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Facilitating Entry through Leasing

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Keywords: Leased capital; Collateral constraint; Total factor productivity (TFP); Extensive margin; Entry; Technology adoption; Capital misallocation
JEL Classification: E22, E32, E44, G32

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1 Introduction

Financial frictions have received much attention as an important source of aggregate productivity losses. Generally speaking, financial frictions reduce total factor productivity (TFP) via two fronts. First, they generate capital misallocation across active entrepreneurs because their marginal products are not equalized (i.e., the intensive margin). Second, they distort the extensive margin, in which the set of active entrepreneurs who adopt technologies are suppressed. A notable finding is that the impact at the extensive margin is more sizable (Midrigan and Xu, 2014; Buera, Kaboski, and Shin, 2015). In this paper, we study a novel role that leasing plays at this extensive margin: the possibility for entrepreneurs to rent capital when they are financially constrained improves aggregate efficiency by facilitating entry of entrepreneurs into a more productive sector.[1] We analytically show the role of leasing in generating positive efficiency gains in a stylized two-period model, and then quantify this channel in our dynamic general equilibrium model.

We start by presenting motivating facts related to leasing activities on the aggregate level and across firms in the US economy. Our focus is operating leases: in a typical operating lease contract, the owner of the capital (lessor) grants to the borrower (lessee) the exclusive right to use capital for an agreed period in exchange for periodic payments; at the end of the lease's term, the capital reverts to the lessor.[2] As shown in Table 1, we document that the operating lease-induced capital accounts for over 13% of overall productive assets used by US public firms, and this proportion is even higher among small and financially constrained firms - around 30%. On the liability side, we find that the overall debt to output ratio increases by 20% after lease-adjustment, and this ratio increases more for small and financially constrained firms. These findings highlight the importance of leasing as a source of physical capital and as a source of external finance, especially for small and financially constrained firms.

[1] We use "lease" and "rent," "purchase" and "own" synonymously in this paper.

[2] There is another type of lease – capital lease (also known as financial lease), in which the lessee acquires ownership of the asset at the end of the lease's term. In this paper, we mainly focus on operating lease, since 87% of leased assets are recorded as operating leases in the US (Graham and Lin, 2018).

Though being extensively used in capital markets and production, operating leases were treated as off-balance-sheet items before the recent lease accounting rule changes in ASC 842.[3] Therefore, leased capital is an important source of "unmeasured" capital and largely ignored in the macroeconomics and macro-finance literature, especially for economies with financial frictions.

In a typical operating lease contract, if the lessee fails to make the specified payments in the middle of the contract, the capital must be returned to the lessor and the contract ends. This asset regaining process emphasizes a major benefit of leasing, i.e., the repossession advantage when the lessee goes bankrupt. However, the capital is under the control of a user who is not the owner, which makes leasing costly due to agency problems (Eisfeldt and Rampini, 2009; Rampini and Viswanathan, 2013). The high debt capacity associated with leasing naturally impacts the two margins induced by financial frictions. Our goal in this paper is to investigate and quantify the novel role that leasing plays in reshaping the extensive margin by facilitating entry in terms of efficiency gains. As a prominent example, Ma, Murfin, and Pratt (2022) provide suggestive evidence showing that leasing contracts allow young entrepreneurs to make less distorted decisions on start-up formation, as well as on the extensive margin of investment.

To formalize our intuition, we develop a general equilibrium model with sectoral choices, collateral constraints, and an explicit buy versus lease decision. In our model, heterogeneous entrepreneurs produce a homogeneous consumption good by operating in one of two sectors: traditional and modern. Entrepreneurs in the traditional sector do not require financing

[3] Effective from 2019, firms are required to recognize lease assets and lease liabilities from off-balance-sheet activities on their balance sheets (Accounting Standards Update No. 2016-02, Leases (ASU 2016-02, Topic 842)). These items were absent before the adoption of the new operating lease accounting rule. After adopting the new accounting rule, firms now report "Lease right-of-use asset" on the asset side, and both short-term and long-term lease liabilities on the liability side. Additionally, firms are required to report the estimates of their operating leases, including the value, average regaining life, and discount rate, and disclose the possibility of renewing or extending existing leases. This rule increases the transparency and comparability among organizations. Similar new lease standards are adopted in IFRS 16 for annual periods beginning on or after January 1, 2019. Note that unlike operating lease, capital lease was already on balance sheets before the lease accounting rule change.

and they have access to an unproductive production technology, which does not use capital. Entrepreneurs in the modern sector can produce with a more productive technology, which requires capital as an input. In addition, entrepreneurs have to pay an entry fixed cost to produce in the productive modern sector. These two forms of investment in the modern sector (i.e., building up capital and paying the fixed cost) both require financing, and such financing is subject to collateral constraints. We model collateral constraints as a limit on the amount borrowing that an entrepreneur can obtain, and only owned capital can be used as collateral, following the standard macro models (e.g., Kiyotaki and Moore (1997) and Gertler and Kiyotaki (2010)). This only affects the technology in the modern sector, as the traditional one abstracts from capital. These financial frictions then prevent entrepreneurs from entering the modern sector.

The entry barriers can be mitigated by leasing, which is our new insight in this paper. First, leasing can fulfill the capital requirement of the modern technology when entrepreneurs are financially constrained and cannot afford the direct purchase of capital. Second, leasing provides additional external financing for the fixed cost payment. Importantly, leasing increases the total amount of capital stock that a modern entrepreneur may possibly utilize, which generates higher output.[4] The higher output in the modern sector under economies with leasing (relative to no-leasing economies) raises the expected payoff of being a modern entrepreneur, especially for those agents with high productivity but low net worth. The above elements jointly induce more high-productivity individuals to enter the modern sector and to produce with a more advanced technology.

Our analysis is organized in two parts. First, we consider a two-period model of sectoral choices and lease versus buy decisions. We characterize the stationary equilibrium analytically. In equilibrium, we find that entrepreneurs with high net worth (relative to productivity) choose to produce in the modern sector, and entrepreneurs with low net worth

[4] This is related to the intensive margin effect, where the dispersion of return to capital across modern entrepreneurs can also be narrowed, as highlighted in Hu, Li, and Xu (2020).

(relative to productivity) choose to produce in the traditional sector; within the modern sector, entrepreneurs that lease capital are the most financially-constrained entrepreneurs (i.e., entrepreneurs with low net worth to productivity ratio), whereas entrepreneurs that purchase capital and borrow against its collateral value are less financially constrained or unconstrained (i.e., entrepreneurs with a high net worth to productivity ratio). We then artificially shut down the leasing market and compare it to our economy with leasing. We analytically show that the measure of total modern-sector entrepreneurs is higher in the economy with leasing; that is, the ability to lease facilitates entrepreneurs' entry into the modern sector (the extensive margin). Next, we conduct an efficiency analysis and provide a closed-form solution for the aggregate efficiency of economies both with and without leasing. We come up with several propositions and obtain a formal sign of efficiency changes from leasing through the extensive margin. Such efficiency changes are positive. This is because a larger number of modern entrepreneurs induced by leasing implies that entrepreneurs now operate at smaller scales on average. Given that we have diminishing returns to scale technology, the economy shows up as higher TFP. Importantly, these results hold independently of specific assumptions about the distribution of net worth.

Second, we consider a richer dynamic quantitative model with persistent idiosyncratic productivity shocks and a growing number of entrepreneurs, which nests our stylized two-period model. We associate the modern sector with the manufacturing industry and the traditional sector with the agriculture industry. We calibrate the quantitative model using the US firm-level dataset by fitting the model-implied moments with their empirical counterparts. The model can tightly match the non-targeted moments observed in the data, such as the interest rate, the level of debt to output ratio, the consumption to investment ratio, as well as within firm and cross-sectional moments of firm-level output, capital, employment, and leased capital ratio. Based on the calibrated model, we perform a quantitative efficiency analysis, with a main focus on the balanced growth equilibrium. Our quantitative exercise indicates that the lease-induced efficiency gain at the extensive margin is about 5%. Our dynamic model also allows us to quantitatively compare the extensive margin with the intensive margin. We find that the role of leasing in attracting more entrepreneurs to enter the modern sector (i.e., the extensive margin) is more sizable than that in mitigating the capital misallocation (i.e., the intensive margin), in terms of efficiency gains. We conduct several exercises in order to gauge the robustness of our benchmark results. The quantitative results show that the role of leasing at the extensive margin becomes more pronounced in a more financially-constrained economy, in an economy with a higher entry fixed cost, and in an economy with a higher productivity gap between different sectors. Therefore, borrowing constraints, fixed costs, and productivity gaps strengthen the entry promotion mechanism in amplifying the positive impacts of leasing on entrepreneurial activities and the aggregate economy.

Table 1

SUMMARY STATISTICS

	Aggregate	**Size**			**WW index**		
Variables	**Mean**	**S**	**M**	**L**	**C**	**MC**	**UC**
Leased capital ratio	0.13	0.29	0.22	0.12	0.29	0.21	0.12
Rental share	0.18	0.31	0.27	0.17	0.32	0.27	0.17
Debt to output	0.81	0.39	0.59	0.82	0.49	0.68	0.81
Lease-adjusted debt to output	0.93	0.56	0.75	0.95	0.67	0.84	0.93

This table presents summary statistics for variables of interest in our sample. Leased capital ratio is the ratio of leased capital over the sum of leased capital and owned capital (PPENT). Leased capital is calculated as the sum of current rental expense and the present value of future lease commitments, following Li, Whited, and Wu (2016). Rental share is defined as the ratio between rental expense over the sum of capital expenditure (CAPX) plus rental expense. Debt to output ratio is the ratio of the sum of long-term debt (DLTT) and debt in current liabilities (DLC) over value-added, where value-added is estimated following Ai, Croce, and Li (2013). Lease-adjusted debt to output ratio is the sum of debt to output ratio and lease to output ratio, the latter of which is defined as the ratio of leased capital over value-added. On the right panel, we split the whole sample into subgroups according to their size, and by financial constraint level. Size is defined by total assets, while the financial constraint level is classified by the WW index, according to Whited and Wu (2006). We use "S," "M," and "L" to denote small, medium, and large firm groups, respectively. We use "UC," "MC," and "C" to denote unconstrained, mildly constrained, and constrained firm groups, respectively. We report time series averages of the cross-section averages in the table. See Appendix C for more details.

Related literature Our paper relates to the literature that explores aggregate productivity losses caused by financial frictions at the micro level (e.g., Buera, Kaboski, and Shin (2011), Moll (2014), Midrigan and Xu (2014), and Buera and Moll (2015)).[5] These papers provide theoretical and quantitative insights on what efficiency gains could be achieved by removing financial frictions. They predict fairly small losses from misallocation and emphasize the role that financial constraints play on the extensive margin. Lanteri and Rampini (2021) examine the gains that could be achieved if a benevolent social planner were to face the same set of financial constraints as private agents. In contrast, we focus on the competitive equilibrium and study what efficiency gains could be achieved if the agents are allowed to lease capital when they are financially constrained. In so doing, we build a bridge between the quantitative literature on capital misallocation, the theoretical literature on efficiency, and the finance literature on leased capital and capital structure.

Our study builds on the theories of corporate leasing decisions. The papers most related to ours are Eisfeldt and Rampini (2009), Rampini and Viswanathan (2013), Zhang (2012), Gal and Pinter (2017), Li and Tsou (2019) and Hu, Li, and Xu (2020).[6] We draw elements from these papers to construct both collateral constraints and a firm's decision to buy versus lease. The differences lie in the following dimensions. First, with respect to model framework, Eisfeldt and Rampini (2009) is a static model, and Rampini and Viswanathan (2013) and Zhang (2012) are dynamic models in a partial equilibrium framework. Gal and Pinter (2017) adopt a representative firm general equilibrium framework, while Li and Tsou (2019) and Hu, Li, and Xu (2020) study general equilibrium models with heterogeneous firms. For our study, we differ from above by directly allowing for entry and exit decisions in our dynamic general equilibrium framework. Second, with respect to research questions, our emphasis is

[5]A series of studies focus on how within-sector capital misallocation suppresses aggregate efficiency include, seminal examples include Hsieh and Klenow (2009) and Restuccia and Rogerson (2008). The role of financial frictions in capital misallocation are examined in Gopinath et al. (2017), Kehrig and Vincent (2017), Ai et al. (2019), David and Venkateswaran (2019), and Cavalcanti et al. (2021), among others. See David, Hopenhayn, and Venkateswaran (2016); David, Schmid, and Zeke (2022); Asker, Collard-Wexler, and De Loecker (2014); Haltiwanger, Kulick, and Syverson (2018); Edmond, Midrigan, and Xu (2018); Whited and Zhao (2021) for other sources in generating capital misallocation.

[6]Eisfeldt and Rampini (2009) provide a comprehensive review of this literature.

on the extensive margin: the ability to lease has sizable positive effects on the number of entrepreneurs that operate as well as the level of technology that these entrepreneurs adopt. Li and Tsou (2019) study the cross-secional asset pricing implications, and Hu, Li, and Xu (2020) focus on the mitigation role of leasing on the intensive-margin capital misallocation, both empirically and economically. Our paper quantitatively analyzes the efficiency gains of leasing by facilitating entry and technology adoption using dynamics of output, capital, and productivity at the firm level, which also allows us to contrast the new extensive-margin role with that on the intensive margin. To the best of our knowledge, our study is the first to examine the role of leasing on entry and exit distortions in the presence of financial frictions.

Our study further belongs to the macro-finance literature based on financial frictions. Quadrini (2011) and Brunnermeier, Eisenbach, and Sannikov (2012) provide excellent surveys. More specifically, the most relevant papers include Kiyotaki and Moore (1997), Gertler and Kiyotaki (2010), Kiyotaki and Moore (2012), Brunnermeier and Sannikov (2014), He and Krishnamurthy (2013), and Elenev, Landvoigt, and Van Nieuwerburgh (2017). These studies all emphasize the importance of borrowing constraints and limited contract enforceability. Through studying the shadow cost of financial frictions for life insurance, Koijen and Yogo (2015) provide micro evidence for macro models with financial frictions, as described earlier. Jermann and Quadrini (2012) emphasize the macroeconomic effects of financial shocks. Gomes, Yamarthy, and Yaron (2015) develop a production-based asset pricing model to discuss the impact of financial frictions on risk premia. Our study differs from these in that we introduce leasing as a strongly collateralized costly financing tool and explore the implications for increasing the aggregate efficiency in the real economy.

The rest of our paper is organized as follows. Section 2 presents our main theoretical results in a two-period model of collateral constraints, lease versus buy capital decisions, and sectoral choices. Section 3 introduces the model with idiosyncratic productivity shocks and characterizes the competitive equilibrium and efficiency gain decomposition. Section 4 presents our quantitative results and Section 5 concludes.

2 A two-period model

In this section, we present a two-period general equilibrium model with sectoral choices, collateral constraints, and leased capital. We analytically characterize the choices of sectors and the allocation of capital in the presence of financial frictions that induce losses in aggregate TFP. We show that the ability for entrepreneurs to lease facilitates more entry into a productive sector, which generates efficiency gains.

2.1 Environment

Time is discrete and the horizon contains two periods. The economy is populated by a representative household and a measure one of heterogeneous entrepreneurs. These agents in the economy have identical preferences.

Entrepreneurs can operate either in a traditional sector or in a modern sector. The traditional sector requires no capital to produce using an unproductive technology, whereas the modern sector produces with capital using a more productive technology. We associate the modern sector with the manufacturing sector and the traditional sector with the agriculture sector.

At period 0, entrepreneurs choose one of two sectors in which to operate. Entry into the modern sector requires an up-front fixed cost. Entrepreneurs also make borrowing and saving decisions with the household (i.e., the household nests the role of financial intermediaries). The amount that an entrepreneur can borrow is subject to a collateral constraint. Entrepreneurs can use their net worth and borrowings to buy owned capital. Additionally, they can lease capital (from the household) to produce by paying leasing fees. At period 1, entrepreneurs produce using both types of capital. The world ends at the end of period 1.

2.1.1 Household

The representative household has risk-neutral preferences and maximizes utility in a linear function of consumption at period 0 and period 1: $\sum_{t=0}^{1} \beta^t C_t^w$, where $\beta \in (0, 1)$ is the discount factor and C_t^w is consumption.[7]

The household is endowed with an initial wealth W_0 and nests the role of the lessor. That is, the household is able to accumulate K_1^l amount of leased capital out of its net worth in order to rent to entrepreneurs at a rate of τ_l (i.e., the leasing fee per unit of leased capital).[8] The household also chooses how much to save (through a one-period bond) and how much to consume. Accordingly, the budget constraints the household faces are given by:

$$C_0^w + \int B_{i1} di + K_1^l = W_0, \tag{1}$$

$$\tau_l K_1^l + R_f \int B_{i1} di + (1 - \delta - h) K_1^l = C_1^w, \tag{2}$$

where B_{i1} is the household's purchase of entrepreneur i's risk-free bond (which pays a gross interest rate of R_f), δ is the rate of capital depreciation, and h is the monitoring cost for leased capital, capturing the disadvantages of leased capital related to the separation of ownership and control (Eisfeldt and Rampini, 2009). $(1 - \delta - h)K_1^l$ is the resale value of the leased capital returned by entrepreneurs after production.

