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Leasing, Pecuniary Externality, and Aggregate Efficiency

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JEL Classification: E22, E23, G32.

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I Introduction

With collateral constraints in the credit market, individual firms in a competitive equilibrium do not internalize their investment decisions on collateral values through asset prices, thus inefficiency may exist, and it is typically referred as pecuniary externality.[1] However, this strand of literature typically misses an important input for firms' production – leased capital, which accounts for approximately 20% to 30% in capital expenditures for U.S. public firms.[2] Unlike other types of capital investing, leasing is a more collateralizable but costly financing tool, since it is easier for lessors to repossess but involves a separation of ownership and control.[3]

We fill this void and investigate the impacts of leasing on the aggregate inefficiency which is due to pecuniary externalities when firms face collateral constraints. We emphasize how the missing leased capital in the previous literature may have significant welfare impacts. Specifically, we show that without leasing, a constrained social planner can improve the economy quite significantly (by about 8% of aggregate consumption); however, with leasing options for firms, the inefficiency due to pecuniary externalities can be largely mitigated, only about 2% of consumption.[4] As a result, missing leasing capital in the previous literature may have biased welfare implications. We provide both theoretical and quantitative analysis through an intuitive model and also a dynamic structural model. We also recommend potentially useful policies (i.e., taxes and subsidies) and government

[1]For the literature, among others, see Kiyotaki and Moore (1997), Buera, Kaboski and Shin (2011), Moll (2014), Midrigan and Xu (2014), Lorenzoni (2008), Nuño and Moll (2018), Dávila and Korinek (2018), Lanteri and Rampini (2023). See more discussions in the literature review.

[2]See Section A in the Appendix for more details: Tables A.1 and A.2 for summary statistics, and Table A.3 for regression results. Moreover, variables that indicate a firm is more likely to be financially constrained have a positive correlation with leased capital usages. These are also consistent with previous findings, such as Eisfeldt and Rampini (2009). Thus, leased capital is quantitatively important for firms, and more so for financially constrained firms. Our main focus is then to explore the detailed efficiency implications due to leased capital with pecuniary externalities. For studies on leasing, e.g., see Eisfeldt and Rampini (2009), Rampini and Viswanathan (2013), Li and Tsou (2019) and Hu, Li and Xu (2020); see the literature review for more details.

[3]In a typical operating lease contract, the owner of the asset (lessor) grants to a capital borrower (lessee) the exclusive right to use the capital for an agreed period of time in return for periodic payments, and the capital reverts to the lessor at the end of the lease term. There is another type of lease: capital lease, in which the lessee owns the assets at the end of the lease's term. However, because the operating lease is much larger in magnitude than capital lease in the U.S. data, the operating lease is our main focus here.

[4]All numbers are relative to the aggregate consumption in the competitive equilibrium with leasing options. See more details in Section IV.E.

regulations on leasing markets that could improve welfare significantly.

Before we provide more details below, we illustrate the main intuition here. Consider the case without leasing options first. Like standard literature (such as Lorenzoni (2008), Dávila and Korinek (2018) and Lanteri and Rampini (2023)), when firms borrow in the credit market, the collateral values of their assets will be affected by the market price. Since an individual firm's investment decision does not internalize its effect on aggregate asset prices, the aggregate supply of capital is typically too low and its price is inefficiently too high. Further consider when the leasing option is available. Firms that are sufficiently financially constrained (productive but with little net worth) will find it beneficial to use leased capital, which tends to reduce the demand of old capital. Also, since leased capital turns into old capital after firm production, the supply of old capital increases. As a result, there will be a drop of the inefficiently high equilibrium price of old capital, and hence alleviates the pecuniary externalities-induced inefficiencies due to financial frictions. Meanwhile, using leased capital means that sufficiently constrained firms are much less affected by their collateral values. Therefore, the channel through collateral values that market prices can affect these constrained firms becomes limited. Intuitively, any inefficient deviation of asset prices due to other firms' decisions then have less "harmful" externality.[5]

Specifically, the road map for our analysis is as follows. We first study a simple two-period model. We analytically characterize firms' solutions, highlight how leasing mitigates financial frictions, and measure different types of pecuniary externalities due to collateral constraints (see more discussions below). Both theoretical and numerical analyses show that leasing can improve the aggregate efficiency in the competitive equilibrium. Next we study a dynamic structural model for more realistic quantitative properties. We show the main results of the simple model still hold. Quantitatively, we find that the constrained social planner can further improve the market economy. We then analyze and quantify the implied optimal taxes that can implement the second best. We

