{t+1}\right)$ to obtain the simplified first-order condition

$$\mathbb{E}_t\left\{M_{t+1}\left(R^D_{t+1} - G\left(\bar{\omega}^0_{t+1}, H_t, R^B_{t+1}\right)\right)\right\} = \psi_t \mathbb{E}_t\left[M_{t+1}\left(T\left(\bar{\omega}^0_{t+1}, H_t, R^B_{t+1}\right) - R^D_{t+1} H_t\right)\right]. \tag{A.60}$$

B Appendix to Empirical Section

B.1 Measuring Distance to Technical Default

In this section, we describe our main measure of distance to technical default. This quarterly firm-level measure, which in the spirit of Murfin (2012) we name "strictness," represents the probability that the firm might breach one of its covenant terms in the next period. In this sense, strictness is a quarterly ex-ante measure of technical default.

To construct our strictness measure, we first make some assumptions with respect to the process generating financial ratios on which loan covenants are written (e.g., the interest coverage ratio and the leverage ratio). Similar to Murfin (2012), we assume that, for a given firm i, the log-growth of a single financial ratio r between quarter t and quarter $t+1$ is equal to a constant plus a normally-distributed noise term, i.e.,

$$\ln(r_{i,t+1}) - \ln(r_{i,t}) \quad = \quad \mu_i + \varepsilon_{i,t+1}, \tag{B.1}$$

in which μ_i is a firm-specific constant and $\varepsilon \sim N(0, \sigma^2)$.[B.1] If a covenant for r is written such that control rights are allocated to the lender if $r < \underline{r}$, (or, equivalently, if $\ln(r) < \ln(\underline{r})$), then the probability that the lender will be allocated control rights between t and $t+1$ is equal to

$$\Pr(r < \underline{r})_{i,t} \quad = \quad 1 - \phi\left(\frac{\widehat{\ln(r_{i,t+1})} - \ln(\underline{r})}{\sigma}\right), \tag{B.2}$$

in which ϕ is the standard normal cdf, and $\widehat{\ln(r_{i,t+1})} = \ln(r_{i,t}) + \mu_i$ is the forecasted value based on process specified in Equation (B.1).[B.2]

We consider the case when more than a single financial covenant is active for firm i at time t. In such a case, (B.1) becomes

$$\ln(\mathbf{r}_{i,t+1}) - \ln(\mathbf{r}_{i,t}) \quad = \quad \boldsymbol{\mu}_i + \boldsymbol{\varepsilon}_{i,t+1}, \tag{B.3}$$

[B.1] In the Appendix, we show that our main portfolio sorting results hold if we instead consider an AR(1) process to describe the time series evolution of a firm's (log) financial ratios.

[B.2] Similarly, our strictness measure takes into account covenants breaches when $r > \underline{r}$ (e.g., covenant on maximum Debt-to-EBITDA ratios, see Murfin (2012)).

in which $\mathbf{r}$ is a $N \times 1$ vector of financial ratios, $\boldsymbol{\mu}_i$ is a $N \times 1$ vector of firm-specific constants, and $\varepsilon \sim N_N(\mathbf{0}, \Sigma)$. Strictness is then the probability that *any* active covenant will be violated, and the lender will be allocated control rights between t and $t+1$. Formally, strictness is equal to

$$\text{Strictness}_{i,t} = 1 - \Phi\left(\widehat{\ln(\mathbf{r}_{i,t+1})} - \ln(\underline{\mathbf{r}})\right), \tag{B.4}$$

in which Φ is the multivariate standard normal cdf with mean $\mathbf{0}$ and variance Σ, and $\underline{\mathbf{r}}$ is a $N \times 1$ vector of active covenants.

B.2 Additional Results

Table B1

Covenant Strictness and Future Covenant Violations

In this table we show that covenant strictness is positively correlated with future covenant violations by regressing firm-level indicators for covenant violations on past-quarter strictness. The sample period starts with the first quarter of 1996 and ends with the last quarter of 2016. The loan covenant violation data for our sample period come from Greg Nini, and are an updated version of the covenant violation data in Nini et al. (2012).

	Dependent Variable: Covenant Violation		
	(1)	(2)	(3)
One-Quarter Lag Strictness	0.109***	0.065***	0.058***
	(0.00)	(0.01)	(0.01)
Firm FE	No	Yes	Yes
Year-Quarter FE	No	No	Yes
R-Squared	0.069	0.249	0.257
Observations	72,781	72,639	72,639

Note: Standard errors (in parentheses) are clustered at the firm level. ***, **, and * respectively denote statistical significance at the 1%, 5%, and 10% levels.