Since there is no aggregate risk in this economy, we have the gross interest rate equal to $R_f = \frac{1}{\beta}$. Let r_f denote the net interest rate, $r_f = R_f - 1$. The optimality condition of the household implies that:

$$\tau_l = r_f + \delta + h, \tag{3}$$

which reveals a no-arbitrage condition: the return on B_{i1} is equal to the return on investing into leased capital. Eq. (3) shows the cost per unit of leased capital, in the notion of

[7] In our dynamic model, we feature risk-averse agents by extending the linear utility to log utility.

[8] We abstract from labor and wage income in the two-period model but include them in the full dynamic model in Section 3.

Jorgenson (1963). The only difference to the rental cost in the frictionless neoclassical model is the positive monitoring cost h.

2.1.2 Entrepreneur

The economy is inhabited by a unit measure of entrepreneurs, and we index each entrepreneur with i. To save notations, we suppress the index i wherever appropriate. At period 0, entrepreneurs are endowed with net worth N, and choose to either enter the traditional sector or enter the modern sector by paying an entry fixed cost. The entrepreneurs have preferences identical to the household.

Traditional sector Entrepreneurs that choose the traditional sector u face a technology that produces output Y_1^u without capital at period 1:

$$Y_1^u = z_1^{1-\alpha}, \tag{4}$$

where $\alpha < 1$ is the degree of returns to scale, and z_1 is an entrepreneur's individual productivity at period 1.[9] We can interpret Eq. (4) in the way that entrepreneurs produce with inelastic labor, or with productive materials (other than capital), which are normalized to 1.

We use C_0^u and C_1^u to denote the entrepreneur's consumption at period 0 and period 1 in the traditional sector, respectively. We use B_1^u to denote the debt position of the entrepreneur in the traditional sector at period 0. Each entrepreneur in the traditional sector chooses consumption C_0^u and C_1^u, as well as borrowing B_1^u to maximize its utility:

$$\max E\left[C_0^u + \beta C_1^u\right], \tag{5}$$

[9]The return to scale can be more clearly seen in the production technology associated with the modern sector.

subject to:

$$C_0^u = N + B_1^u, \tag{6}$$

$$C_1^u = Y_1^u - R_f B_1^u, \tag{7}$$

$$B_1^u \leq 0, \tag{8}$$

$$C_0^u, C_1^u \geq 0, \tag{9}$$

where R_f is the gross interest rate, as discussed in the household problem. These entrepreneurs are unable to borrow, so $B_1^u \leq 0$; thus, they are unconstrained.

Modern sector Entrepreneurs that choose the modern sector m have access to a production technology which differs from that of the traditional sector in two ways: it requires capital, and it also uses inputs more efficiently. Hence, the modern entrepreneur's output at period 1, Y_1^m, is:

$$Y_1^m = \kappa^{1-\alpha} z_1^{1-\alpha} \left(K_1^o + K_1^l\right)^\alpha, \tag{10}$$

where $\kappa > 1$ is the relative productivity gap between modern and traditional sectors, K_1^o is the amount of owned capital, and K_1^l is the amount of leased capital. Owned capital K_1^o and leased capital K_1^l are perfect substitutes in production, following Eisfeldt and Rampini (2009), Rampini (2019) and other studies in the literature.[10]

If an entrepreneur chooses to enter the modern sector, it must pay an entry fixed cost f at period 0. The entrepreneur uses its initial net worth N and borrowing B_1^m in period 0 to finance the entry cost f, to accumulate owned capital K_1^o, and to consume C_0^m. The borrowing of the modern entrepreneur is limited by: $B_1^m \leq \theta K_1^o$, where $\theta \in [0, 1]$ captures the tightness of the financial constraint in this economy. This constraint requires that a modern

[10] Apart from being consistent with previous studies, our assumption on the perfect substitution between two types of capital is innocuous. According to the new accounting rule starting in 2019, firms must report operating leases as "Lease right-of-use asset," and firms' fixed assets (PPENT) now include "Lease right-of-use asset" with an assumption of perfect substitution. Moreover, in untabulated results, we directly estimate the degree of elasticity between these two types of capital in the data, and we find a very high estimate, which favors the perfect substitute assumption.

entrepreneur's borrowing amount should not exceed a fraction of its owned capital stock.

Meanwhile, the modern entrepreneur can lease capital. At period 0, the entrepreneur discusses with the lessor for the amount of leased capital K_1^l in need at period 1. Without loss of generality, we assume that the payment of leasing fee occurs at period 1.[11] At period 1, production happens, and the modern entrepreneur consumes C_1^m after paying back bond $R_f B_{i1}^m$ and leasing fees $\tau_l K_1^l$. The entrepreneur also resells the depreciated owned capital $(1-\delta)K_1^o$ and gives back the depreciated leased capital K_1^l to the lessor.

We summarize the optimization problem of the modern entrepreneur below. Each modern entrepreneur chooses consumption C_0^m and C_1^m, as well as borrowing B_1^m to maximize:

$$\max E\left[C_0^m + \beta C_1^m\right], \tag{11}$$

subject to:

$$C_0^m + K_1^o + f = N + B_1^m, \tag{12}$$

$$C_1^m = Y_1^m - \tau_l K_1^l - R_f B_{i1}^m + (1-\delta)K_1^o, \tag{13}$$

$$B_1^m \leq \theta K_1^o, \tag{14}$$

$$K_1^o, K_1^l, C_0^m, C_1^m \geq 0. \tag{15}$$

2.1.3 Market clearing conditions

To complete the specification of the model, we list the market clearing conditions as follows:

$$C_0^w + \int D_{i0} di + \int K_{i1}^o di + K_1^l + \int f di = W_0 + \int N_i di, \tag{16}$$

$$\int Y_{i1} di + \int (1-\delta) K_{i1}^o di + (1-\delta-h) K_1^l = C_1^w + \int D_{i1} di, \tag{17}$$

[11] For simplicity, we assume that the leasing fee is paid after production, consistent with Gal and Pinter (2017). We are effectively providing an upper bound of the role of leasing in relaxing collateral constraints. In real life, 100% financing of leasing is not uncommon.

$$K_1^l = \int K_{i1}^l di, \tag{18}$$

where i represents an individual entrepreneur in one of two sectors. The first two equations are the market clearing conditions for output at period 0 and period 1, respectively. The last equation is the leased capital market clearing condition.

2.2 Equilibrium characterization

For simplicity's sake, we assume that entrepreneurs are heterogeneous only in their initial net worth N. Hence, z_1 is identical across entrepreneurs and we denote $z_1 = z$. N is distributed over the interval $[N_{\min}, N_{\max}]$ according to an exogenous non-degenerate distribution $\Pi(N)$. Further, we assume that at the end of period 0, each entrepreneur can observe its next period's idiosyncratic productivity z in advance and then make decisions on borrowing, sectoral choices, and investment before z is realized. This assumption of "observing idiosyncratic shock ahead of time" is standard in the investment literature, as in Moll (2014) and Midrigan and Xu (2014). It is also consistent with the view that entrepreneurs enjoy information advantages because of their access to potential insider information. Without loss of generality, we assume that entrepreneurs only consume at period 1, i.e., $C_0^u = C_0^m = 0$. These assumptions enable us to derive our analytical results.

Our main focus in this section is to analytically characterize the novel role of leasing along the extensive margin. To do so, in the following, we first briefly discuss modern entrepreneurs' buy versus lease decisions as a preparation step. Then we proceed to the extensive margin analysis. Finally, we use a numerical example to illustrate the properties of the stationary competitive equilibrium.

2.2.1 Buy versus lease decisions

We first analyze entrepreneurs that *already* choose to enter the modern sector and characterize their lease versus buy capital decisions. We study the user costs of these two options to incorporate the benefit and cost of leasing. We set up the Lagrangian of an entrepreneur in the modern sector and outline these details in Appendix A.1.

Denote the multipliers on the constraints (12) to (14) by η_0, η_1, and $\xi_0\eta_0$, respectively. Let $\bar{\nu}_0\eta_0$, $\underline{\nu}_0\eta_0$, and $\bar{\nu}_{d1}$ be the multipliers on the constraints $K_1^o \geq 0$, $K_1^l \geq 0$, and $C_1^m \geq 0$. In terms of consumption at period 0, the user cost of leased capital is:

$$\tilde{\tau}_l = \beta\frac{\tau_l}{\eta_0} = \frac{\beta}{\eta_0}\tau_l = \tilde{\beta}(r_f + \delta + h), \tag{19}$$

that is, the discounted leasing fee in terms of the marginal value of net worth for the modern entrepreneur. η_0 is the marginal value of net worth at period 0 for the modern entrepreneur, and β is the discount factor. Hence, $\tilde{\beta} = \frac{\beta}{\eta_0}$ represents the modern entrepreneur's specific stochastic discount factor.

We define the user cost of owned capital as:

$$\tilde{\tau}_o = 1 - \tilde{\beta}(1-\delta) - \theta\xi_0. \tag{20}$$

The interpretation is that the user cost of owned capital is equal to the current price, 1, minus the discounted resale value, and also minus the marginal value of relaxing the collateral constraint for owning this capital.

To discuss the trade-off through comparing the user costs of buying owned versus leasing capital, we start by defining a shadow interest rate R_I for the borrowing and lending among entrepreneurs:

$$R_I = \frac{1}{\tilde{\beta}},$$

and hence we derive a wedge $\Upsilon_I = R_I - R_f = R_f(\eta_0 - 1) = R_f \frac{\xi_0}{1-\xi_0} \equiv \Upsilon_I(\xi_0)$, which is an increasing function of the tightness of the constraint ξ_0. When the collateral constraint is binding, this wedge becomes strictly positive. Specifically, it reflects a premium that entrepreneurs must pay for the loans among themselves, when cheaper household loans become inaccessible due to a binding collateral constraint.

Using this wedge, we can re-write the user costs as:

$$\tilde{\tau}_l = \tilde{\beta}\tau_l = \frac{r_f + \delta + h}{R_f + \Upsilon_I}, \tag{21}$$

and

$$\tilde{\tau}_o = \frac{r_f + \delta + \Upsilon_I}{R_f + \Upsilon_I} - \theta\xi_0. \tag{22}$$

The difference between two user costs (lease - own) is thus:

$$\tilde{\tau}_l - \tilde{\tau}_o = \frac{h}{R_f + \Upsilon_I} - \frac{\Upsilon_I}{R_f + \Upsilon_I} + \theta\xi_0 = \frac{\eta_1}{\eta_0}h + \xi_0(\theta - 1). \tag{23}$$

The benefit of leasing is the premium saved on internal funds due to constraints, while the cost of leasing includes the additional monitoring cost and the cost of giving up the marginal value of relaxing the collateral constraint when buying this capital. In the environment of collateral constraint, ξ_0 is non-negative and $\theta < 1$. When modern entrepreneurs become sufficiently constrained (ξ_0 sufficiently large), the benefit of leasing dominates its cost, and they start to lease.

In the following proposition, we summarize entrepreneurs' buy versus lease decisions.

Proposition 1. *Conditional on entrepreneurs entering the modern sector, there exist cutoff values N_{uc} and N_l, such that:*

- *Entrepreneurs with $N > N_{uc}$ are unconstrained.*
- *Entrepreneurs with $N \in [N_l, N_{uc}]$ are constrained but do not lease capital.*

- *Entrepreneurs with $N < N_l$ lease capital.*[12]

Proof: See Appendix A.2.1.

Proposition 1 implies that given entrepreneurs *already* in the modern sector, their buy versus lease decisions are completely determined by the trade off between the benefit and cost of leasing. When entrepreneurs' initial wealth N is higher than N_{uc}, they are rich enough to borrow less than the limit. These entrepreneurs are unconstrained and naturally will not use the leased capital, because of the expensive agency costs. For entrepreneurs with net worth $N \in [N_l, N_{uc}]$, they are financially constrained. However, since they are not sufficiently constrained, they don't lease capital either. For the remaining entrepreneurs in the modern sector (i.e., entrepreneurs with $N < N_l$), they will lease. This result is consistent with Eisfeldt and Rampini (2009) and Hu, Li, and Xu (2020).

Meanwhile, we have a general form of marginal product of capital (MPK):

$$MPK = \alpha(\kappa z)^{1-\alpha} K_1^{\alpha-1}, \tag{24}$$

where $K_1 = K_1^o + K_1^l$, representing the total utilized capital with which a modern entrepreneur operates. Using the thresholds outlined in Proposition 1, we can write MPK in detail and calculate each modern entrepreneur's total utilized capital, which we summarize below:

Lemma 1. *The MPK of entrepreneurs in the modern sector can be calculated as:*

$$MPK = \begin{cases} r_f + \delta, & \text{if} \quad N > N_{uc}, \\ \alpha(\kappa z)^{1-\alpha} \left(\frac{N-f}{1-\theta}\right)^{\alpha-1}, & \text{if} \quad N \in [N_l, N_{uc}], \\ r_f + \delta + h, & \text{if} \quad N < N_l. \end{cases}$$

[12] There exists another threshold value, which ensures that entrepreneurs must choose the modern sector to produce. This argument also applies to Lemma 1. We discuss this in detail in the next section.

Each entrepreneur's utilized capital can be calculated as:

$$K_1 = \begin{cases} \left[\frac{r_f+\delta}{\alpha(\kappa z)^{1-\alpha}}\right]^{\frac{1}{\alpha-1}}, & \textit{if} \quad N > N_{uc}, \\ \frac{N-f}{1-\theta}, & \textit{if} \quad N \in [N_l, N_{uc}], \\ \left[\frac{r_f+\delta+h}{\alpha(\kappa z)^{1-\alpha}}\right]^{\frac{1}{\alpha-1}}, & \textit{if} \quad N < N_l. \end{cases}$$

From Lemma 1, we note that unconstrained entrepreneurs (with $N > N_{uc}$) equalize the MPK to $r_f+\delta$, the lowest MPK among all three cases. They utilize the optimal level of capital, $\left[\frac{r_f+\delta}{\alpha(\kappa z)^{1-\alpha}}\right]^{\frac{1}{\alpha-1}}$, which consists of only owned capital. Sufficiently constrained entrepreneurs (with $N < N_l$) equalize the MPK to $r_f+\delta+h$, the highest MPK among all three cases. Their total utilized capital is the lowest among all cases, among which $\frac{N-f}{1-\theta}$ is owned capital and the rest is leased capital. For entrepreneurs with net worth $N \in [N_l, N_{uc}]$, their MPK varies with N and is between the two MPK values discussed earlier. They can finance owned capital only equal to $\frac{N-f}{1-\theta}$ and use no leased capital. Please refer to Figure 2 for an illustration.

2.2.2 Traditional sector versus modern sector decisions

Having characterized the optimal behavior of entrepreneurs already in the modern sector, we now turn to the decision on which sector an entrepreneur chooses to enter in the first place, i.e., the extensive margin. Importantly, we will illustrate the facilitation of leasing on entry into the modern sector, which is the main focus of our paper.

Upon being born, if the entrepreneur chooses the traditional sector, its consumption is given by Eqs. (6) and (7). As discussed before, we have assumed $C_0^u = 0$ without loss of generality. This gives:

$$C_1^u = z^{1-\alpha} + R_f N, \tag{25}$$

which indicates the entrepreneur's utility at period 0 is βC_1^u if it chooses the traditional sector.

If the entrepreneur chooses the modern sector, its consumption at period 1 is given by Eqs. (12) and (13):

$$C_1^m = (\kappa z)^{1-\alpha} \left(K_1^o + K_1^l\right)^\alpha - \tau_l K_1^l - (r_f + \delta) K_1^o - R_f f + R_f N, \tag{26}$$

with the entrepreneur's utility at period 0 being βC_1^m.

To determine whether the entrepreneur chooses the traditional sector or the modern sector, we compare the above two utility values, i.e., βC_1^u and βC_1^m.[13] The entrepreneur enters the modern sector if the utility of choosing the modern sector exceeds the utility of choosing the traditional sector. The utility comparison is equivalent to the comparison between consumption at period 1. We define Δ as the value difference between choosing the modern sector and choosing the traditional sector, i.e., $\Delta = C_1^m - C_1^u$. Obviously, Δ is a function of entrepreneurs' net worth N.