[5]Hypothetically, when leasing capital does not have any additional monitoring cost in the extreme case, first best can be restored and any inefficiency can be completely eliminated. With calibrated, realistic leasing cost, however, inefficiency still exists, but quantitatively we can show it is largely mitigated as compared with a counterfactual economy where firms do not have option to lease.

find the benefit from taxes and subsidies is relatively larger if the firm is more likely to be financially constrained and its investment is more elastic. Lastly, we also quantify the impacts of government regulations that may reduce monitoring costs in the leasing market and we find sizable welfare improvements.

Consider the intuitive, two-period model first. Firms are heterogeneous in initial net worth, and they live for two periods. To produce, firms must either purchase old capital goods from the market, or they must invest in new capital themselves; however, firms are subject to collateral constraints (as in Kiyotaki and Moore (1997)). Importantly, the price of capital goods in the market in the next period may affect today's collateral values. As a result of limited net worth and collateral constraints, some firms may be financially constrained in equilibrium. So far all these elements are quite standard, as in Eisfeldt and Rampini (2007) and Lanteri and Rampini (2023). Our paper is different, however. In particular, we explicitly introduce leased capital and assume it is supplied by unconstrained agents in the economy (the risk-neutral, representative lessor, or household).[6] An important feature of leased capital is that it has strongly collateralizable but costly financing, due to its repossession advantage and separation of ownership and control (Eisfeldt and Rampini (2009)). Constrained firms find it particularly attractive to use leased capital, even though they have to pay the expensive monitoring costs. We then analytically characterize these optimal capital choices.

We further explore the sources of inefficiency (and their changes) due to pecuniary externalities in the competitive equilibrium. To do so, we introduce the constrained social planners' allocations (second best) as useful benchmarks. That is, we focus on constrained (in)efficiency. In a competitive equilibrium, firms will not internalize the effects of their borrowing, investing, and selling behavior on market prices, and thus inefficiency and pecuniary externality may arise.[7] Equipped with

[6]This assumption is reasonable, as in the data lessor firms are indeed less financially constrained than lessee firms (Li and Tsou, 2019). It is also consistent with prior studies, including Eisfeldt and Rampini (2009) and Rampini and Viswanathan (2013).

[7]For constrained efficiency, for example, seminal papers such as Lorenzoni (2008) describe how asset sales by financially constrained agents generate pecuniary externalities and hence lead to constrained inefficient allocations (also see Dávila and Korinek (2018), Jeanne and Korinek (2019), Lanteri and Rampini (2023)). The constrained planner, by definition, must respect all firms' constraints (individual-specific budget constraints, collateral constraints, and dividend constraints) and internalize the effects of individual behaviors on the price of old capital (and related rental fees); at the same time, the planner takes into account market clearing conditions. She will simply command

the constrained planner's optimality conditions, we can further formally characterize the inefficiencies in the competitive equilibrium. More specifically, two distinct types of pecuniary externalities arise. The first one is a distributive externality, which is best illustrated as follows. Consider the demand and supply of old capital. When the price for old capital in the market increases by one unit, this may benefit sellers but hurt buyers overall; Often, there are many financially constrained firms that have strong incentives to buy capital, and these firms' marginal benefits from price decreases outweigh the marginal costs to other firms who sell. A planner can thus possibly improve aggregate efficiency by lowering the old capital price. The second type of pecuniary externality is referred to as a collateral externality, which arises when financial constraints for individual firms depend on the market value of capital assets that serve as collateral. We prove that the first one is always larger than the second, and it is reduced with leasing options (see more details in Section II.D), which is also consistent with our intuitions.[8]

Combining theoretical and numerical analysis, we find several important lessons. In a competitive equilibrium with leased capital, sufficiently constrained firms still exist and they use leased capital; therefore, leased capital directly mitigates financial frictions for these firms relative to the case without leasing options. With better leasing market conditions, both the competitive equilibrium and the constrained social planner have higher output and consumption, use more leased capital, and have more capital for production in the economy. However, even for competitive equilibrium with leased capital, there are still inefficiencies due to pecuniary externalities from the constrained social planner's perspective. Numerically, we find the room for social planner to further improve competitive equilibrium decreases in leasing costs. We show this is the case by examining externalities measurement and also for a range of other indicators of inefficiencies.[9] Intuitively, as

different levels of new, old, and leased capital, as well as debt, on behalf of individual firms. That is, the social planner is constrained to consider allocations with zero net transfers across any firm.