Table B2

Robustness: q-Factor Regressions

In this table we show that our main results from Table 5 are robust to q-factors in Hou et al. (2015). The sample period starts with the first quarter of 1996 and ends with the last quarter of 2016.

	Low	2	3	4	High	High-4	High-Low	4-Low
α^{HXZ}	-1.70	-0.35	-1.82	0.41	-5.54**	-5.95**	-3.84*	2.11
t-stat.	-0.91	-0.18	-0.78	0.19	-2.21	-2.20	-1.77	1.36
β^{MKT}	1.04***	0.98***	1.05***	1.06***	1.16***	0.10*	0.12**	0.02
t-stat.	30.28	27.38	24.92	20.45	23.97	1.91	2.12	0.41
β^{ME}	0.01	0.11	0.07	0.19**	0.24**	0.04	0.23***	0.18***
t-stat.	0.19	1.18	0.57	1.96	2.58	0.58	2.79	2.81
$\beta^{I/A}$	0.16	0.21*	0.24**	0.37***	0.03	-0.34*	-0.14	0.21**
t-stat.	1.53	1.71	2.02	3.29	0.23	-1.86	-0.91	2.40
β^{ROE}	0.14**	0.20***	0.10*	0.18*	-0.26***	-0.44***	-0.40***	0.04
t-stat.	2.57	2.64	1.68	1.93	-3.48	-4.62	-5.25	0.58

Table B3

Fama-MacBeth Regressions on Strictness Portfolio Indicators

This table shows that the negative relationship between expected returns and strictness comes from high-strictness firms. We repeat the same exercise as in Table 6, but we replace our continuous measure of lagged strictness with indicators for whether a firm belongs to a different quintile of the strictness distribution two quarters before the excess returns' realization. As in Table 5, strictness portfolios are constructed quarterly. The low-strictness portfolio represents our baseline portfolio, and hence is omitted from our regressions. The sample starts in January 1996 and ends in December 2016.

	Dependent Variable: Monthly Excess Returns			
	(1)	(2)	(3)	(4)
Str. Portfolio 2	0.013	0.011	0.038	0.027
	(0.09)	(0.09)	(0.08)	(0.08)
Str. Portfolio 3	-0.026	-0.034	0.002	-0.008
	(0.10)	(0.09)	(0.08)	(0.08)
Str. Portfolio 4	-0.162	-0.169	-0.147	-0.147
	(0.11)	(0.11)	(0.11)	(0.10)
High Str. Portfolio	-0.310**	-0.296**	-0.317**	-0.298**
	(0.12)	(0.12)	(0.13)	(0.13)
Pr(Failure)		-84.430***		-94.865***
		(32.18)		(28.83)
EDF			0.119	2.216
			(2.52)	(2.42)
Other Controls	Yes	Yes	Yes	Yes
R-Squared	0.044	0.050	0.052	0.057
Observations	219,247	218,872	214,669	214,619

Note: Newey-West standard errors in parentheses. ***, **, and * respectively denote statistical significance at the 1%, 5%, and 10% levels.

Table B4

Fama-MacBeth Regressions: Non-Distressed Firms

This table shows that the negative, non-monotonic relationship between expected returns and strictness is not generated by financially-distressed firms (Garlappi and Yan (2011)). In practice, we repeat the same exercise as in Table B3, but we drop firms above the 90^{th} percentile of the EDF distribution (Columns (1) and (2)) and above the 90^{th} percentile of the failure probability distribution (Columns (3) and (4)). In Panel A, we compute the strictness-based portfolios using cutoffs from the unconditional strictness distribution in each quarter (i.e., including distressed firms). In Panel B, we first drop distressed firms and then compute the strictness-based portfolios in the resulting sample. The sample starts in January 1996 and ends in December 2016.