We now examine the Δ function in detail using the thresholds of net worth in Proposition 1. Proposition 1 suggests that there are a total of three cases to consider if the entrepreneur chooses the modern sector:

1) The entrepreneur is unconstrained in the modern sector (i.e., $N > N_{uc}$). The corresponding consumption at period 1 is:

$$C_1^{m,uc} = (\kappa z)^{1-\alpha} \left[\frac{r_f + \delta}{\alpha (\kappa z)^{1-\alpha}}\right]^{\frac{\alpha}{\alpha-1}} - (r_f + \delta) \left[\frac{r_f + \delta}{\alpha (\kappa z)^{1-\alpha}}\right]^{\frac{1}{\alpha-1}} - R_f f + R_f N.$$

2) The entrepreneur is financially constrained but doesn't lease in the modern sector (i.e., $N \in [N_l, N_{uc}]$). The corresponding consumption at period 1 is:

$$C_1^{m,c} = (\kappa z)^{1-\alpha} \left[\frac{N - f}{1-\theta}\right]^\alpha - (r_f + \delta) \frac{N - f}{1-\theta} - R_f f + R_f N.$$

[13]It is noteworthy that an individual entrepreneur's decision will not affect the interest rate under the linear utility assumption.

3) The entrepreneur is sufficiently constrained and leases capital in the modern sector (i.e., $N < N_l$). From the MPK formula in Lemma 1, the entrepreneur's total utilized capital is equal to:

$$\left[\frac{r_f+\delta+h}{\alpha\left(\kappa z\right)^{1-\alpha}}\right]^{\frac{1}{\alpha-1}},$$

among which the owned capital amount is $\frac{N-f}{1-\theta}$, and the rest is leased capital amount K_1^l. The corresponding consumption at period 1 is:

$$C_1^{m,l}=\left(\kappa z\right)^{1-\alpha}\left[\frac{r_f+\delta+h}{\alpha\left(\kappa z\right)^{1-\alpha}}\right]^{\frac{\alpha}{\alpha-1}}-\left(r_f+\delta\right)\frac{N-f}{1-\theta}-R_f f+R_f N-\tau_l K_1^l.$$

We define $\Delta_{uc}=C_1^{m,uc}-C_1^u$, $\Delta_c=C_1^{m,c}-C_1^u$, and $\Delta_l=C_1^{m,l}-C_1^u$. Δ_{uc} is the value difference when the entering-modern entrepreneur is unconstrained in the modern sector, Δ_c is the value difference when the entering-modern entrepreneur is constrained but doesn't lease in the modern sector, and Δ_l is the value difference when the entering-modern entrepreneur is sufficiently constrained and leases. From observing the functional forms, we note that Δ_{uc} is constant regardless of the value of N, Δ_c initially increases and then decreases with net worth N (i.e., an inverse-U shape), and Δ_l increases linearly with N. Therefore, the Δ function consists of the above three parts in their corresponding net worth regions:

$$\Delta=\begin{cases}\Delta_{uc}, & \text{if} \quad N>N_{uc},\\ \Delta_c, & \text{if} \quad N\in[N_l,N_{uc}],\\ \Delta_l, & \text{if} \quad N<N_l.\end{cases} \tag{27}$$

Before we further analyze the properties of Δ and characterize the sectoral decisions, we present a discussion on reasonable parameter regions in Lemma 2.

Lemma 2. *To make our model economy reasonable, we must have:*

- *Given the productivity z, the entry fixed cost f must satisfy: $f\in(N_{min},f_{max})$, so that entrepreneurs with net worth N_{min} will choose the traditional sector, whereas* *entrepreneurs with huge net worth will choose to enter the modern sector, in which* $f_{max} = \frac{1}{R_f}\left((\kappa z)^{1-\alpha}\left[\frac{r_f+\delta}{\alpha(\kappa z)^{1-\alpha}}\right]^{\frac{\alpha}{\alpha-1}} - (r_f+\delta)\left[\frac{r_f+\delta}{\alpha(\kappa z)^{1-\alpha}}\right]^{\frac{1}{\alpha-1}} - z^{1-\alpha}\right)$.

- *Given the productivity* z *and fixed cost* $f \in (N_{min}, f_{max})$*, the monitoring cost* h *must be smaller than* h^{upper}*, in which* h^{uppper} *is the root for* $(\kappa z)^{1-\alpha}\left[\frac{r_f+\delta+h}{\alpha(\kappa z)^{1-\alpha}}\right]^{\frac{\alpha}{\alpha-1}} - (r_f+\delta)\left[\frac{r_f+\delta+h}{\alpha(\kappa z)^{1-\alpha}}\right]^{\frac{1}{\alpha-1}} - R_f f - z^{1-\alpha} = 0$ *in the region where* $h > 0$.

Proof. See Appendix A.2.2.

Lemma 2 states that the fixed cost cannot be larger than f_{max}; otherwise no entrepreneurs would choose to enter the modern sector. The fixed cost cannot be lower than N_{min}; otherwise traditional entrepreneurs might not exist. Additionally, Lemma 2 specifies an upper bound of the monitoring cost. If the monitoring cost is too expensive, entrepreneurs will find it non-profitable to lease at all; hence, it is meaningless to discuss leasing and inconsistent with the reality.

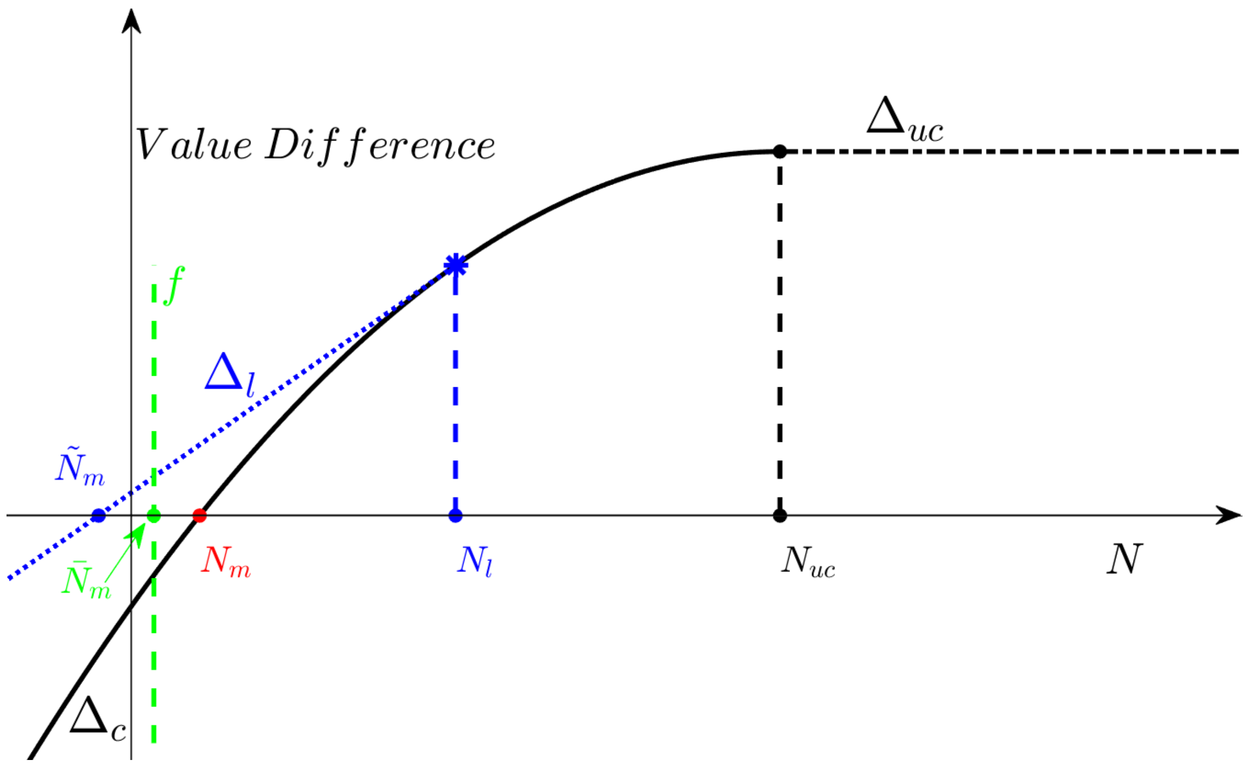
With the parameter regions in hand, we now turn our attention back to the Δ function. We use the horizontal axis (x-axis) to represent net worth and the vertical axis (y-axis) to represent the value of our Δ functions (i.e., the value difference between different options). When parameters satisfy the conditions in Lemma 2, we denote the x-intercept of Δ_l as $\tilde{N}_m$, the x-intercept of Δ_c as N_m (before N_i reaches N_{uc}), the x-coordinate of the intersection point between Δ_c and Δ_l as N_l, and the x-coordinate of the maximum point for Δ_c as $\hat{N}$. Using these notations, we plot our defined functions of value difference (Δ_l , Δ_{uc}, and Δ_c) in Figure 1. Clearly, Δ_l , Δ_{uc}, and Δ_c jointly determine Δ, following Eq. (27).

The final block is to fully connect the positive Δ to the decision of choosing the modern sector. In specific, if Δ is positive for entrepreneurs with $N < f$, we must restrict these entrepreneurs from choosing the modern sector; otherwise their owned capital amount would be negative, which is unrealistic. That is, Δ is left truncated by the entry cost f. Hence, we denote $\bar{N}_m = max(f, \tilde{N}_m)$.

The following proposition summarizes the sectoral decisions and contains the properties

Figure 1

ILLUSTRATION OF Δ FUNCTIONS



This figure shows the value difference between entering modern and traditional sectors, i.e., Δ functions in the two-period model.

of the Δ function, as displayed in Figure 1:

Proposition 2. *When parameters satisfy the conditions in Lemma 2, we obtain the following.*

1. *With the leasing market, we have:*

- *Entrepreneurs with* $N < \bar{N}_m$ *choose the traditional sector.*
- *Entrepreneurs with* $N \in [\bar{N}_m, N_{max}]$ *choose the modern sector.*
- *Whether entrepreneurs in the modern sector are financially constrained, and whether they lease capital, will follow Proposition 1.*

2. *Without the leasing market, we have:*

- *Entrepreneurs with* $N < N_m$ *choose the traditional sector.*
- *Entrepreneurs with* $N \in [N_m, N_{uc}]$ *choose the modern sector. They are financially constrained but don't lease capital.*
- *Entrepreneurs with* $N > N_{uc}$ *choose the modern sector and are unconstrained.*

3. *We can prove:*

- $f \leq \bar{N}_m < N_m < N_l < N_{uc} = \hat{N}$.
- Δ_l *is a tangent line to* Δ_c*, and* N_l *is the x-coordinate of the tangent point.*

Proof. See Appendix A.2.3.

Proposition 2 indicates that given reasonable parameters, whether an entrepreneur chooses the modern sector or the traditional sector is completely determined. Through comparing the utility of different entering options, entrepreneurs make optimal sectoral choices. The net worth threshold for choosing the modern sector varies across economies with and without leasing. When the leasing market exists, entrepreneurs with $N < \bar{N}_m$ find it unprofitable to choose the modern sector. They produce in the traditional sector and don't invest in any capital. Entrepreneurs with $N > \bar{N}_m$ choose to produce in the modern sector. For these modern entrepreneurs, Proposition 1 clearly describes their capital and borrowing choices. If the leasing market is shut down, entrepreneurs with $N < N_m$ find it more profitable to choose the traditional sector. For entrepreneurs with $N > N_m$, they choose to produce in the modern sector. In this case, modern entrepreneurs are only able to purchase capital.

Moreover, Proposition 2 outlines the properties of the Δ function. Here we emphasize three of them. First, $N_m > \bar{N}_m$, i.e., the net worth threshold for entrepreneurs to choose the modern sector is lower when leasing is in play. Intuitively, given the same net worth, leasing allows entrepreneurs to produce with more capital, generating higher output and higher utility. Entering the modern sector hence becomes more attractive and feasible, which gives a lower net worth threshold for entering the modern sector. In another word, leasing facilitates entry into the modern sector, and the set of modern sectors is larger when there is leasing. Moreover, these entering entrepreneurs will have MPK equal to $r_f + \delta + h$. Shutting down the leasing market effectively removes the blue dotted line Δ_l as a part of the final Δ function. Second, $N_{uc} = \hat{N}$. This equalization indicates that the net worth threshold in which a modern entrepreneur becomes unconstrained is exactly when Δ_c achieves the highest value. This is intuitive as the first-best level of welfare can only be achieved by the unconstrained entrepreneurs. Lastly, Δ_l is a tangent line to Δ_c, and N_l is the x-coordinate of the tangent point. This ensures that when $N < N_l$, Δ_l is always above Δ_c; that is, leasing is more beneficial for these sufficiently constrained agents, corresponding to point 1.

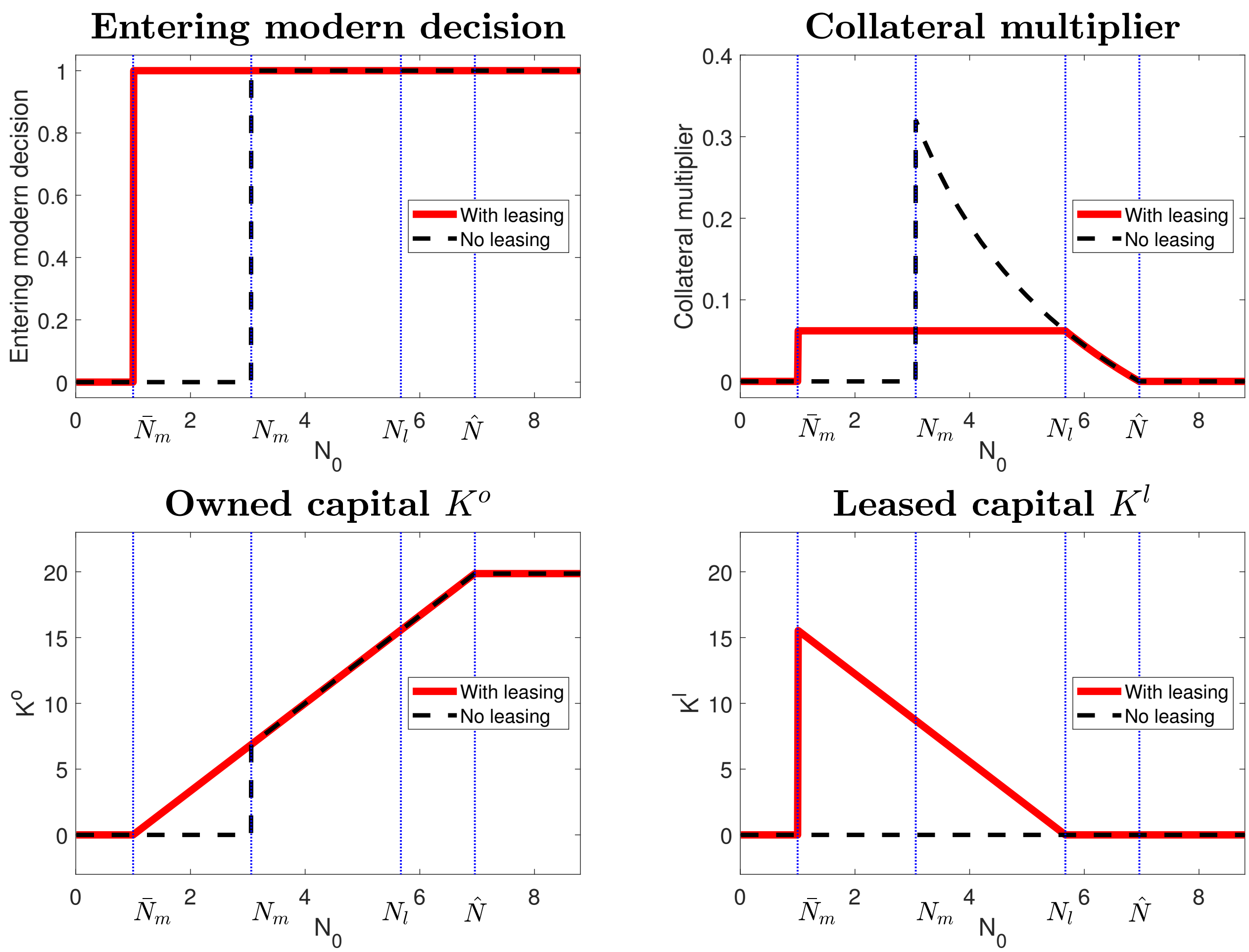
2.2.3 Numerical example

We now compute a numerical example and use it to illustrate the mechanism and the main properties of our model equilibrium. The entrepreneur's initial net worth is assumed to be uniformly distributed on $[N_{min}, N_{max}]$. To solve our model, we fit the parameters described in the caption of Figure 2.