[8] Also see Dávila and Korinek (2018) and Lanteri and Rampini (2023) for related discussions on these two types of externalities.

[9] As leasing options become more favorable, the competitive equilibrium will have higher aggregate output, higher aggregate consumption, and smaller inefficiency in terms of externalities measurement. In addition, the old capital price is also lower in the equilibrium due to lower demand, as more firms switch their capital demand to leased capital, and also due to higher supply, since we assume that leased capital supplied in the current period will become old capital in the next period.

pointed out before, those very constrained firms now have some good options from leasing, which leads to lower market price of old capital due to a higher supply and old demand of old capital. Firms also become less affected by market prices; in turn, the planner/policymaker cannot improve the situation much further.

We then consider a dynamic stochastic general equilibrium model, which endogenizes firm distributions in the two-period model, and we are able to conduct further quantitative analysis. In our dynamic model, firms have heterogeneous but stochastic productivity over time. Over time, firms' net worth is endogenously determined. They can invest in new capital or purchase old capital from other firms; different from the firm dynamics literature, firms can lease capital from the lessor. The quantitative model provides a good fit for several important firm-level moments, including firm leverage, the correlation between firm productivity, firm output and financing conditions. Importantly, the average level of leased capital used by firms is matched very well. In addition, the correlations between firm productivity and leasing, the correlations between firms' financial conditions and leasing, are all matched reasonably well.

Using our dynamic model as a laboratory, we first confirm that the welfare implications of leasing markets in the two-period model still hold. In the benchmark model, there are still inefficiencies in the competitive market economy, since leasing capital can only mitigate but not completely eliminate the financial frictions and the associated externalities (leasing itself has some agency costs). The constrained social planner can further improve the market economy: output can be further increased by 1.7%, and consumption by 2.4%. With better leasing options, the market economy will feature higher output and welfare, but we find that the constrained social planner now improves less upon the market economy; or, better leasing options greatly reduce inefficiencies in the economy. Therefore, in contrast to the standard literature, the welfare implications are more complete in our setting with leasing capital.

Closely related, we study the implications on optimal taxation and subsidies when leasing market still features inefficiency. That is, we consider what the implied tax and subsidy rates for the constrained social planner would be. In the equilibrium, the benefit is relatively larger if the firm is more likely to be financially constrained and hence its investment is more elastic for additional marginal benefit from subsidies. On average, the subsidy rates are around 2–3% for new capital purchases, and about 15–25% for leased capital. Higher productivity firms are associated with lower subsidies since they are on average less constrained. With better leasing conditions, the required subsidy rates decrease. For example, when the monitoring cost is doubled exogenously, the equilibrium rental fee increases by about 6%, and the optimal subsidy rate on leased capital will increase on average by about 4.5 percentage points. We also show that, if the planner mistakenly ignores leasing market, the cost would be 4.6% of steady state consumption, which is economically significant; the planner would induce too low asset price and too much investment into new capital (for more details see Section IV.E). Therefore, a lesson is that ignoring leasing market may lead to incomplete and biased welfare implications; and in turn, the implied optimal tax/subsidy policies should take into account the developments of both the leasing market and financial market.

In addition, we find that the impacts of government regulation policy in leasing markets can be large. These policies can help reducing frictions for leasing activities. For example, improvements in regulations may change monitoring costs/depreciation costs that firms must pay in the process of leasing. Quantitatively, if the monitoring cost for leasing capital is reduced (by half), compared to our benchmark model, the rental fee drops by about 2.4%, the total expenditure on leasing capital increases by about 3.5%, aggregate consumption and output increase by about 2.1% and 1.9%, respectively (see more examples in Section IV.F).[10] These policy improvements are not only relevant for leasing markets in developed countries (e.g., U.S.), but also potentially important for developing countries as well, as financial market development is more limited and firms could use leased capital more. Using our quantitative model, we also experimented with different levels of monitoring costs associated with different degrees of financial frictions, and we confirm it is robust that improving leasing market conditions can have sizable welfare improvements.