Panel A: Unconditional Portfolio Dummies

	EDF $\leq$ 90th Percentile		Pr(Failure) $\leq$ 90th Percentile	
	(1)	(2)	(3)	(4)
Str. Portfolio 2	-0.005	0.011	0.022	0.031
	(0.08)	(0.08)	(0.08)	(0.08)
Str. Portfolio 3	-0.097	-0.072	-0.096	-0.085
	(0.08)	(0.08)	(0.08)	(0.08)
Str. Portfolio 4	-0.203**	-0.171*	-0.156*	-0.139
	(0.10)	(0.10)	(0.09)	(0.09)
High Str. Portfolio	-0.328**	-0.308**	-0.347***	-0.324**
	(0.14)	(0.14)	(0.13)	(0.14)
Distress Controls	No	Yes	No	Yes
Other Controls	Yes	Yes	Yes	Yes
R-Squared	0.044	0.052	0.041	0.051
Observations	193,327	193,281	197,033	193,338

Panel B: Conditional Portfolio Dummies

	EDF $\leq$ 90th Percentile		Pr(Failure) $\leq$ 90th Percentile	
	(1)	(2)	(3)	(4)
Str. Portfolio 2	0.028	0.040	0.049	0.056
	(0.09)	(0.09)	(0.08)	(0.08)
Str. Portfolio 3	-0.094	-0.072	-0.122	-0.109
	(0.08)	(0.08)	(0.09)	(0.09)
Str. Portfolio 4	-0.127	-0.096	-0.081	-0.060
	(0.10)	(0.09)	(0.09)	(0.09)
High Str. Portfolio	-0.286**	-0.266**	-0.311**	-0.299**
	(0.13)	(0.13)	(0.12)	(0.13)
Distress Controls	No	Yes	No	Yes
Other Controls	Yes	Yes	Yes	Yes
R-Squared	0.044	0.052	0.041	0.051
Observations	193,327	193,281	197,033	193,338

Note: Newey-West standard errors in parentheses. ***, **, and * respectively denote statistical significance at the 1%, 5%, and 10% levels.

Table B5

Fama-MacBeth Regressions: Additional Controls

In this table, we add the SA Index and the WW Index to our main specification from Table 6 to control for financial constraints. All the variables in this table are defined as in Table 3. The sample period starts with the first quarter of 1996 and ends with the last quarter of 2016.

	Dependent Variable: Monthly Excess Returns			
	(1)	(2)	(3)	(4)
Strictness	-0.342***	-0.312**	-0.356***	-0.320**
	(0.12)	(0.12)	(0.12)	(0.12)
Size	-0.079	-0.101	-0.020	-0.042
	(0.09)	(0.09)	(0.09)	(0.08)
Log B/M	0.123	0.116	0.092	0.079
	(0.10)	(0.10)	(0.10)	(0.10)
Reversal	-0.015**	-0.016**	-0.016**	-0.016**
	(0.01)	(0.01)	(0.01)	(0.01)
Book Leverage	-0.101	-0.057	-0.338	-0.337
	(0.40)	(0.39)	(0.40)	(0.39)
ROA	5.105	3.518	5.252	3.550
	(3.79)	(3.48)	(3.59)	(3.39)
SA Index	-0.170**	-0.149*	-0.206***	-0.188**
	(0.08)	(0.08)	(0.08)	(0.08)
WW Index	1.169	0.890	2.102	1.828
	(1.70)	(1.64)	(1.61)	(1.57)
Pr(Failure)		-82.739***		-93.551***
		(31.43)		(26.94)
EDF			0.121	2.179
			(2.61)	(2.48)
R-Squared	0.046	0.051	0.054	0.058
Observations	214,844	214,595	210,775	210,729

Note: Newey-West standard errors in parentheses. ***, **, and * respectively denote statistical significance at the 1%, 5%, and 10% levels.

Table B6

Robustness: AR(1) Process for Financial Ratios

This table reports the results of a robustness test in which we use an AR(1) process to describe the time-series evolution of a firm's (log) financial ratios (see Equation (B.1) in the main text). The sample period starts with the first quarter of 1996 and ends with the last quarter of 2016.