Figure 2 displays the policy functions for entering modern sector (top left), for the value

Figure 2

EQUILIBRIUM IN A NUMERICAL EXAMPLE



The top left figure shows whether entrepreneurs choose to enter the modern sector (value=1) or to enter the traditional sector (value=0). The top right figure plots the value of the collateral constraint multiplier. The bottom figures show entrepreneurs' choices on owned capital (bottom left) and leased capital (bottom right), respectively. Red solid lines denote economies with leasing, whereas black dashed lines denote economies without leasing. Parameter values: Discount factor: $\beta = 0.93$; Curvature of production: $\alpha = 0.6$; Depreciation rate: $\delta = 0.12$; Monitoring cost for leased capital due to the separation of ownership and control: $h = 0.02$; Collateralizability in the collateral constraint: $\theta = 0.7$; Entrepreneurs' productivity: $z = 1$; The productivity gap between two sectors: $\kappa = 1.2$, following Midrigan and Xu (2014); Fixed cost: $f = 1$; and Support of net worth distribution: $N_{min} = 0.01$ and $N_{max} = 9$.

of the collateral constraint multiplier (top right), as well as for owned capital (bottom left) and leased capital (bottom right) in stationary equilibrium. We compare economies both with (red solid lines) and without the leasing market (black dashed lines). Consistent with our propositions, there exists four thresholds, $\bar{N}_m < N_m < N_l < N_{uc}$, which are highlighted with vertical lines in the figure.[14]

We first look at the red solid lines. Entrepreneurs with $N < \bar{N}_m$ choose the traditional sector. They don't borrow (and hence are unconstrained), and don't invest in any capital. Entrepreneurs with $N \in [\bar{N}_m, N_l]$ choose the modern sector and are sufficiently constrained. They invest in both leased capital and owned capital, facing a high and constant level of collateral constraint multiplier. Entrepreneurs with $N \in (N_l, N_{uc}]$ choose the modern sector and are less constrained: they only invest in owned capital and their collateral multiplier decreases in net worth. Entrepreneurs with $N \in (N_{uc}, N_{max}]$ choose the modern sector and are unconstrained: their investment of owned capital achieves the first best level.

We next study the black dashed lines, in which the leasing market is shut down. In this economy, we note that a larger number $(N_m - N_{min} > \bar{N}_m - N_{min})$ of entrepreneurs choose the traditional sector. Entrepreneurs with $N \in (N_m, N_{uc}]$ choose the modern sector and are constrained in their investment in owned capital. Entrepreneurs with $N \in (N_{uc}, N_{max}]$ have similar patterns with the policy functions with leasing.

All told, we conclude that the ability for entrepreneurs to lease not only relaxes financial constraints among entrepreneurs within the modern sector by inducing a lower collateral multiplier (the intensive margin), but also facilitates the entry of financially-constrained entrepreneurs into the modern sector (the extensive margin). We next present our efficiency analysis in this economy.

[14] We have $\bar{N}_m = f$ in this numerical example.

2.3 Efficiency analysis

2.3.1 The effect of leasing on TFP in the modern sector

In our study, the modern sector is associated with the manufacturing sector. Consequently, our analysis focuses on the TFP of the modern sector. Focusing on the modern sector is also consistent with the literature (Midrigan and Xu, 2014).

The TFP in the modern sector is calculated as:

$$TFP = \frac{\int Y_{i1}^{m} di}{(\int K_{i1} di)^{\alpha}}, \tag{28}$$

where Y_{i1}^{m} is the output and K_{i1} is the total utilized capital of the entrepreneur i, as defined before.

Combining this formula with the net worth thresholds derived earlier, we can obtain TFP with the leasing market ($\overline{TFP}$) as well as TFP without the leasing market ($\widetilde{TFP}$), as listed in Eqs. (A16) and (A17), respectively.

We next define the total TFP gain from the option to lease as the difference of Eqs. (A16) and (A17):

$$G^{TFP} = log\left(\overline{TFP}\right) - log\left(\widetilde{TFP}\right). \tag{29}$$

We can obtain the following proposition:

Proposition 3. *When parameters satisfy the conditions outlined in Lemma 2, opening up the leasing market improves total TFP compared to the economy without the leasing market, i.e., $G^{TFP} > 0$.*

Proof. See Appendix A.2.4.

Proposition 3 states that leasing increases productivity in the modern sector. Intuitively, leasing helps more entrepreneurs enter the more productive modern sector. These new entrants are entrepreneurs with high productivity over net worth ratios. That is, they are more productive but poorer. They are unlikely to enter the modern sector without the additional external financing such as leasing.

2.3.2 TFP gain decomposition

Leasing affects TFP via two channels: by inducing entry into the modern sector, as well as by affecting losses from misallocation in the modern sector. This suggests that the TFP gain of leasing can be decomposed into an accounting identity, which is the sum of " G^{TFP}_{entry}" and " G^{TFP}_{misall}:"

$$G^{TFP} = G^{TFP}_{entry} + G^{TFP}_{misall}. \tag{30}$$

To obtain the TFP gain from facilitating entry, we conduct a decomposition analysis below.

Our first step is to obtain " G^{TFP}_{misall}." We first calculate the efficient TFP (or first best TFP), TFP^e. To compute the efficient level of TFP given the set of modern entrepreneurs that operate in the original economy, we consider the problem of allocating capital across these entrepreneurs in order to maximize total output in the modern sector:

$$\max_{K_{i1}} \int_{i\in m} (\kappa z)^{1-\alpha} \left(K_{i1}^{\alpha}\right) di, \tag{31}$$

subject to the constraint that the planner uses the same amount of aggregate capital as in the original economy. The solution to this problem requires that the MPK is equalized across entrepreneurs, and the efficient level of TFP is given by:

$$TFP^e = \left(\int_{i\in m} (\kappa z)\, di\right)^{1-\alpha}.\text{[15]} \tag{32}$$

Therefore, when there is a leasing market, the TFP losses (in logs) from capital misallo-

[15] We provide details in Appendix A.2.4.

cation in the benchmark economy are:

$$\overline{\Gamma_{misall}^{TFP}} = log\left(\overline{TFP^e}\right) - log\left(\overline{TFP}\right). \tag{33}$$

If we shut down the leasing market, we can similarly obtain the TFP losses (in logs) from capital misallocation:

$$\widetilde{\Gamma_{misall}^{TFP}} = log\left(\widetilde{TFP^e}\right) - log\left(\widetilde{TFP}\right). \tag{34}$$

Therefore, we can calculate the TFP gain of leasing on reducing misallocation as:

$$G_{misall}^{TFP} = \widetilde{\Gamma_{misall}^{TFP}} - \overline{\Gamma_{misall}^{TFP}}. \tag{35}$$

This suggests that the TFP gain of leasing from facilitating entry is then:

$$\begin{aligned} G_{entry}^{TFP} &= G^{TFP} - G_{misall}^{TFP} \\ &= log\left(\overline{TFP^e}\right) - log\left(\widetilde{TFP^e}\right). \end{aligned} \tag{36}$$

We summarize the signs of " G_{entry}^{TFP} " and " G_{misall}^{TFP} " in following propositions.

Proposition 4. *When parameters satisfy the conditions outlined in Lemma 2, we have:*

$$G_{entry}^{TFP} > 0. \tag{37}$$

Proof. See Appendix A.2.5.

Opening up the leasing market allows more entrepreneurs to enter the modern sector. Along with the decreasing returns to scale we have assumed at the individual level, we see a love-for-variety effect: TFP increases with the number of entrepreneurs operating in the modern sector.

Proposition 5. *When parameters satisfy the conditions outlined in Lemma 2, enabling entrepreneurs with $N > N_m$ to lease reduces capital misallocation among them.*

Proof. See Appendix A.2.6.

Compared to the case without leasing, the ability to lease capital enables entrepreneurs that are already in the modern sector (i.e., entrepreneurs with net worth larger than N_m) to relax their financial constraints, which reduces capital misallocation (Hu, Li, and Xu, 2020). However, the ability to lease brings new entrants, and the new entrants are sufficiently constrained entrepreneurs, which has the tendency to lift capital misallocation. These are competing forces. Nevertheless, our reasonable parameterization in the numerical example as well as our calibration in the quantitative exercise (Section 4) both confirm that the sign of G^{TFP}_{misall} is indeed positive.

In the next section, we introduce a fully dynamic general equilibrium model with heterogeneous entrepreneurs to quantitatively evaluate these two channels.

3 The quantitative model

In this section, we describe the ingredients of our dynamic quantitative model of investment with an explicit leasing option. The general spirit of the model is the same as that of the two-period model presented in the previous section. The key additional elements are: i) heterogeneous entrepreneurs with persistent idiosyncratic productivity shocks; ii) heterogeneous households with labor efficiency and wage income; iii) risk-averse preferences for both entrepreneurs and households; and iv) a constant growth rate of the measure of entrepreneurs and labor efficiency. These features allow us to generate quantitatively plausible firm dynamics and heterogeneity in entrepreneurs' capital stocks in order to conduct a quantitative analysis on the efficiency implications of leasing.

3.1 Household

Time is infinite and discrete. The economy is populated by a measure one of households, and we index each household with j, which we suppress wherever appropriate. Each household has log utility preferences and maximizes its lifetime utility:

$$E_0 \sum_{t=0}^{\infty} \beta^t \log\left(C_t^w\right), \tag{38}$$

where β is the time discount factor and C_t^w is consumption at time t.

The household maximization problem is subject to the following intertemporal budget constraint:

$$C_t^w + B_{t+1}^w + K_{t+1}^l = W_t \gamma^t \nu_t + R_{ft} B_t^w + \tau_{lt} K_t^l + (1 - \delta - h) K_t^l. \tag{39}$$

At time t, the household consumes C_t^w and preserves B_{t+1}^w amount of cash for purchasing risk-free bonds. The household also serves as the lessor: it can transform net worth into K_{t+1}^l amount of leased capital and rent to entrepreneurs. The income of the household consists of the following. First, the household gets the leasing payment $\tau_{lt} K_t^l$ for the capital rented out to entrepreneurs, where τ_{lt} is the leasing fee per unit of leased capital. Second, the household gets the resale value of leased capital $(1 - \delta - h) K_t^l$ returned by entrepreneurs after production. δ is the rate of capital depreciation, and h is the monitoring cost of leased capital due to the separation of ownership and control, in line with Eisfeldt and Rampini (2009) and our two-period model. Eventually, the household receives the debt repayment $R_{ft} B_t^w$ and the labor income $W_t \gamma^t \nu_t$, where R_{ft} is the gross risk-free interest rate, and ν_t is the (idiosyncratic) labor efficiency, which grows over time at a constant rate γ. ν_t reflects the uninsurable idiosyncratic labor income risk faced by the household.

In this setup, the first-order condition of K_t^l implies that $R_{ft} = \tau_{lt} + 1 - \delta - h$. That is, the household faces a no-arbitrage condition between the returns on supplying risk-free bonds and on supplying leased capital.

3.2 Entrepreneur

The economy is also populated by a measure Θ_t of entrepreneurs. At each period t, a measure $(\gamma - 1)\Theta_t$ of new entrepreneurs are born. That is, the measure of entrepreneurs grows over time at a constant rate γ. The newly-born entrepreneurs are endowed with zero net worth, and they start with a traditional sector that is associated with an unproductive technology using labor as the only input.[16] Over time, they have the option to choose to enter a modern sector that uses capital and labor under a more productive technology. Again, we index each entrepreneur with i and suppress the index i wherever appropriate.

3.2.1 Traditional sector

Entrepreneurs in the traditional sector u face a decreasing returns technology that produces output Y_t^u using labor L_t^u:

$$Y_t^u = (z^p z_t)^{1-\eta} (L_t^u)^\eta . \tag{40}$$

Here, $\eta < 1$ is the degree of returns to scale, z^p is a permanent component of the entrepreneur's productivity, and z_t is a transitory productivity component that evolves over time.

Let B_t^u denote the entrepreneur's debt position in the traditional sector. The problem of a traditional entrepreneur is to maximize its life-time utility $U(C_t^u)$ given by:

$$E_0 \sum_{t=0}^{\infty} \beta^t \log(C_t^u) , \tag{41}$$

where C_t^u is the traditional entrepreneur's consumption at time t.

[16]This slightly differs from our two-period setting, in which entrepreneurs decide which sector to enter upon being born and are endowed with positive net worth.

The budget constraint that the entrepreneur in the traditional sector faces depends on whether it decides to remain in the traditional sector or switch to the modern sector. The entrepreneur who stays in the traditional sector earns profits $Y_t^u - W_t L_t^u$, and chooses how much of their income to save and to consume. Its budget constraint is:

$$C_t^u = Y_t^u - W_t L_t^u - R_{ft} B_t^u + B_{t+1}^u, \tag{42}$$

where W_t is the equilibrium wage in this economy. These entrepreneurs are unable to borrow and so $B_{t+1}^u \leq 0$.

We next consider the problem of traditional entrepreneurs that enter the modern sector. Entering the modern sector requires an up-front investment equal to $z^p f$ units of output. We assume that this sunk cost is proportional to the permanent productivity component so that even the most productive entrepreneurs face a non-trivial cost of entering the modern sector.

The entrepreneur who enters the modern sector finances expenditures on its owned physical capital, K_{t+1}^o, and the fixed cost, $z^p f$, using either its internal funds, or by borrowing from one-period risk-free debt B_{t+1}^m. Hence the budget constraint is:

$$C_t^m + K_{t+1}^o + z^p f = Y_t^u - W_t L_t^u - R_{ft} B_t^u + B_{t+1}^m. \tag{43}$$

The amount the entrepreneur can borrow is limited by a collateral constraint which requires that its debt does not exceed a fraction of the sum of its owned physical capital and the fixed cost:

$$B_{t+1}^m \leq \theta K_{t+1}^o, \tag{44}$$

where $\theta \in [0, 1]$ characterizes the strength of financial frictions in this economy.

3.2.2 Modern sector

For entrepreneurs in the modern sector m, the technology with which they operate is:

$$Y_t^m = \kappa^{1-\eta} \left(z^p z_t\right)^{1-\eta} \left(K_t^o + K_t^l\right)^{\alpha\eta} \left(L_t^m\right)^{(1-\alpha)\eta}, \tag{45}$$

where α governs the share of total capital in production, K_t^o is the amount of owned capital, K_t^l is the amount of leased capital, and $\kappa \geq 0$ determines the relative productivity of the modern sector.

Entrepreneurs' utility in the modern sector $U(C_t^m)$ takes the same form as that of entrepreneurs in the traditional sector. Modern entrepreneurs can save and borrow at the risk-free rate R_{ft}, subject to the collateral constraint in Eq. (44). The budget constraint can be summarized as:

$$C_t^m + K_{t+1}^o - (1-\delta)K_t^o = Y_t^m - \tau_{lt}K_t^l - R_{ft}B_t^m - W_t L_t^m + B_{t+1}^m. \tag{46}$$

The right-hand side of this constraint states that at each time t, the entrepreneur in the modern sector produces output Y_t^m, pays back wage $W_t L_t^m$, bond and interest $R_{ft}B_t^m$, as well as the leasing fees $\tau_{lt}K_t^l$. Combined with the borrowing B_{t+1}^m, the modern entrepreneur determines its consumption C_t^m and the new capital amount to be purchased $K_{t+1}^o - (1-\delta)K_t^o$, as the left-hand side suggests. For the depreciated leased capital, the entrepreneur will return it to the household after production.

3.3 Equilibrium

In this part, we consider the competitive equilibrium of our dynamic model. We define $N_t = K_t^o - B_t^m$ as the entrepreneur's net worth at time t in the modern sector m, and define $N_t = -B_t^u$ as the entrepreneur's net worth at time t in the traditional sector u.[17] Net worth is endogenously determined in the dynamic model. Consistent with our two-period model, we assume that capital and borrowing decisions are made after observing the next period's productivity for modern-sector entrepreneurs. This timing assumption allows us to reformulate the entrepreneur's problem into a two-step procedure, and conveniently reduces the state-space by only containing entrepreneurs' net worth and productivity.[18] Because all of our key variables are homogeneous of degree one in net worth and z^p, we rescale them by dividing z^p. We rewrite the entrepreneur's problem in different sectors in recursive form.[19] We provide details in Appendix B.1.