[10]From another perspective, if the leasing market is improved and there are more expenditures on leased capital; for one more dollar expended, the aggregate consumption will increase by about 4.1 dollars.

Related literature This paper contributes to several strands of literature, specifically on the pecuniary externalities with collateral constraints for firms, on possible welfare improvement from constrained social planners, on firms' leasing decisions in partial and general equilibrium, and also on capital allocations with financial frictions. We highlight our contribution with details below.

Since the seminal work by Kiyotaki and Moore (1997), researchers have long been interested in understanding collateral-constraint-related pecuniary externalities. Lorenzoni (2008) shows that there could be inefficient borrowing and investment due to financial frictions since individuals do not take into account their investment in aggregate asset prices, especially prices in a downturn. He and Kondor (2016) consider the role of pecuniary externalities in liquidity management. Dávila and Korinek (2018) further highlight different types of externalities, distributive externalities, and collateral externalities, and we are motivated by their analysis. Bianchi and Mendoza (2018) and Jeanne and Korinek (2019) provide quantitative analysis in different business cycle models. Building on the work of Dávila and Korinek (2018), Lanteri and Rampini (2023) make significant quantitative contributions to the literature, and show that the distributive externality is larger than the collateral externality in the stationary equilibrium typically. In their framework, the price of collateral is too high from the perspective of a constrained social planner, because the most financially constrained firms are net buyers of the old capital. In comparison, our model shows that with leased capital, the inefficiencies due to pecuniary externalities when we have financial frictions will be mitigated, and the extent to which a social planner can further improve the market economy could be much smaller than is the case without leased capital (or leased capital is very costly).

Our modeling of leasing builds on the theories of corporate leasing decisions, mostly following seminal papers such as Eisfeldt and Rampini (2009), Rampini and Viswanathan (2010, 2013). However, our setup and framework differ in several important ways: Eisfeldt and Rampini (2009) is a static model, and Rampini and Viswanathan (2010, 2013) are dynamic models in partial equilibrium frameworks and they mostly focus on individuals' optimal choices. Our framework however is on general equilibrium, both for the two-period model and for the dynamic quantitative model.

For studying equilibrium capital allocation/reallocation and related efficiency properties, it is necessary to have a fully specified, general equilibrium model such as ours. Several other papers in the literature also consider leasing choices but emphasize different aspects and implications.[11] Different from these papers, we explicitly model individual firms' dynamic capital choices, including leasing, purchasing capital from markets or investing by themselves. This framework allows us to highlight and analyze in detail how the inefficiency due to pecuniary externalities is impacted by leasing option both in the market economy and for social planner.[12]

Our paper further relates to the literature that links aggregate efficiency to capital misallocation caused by financial frictions at the firm level.[13] We highlight and analyze the possible gains if a benevolent social planner were to face the same set of financial constraints as individual firms (constrained efficient allocation) when leasing options differ. Different from Lanteri and Rampini (2023) who also analyzed constrained efficiency with financial frictions, we introduce, analyze, and quantify how the leasing market could impact both the market economy and constrained efficiency.

The paper proceeds as follows. Section II presents the two-period model with capital reallocation of new, leased, and old capital, for which we characterize the equilibrium and discuss our main results intuitively. In Section III, we then introduce a dynamic, quantitative, general equilibrium model. We then study the impacts of leased capital on individual firms' investment, capital reallocation, and inefficiencies in markets, and explore how different regulations in the leasing market

[11]For example, Li and Tsou (2019) mostly focus on the cross-sectional asset pricing implications of leasing; Hu, Li and Xu (2020) analyze the mitigation of leasing on capital misallocation but do not study the role of leasing on pecuniary externalities-induced inefficiencies as this paper does.

[12]Our paper is also connected to the capital reallocation literature (e.g., among others, Eisfeldt and Rampini (2006); also see Eisfeldt and Shi (2018) for an excellent review). Eisfeldt and Rampini (2007) analyze investment in new and used capital when firms face financial constraints. Lanteri (2018) introduces idiosyncratic productivity shocks and quantitatively studies the business-cycle model with an explicit market for used capital and heterogeneous firms. Ai, Li and Yang (2020) link financial intermediation and capital reallocation. Rampini (2019) meanwhile analyzes the effects of asset durability on the financing of investment with collateral constraints. In addition, Lanteri and Rampini (2023) build a quantitative model and focus on quantifying the pecuniary efficiency. Empirically, Eisfeldt and Rampini (2006) and Ai, Li and Yang (2020) document that the amount of capital reallocation is procyclical. Li and Xu (2022) propose a measure of lease-adjusted capital reallocation, and find that it is less procyclical, especially for small and financially constrained firms. Our paper also accounts for leasing as an important reallocation channel.