	Low	2	3	4	High	High-4	High-Low	4-Low
Excess Return (pp)	6.73*	8.55**	6.31	10.05**	1.58	-8.47**	-5.15*	3.32*
t-stat.	1.89	2.36	1.60	2.57	0.28	-2.48	-1.86	1.80
α^{FF5}	-2.44*	-2.24	-3.55*	-1.15	-7.56***	-6.41**	-5.12**	1.29
t-stat.	-1.79	-1.25	-1.70	-0.64	-2.91	-2.20	-2.03	0.90
β^{MKT}	1.05***	1.04***	1.06***	1.11***	1.17***	0.06	0.13*	0.06
t-stat.	32.75	32.42	26.12	23.47	21.25	0.92	1.76	1.43
β^{SMB}	0.05	0.25***	0.22***	0.28***	0.37***	0.09	0.32***	0.23***
t-stat.	1.04	3.76	3.55	3.94	6.34	0.98	3.81	4.25
β^{HML}	0.04	-0.00	0.18**	0.12	0.23**	0.10	0.19**	0.08
t-stat.	0.55	-0.02	2.17	1.10	2.15	1.04	1.97	1.24
β^{RMW}	0.25***	0.47***	0.32***	0.38***	-0.04	-0.42***	-0.29*	0.13
t-stat.	4.24	4.60	4.05	4.21	-0.33	-3.32	-1.96	1.37
β^{CMA}	0.07	0.19**	-0.09	0.15	-0.31**	-0.47***	-0.38**	0.09
t-stat.	0.92	1.97	-0.54	1.49	-2.09	-2.95	-2.42	0.82

Table B7

Pooled OLS Regression on Strictness

This table presents the results of a pooled OLS regression to study the relation between strictness and future excess returns. The specifications are identical to the specifications in Table 6, but here we use a firm-month panel instead of computing the average of the monthly cross-sectional regressions. The sample period starts with the first quarter of 1996 and ends with the last quarter of 2016.

	Dependent Variable: Monthly Excess Returns			
	(1)	(2)	(3)	(4)
Strictness	-0.440***	-0.445***	-0.482***	-0.480***
	(0.16)	(0.17)	(0.17)	(0.17)
Size	-0.119**	-0.111*	-0.091	-0.088
	(0.06)	(0.06)	(0.06)	(0.06)
Log B/M	0.159	0.149	0.071	0.078
	(0.13)	(0.13)	(0.13)	(0.13)
Reversal	-0.032**	-0.032**	-0.033**	-0.033**
	(0.01)	(0.01)	(0.02)	(0.02)
Book Leverage	-0.116	-0.176	-0.505	-0.478
	(0.51)	(0.51)	(0.55)	(0.55)
ROA	-1.235	-0.789	-0.500	0.071
	(6.11)	(5.50)	(6.16)	(5.73)
Pr(Failure)		3.513		2.600
		(2.77)		(2.95)
EDF			2.226*	1.959
			(1.16)	(1.23)
R-Squared	0.151	0.151	0.151	0.151
Observations	219,331	218,952	214,750	214,699

Note: Standard errors (in parentheses) are clustered at the year-quarter level. ***, **, and * respectively denote statistical significance at the 1%, 5%, and 10% levels.

Table B8

Pooled OLS Regression on Strictness Portfolio Indicators

This table presents the results of a pooled OLS regression to study the relation between strictness and future excess returns. The specifications are identical to the specifications in Table B3, but here we use a firm-month panel instead of computing averages of monthly cross-sectional regressions' estimates. The sample period starts with the first quarter of 1996 and ends with the last quarter of 2016.

	Dependent Variable: Monthly Excess Returns			
	(1)	(2)	(3)	(4)
Str. Portfolio 2	-0.014	0.006	0.018	0.024
	(0.09)	(0.09)	(0.08)	(0.08)
Str. Portfolio 3	-0.030	-0.008	0.010	0.014
	(0.10)	(0.09)	(0.09)	(0.09)
Str. Portfolio 4	-0.177	-0.156	-0.147	-0.141
	(0.11)	(0.11)	(0.11)	(0.11)
High Str. Portfolio	-0.434***	-0.428***	-0.455***	-0.451***
	(0.15)	(0.15)	(0.16)	(0.15)
Pr(Failure)		3.630		2.707
		(2.81)		(2.99)
EDF			2.272*	1.996
			(1.16)	(1.23)
Other Controls	Yes	Yes	Yes	Yes
R-Squared	0.151	0.151	0.151	0.151
Observations	219,247	218,872	214,669	214,619

Note: Standard errors (in parentheses) are clustered at the year-quarter level. ***, **, and * respectively denote statistical significance at the 1%, 5%, and 10% levels.

Table B9

Pooled OLS Regressions: Non-Distressed Firms

This table presents the results of a pooled OLS regression to study the relation between strictness and future excess returns. The specifications are identical to the specifications in Table B4, but here we use a firm-month panel instead of computing averages of monthly cross-sectional regressions' estimates. The sample period starts with the first quarter of 1996 and ends with the last quarter of 2016.