An equilibrium in this full dynamic economy is defined in the usual way. That is, a balanced growth equilibrium is a set of prices W, R_f, and τ_l, policy functions for households' consumption $C_t^w(N,\nu)$, saving $B_{t+1}^w(N,\nu)$, and accumulated leased capital $K_{t+1}^l(N,\nu)$, for modern entrepreneurs' consumption $C_t^m(N,z)$, net worth $N_{t+1}^m(N,z)$, output $Y^m(N,z)$, labor $L^m(N,z)$, owned capital $K^o(N,z)$, and leased capital $K^l(N,z)$, for traditional entrepreneurs' consumption $C_t^u(N,z)$, net worth $N_{t+1}^u(N,z)$, output $Y^u(N,z)$, labor $L^u(N,z)$, as well as a decision on whether to enter the modern sector $\psi(N,z)$ that: (i) solve the entrepreneurs' and households' optimization problems; (ii) satisfy the market clearing conditions for the bond market, the leased capital market, and the labor market, respectively; and (iii) the law of motion for measure of entrepreneurs in both traditional and modern sectors. That is, the measure of modern entrepreneurs is the sum of the original measure of modern entrepreneurs and those entrepreneurs in the traditional sector that decide to enter; the measure of traditional entrepreneurs is the sum of the measure of entrepreneurs that decide to stay in the traditional sector and newly-born entrepreneurs; and the total measures of entrepreneurs

[17]We unify "net worth" in two sectors since the traditional entrepreneurs can be considered as having zero capital, $K_t^o = 0$.

[18]This assumption simplifies our analysis by rendering the choice of capital and labor static, and also allows us to focus solely on the role that financial frictions play in distorting the allocation of capital among entrepreneurs.

[19]To save notations, we use the original notations to denote variables after rescaling (i.e., relative to permanent productivity). Accordingly, the state variables for each entrepreneur become net worth N and transitory productivity z.

across two sectors are γ^t. The details are provided in Appendix B.2.

The variables with time subscripts all grow at a constant rate γ, while the remaining variables are time-invariant. To solve for the balanced growth equilibrium, we rescale the growing variables by the growth rate and solve the resulting stationary system based on Aiyagari (1994).

4 Calibration and quantitative analysis

In this section, we first calibrate our model and evaluate its ability to account for key features of the economy. We then provide a quantitative analysis of how leasing relaxes financial frictions through the extensive margin, and evaluate its implications for TFP. We focus on a long sample of US annual data for our calibration. Appendix C provides more details of the data we use.

4.1 Calibration

We calibrate our model at the annual frequency and present the parameters in Table 2. We group our parameters into three blocks. The parameters in the first block can be determined based on the literature. We set $\gamma = 1.025$ to roughly match the growth of US real output. We choose a standard value of β equal to 0.92γ. The capital share α is assumed to be 0.39, slightly higher than the standard value, reflecting the fact that factoring in leased capital drives up the capital share (Hu, Li, and Xu, 2022). The span of control parameter η is set to be 0.85, consistent with Basu and Fernald (1997) and Atkeson and Kehoe (2005). Since we assume a period length of one year, we set the rate of depreciation to $\delta = 0.06$.

We set $\theta = 0.37$ using the collateralizability score estimated in Ai et al. (2020) and Li, Whited, and Wu (2016). We follow Midrigan and Xu (2014) and assume that the labor's efficiency can be on and off, which allows us to feature incomplete markets and precautionary saving motives. We assign the probability of remaining on (λ_1) and remaining off (λ_0), as well as the value of labor efficiency ν to (i) roughly match the employment to population ratio in the US, and to (ii) normalize the total labor supply to one. In line with Midrigan and Xu (2014) and Fried and Lagakos (2020), we assume that an entrepreneur's efficiency in the modern sector is on average 20 percent larger than the efficiency of the traditional sector, i.e., the productivity gap satisfies $(1-\eta)\log(\kappa) = 0.20$. We also consider alternative values of κ and show how our results vary in robustness checks.

Table 2

PARAMETER VALUES IN THE DYNAMIC MODEL

Description	**Parameter**	**Value**
Block A: Assigned parameters		
Growth rate	γ	1.03
Discount factor	β	0.92
Capital share	α	0.39
Capital depreciation	δ	0.06
Span of control	η	0.85
Collateralizability	θ	0.37
Persistence unit worker state	λ_1	0.80
Persistence zero worker state	λ_0	0.50
Labor efficiency	ν	1.40
Productivity gap	$(1-\eta)\log(\kappa)$	0.20
Block B: Calibrated parameters		
Monitoring cost	h	0.08
Fixed cost of entering	f	0.76
Block C: Productivity parameters		
Persistence of idiosyncratic transitory shocks	ρ	0.79
Std. Dev. of idiosyncratic transitory shocks	σ_z	0.42
Std. Dev. of exogenous permanent component	σ_{z^p}	1.69

This table reports the parameter values we used in the calibration procedure. We calibrate the model at annual frequency.

The parameters in the second block are determined by matching a set of first moments to their empirical counterparts. In particular, we calibrate the monitoring cost h to match the average leased capital ratio in the US. The fixed cost of entering the modern sector is chosen to ensure that the total output in the traditional sector to the total output in both sectors is equal to 10 percent.[20]

The last block contains the parameters related to the idiosyncratic transitory productivity shocks, ρ and σ_z, as well as the volatility of the permanent component, σ_{z^p}. We set $\rho = 0.79$, consistent with typical estimates in the literature (Zhang, 2005; Gopinath et al., 2017; David, Schmid, and Zeke, 2022). We choose σ_z and σ_{z^p} to jointly match the standard deviation of output and output growth in the US.[21]

4.2 Model fit

Below we turn to the quantitative performance of our model. Table 3 reports the model-simulated moments and compares them with the counterparts in the data. The upper panel shows the targeted moments, and the bottom panel shows the results for non-targeted moments.

We first look at the moments related to levels. Our calibration produces a low risk-free rate (2%), which is close to the value of 2.2% in the data. The debt to output ratio is 0.81 and consumption to investment ratio is 4.1, both in line with the data. The fraction of modern labor 0.85 also closely matches with the data (0.89).[22]

Turning the attention to the distributional moments, our benchmark model accounts well for the variability of the levels and growth rates of total capital and employment in the data. In the model, if we eliminate financial constraints, capital and employment are proportional

[20] In our study, the modern sector is associated with the manufacturing sector and the traditional sector is associated with the agriculture sector. We measure output using value-added, and rely on BEA real GDP data at the industry level to obtain this moment.

[21] Setting ρ arbitrarily and then finding the doublet (σ_z, σ_{z^p}) that matches targets is always a feasible strategy (Clementi and Palazzo, 2016).

[22] We utilize the employment data from BEA in calculating this moment.

Table 3

MODEL FIT

Description	Data	Model
Calibrated moments		
Std. Dev. of output	1.77	1.77
Std. Dev. of output growth	0.27	0.27
Fraction output, modern	0.90	0.90
Leased capital ratio	0.13	0.13
Untargeted moments		
Interest rate, %	2.2	2.0
Debt to output	0.81	0.81
Consumption to investment	4	4.1
Fraction labor, modern	0.89	0.85
Std. Dev. of total capital	1.80	1.78
Std. Dev. of total capital growth	0.23	0.23
Std. Dev. of labor	1.65	1.77
Std. Dev. of labor growth	0.20	0.27
Std. Dev. of leased capital ratio	0.24	0.31
Autocorrelation output	0.98	0.99
Autocorrelation labor	0.98	0.99
Autocorrelation total capital	0.99	0.99
Autocorrelation leased capital ratio	0.90	0.92
Correlation of leased capital ratio and MPK	0.11	0.62
Correlation of total capital and MPK	-0.24	-0.41
Correlation of labor and MPK	-0.07	-0.08

This table reports additional statistics in the data and from the model simulation. The "Data" column reports the empirical moments. The "Model" column reports the model-implied moments. We simulate the economy at annual frequency, based on the calibration parameters in Table 2.

to output and thus should be equally volatile. This pattern is consistent with the data in which all output moments are broadly similar for capital and employment. Also, our model can reproduce the volatility of leased capital ratio (0.31), a value broadly consistent with its counterpart in the data across US Compustat firms.

Finally, we report the correlations in the time series and in the cross-section. We note that our model is successful in replicating the auto-correlations of key quantities. This jointly reflects our parameters on the persistence of the transitory productivity component and the variance of the permanent component. With respect to the untargeted cross-sectional moments, we find that our model is able to reproduce the negative correlations of MPK with total capital, employment, and leased capital ratio in the data. The magnitudes of the above correlations generated from our model are also broadly consistent with the data.

To understand the signs of the cross-sectional correlations, first we note that MPK is uncorrelated with capital and employment in a model without financial frictions, since entrepreneurs can optimally adjust (up) their capital and employment to a high productivity while achieving a constant unconstrained MPK. Adding financial frictions effectively makes entrepreneurs less responsive to a higher productivity. These high productivity entrepreneurs become constrained - they tend to have high MPK but insufficient capital and employment, which generates the negative correlations. Financially constrained entrepreneurs also tend to have high leased capital ratios, which implies that MPK is positively correlated with the leased capital ratio. In sum, our model is successful in generating outcomes that resemble those observed in the data.

4.3 Quantitative results

4.3.1 Benchmark economy and the role of leasing

Given our calibration, we evaluate the role of leasing in reducing financial friction-induced inefficiency through the extensive and intensive margins quantitatively. As our dynamic model features richer ingredients, we extend the decomposition analysis in Section 2.3.2 of our two-period setting accordingly.

In specific, TFP losses (in logs) from capital misallocation are:

$$\Gamma^{TFP}_{misall} = \underbrace{log\left(\kappa\int z_i di\right)^{1-\eta}}_{TFP^e} - \underbrace{log\left(\kappa^{1-\eta}\frac{\left[\int z_i\left(MPK_i\right)^{\frac{\alpha\eta}{\eta-1}} di\right]^{1-(1-\alpha)\eta}}{\left\{\int z_i\left(MPK_i\right)^{\frac{(1-\alpha)\eta-1}{1-\eta}} di\right\}^{\alpha\eta}}\right)}_{TFP}. \tag{47}$$

Here, TFP is obtained by integrating the decision rules for labor and capital across the existing modern entrepreneur i, and TFP^e is obtained by asking a social planner to maximize total output subject to the total amount of resourced already allocated to the set of entrepreneurs that operate in the original economy.[23]

We compare the leasing-economy with the no-leasing economy. This gives: i) the total TFP gains (G^{TFP}): $G^{TFP} = log\left(\overline{TFP}\right) - log\left(\widetilde{TFP}\right)$, where $\overline{TFP}$ is the TFP with leasing and $\widetilde{TFP}$ is the TFP without leasing; ii) the gain of leasing from affecting misallocation (G^{TFP}_{misall}): $G^{TFP}_{misall} = \widetilde{\Gamma^{TFP}_{misall}} - \overline{\Gamma^{TFP}_{misall}}$, where $\overline{\Gamma^{TFP}_{misall}}$ and $\widetilde{\Gamma^{TFP}_{misall}}$ denote the TFP losses from misallocation when there is a leasing market and when the leasing market is shut down, respectively; and iii) most importantly, the extensive-margin gain from leasing (G^{TFP}_{entry}): $G^{TFP}_{entry} = G^{TFP} - G^{TFP}_{misall} = log\left(\overline{TFP^e}\right) - log\left(\widetilde{TFP^e}\right)$, where $\overline{TFP^e}$ is the efficient TFP with leasing and $\widetilde{TFP^e}$ is the efficient TFP without leasing. G^{TFP}_{entry} is obtained according to an accounting identity (as in Eq. (30)) and is our focus in this paper.

Using Eq. (47), we find that in our benchmark economy (Column "Benchmark" with label "W" of Table 4), the TFP losses from misallocation in the modern sector are about 2 percent, consistent with Hu, Li, and Xu (2020) and Gilchrist, Sim, and Zakrajšek (2013).[24]

[23] We can further simplify Eq. (47) to $\Gamma^{TFP}_{misall} = \frac{1}{2}\frac{\alpha\eta(1-(1-\alpha)\eta)}{1-\eta}\,\mathrm{var}\left(\log MPK_i\right)$, under the assumption that MPK_i and z_i are jointly log normal distributed. This simplification indicates that TFP losses increase in MPK dispersion (Hsieh and Klenow, 2009). Also, TFP losses depend on the curvature in the production function, and the relative shares of capital and labor, as manifested in α and η.

[24] For a comparison, our estimate is within the same ballpark of prior studies (Buera, Kaboski, and Shin, 2011; Midrigan and Xu, 2014). Gilchrist, Sim, and Zakrajšek (2013) use direct measures of firms' borrowing

In addition, we observe that 97 percent of modern entrepreneurs are financially constrained, whereas 30 percent of them lease in this benchmark. In Column "Benchmark" with label "W/O" of Table 4, we report the key statistics in which the leasing market is artificially shut down.[25] Now, all modern entrepreneurs are financially constrained, and none of them lease. Naturally, the leased capital ratio becomes zero.

Table 4

AGGREGATE IMPLICATIONS OF LEASING

	Benchmark		$\theta = 0.75$		$\theta = 0.50$		$\theta = 0.25$	
Statistics	W	W/O	W	W/O	W	W/O	W	W/O
Fraction constrained	0.97	1	0.47	0.48	0.85	0.90	0.99	1
Debt to output (modern)	0.81	1.06	1.47	1.54	1.11	1.27	0.49	0.73
Fraction leased	0.30	0	0.09	0	0.20	0	0.43	0
Leased capital ratio	0.13	0	0.02	0	0.07	0	0.24	0
TFP (modern)	1.13	1.04	1.15	1.14	1.14	1.10	1.13	0.99
Losses from misallocation, %	2.0	5.9	1.3	1.9	2.1	4.2	1.4	6.8
Leasing in reducing misall, %	3.9		0.7		2.1		5.4	
Leasing in inducing entry, %	4.6		0.5		1.4		7.8	
Fraction output modern	0.90	0.82	0.92	0.92	0.91	0.88	0.89	0.73
Output	1.69	1.40	1.80	1.76	1.73	1.60	1.65	1.16
Consumption	1.43	1.35	1.51	1.50	1.47	1.43	1.40	1.25

This table reports the implications of our model simulations. We consider the benchmark model, as well as models under different values of the collateralizability parameter θ. "W" denotes the model economy with leasing, whereas "W/O" denotes the model economy without leasing.

The difference between these two economies reveals the effect of leasing, which includes higher output, consumption, and TFP. Specifically, the ability for entrepreneurs to lease increases output, consumption, and TFP by 19 percent, 6 percent, and 9 percent, respectively. Out of the total TFP gains, we find that approximately 4 percentage points come from the role of leasing in reducing intensive margin misallocation of capital among entrepreneurs,

costs to infer TFP losses from financial frictions - on the order of 2 to 4 percent. This number corresponds to (and matches) the TFP losses from misallocation under our benchmark model with leasing.

[25] We use the parameter values from the benchmark experiment here and solve the new equilibrium wage and interest rate.

while the rest (5 percentage points) come from the extensive margin through facilitating entry and technology adoption. The latter accounts for 55%, indicating the role that leasing plays at the extensive margin is more important than the intensive margin in improving TFP. This extensive-channel effect is manifested by a larger fraction of modern-sector output in the economy with leasing (90 %) relative to the economy without leasing (82 %).[26]

4.3.2 Implications under alternative parameters

We now discuss how our quantitative results change with respect to changes in three parameters: the entrepreneur's ability to borrow θ (equal to 0.37 in the benchmark calibration), the fixed cost of entering f (equal to 0.76 in the benchmark calibration), as well as the productivity gap between the modern and traditional sectors $log(\kappa)$ (equal to $\frac{0.2}{1-\eta} = 1.33$ in the benchmark calibration).

Variations in collateral constraint We solve the model for $\theta = 0.75, 0.5, \text{and}\, 0.25$. We compare the economies with and without leasing under these θs, and report the results in the last six Columns of Table 4.

For economies without leasing (Columns with label "W/O"), we note that when θ declines, the debt to output ratio reduces and the fraction of modern entrepreneurs that are constrained increases. This is intuitive, as a lower θ is associated with a tighter financial constraint, which limits the entrepreneur's ability to borrow. Also, in response to a decline of θ, we observe a larger dispersion in MPK: misallocation losses increase from 1.9% to 6.8% when θ decreases from 0.75 to 0.25. The rising pattern originates from two sources: on the one hand, a tighter financial constraint lifts the average shadow costs of fund and generates a larger dispersion in shadow costs; on the other hand, the interest rate decreases in response to the decline

[26]Comparing across the factor prices (i.e., interest rate and wage rate that clear the market), we find that both factor prices leasing economies are consistently higher than their counterparts in no-leasing economies. The reason is that leasing brings more production sources back. Higher factor prices may, however, weaken our results. Nonetheless, our quantification shows that such effects are very trivial.

of θ, which leads to a greater desired level of capital stock for entrepreneurs and a longer time for constrained entrepreneurs to catch up to unconstrained entrepreneurs. In Row 5, we observe a reduced TFP of the modern sector (from 1.14 to 0.99 when θ decreases from 0.75 to 0.25). Comparing this TFP decline (14%) to the rise of misallocation losses, we conclude that the bulk of the TFP decline is due to the considerable drop in the fraction of modern entrepreneurs, whose modern-sector output fraction is close to 0.92 when $\theta = 0.75$ and drops to 0.73 when $\theta = 0.25$.