[13]After the seminal framework by Hsieh and Klenow (2009), Restuccia and Rogerson (2008), this literature has been growing fast (see, among others, Buera, Kaboski and Shin (2011), Moll (2014), Buera and Moll (2015), Midrigan and Xu (2014), Ai et al. (2019), and Gopinath et al. (2017)).

affect aggregate welfare, how constrained social planners can improve the market economy, among others. These are in Section IV. Finally, we conclude the paper in Section V.

II Two-period Model

Time is discrete. The economy is populated with a representative household and over-lapping generations of heterogeneous firms (firms only live for two periods). Comparing to the framework used in seminal papers such as Eisfeldt and Rampini (2007) and Lanteri and Rampini (2023), we introduce leased capital. Our model allows us to highlight theoretically the role of leased capital in mitigating inefficiencies due to pecuniary externalities in competitive equilibrium. We capture capital reallocation in the equilibrium through three different types of capital: new, old, and leased capital (denoted as k_t^n, k_t^o and k_t^l, respectively).

II.A Model Setup

1 Household

The representative household has a risk-neutral preference,

$$\sum_{t=0}^{\infty} \beta^t C_t, \tag{1}$$

in which $\beta \in (0,1)$ is the discount factor, and C_t is consumption. The household owns firms directly (detailed below) and also serves as the lessor: she can accumulate leased capital K_{t+1}^l and rent it to firms. Both new and leased capital are productive for two periods, and they will turn into "old" capital going to the next period; old capital is productive only for one period and then fully depreciates. Her budget constraint follows:

$$C_t + K_{t+1}^l + \beta^{-1} B_t^H = B_{t+1}^H + q_t(1-h)K_t^l + \tau_t^l K_t^l + \int d_t^i di.$$

That is, at time t, the household collects the dividend from all owning firms $\int d_t^i di$ and collects the rental fees $\tau_t K_t^l$ for the leased capital rented out in the previous period. In addition, as the owner of the leased capital, the household is able to resell the returned leased capital $(1-h)K_t^l$ in the secondary market at price q_t (the price for old capital). $h > 0$ is the monitoring cost (or, additional depreciation cost) to avoid agency problems caused by the separation of ownership and control. At time t, the household repays all the debt $\beta^{-1}B_t^H$ at the risk-free gross interest rate β^{-1} (B_t^H is debt if $B_t^H > 0$ and is saving if negative), and borrows B_{t+1}^H for the next period. The household combines the above net worth and uses it for accumulating leased capital K_{t+1}^l and for her consumption C_t.

The first-order conditions for the household imply that the per unit leasing fee τ_t should satisfy:

$$\tau_t = \beta^{-1} - q_t(1-h).$$

That is, the user cost for leased capital is equal to the discounted price for new capital minus the resale value as old capital, plus the required monitoring cost due to the agency problem.[14] it does not change our implications (also see more general setting in our dynamic quantitative model).

2 Firms and Financial Frictions

Next, we describe firms' problems and the financial frictions they face. There are over-lapping generations of firms, and each firm has a life period of two. At each date, a continuum of firms with a measure of one is born. Each firm is endowed with an exogenous net wealth w, which is distributed over the interval $[w_{min}, w_{max}]$, following an exogenous distribution $\pi(w)$. Naturally, w_{min} and w_{max} are both positive since otherwise a firm is unrealistically born with negative net worth. We assume that w_{max} is sufficiently large and w_{min} is sufficiently small. We suppress the dependence of firms on w wherever appropriate.

Upon birth, firms make decisions with respect to three types of capital and production in the

[14] For simplicity and illustration purpose, we assume the rental fee is paid after production, consistent with Gal and Pinter (2017). The key intuition remains when we consider the alternative economy in which period-by-period rental fee is paid in advance.

next period. That is to say, after one period, firms become old and produce. Each firm has a production technology f, which is a function of total utilized capital k. Moreover, the production technology satisfies $f_k > 0$, $f_{kk} < 0$. Without a loss of generality, we assume $f(k_t) = k_t^{\alpha}$, in which k_t is the total capital input in the production under a perfect substitution assumption (i.e., $k_t \equiv k_t^n + k_t^l + k_t^o$).[15] Figure A.1 in the Appendix illustrates the timing of the model.