Panel A: Unconditional Portfolio Dummies

	EDF $\leq$ 90th Percentile		Pr(Failure) $\leq$ 90th Percentile	
	(1)	(2)	(3)	(4)
Str. Portfolio 2	0.039	0.053	0.054	0.074
	(0.08)	(0.08)	(0.09)	(0.08)
Str. Portfolio 3	-0.089	-0.075	-0.123	-0.098
	(0.09)	(0.09)	(0.09)	(0.09)
Str. Portfolio 4	-0.105	-0.096	-0.089	-0.070
	(0.10)	(0.10)	(0.10)	(0.11)
High Str. Portfolio	-0.324**	-0.330**	-0.342**	-0.340**
	(0.14)	(0.14)	(0.14)	(0.14)
Distress Controls	No	Yes	No	Yes
Other Controls	Yes	Yes	Yes	Yes
R-Squared	0.167	0.168	0.169	0.170
Observations	193,327	193,281	197,033	193,338

Panel B: Conditional Portfolio Dummies

	EDF $\leq$ 90th Percentile		Pr(Failure) $\leq$ 90th Percentile	
	(1)	(2)	(3)	(4)
Str. Portfolio 2	-0.009	0.005	0.006	0.022
	(0.08)	(0.08)	(0.08)	(0.08)
Str. Portfolio 3	-0.073	-0.059	-0.088	-0.066
	(0.08)	(0.08)	(0.09)	(0.09)
Str. Portfolio 4	-0.180	-0.172	-0.165	-0.152
	(0.11)	(0.11)	(0.10)	(0.11)
High Str. Portfolio	-0.383**	-0.393**	-0.395***	-0.392**
	(0.16)	(0.16)	(0.15)	(0.15)
Distress Controls	No	Yes	No	Yes
Other Controls	Yes	Yes	Yes	Yes
R-Squared	0.167	0.168	0.169	0.170
Observations	193,327	193,281	197,033	193,338

Note: Standard errors (in parentheses) are clustered at the year-quarter level. ***, **, and * respectively denote statistical significance at the 1%, 5%, and 10% levels.

Table B10

RDD Regressions: Bandwidth Robustness

This table presents results of robustness tests on Columns (1) and (2) of Table 7 using different sample bandwidths. In the table, $\pm$ 0.3 $\times$ Threshold indicates the sub-sample of firms whose Debt-to-EBITDA ratio falls into the $[0.7 \times \text{threshold}, 1.3 \times \text{threshold}]$ range. Similarly, $\pm$ 0.2 $\times$ Threshold and $\pm$ 0.1 $\times$ Threshold indicate the sub-samples with range $[0.8 \times \text{threshold}, 1.2 \times \text{threshold}]$ and $[0.9 \times \text{threshold}, 1.1 \times \text{threshold}]$, respectively. In all specifications, year-quarter fixed effects are included. The sample starts in January 1996 and ends in December 2016.

	$\pm$0.3 $\times$ Threshold		$\pm$0.2 $\times$ Threshold		$\pm$0.1 $\times$ Threshold	
	(1)	(2)	(3)	(4)	(5)	(6)
Violation	-0.594***	-0.689***	-0.588**	-0.588**	-0.712*	-0.736*
	(0.21)	(0.22)	(0.26)	(0.27)	(0.37)	(0.38)
Distance	0.259	0.312*	0.020	-0.093	1.852*	1.049
	(0.17)	(0.17)	(0.34)	(0.33)	(1.03)	(1.06)
Violation $\times$ Distance	0.179	0.331	0.494	0.663	-2.251	-1.017
	(0.32)	(0.34)	(0.60)	(0.64)	(1.77)	(1.91)
Size		-0.052		-0.060		-0.025
		(0.04)		(0.05)		(0.08)
Log B/M		0.036		-0.013		-0.036
		(0.09)		(0.12)		(0.17)
Book Leverage		-0.278		-0.565		-0.894
		(0.51)		(0.65)		(0.90)
ROA		13.533***		14.743**		7.918
		(4.87)		(6.32)		(9.73)
Year-Quarter FE	Yes	Yes	Yes	Yes	Yes	Yes
R-Squared	0.215	0.220	0.215	0.218	0.218	0.223
Observations	20,293	18,873	13,291	12,317	6,587	6,079

Note: Standard errors (in parentheses) are clustered at the firm level. ***, **, and * respectively denote statistical significance at the 1%, 5%, and 10% levels.