Similar patterns can be found in economies with leasing (Columns with label "W"). More importantly, the leased capital ratio rises when θ drops, reflecting the fact that leasing is valued more by financially constraint borrowers. Relative to the no-leasing economies, we find a smaller drop in the interest rate when θ declines. This is due to the presence of leasing, which channels resources back into production and takes over some of the negative interest rate adjustment when collateral constraints tighten. Next we focus on the role of leasing in improving modern-sector TFP. When θ is high, in which financial constraint almost disappears, it is intuitive that leasing plays a trivial role (TFP gains are only 1.2% when θ is 0.75). Output and consumption are also close to those in the economy allocation without leasing. For a rather low θ, in which financial constraints distort the economy most, we clearly see that leasing could improve the modern-sector TFP by over 13 percent. Such an increasing pattern is present at both the intensive and extensive margins. Importantly, the bulk of the improvement from leasing is documented in the extensive margin: its proportion increases from 40% to 60% when θ decreases from 0.75 to 0.25. Intuitively, when θ is lower, the proportion of entrepreneurs that are prevented from entrance into the modern sector is substantially higher, and the economy suffers greater losses from the extensive margin. Hence, leasing becomes more crucial in facilitating entry at the extensive margin.

Variations in entry fixed cost We next consider how changes in the entry fixed cost affect our results. The first four columns in Table 5 suggest that a low entry fixed cost weakens the effect of leasing at the extensive margin: TFP gains drop from 12 percent to 1 percent. This is intuitive because a low entry cost itself already facilitates entry of many entrepreneurs into the modern sector. In fact, in the no-leasing economy associated with a low entry cost, the modern output fraction is already very high (93%), which is close to the number when there is a leasing market (95%).

Table 5

ALTERNATIVE PARAMETERS

	Benchmark		$f = 0.5$**x**		$f = 1.5$**x**		$\log(\kappa) = 2$**x**		$\log(\kappa) = 0$**x**	
Statistics	W	W/O	W	W/O	W	W/O	W	W/O	W	W/O
Fraction constrained	0.97	1	0.94	0.99	0.99	1	0.96	1	1	1
Debt to output (modern)	0.81	1.06	0.78	0.98	0.83	1.26	0.79	1.01	0.84	1.27
Fraction leased	0.30	0	0.29	0	0.33	0	0.29	0	0.36	0
Leased capital ratio	0.13	0	0.12	0	0.16	0	0.13	0	0.19	0
TFP (modern)	1.13	1.04	1.18	1.13	1.05	0.89	1.25	1.16	0.94	0.83
Losses from misallocation, %	2.0	5.9	2.1	5.4	1.8	7.2	2.0	6.5	1.6	5.2
Leasing in reducing misall, %	3.9		3.3		5.4		4.5		3.6	
Leasing in inducing entry, %	4.6		1.3		11.6		3.2		8.6	
Fraction output modern	0.90	0.82	0.95	0.93	0.80	0.60	0.94	0.89	0.72	0.53
Output	1.69	1.40	2.01	1.92	1.70	1.45	2.20	1.99	1.45	1.33
Consumption	1.43	1.35	1.53	1.49	1.32	1.18	1.67	1.55	1.14	1.09

This table reports the implications of our model simulations under alternative parameters of fixed cost f and productivity gap κ. "W" denotes the model economy with leasing, whereas "W/O" denotes the model economy without leasing.

Variations in productivity gap Finally, we explore how the gains from leasing vary with the size of the productivity gap between two sectors. We study two experiments: a 40 percent productivity gap and a 0 percent productivity gap. The last four columns of Table 5 report our results. There are two noteworthy messages. First, leasing increases the fraction of output in the modern sector under different gaps. Second, the role of leasing in improving TFP by facilitating entry becomes more important relative to the intensive margin when the productivity gap is low. All else constant, a higher productivity gap increases the output per modern entrepreneur, making the modern sector more attractive, which effectively makes the fixed cost less of a concern. Hence, financial friction-induced losses at the extensive margin are already small, resulting in a smaller role of leasing in facilitating entry.

5 Conclusion

As an important proportion of productive assets, leased capital has been largely ignored in the macro-finance literature, due to the fact that it does not show up on firms' balance sheets under previous lease accounting standards. In this paper, we study a novel role that leasing plays in improving TFP through the extensive margin: leasing has positive effects on the number of entrepreneurs that operate and on the technology level these entrepreneurs adopt. As a strong form of collateralizable financing, leasing helps entrepreneurs overcome the barriers to entry and to technology adoption in the presence of financial constraints. We develop a general equilibrium model with collateral constraints, sectoral choices and buy versus lease decisions to formalize our intuitions. First, in a simplified two-period setting, we analytically characterize leasing's role in facilitating entry and in generating efficiency gains. Second, we analyze a full dynamic general equilibrium model that closely matches various moments estimated from production and balance sheet data. Our quantitative analysis indicates that the extensive-margin channel induced from leasing can generate 5% TFP gains. Also, our calibrated model allows us to contrast this extensive-margin channel with the intensive-margin channel, in which leasing mitigates capital misallocation through reducing the inefficient dispersion of entrepreneurs' marginal product of capital. We find that the former (extensive-margin) channel has potentially more sizable positive effects in terms of efficiency gains when compared to the other.

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Online Appendix

A Derivations for the two-period model

A.1 Lagrangian

To facilitate discussion, we bring back the index i for entrepreneurs. We present the Lagrangian of entrepreneur i under our simplifying assumptions in the two-period model.

A.1.1 Modern sector

If the entrepreneur i chooses the modern sector, its Lagrangian is:

$$\begin{array}{rcl} L_i & = & \max\left[\beta C_{i1}^m\right] \\ + & \eta_{i0} & \left[\ N_0 + B_{i1}^m - f - K_{i1}^o\ \right] \\ + & \eta_{i1} & \left[(\kappa z)^{1-\alpha}\left(K_{i1}^o + K_{i1}^l\right)^{\alpha} - \tau_l K_{i1}^l - R_f B_{i1}^m + (1-\delta)K_{i1}^o - C_{i1}^m\right] \\ + & \xi_{i0}\eta_{i0} & \left[\theta K_{i1}^o - B_{i1}^m\right] \\ + & \bar{\nu}_{i0}\eta_{i0} & \left[K_{i1}^o\right] \\ + & \underline{\nu}_{i0}\eta_{i0} & \left[K_{i1}^l\right] \\ + & \bar{\nu}_{d1}^i & \left[C_{i1}^m\right]. \end{array}$$

FOC:

$$[C_{i1}^m] : \beta - \eta_{i1} = 0.$$

$$\begin{aligned} [K_{i1}^o] \quad : \quad & -\eta_{i0} + \eta_{i1}\left[(\kappa z)^{1-\alpha}\alpha\left(K_{i1}^o + K_{i1}^l\right)^{\alpha-1} + (1-\delta)\right] \\ & +\xi_{i0}\eta_{i0}\theta + \bar{\nu}_{i0}\eta_{i0} = 0. \end{aligned}$$

$$
\begin{aligned}
[K^l_{i1}] \quad : \quad & \eta_{i1}\left[(\kappa z)^{1-\alpha}\,\alpha\left(K^o_{i1}+K^l_{i1}\right)^{\alpha-1}-\tau_l\right] \\
& +\underline{\nu}_{i0}\eta_{i0}=0.
\end{aligned}
$$

$$
[B_{i1}] : \eta_{i0} - R_f\eta_{i1} - \xi_{i0}\eta_{i0} = 0.
$$

A.1.2 Traditional sector

If the entrepreneur i chooses the traditional sector, its Lagrangian is:

$$
\begin{aligned}
L_i \quad & = \quad \max\left[\beta C^u_{i1}\right] \\
+ \quad \eta^u_{i0} \quad & \left[\; N_i + B^u_{i1} \;\right] \\
+ \quad \eta^u_{i1} \quad & \left[\; z^{1-\alpha} - R_f B^u_{i1} - C^u_{i1} \;\right] \\
+ \quad \bar{\nu}^i_{u,d1} \quad & \left[C^u_{i1}\right].
\end{aligned}
$$

FOC:

$$
[C^u_{i1}] : \beta - \eta^u_{i1} + \bar{\nu}^i_{u,d1} = 0.
$$

$$
[B_{i1}] : \eta^u_{i0} - R_f\eta^u_{i1} = 0.
$$

$$
[C^u_{i1}] : \bar{\nu}^i_{u,d1} = 0.
$$

Hence,

$$
\eta^u_{i0} = 1.
$$

A.2 Propositions

A.2.1 Proposition 1

Conditional on entrepreneurs *already* in the modern sector, we combine the Lagrangian and the entrepreneur's buy versus lease decisions. Under the collateral constraint, the maximum owned capital amount is $\frac{N_i - f}{1-\theta}$.

The MPK formulas suggest that:

- When entrepreneur i is unconstrained, it will not borrow up to the limit nor use the costly leased capital. All of its capital is owned capital and its MPK is:

$$\alpha(\kappa z)^{1-\alpha}(\text{ owned capital })^{\alpha-1} = r_f + \delta.$$

Hence the amount of owned capital is:

$$\left[\frac{r_f + \delta}{\alpha\,(\kappa z)^{1-\alpha}}\right]^{\frac{1}{\alpha-1}}.$$

Therefore, when $\left[\frac{r_f+\delta}{\alpha(\kappa z)^{1-\alpha}}\right]^{\frac{1}{\alpha-1}} \leq \frac{N_i-f}{1-\theta}$, i.e., when $N_i > N_{uc} = \left[\frac{r_f+\delta}{\alpha(\kappa z)^{1-\alpha}}\right]^{\frac{1}{\alpha-1}}(1-\theta)+f$, entrepreneur i is unconstrained.

- If entrepreneur i is constrained and uses leased capital, it borrows up to the limit so owned capital is $\frac{N_i-f}{1-\theta}$. Its MPK is:

$$\alpha(\kappa z)^{1-\alpha}(\text{ total utilized capital })^{\alpha-1} = r_f + \delta + h.$$

Hence the total utilized capital is $\left[\frac{r_f+\delta+h}{\alpha(\kappa z)^{1-\alpha}}\right]^{\frac{1}{\alpha-1}}$. When the total utilized capital is larger than owned capital, i.e., when $N_i < N_l = \left[\frac{r_f+\delta+h}{\alpha(\kappa z)^{1-\alpha}}\right]^{\frac{1}{\alpha-1}}(1-\theta)+f$, the entrepreneur leases capital. It is worth noticing that our thresholds are conditional on an entrepreneur's entering the modern sector. This has additional requirements, which we present in Proposition 2 and Appendix A.2.3.

- Naturally, if entrepreneur i is constrained but doesn't lease, its initial net worth N_i satisfies:

$$N_i \in (N_l, N_{uc}].$$

A.2.2 Lemma 2

First, we want to ensure that entrepreneurs with sufficient net worth will choose to enter the modern sector; otherwise, no entrepreneurs will choose the modern sector and the problem becomes meaningless. Utilizing the total capital in Lemma 1, we thus have the following requirement:

$$(\kappa z)^{1-\alpha}\left[\frac{r_f+\delta}{\alpha(\kappa z)^{1-\alpha}}\right]^{\frac{\alpha}{\alpha-1}}-(r_f+\delta)\left[\frac{r_f+\delta}{\alpha(kz)^{1-\alpha}}\right]^{\frac{1}{\alpha-1}}-R_f f-z^{1-\alpha}>0. \tag{A1}$$

That is to say, given z and other commonly used parameters, we must have:

$$f<f_{max}=\frac{1}{R_f}\left((\kappa z)^{1-\alpha}\left[\frac{r_f+\delta}{\alpha\left(\kappa z\right)^{1-\alpha}}\right]^{\frac{\alpha}{\alpha-1}}-(r_f+\delta)\left[\frac{r_f+\delta}{\alpha\left(\kappa z\right)^{1-\alpha}}\right]^{\frac{1}{\alpha-1}}-z^{1-\alpha}\right). \tag{A2}$$

Next, we want to ensure that there always exist traditional entrepreneurs. This requires that entrepreneurs with the lowest net worth must choose the traditional sector. In other words, they are unable to pay the fixed cost from their net worth, which leads to $f>N_{min}$. QED.

Now, we must ensure that it would be beneficial for entrepreneurs to lease; that is, given z and $f\in(N_{min},f_{max})$, we must have positive $\Delta_l(N_l)$. This means that:

$$(\kappa z)^{1-\alpha}\left[\frac{r_f+\delta+h}{\alpha(\kappa z)^{1-\alpha}}\right]^{\frac{\alpha}{\alpha-1}}-(r_f+\delta)\left[\frac{r_f+\delta+h}{\alpha(kz)^{1-\alpha}}\right]^{\frac{1}{\alpha-1}}-R_f f-z^{1-\alpha}>0. \tag{A3}$$

Denote a function of the LHS of the above equation. We take the FOC of this function with respect to h and obtain:

$$\kappa z \alpha^{\frac{1}{1-\alpha}} \frac{1}{\alpha-1} \left(r_f+\delta+h\right)^{\frac{1}{\alpha-1}-1} \frac{h}{r_f+\delta+h}. \tag{A4}$$

In the region where $h>0$, we can easily see its FOC is negative since $\alpha<1$; hence, $\Delta_l(N_l)$ is decreasing in h. Suppose h^{uppper} is the root for $(\kappa z)^{1-\alpha}\left[\frac{r_f+\delta+h}{\alpha(\kappa z)^{1-\alpha}}\right]^{\frac{\alpha}{\alpha-1}} - (r_f+\delta)\left[\frac{r_f+\delta+h}{\alpha(\kappa z)^{1-\alpha}}\right]^{\frac{1}{\alpha-1}} - R_f f - z^{1-\alpha} = 0$ in the positive h region, we must satisfy $h<h^{upper}$. QED.

A.2.3 Proposition 2

(1): $N_{uc}=\hat{N}$:

We know that when entrepreneur i in the modern sector is indifferent from being constrained (then of course no leasing) and unconstrained, it satisfies:

$$\frac{N_{uc}-f}{1-\theta} = \left[\frac{r_f+\delta}{\alpha\left(\kappa z\right)^{1-\alpha}}\right]^{\frac{1}{\alpha-1}}. \tag{A5}$$

For one particular entrepreneur, we look at:

$$\Delta_c = (\kappa z)^{1-\alpha}\left(\frac{N_i-f}{1-\theta}\right)^{\alpha} - (r_f+\delta)\frac{N_i-f}{1-\theta} \quad -R_f f- \quad z^{1-\alpha},$$

and

$$\Delta_{uc} = (\kappa z)^{1-\alpha}\left[\frac{r_f+\delta}{\alpha\left(\kappa z\right)^{1-\alpha}}\right]^{\frac{\alpha}{\alpha-1}} - (r_f+\delta)\left[\frac{r_f+\delta}{\alpha\left(\kappa z\right)^{1-\alpha}}\right]^{\frac{1}{\alpha-1}} - R_f f \quad - \quad z^{1-\alpha}.$$

FOC of Δ_c wrt to N_i is:

$$FOC(\Delta_c) = (\kappa z)^{1-\alpha}\left(\frac{1}{1-\theta}\right)^{\alpha}\alpha\left(N-f\right)^{\alpha-1} - (r_f+\delta)\frac{1}{1-\theta}. \tag{A6}$$

Since $\alpha<1$, the shape of Δ_c indicates that we will obtain the highest value for Δ_c when the FOC of Δ_c is 0. This means the point $\hat{N}$ with highest Δ_c is:

$$\hat{N} = \left[\frac{r_f + \delta}{\alpha\left(\kappa z\right)^{1-\alpha}}\right]^{\frac{1}{\alpha-1}}(1-\theta) + f. \tag{A7}$$

We can easily see $N^{uc} = \hat{N}$.

(2): Δ_l is a tangent line to Δ_c, and the x-coordinate of the tangent point is net worth N_l:

When $N_i = N_l$, the slope of Δ_c is:

$$(\kappa z)^{1-\alpha}\left(\frac{1}{1-\theta}\right)^{\alpha}\alpha\left[\left(\frac{r_f+\delta+h}{\alpha\left(\kappa z\right)^{1-\alpha}}\right)^{\frac{1}{\alpha-1}}(1-\theta)+f-f\right]^{\alpha-1} - (r_f+\delta)\frac{1}{1-\theta}, \tag{A8}$$

which can be reduced to:

$$h \times \frac{1}{1-\theta}. \tag{A9}$$

We know Δ_l is: $\Delta_l = (\kappa z)^{1-\alpha}\left[\frac{r_f+\delta+h}{\alpha(\kappa z)^{1-\alpha}}\right]^{\frac{\alpha}{\alpha-1}} - (r_f+\delta+h)\left[\frac{r_f+\delta+h}{\alpha(\kappa z)^{1-\alpha}}\right]^{\frac{1}{\alpha-1}} - R_f f + h\frac{N_i-f}{1-\theta} - z^{1-\alpha}$, which has a slope equal to:

$$h \times \frac{1}{1-\theta}. \tag{A10}$$

We can see at point N_l, Δ_l and Δ_c have equal slopes. Also, we note that Δ_l and Δ_c have the same values when net worth is N_l. We conclude that N_l is the x-coordinate of the tangent point between Δ_l and Δ_c.