The firm can borrow through the bonds market at the gross interest rate β^{-1}; the borrowing amount at time t is subject to a classic form of collateral constraint: $\theta q_{t+1} k_{t+1}^n \geq \beta^{-1} b_{t+1}$. The collateral constraint requires that debt repayments cannot exceed a fraction $\theta \in [0, 1)$ of the future resale value of new capital invested. The collateral value of new capital is the new capital amount determined at time t, k_{t+1}^n, multiplied by the future price of the old capital, because k_{t+1}^n becomes old capital after production at time $t+1$. Note that here old capital purchases have no future resale value and don't serve as collateral, as old capital fully depreciates at the end of the period.[16]

Given the initial net wealth w and the price of old capital q_t, a firm maximizes the present discounted value of dividends using the same discount factor of the household, and decides on dividends d_{0t} and $d_{1,t+1}$, new, old, and leased capital, k_{t+1}^n,k_{t+1}^o,k_{t+1}^l, as well as the debt level b_{t+1}, to solve:

$$\max_{\left\{d_{0,t}, d_{1t+1}, b_t, k_{t+1}^n, k_{t+1}^o, k_{t+1}^l\right\}} d_{0,t} + \beta d_{1,t+1}, \tag{2}$$

subject to the budget constraints for the current and next period, the collateral constraint, and the

[15] This assumption is consistent with prior studies (Lanteri and Rampini, 2023, Rampini and Viswanathan, 2013). Nevertheless, we consider a more general form relaxing this assumption in the dynamic quantitative analysis.

[16] We extend this in our dynamic model.

non-negativity constraints:

$$
\begin{aligned}
w_{0t} + b_{t+1} =& d_{0t} + k^n_{t+1} + q_t k^o_{t+1}, && (3)\\
f\left(k^n_{t+1} + k^o_{t+1} + k^l_{t+1}\right) + q_{t+1} k^n_{t+1} =& d_{1,t+1} + \tau_{t+1} k^l_{t+1} + \beta^{-1} b_{t+1}, && (4)\\
\theta q_{t+1} k^n_{t+1} \geq& \beta^{-1} b_{t+1}, && (5)\\
k^n_{t+1}, k^l_{t+1}, k^o_{t+1} \geq& 0, && (6)\\
d_{0,t}, d_{1,t+1} \geq& 0. && (7)
\end{aligned}
$$

II.B Competitive Equilibrium

In the closed economy, there are three markets: a bond market, a market for leased capital, and a market for old capital. The condition for old capital at any time t is:

$$\int k^n_t(w)\, d\pi(w) + (1-h) K^l_t = \int k^o_{t+1}(w) d\pi(w).$$

The left-hand side is the total supply of old capital in time t: it includes all firms' new capital invested in time $t-1$ and the remaining leased capital from time $t-1$; the right-hand side is the total demand for old capital in time t (the supply and demand are from different generations of firms). The market-clearing condition for leased capital at any time t is:

$$K^l_{t+1} = \int k^l_{t+1}(w)\, d\pi(w).$$

The market-clearing condition for bonds across all firms and households in any time t is:

$$\int b_{t+1}(w)\, d\pi(w) + B^H_{t+1} = 0.$$

We focus on the stationary competitive equilibrium: it consists of a set of heterogeneous firms' policy functions, $\{d_1(w), k^n(w), k^o(w), k^l(w), b(w)\}$, and a price of old capital q, such that the firms and the household solve their maximization problems in Eqs. (1) and (2), and the market for old capital clears, $\int k^n(w)d\pi(w) + \int k^l(w)(1-h)d\pi(w) = \int k^o(w)d\pi(w)$.

II.C Characterizing Competitive Equilibrium

1 User Costs of Different Capital

To facilitate our analysis, we define the user costs for different capital. We first set up the lagrangian for a typical firm. Without a loss of generality, we assume that firms only pay dividends when they are old (i.e., $d_{0,t}$ is 0). We denote the multipliers on Eqs. (3) to (5) by μ_{0t}, $\mu_{1,t+1}$ and $\beta\lambda_t$, respectively. Further, we denote the multipliers on the non-negativity constraint for k^n_{t+1}, k^o_{t+1}, k^l_{t+1}, and $d_{1,t+1}$ by $\underline{\nu}^n_t$, $\underline{\nu}^o_t$, $\underline{\nu}^l_t$, and γ^d_t, respectively.

Taking first-order conditions gives the optimal demand for investment in new capital, the old capital, leased capital, and also bonds as follows:

$$1 + \mu_{0t} = \beta(f_k(k_{t+1}) + q_{t+1}) + \beta\lambda_t\theta q_{t+1} + \underline{\nu}^n_t, \tag{8}$$

$$q_t(1 + \mu_{0t}) = \beta f_k(k_{t+1}) + \underline{\nu}^o_t, \tag{9}$$

$$\beta\tau_{t+1} = \beta f_k(k_{t+1}) + \underline{\nu}^l_t, \tag{10}$$

$$1 + \mu_{0t} = 1 + \lambda_t. \tag{11}$$

That is, the marginal value of net worth at date t is $1 + \mu_{0t} = 1 + \lambda_t \geq 1$. This reflects the additional value due to the collateral constraint. A firm's marginal value of net wealth at date $t+1$ is $1 + \mu_{1,t+1} = 1$, or $\mu_{1,t+1} = 0$, as the firm pays all its remaining, positive net wealth out as dividends, and it is unconstrained at $t+1$. Eqs. (8), (9), and (10) indicate the following user costs of different capital (in terms of consumption goods at time t that must be paid). Specifically, the user cost of new capital u^{nn}_t is defined as:

$$u^{nn}_t(w) = 1 - \frac{\beta}{1+\lambda_t(w)}q_{t+1} - \frac{\lambda_t(w)}{1+\lambda_t(w)}\beta\theta q_{t+1},$$

that is, the user cost of new capital is equal to the current price of new capital, 1, minus the discounted resale value, $\frac{\beta}{1+\lambda_{t+1}(w)}q_{t+1}$, and we subtract the marginal value of relaxing the collateral constraint for owning this capital. The user cost of old capital is $u_t^{oo}(w) = q_t$, which is the price of the old capital since it is only productive for one period. The user cost of leased capital u_t^{ll} (combined with the leasing fee inferred from the first-order condition of the household) is:

$$u_t^{ll}(w) = \frac{1}{1+\lambda_t(w)}\beta\tau_{t+1} = \frac{1}{1+\lambda_t(w)}\left(1-\beta q_{t+1}(1-h)\right),$$

that is, the leasing fee in terms of the marginal value of the firm with a net worth w.

For ease of comparison, we multiply these three user costs by $1+\lambda_t(w)$ and relabel them as:

$$u_t^n(w) = 1-\beta q_{t+1} + \lambda_t\left(1-\beta\theta q_{t+1}\right), \tag{12}$$

$$u_t^o(w) = q_t + \lambda_t q_t, \tag{13}$$

$$u_t^l(w) = 1-\beta q_{t+1}(1-h). \tag{14}$$

Comparing u_t^l and u_t^n, we can see that the cost of leasing includes the monitoring cost incurred due to the agency problem and the cost of giving up the marginal value of relaxing the collateral constraint by owning the capital, whereas the benefit of leasing is the premium saved on the borrowing cost when cheaper household loans become unaccessible due to a binding collateral constraint(i.e., λ_t becomes positive). In other words, leased capital is “highly collateralizable but expensive” as in Eisfeldt and Rampini (2009), Rampini and Viswanathan (2013). For ease of comparisons, also note that in a frictionless economy (first best), there are no financial frictions, all λ will be zero, and we have $q = 1/(1+\beta)$. The firm will not lease due to the monitoring cost h (see Section B.1 in the Appendix for more details on first best).

2 Characterizations

We summarize the characterization in the following proposition (for detailed technical assumptions, proofs, solutions and thresholds, please see Appendix B.1).