(3): $f \leq \bar{N}_m < N_m < N_l < N_{uc}$:

When $N_i < N_l$, at the tangent point, the slope of Δ_c is greater than the slope of Δ_l. This indicates Δ_c has a smaller intersection point with Line 0 $F(x) = 0$ than Δ_c does, i.e., $\tilde{N}_m < N_m$.

Since $\Delta_c(N_m) = 0$ before N_i reaches N_{uc}, i.e., N_m is the net worth value when Δ_c intersects with Line 0 $F(x) = 0$ before N_i reaches N_{uc}, it must be that N_m is in the increasing proportion of Δ_c.

When $N_i = f$,

$$\Delta_c(f) = (\kappa z)^{1-\alpha} \left(\frac{f-f}{1-\theta} \right)^{\alpha} - (r_f + \delta) \frac{f-f}{1-\theta} - R_f f - z^{1-\alpha} < 0. \tag{A11}$$

Therefore, we must have $f < N_m$ due to the increasing pattern of Δ_c when $N_i < N_{uc}$.

Combined with $\bar{N}_m = max(f, \tilde{N}_m)$, we hence have $f \leq \bar{N}_m < N_m$.

We already know that $\hat{N}$ is:

$$\hat{N} = N_{uc} = \left[\frac{r_f + \delta}{\alpha (\kappa z)^{1-\alpha}} \right]^{\frac{1}{\alpha-1}} (1-\theta) + f. \tag{A12}$$

N_l is calculated as:

$$N_l = \left[\frac{r_f + \delta + h}{\alpha (\kappa z)^{1-\alpha}} \right]^{\frac{1}{\alpha-1}} (1-\theta) + f. \tag{A13}$$

Since $h > 0$ and $\alpha < 1$, we have $N_l < \hat{N}$.

Additionally, with reasonable parameter choices, we have $N_m < N_l$. QED.

A.2.4 Proposition 3

Proposition 3 states that we have smaller $\widetilde{TFP}$ and larger $\overline{TFP}$.

We first derive an expression for the aggregate TFP.

Inferred from the MPK formulas, we have:

$$K_{i1} = \left[\frac{MPK_{i1}}{\alpha (\kappa z)^{1-\alpha}} \right]^{\frac{1}{\alpha-1}}. \tag{A14}$$

We do an integration across modern entrepreneurs:

$$\begin{aligned}
\int_i K_{i1} di &= \int_i \left[\frac{MPK_{i1}}{\alpha\left(\kappa z\right)^{1-\alpha}}\right]^{\frac{1}{\alpha-1}} di \\
\Rightarrow \frac{K_{i1}}{\int_i K_{i1} di} &= \frac{\left[\frac{MPK_{i1}}{\alpha(\kappa z)^{1-\alpha}}\right]^{\frac{1}{\alpha-1}}}{\int_i \left[\frac{MPK_{i1}}{\alpha(\kappa z)^{1-\alpha}}\right]^{\frac{1}{\alpha-1}} di} \\
\Rightarrow Y_{i1} &= (\kappa z)^{1-\alpha} \left\{ \frac{\left[\frac{MPK_{i1}}{\alpha(\kappa z)^{1-\alpha}}\right]^{\frac{1}{\alpha-1}}}{\int_i \left[\frac{MPK_{i1}}{\alpha(\kappa z)^{1-\alpha}}\right]^{\frac{1}{\alpha-1}} di} \right\}^{\alpha} \left(\int_i K_{i1} di\right)^{\alpha} \\
\Rightarrow Y &= \int_i Y_{i1} di \\
&= \frac{\int_i \left[(\kappa z)^{1-\alpha} \left(\frac{MPK_{i1}}{\alpha(\kappa z)^{1-\alpha}}\right)^{\frac{\alpha}{\alpha-1}} di\right]}{\left[\int_i \left[\frac{MPK_{i1}}{\alpha(\kappa z)^{1-\alpha}}\right]^{\frac{1}{\alpha-1}} di\right]^{\alpha}} \left(\int_i K_{i1} di\right)^{\alpha} \\
&= \frac{\int_i \left[(\kappa z) \left(\frac{MPK_{i1}}{\alpha}\right)^{\frac{\alpha}{\alpha-1}} di\right]}{\left[\int_i \left[\frac{MPK_{i1}}{\alpha(\kappa z)^{1-\alpha}}\right]^{\frac{1}{\alpha-1}} di\right]^{\alpha}} \left(\int_i K_{i1} di\right)^{\alpha} \\
&= \frac{\int_i \left[(\kappa z) \left(MPK_{i1}\right)^{\frac{\alpha}{\alpha-1}} di\right]}{\left[\int_i (\kappa z) \left(MPK_{i1}\right)^{\frac{1}{\alpha-1}} di\right]^{\alpha}} \left(\int_i K_{i1} di\right)^{\alpha}
\end{aligned}$$

This implies that TFP is:

$$TFP = \frac{\int_i \left[(\kappa z) \left(MPK_i\right)^{\frac{\alpha}{\alpha-1}} di\right]}{\left[\int_i (\kappa z) \left(MPK_i\right)^{\frac{1}{\alpha-1}} di\right]^{\alpha}}. \tag{A15}$$

We then combine the thresholds outlined in the Proposition for equilibrium characterizations and obtain equations for the TFP both with and without leasing.[27]

[27] There is no need for specific distribution.

The TFP with leasing, $\overline{TFP}$, is:

$$\overline{TFP} = (\kappa z)^{1-\alpha} \frac{\int_{\bar{N}_m}^{N_l} \left[\frac{r_f+\delta+h}{\alpha(\kappa z)^{1-\alpha}}\right]^{\frac{\alpha}{\alpha-1}} \Pi(N_i)dN + \int_{N_l}^{\hat{N}} \left(\frac{N_i-f}{1-\theta}\right)^{\alpha} \Pi(N_i)dN + \int_{\hat{N}}^{N_{max}} \left(\frac{\hat{N}-f}{1-\theta}\right)^{\alpha} \Pi(N_i)dN}{\left[\int_{\bar{N}_m}^{N_l} \left[\frac{r_f+\delta+h}{\alpha(\kappa z)^{1-\alpha}}\right]^{\frac{1}{\alpha-1}} \Pi(N_i)dN + \int_{N_l}^{\hat{N}} \frac{N_i-f}{1-\theta}\Pi(N_i)dN + \int_{\hat{N}}^{N_{max}} \frac{\hat{N}-f}{1-\theta}\Pi(N_i)dN\right]^{\alpha}}. \tag{A16}$$

The TFP without leasing, $\widetilde{TFP}$, is:

$$\widetilde{TFP} = (\kappa z)^{1-\alpha} \frac{\int_{N_m}^{N_l} \left(\frac{N_i-f}{1-\theta}\right)^{\alpha} \Pi(N_i)dN + \int_{N_l}^{\hat{N}} \left(\frac{N_i-f}{1-\theta}\right)^{\alpha} \Pi(N_i)dN + \int_{\hat{N}}^{N_{max}} \left(\frac{\hat{N}-f}{1-\theta}\right)^{\alpha} \Pi(N_i)dN}{\left[\int_{N_m}^{N_l} \frac{N_i-f}{1-\theta}\Pi(N_i)dN + \int_{N_l}^{\hat{N}} \frac{N_i-f}{1-\theta}\Pi(N_i)dN + \int_{\hat{N}}^{N_{max}} \frac{\hat{N}-f}{1-\theta}\Pi(N_i)dN\right]^{\alpha}}. \tag{A17}$$

To compare these two, we consider an intermediate statistic, which is:

$$\overline{\widetilde{TFP}} = (\kappa z)^{1-\alpha} \frac{\int_{N_m}^{N_l} \left[\frac{r_f+\delta+h}{\alpha(\kappa z)^{1-\alpha}}\right]^{\frac{\alpha}{\alpha-1}} \Pi(N_i)dN + \int_{N_l}^{\hat{N}} \left(\frac{N_i-f}{1-\theta}\right)^{\alpha} \Pi(N_i)dN + \int_{\hat{N}}^{N_{max}} \left(\frac{\hat{N}-f}{1-\theta}\right)^{\alpha} \Pi(N_i)dN}{\left[\int_{N_m}^{N_l} \left[\frac{r_f+\delta+h}{\alpha(\kappa z)^{1-\alpha}}\right]^{\frac{1}{\alpha-1}} \Pi(N_i)dN + \int_{N_l}^{\hat{N}} \frac{N_i-f}{1-\theta}\Pi(N_i)dN + \int_{\hat{N}}^{N_{max}} \frac{\hat{N}-f}{1-\theta}\Pi(N_i)dN\right]^{\alpha}}. \tag{A18}$$

There are two steps. In the first step, we do the comparison between $\overline{\widetilde{TFP}}$ and $\widetilde{TFP}$. In the second step, we do the comparison between $\overline{\widetilde{TFP}}$ and $\overline{TFP}$:

Step 1: $\overline{\widetilde{TFP}}$ vs. $\widetilde{TFP}$.

When $N_i \in [N_m,\ N_l]$, we have:

$$\begin{cases} \left[\frac{r_f+\delta+h}{\alpha(\kappa z)^{1-\alpha}}\right]^{\frac{1}{\alpha-1}} \geq \frac{N_i-f}{1-\theta}, \\ \left[\frac{r_f+\delta+h}{\alpha(\kappa z)^{1-\alpha}}\right]^{\frac{\alpha}{\alpha-1}} \geq \left(\frac{N_i-f}{1-\theta}\right)^{\alpha}. \end{cases} \tag{A19}$$

We define $F(x)$ as:

$$F(x) = (\kappa z)^{1-\alpha} \frac{\int_{N_m}^{N_l} x^{\alpha}\Pi(N_i)dN + \int_{N_l}^{\hat{N}} \left(\frac{N_i-f}{1-\theta}\right)^{\alpha} \Pi(N_i)dN + \int_{\hat{N}}^{N_{max}} \left(\frac{\hat{N}-f}{1-\theta}\right)^{\alpha} \Pi(N_i)dN}{\left[\int_{N_m}^{N_l} x\Pi(N_i)dN + \int_{N_l}^{\hat{N}} \frac{N_i-f}{1-\theta}\Pi(N_i)dN + \int_{\hat{N}}^{N_{max}} \frac{\hat{N}-f}{1-\theta}\Pi(N_i)dN\right]^{\alpha}}, \tag{A20}$$

where $x \in [\frac{N_i-f}{1-\theta}, \left[\frac{r_f+\delta+h}{\alpha(\kappa z)^{1-\alpha}}\right]^{\frac{1}{\alpha-1}}]$, in which $N_i \in [N_m,\ N_l]$.

We first examine whether the function $F(x)$ is increasing or not.

a. Take log:

$$\begin{aligned} f(x) &= log[F(x)] = log\left[(\kappa z)^{1-\alpha}\right] + log\left[\int_{N_m}^{N_l} x^{\alpha}\Pi(N_i)dN + \int_{N_l}^{\hat{N}} \left(\frac{N_i - f}{1-\theta}\right)^{\alpha} \Pi(N_i)dN + \int_{\hat{N}}^{N_{max}} \left(\frac{\hat{N} - f}{1-\theta}\right)^{\alpha} \Pi(N_i)dN\right] \\ & -\alpha\, log\left[\int_{N_m}^{N_l} x\Pi(N_i)dN + \int_{N_l}^{\hat{N}} \frac{N_i - f}{1-\theta}\Pi(N_i)dN + \int_{\hat{N}}^{N_{max}} \frac{\hat{N} - f}{1-\theta}\Pi(N_i)dN\right]. \end{aligned}$$

b. Take derivatives wrt x:

$$\frac{\int_{N_m}^{N_l} \alpha x^{\alpha-1}\Pi(N_i)dN}{\int_{N_m}^{N_l} x^{\alpha}\Pi(N_i)dN + \int_{N_l}^{\hat{N}} \left(\frac{N_i-f}{1-\theta}\right)^{\alpha} \Pi(N_i)dN + \int_{\hat{N}}^{N_{max}} \left(\frac{\hat{N}-f}{1-\theta}\right)^{\alpha} \Pi(N_i)dN} - \alpha \frac{\int_{N_m}^{N_l} \{1\}\Pi(N_i)dN}{\int_{N_m}^{N_l} x\Pi(N_i)dN + \int_{N_l}^{\hat{N}} \frac{N_i-f}{1-\theta}\Pi(N_i)dN + \int_{\hat{N}}^{N_{max}} \frac{\hat{N}-f}{1-\theta}\Pi(N_i)dN}. \tag{A21}$$

$\Longrightarrow$

$$\frac{\alpha\int_{N_m}^{N_l} \{1\}\Pi(N_i)dN}{\int_{N_m}^{N_l} x\Pi(N_i)dN + \int_{N_l}^{\hat{N}} \frac{\left(\frac{N_i-f}{1-\theta}\right)^{\alpha}}{x^{\alpha-1}} \Pi(N_i)dN + \int_{\hat{N}}^{N_{max}} \frac{\left(\frac{\hat{N}-f}{1-\theta}\right)^{\alpha}}{x^{\alpha-1}} \Pi(N_i)dN} - \frac{\alpha\int_{N_m}^{N_l} \{1\}\Pi(N_i)dN}{\int_{N_m}^{N_l} x\Pi(N_i)dN + \int_{N_l}^{\hat{N}} \frac{N_i-f}{1-\theta}\Pi(N_i)dN + \int_{\hat{N}}^{N_{max}} \frac{\hat{N}-f}{1-\theta}\Pi(N_i)dN}. \tag{A22}$$

We know $x \in [\frac{N_i-f}{1-\theta}, \left[\frac{r_f+\delta+h}{\alpha(\kappa z)^{1-\alpha}}\right]^{\frac{1}{\alpha-1}}]$ in which $N_i \in [N_m,\ N_l]$. Then we must have: $x \leq \frac{N_i-f}{1-\theta} < \frac{\hat{N}-f}{1-\theta}$ when $N_i > N_l$.

Therefore, when $N_i > N_l$, we have $\frac{\frac{N_i-f}{1-\theta}}{x} \geq 1$. This means:

$$\frac{\left(\frac{N_i-f}{1-\theta}\right)^{\alpha-1}}{x^{\alpha-1}} \leq 1 \Longrightarrow \frac{\left(\frac{N_i-f}{1-\theta}\right)^{\alpha}}{x^{\alpha-1}} = \frac{N_i - f}{1-\theta}\frac{\left(\frac{N_i-f}{1-\theta}\right)^{\alpha-1}}{x^{\alpha-1}} \leq \frac{N_i - f}{1-\theta}. \tag{A23}$$

Hence, Eq. (A22) is positive as the first term has a lower denominator. Consequently, $F(x)$ and $f(x)$ are increasing in x. This implies that:

$$\overline{\widetilde{TFP}} > \widetilde{TFP}. \tag{A24}$$

Step 2: $\overline{\widetilde{TFP}}$ vs. $\overline{TFP}$.

We define $FF(p)$ as:

$$FF(p) = (\kappa z)^{1-\alpha} \frac{\int_{N_m-p}^{N_l} \left[\frac{r_f+\delta+h}{\alpha(\kappa z)^{1-\alpha}}\right]^{\frac{\alpha}{\alpha-1}} \Pi(N_i)dN + \int_{N_l}^{\hat{N}} \left(\frac{N_i-f}{1-\theta}\right)^{\alpha} \Pi(N_i)dN + \int_{\hat{N}}^{N_{max}} \left(\frac{\hat{N}-f}{1-\theta}\right)^{\alpha} \Pi(N_i)dN}{\left[\int_{N_m-p}^{N_l} \left[\frac{r_f+\delta+h}{\alpha(\kappa z)^{1-\alpha}}\right]^{\frac{1}{\alpha-1}} \Pi(N_i)dN + \int_{N_l}^{\hat{N}} \frac{N_i-f}{1-\theta}\Pi(N_i)dN + \int_{\hat{N}}^{N_{max}} \frac{\hat{N}-f}{1-\theta}\Pi(N_i)dN\right]^{\alpha}}. \tag{A25}$$

We first examine whether $FF(p)$ is increasing in p.

a. Take log:

$$\begin{aligned} ff(p) \quad = \quad & log[FF(p)] = log\left[(\kappa z)^{1-\alpha}\right] + log\left[\int_{N_m-p}^{N_l} \left[\frac{r_f+\delta+h}{\alpha\,(\kappa z)^{1-\alpha}}\right]^{\frac{\alpha}{\alpha-1}} \Pi(N_i)dN + \int_{N_l}^{\hat{N}} \left(\frac{N_i-f}{1-\theta}\right)^{\alpha} \Pi(N_i)dN + \int_{\hat{N}}^{N_{max}} \left(\frac{\hat{N}-f}{1-\theta}\right)^{\alpha} \Pi(N_i)dN\right] \\ & -\alpha\, log\left[\int_{N_m-p}^{N_l} \left[\frac{r_f+\delta+h}{\alpha\,(\kappa z)^{1-\alpha}}\right]^{\frac{1}{\alpha-1}} \Pi(N_i)dN + \int_{N_l}^{\hat{N}} \frac{N_i-f}{1-\theta}\Pi(N_i)dN + \int_{\hat{N}}^{N_{max}} \frac{\hat{N}-f}{1-\theta}\Pi(N_i)dN\right] \end{aligned}$$

b. Take derivatives wrt p:

$$\begin{aligned} & \frac{\left[\frac{r_f+\delta+h}{\alpha(\kappa z)^{1-\alpha}}\right]^{\frac{\alpha}{\alpha-1}} \square}{\int_{N_m-p}^{N_l} \left[\frac{r_f+\delta+h}{\alpha(\kappa z)^{1-\alpha}}\right]^{\frac{\alpha}{\alpha-1}} \Pi(N_i)dN + \int_{N_l}^{\hat{N}} \left(\frac{N_i-f}{1-\theta}\right)^{\alpha} \Pi(N_i)dN + \int_{\hat{N}}^{N_{max}} \left(\frac{\hat{N}-f}{1-\theta}\right)^{\alpha} \Pi(N_i)dN} \\ & -\frac{\alpha\left[\frac{r_f+\delta+h}{\alpha(\kappa z)^{1-\alpha}}\right]^{\frac{1}{\alpha-1}} \square}{\int_{N_m-p}^{N_l} \left[\frac{r_f+\delta+h}{\alpha(\kappa z)^{1-\alpha}}\right]^{\frac{1}{\alpha-1}} \Pi(N_i)dN + \int_{N_l}^{\hat{N}} \frac{N_i-f}{1-\theta}\Pi(N_i)dN + \int_{\hat{N}}^{N_{max}} \frac{\hat{N}-f}{1-\theta}\Pi(N_i)dN}, \end{aligned} \tag{A26}$$

where $\square$ is a function of p, denoting the derivative of the distribution wrt to p. Obviously $\square$ is positive and increasing in p.

We simplify it and get:

$$\begin{aligned} & \frac{\square}{\int_{N_m-p}^{N_l} \{1\}\,\Pi(N_i)dN + \int_{N_l}^{\hat{N}} \frac{\left(\frac{N_i-f}{1-\theta}\right)^{\alpha}}{\left[\frac{r_f+\delta+h}{\alpha(\kappa z)^{1-\alpha}}\right]^{\frac{\alpha}{\alpha-1}}} + \int_{\hat{N}}^{N_{max}} \frac{\left(\frac{\hat{N}-f}{1-\theta}\right)^{\alpha}}{\left[\frac{r_f+\delta+h}{\alpha(\kappa z)^{1-\alpha}}\right]^{\frac{\alpha}{\alpha-1}}}\Pi(N_i)dN} \\ & -\alpha\frac{\square}{\int_{N_m-p}^{N_l} \{1\}\,\Pi(N_i)dN + \int_{N_l}^{\hat{N}} \frac{\left(\frac{N_i-f}{1-\theta}\right)}{\left[\frac{r_f+\delta+h}{\alpha(\kappa z)^{1-\alpha}}\right]^{\frac{1}{\alpha-1}}} + \int_{\hat{N}}^{N_{max}} \frac{\left(\frac{\hat{N}-f}{1-\theta}\right)}{\left[\frac{r_f+\delta+h}{\alpha(\kappa z)^{1-\alpha}}\right]^{\frac{1}{\alpha-1}}}\Pi(N_i)dN}. \end{aligned}$$

When $N_i > N_l$, we have $\frac{\frac{\hat{N}-f}{1-\theta}}{\left[\frac{r_f+\delta+h}{\alpha(\kappa z)^{1-\alpha}}\right]^{\frac{1}{\alpha-1}}} \geq \frac{\frac{N_i-f}{1-\theta}}{\left[\frac{r_f+\delta+h}{\alpha(\kappa z)^{1-\alpha}}\right]^{\frac{1}{\alpha-1}}} \geq 1$. This means:

$$\frac{\left(\frac{\hat{N}-f}{1-\theta}\right)^{\alpha}}{\left[\frac{r_f+\delta+h}{\alpha(\kappa z)^{1-\alpha}}\right]^{\frac{\alpha}{\alpha-1}}} \leq \frac{\left(\frac{N_i-f}{1-\theta}\right)^{\alpha}}{\left[\frac{r_f+\delta+h}{\alpha(\kappa z)^{1-\alpha}}\right]^{\frac{\alpha}{\alpha-1}}} \leq 1, \tag{A27}$$

where $\alpha \in (0, 1)$.

Meanwhile, $\square > \alpha\square$.

Hence we know that Eq. (A26) is positive, meaning that $FF(p)$ and $ff(p)$ are increasing in p. This implies that:

$$\overline{TFP} > \overline{\widetilde{TFP}}. \tag{A28}$$

Eventually, we combine the above two steps, and obtain that $\overline{TFP} > \widetilde{TFP}$. That is, leasing always increases total TFP, i.e, the total gain in positive. QED.

A.2.5 Proposition 4

The proof for Proposition 4 is shown as follows.

We first derive the efficient TFP. Previously, we have shown that the aggregate TFP is:

$$TFP = \frac{\int_i \left[(\kappa z)\left(MPK_i\right)^{\frac{\alpha}{\alpha-1}} di\right]}{\left[\int_i (\kappa z)\left(MPK_i\right)^{\frac{1}{\alpha-1}} di\right]^{\alpha}}. \tag{A29}$$

At the efficient allocation, all MPKs are equalized, hence:

$$TFP^e = \frac{\int_i \left[(\kappa z)\, di\right]}{\left[\int_i (\kappa z)\, di\right]^{\alpha}} = \left[\int_i (\kappa z)\, di\right]^{1-\alpha}. \tag{A30}$$

Since $\bar{N}_m < N_m$, we can easily conclude that:

$$\overline{TFP^e} > \widetilde{TFP^e}. \tag{A31}$$

That is, the efficient TFP when there is a leasing market is larger than the efficient TFP when there is no leasing market. Hence, the gain of leasing from entry is positive, $G^{TFP}_{entry} > 0$.

QED.

A.2.6 Proposition 5

The incumbents refer to entrepreneurs who have net worth larger than N_m. They already choose the modern sector even if there is no leasing market. Enabling them to lease reduces capital misallocation among them. This is because allowing these entrepreneurs to lease relaxes financial constraints and effectively adds an upper bar for their marginal product of capital $(r_f + \delta + h)$, which of course reduces the capital misallocation among these entrepreneurs.

B Derivations for the dynamic model

B.1 Reformulation and decision rules

In the full dynamic model, we define $N_t = K^o_t - B^m_t$ as the entrepreneur's net worth at time t in modern sector m, and $N_t = -B^u_t$ as the entrepreneur's net worth at time t in traditional sector u. Our assumption on "observing idiosyncratic productivity ahead of time" allows us to reformulate our problem into a two-step procedure. We scale all variables by dividing the permanent productivity component z^p. With a slight abuse of notation, we use the original notations to denote variables after the rescaling. We rewrite the entrepreneur's problem in different sectors in recursive form. We use primes to denote next-period variables.

B.1.1 Modern entrepreneurs

With prices W, R_f, and τ_l, the Bellman equation of an entrepreneur with net worth N and transitory idiosyncratic productivity z in the stationary equilibrium is given by:

$$V^m(N, z) = \max_{N', C^m} \log C^m + \beta E\left[V(N', z')\right]. \tag{B32}$$

The budget constraint of the entrepreneur is:

$$C^m + N' = \pi^m(N, z) + R_f N, \tag{B33}$$

where

$$\pi^m(N, z) = \max_{K^o, K^l, L^m} \kappa^{1-\eta} z^{1-\eta} \left(K^o + K^l\right)^{\alpha\eta} (L^m)^{(1-\alpha)\eta} - (r_f + \delta)K^o - \tau_l K^l - W L^m. \tag{B34}$$

The borrowing constraint reduces to:

$$K^o \leq \frac{1}{1-\theta} N. \tag{B35}$$

B.1.2 Traditional entrepreneurs

We next consider the problem of entrepreneurs in the traditional sector. The Bellman equation of such entrepreneurs is:

$$V^u(N, z) = \max_{N', C^u} \log C^u + \beta \max\left\{E\left[V^u(N', z')\right], E\left[V^m(N', z')\right]\right\}, \tag{B36}$$

subject to:

$$C^u + X = \pi^u(z) + R_f N, \tag{B37}$$

where:

$$\pi^u(z) = \max_{L^u} z^{1-\eta} (L^u)^\eta - WL^u \tag{B38}$$

is the profit of an entrepreneur in the traditional sector, and X is its savings. The entrepreneur's continuation value is the envelope over the expected value of the two options it has: either staying in the traditional sector, or switching to the modern sector. The evolution of its net worth is a function of whether the entrepreneur switches. An entrepreneur that stays in the traditional sector simply inherits its past savings, $N' = X$. In contrast, an entrepreneur that enters the modern sector has $N' = X - f$, where f is the entry cost.

B.2 Equilibrium

We define next the equilibrium of this economy. Let $\Theta_t^m(N, z)$ be the measure of modern entrepreneurs and $\Theta_t^u(N, z)$ be the measure of traditional entrepreneurs. Clearly, the measures of entrepreneurs in the two sectors must satisfy:

$$\int_{N\times z} d\Theta_t^m(N, z) + \int_{N\times z} \Theta_t^u(N, z) = \gamma^t.$$

To characterize the evolution of these measures, we let $\psi(N, z)$ be an indicator for whether an entrepreneur in the traditional sector switches to the modern sector. We also let $N = [\underline{N}, \bar{N}]$ denote the compact set of values an entrepreneur's net worth can take and let N denote a family of its subsets.

A balanced growth equilibrium is a set of prices W, R_f, and τ_l, policy functions for households' consumption $C_t^w(N, \nu)$, saving $B_{t+1}^w(N, \nu)$, and accumulated leased capital $K_{t+1}^l(N, \nu)$, for modern entrepreneurs' consumption $C_t^m(N, z)$, net worth $N_{t+1}^m(N, z)$, output $Y^m(N, z)$, labor $L^m(N, z)$, owned capital $K^o(N, z)$, and leased capital $K^l(N, z)$, for traditional entrepreneurs' consumption $C_t^u(N, z)$, net worth $N_{t+1}^u(N, z)$, output $Y^u(N, z)$, labor $L^u(N, z)$, as well as a decision on whether to enter the modern sector $\psi(N, z)$ that: (i) solve the entrepreneurs' and households' optimization problems; (ii) satisfy the market clearing conditions for

- the labor market:

$$\gamma^t = \int_{N\times z} L^u(z)d\Theta_t^u(N,z) + \int_{N\times z} L^m(N,z)d\Theta_t^m(N,z),$$

  where γ^t is the total amount of efficiency units of labor supplied by households (we normalize the mean of ν to unity);

- the (owned) asset market:

$$\int_{N\times\nu} B_{t+1}^w(N,\nu)d\Theta_t^w(N,\nu) + \int_{N\times z} N_{t+1}^u(N,z)d\Theta_{t+1}^u(N,z) + \int_{N\times z} N_{t+1}^m(N,z)d\Theta_{t+1}^m(N,z)$$
$$= \int_{N\times z} K_{t+1}^o(N,z)d\Theta_{t+1}^m(N,z),$$

- the leased capital market:

$$\int_{N\times\nu} K_{t+1}^l(N,\nu)d\Theta_t^w(N,\nu) = \int_{N\times z} K_{t+1}^l(N,z)d\Theta_{t+1}^m(N,z);$$

and (iii) the law of motion for different sectors:

- the measure of entrepreneurs in the modern sector evolves over time according to:

$$\begin{aligned}\Theta_{t+1}^m(N,z_j) = &\int_N \sum_i \phi_{i,j}\mathrm{I}_{\{N^m(N,z_i)\in N\}}d\Theta_t^m(N,z_i) \\ &+ \int_N \sum_i \phi_{i,j}\mathrm{I}_{\{\psi(N,z_i)=1,N^{u,s}(N,z_i)\in N\}}d\Theta_t^u(N,z_i),\end{aligned}$$

  where $N^m(\cdot)$ is the savings decision of an entrepreneur in the modern sector and $N^{u,s}(\cdot)$ is the amount of net worth an entrepreneur that switches sectors carries into the next period. The law of motion simply adds up entrepreneurs in the modern sector and those entrepreneurs in the traditional sector that decide to switch.

- the measure of entrepreneurs in the traditional sector evolves according to:

$$\Theta_{t+1}^{u}(N, z_j) = \int_N \sum_i \phi_{i,j} \mathrm{I}_{\{\psi(N,z_i)=0, N^u(N,z_i)\in N\}} d\Theta_t^u (N, z_i)$$
$$+ (\gamma - 1) N_t \mathrm{I}_{\{0\in N\}} \bar{f}_j,$$

where $\bar{\phi}_j$ is the stationary distribution of the transitory productivity and $N^u(\cdot)$ is the savings decision of an entrepreneur that remains in the traditional sector. The right-hand side simply adds up entrepreneurs that stay in the traditional sector and newly entering entrepreneurs.

C Data

We obtain the firm level data from Compustat. The sample period ranges from 1977 to 2015. We focus on manufacturing firms (firms with 4-digit SIC codes between 2000 and 3999) with non-negative total assets (AT) and sales (SALE). To mitigate the effects of outliers, all firm-level variables are trimmed at the top and bottom 0.1%. We calculate value-added following Ai, Croce, and Li (2013). We follow Li, Whited, and Wu (2016) to construct leased capital. We discount future lease commitments in years 1-5 (MRC1–MRC5) at the BAA bond rate. We similarly discount lease commitments beyond year 5 (MRCTA) by assuming that they are evenly spread out in years six to ten. The leased capital, then, is the sum of current rental payment and the present value of future lease commitments as calculated above.

For the motivating facts presented in Table 1, we use Property, Plant, and Equipment - Total (Net), i.e., PPENT, to measure purchased (owned) tangible capital and further define leased capital ratio as leased capital divided by the sum of leased and owned capital. Similarly, we define rental share as the ratio between rental expense over the sum of capital expenditure plus rental expense. The leased capital ratio and the rental share measure the proportion of total capital input in a firm's production obtained from leasing activities. We use total book assets (AT) to determine size groups. We measure the firm-level constraint by the Whited-Wu index (Whited and Wu (2006), Hansen et al. (2007), WW index hereafter).[28]

At the aggregate level, leased capital accounts for a substantial portion of overall productive assets - over 13%. Using rental share yields a slightly higher proportion of 18%. The magnitude is consistent with Eisfeldt and Rampini (2009) and Rauh and Sufi (2012), illustrating how leased capital might be utilized in production. For the debt to output ratio, considering leased capital will increase its overall level by close to 20% in our sample, which reveals that leasing is an essential source of external finance, which complements financial debt.[29]

In the cross-section, we note that the average leased capital ratio of small firms (0.29) is significantly higher than that of large firms (0.12). Meanwhile, we observe a large dispersion in the debt to output ratio, ranging from 0.39 to 0.82. The lease-adjusted debt to output ratio, however, exhibits a narrower dispersion across different size groups. A similar pattern holds for financial-constraint-sorted groups. These imply that leasing is a more important source of external finance for small and financially constrained firms.[30]

In summary, our findings in Table 1 recognize that leasing can be a more important source of productive asset and external finance for small and financially constrained firms.

[28] The results are very similar when we use other financial constraint measures, such as the SA index.

[29] In untabulated results, we construct an alternative measure of leased capital ratio based on the newly reported lease right-of-use asset (ROUANT) after the lease accounting rule change, and compare it with the method adopted here. The results validate our calculation of leased capital. Additionally, Hu, Li, and Xu (2020) consider other common measures of leased capital, and document similar results.

[30] The sectoral dimension is also important: leasing is more intensive in services-producing industries (Gal and Pinter, 2017; Hu, Li, and Xu, 2022). This is linked to the deep technological differences across industries, which may be further attributed to their varying scopes of flexibility or reversibility (Eisfeldt and Rampini, 2009).

References for Online Appendix

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