Proposition 1. *A stationary competitive equilibrium when* $h \in (\frac{(1-\beta\theta q)[q(1+\beta)-1]}{\beta q(1-q-\beta\theta q)}, \frac{\beta u^o(w_{min})-1+\beta q}{\beta q})$ *is characterized as follows:*

1. *If* $q > \frac{1}{1+\beta}$*, there exists threshold* $\bar{w}_l < \underline{w}_n < \bar{w}^o < \bar{w}$ *such that: firms with* $w \leq \bar{w}_l$ *invest in both old and leased capital; firms with* $w \in (\bar{w}_l, \underline{w}_n]$ *invest only in old capital; firms with* $w \in (\underline{w}_n, \bar{w}^o)$ *invest in new and old capital; firms with* $w \geq \bar{w}^o$ *invest only in new capital; and* $w \geq \bar{w}$*, firms are unconstrained, invest only in new capital, and achieve the optimal marginal product of capital.*
2. *If* $q = \frac{1}{1+\beta}$*, then* $\bar{w}^o = \bar{w}$*, such that firms with* $w \leq \bar{w}_l$ *invest in both old and leased capital; firms with* $w \in (\bar{w}_l, \underline{w}_n]$ *invest only in old capital; firms with* $w > \underline{w}_n$ *invest in the new and old capital, and achieve the optimal scale of production for all firms* $\left(f_k^{-1}\left(\frac{1}{\beta(1+\beta)}\right)\right)$.

In Figure A.2 in the Appendix, we intuitively illustrate user costs and optimal solutions (using parameters specified in the numerical examples we introduce below). For a given q, the left panel plots user costs ($u_t^n(w)$, $u_t^o(w)$, $u_t^l(w)$ as in Eqs. (12), (13) and (14) as linear functions of λ. Since firms' net worth moves monotonically with λ, we can then directly compare these different lines for a given λ. The right panel additionally plots the optimal choices for different firms (the bold, yellow parts). Clearly, the ability to lease limits the maximum possible level of user cost: when the firm is poor and the λ is sufficiently large. These firms are most likely constrained, and they choose both old capital and leased capital (concentrated on the point of $\bar{w}^l$ in the figure). This is the place where leased capital can mitigate financial frictions and alleviate inefficiencies. This role of leasing is also pointed out as in Eisfeldt and Rampini (2009) and Hu, Li and Xu (2020). Lastly, the equilibrium choices for old capital vs. new capital are also consistent with Lanteri and Rampini (2023) (for our paper's main difference from these papers, please see the literature review for more details).

To illustrate the impacts of leased capital, we use a numerical example.[17] In Figure I, we plot firms' optimal choices in the benchmark economy (monitoring cost h=0.3) in blue lines. Consistent with Proposition 1, there exist four thresholds, $\bar{w}_l < \underline{w}_n < \bar{w}^o < \bar{w}$. The firm with $w < \bar{w}_l$ invests both leased and old capital, and their total investment k^{total} is constant in net worth, and marginal benefit for relaxing borrow constraints is also constant. Firms with $\bar{w}_l < w < \underline{w}_n$ invest only old capital, and their total investment increases in wealth and λ decreases in w. Firms with $\underline{w}_n < w < \bar{w}^o$ invest both new and old capital, keeping the total investment constant. Firms with $\bar{w}^o < w < \bar{w}$ invest only in new capital but are still financially constrained. Firms with $w > \bar{w}$ invest only in new capital and are unconstrained.

[Place Figure I about here]

In Figure I, we also present the results for another economy with a slightly higher monitoring cost (h=0.4) in red lines, and contrast them with those in blue lines. For firms with sufficiently small $w < \bar{w}_l$, the amount of leased capital is reduced, and also, the number of firms using leased capital shrinks, since leasing is more expensive now; instead, some of these firms must fully rely on old capital. For other firms with $w > \bar{w}_l$, the main pattern for capital choice is very similar, except that since equilibrium price q increases with higher h, firms within $\underline{w}_n < w < \bar{w}^o$ use more compositions toward new capital.

II.D Analyzing the Inefficiency in Competitive Equilibrium

1 Inefficiency in C.E. with Leasing Market

Having described the competitive equilibrium, it is natural to ask what the inefficiencies in the market economy are and upon which dimension the social planner can improve. Thus, we next

[17]The details of the example include: $[w_{\min}, w_{\max}] = [0.1, 5]$, and we assume that a firm's net worth is distributed such that smaller firms have larger probability measures. For example, with $N = 5000$ firms in $[w_{\min}, w_{\max}]$, the probability of π for firm i is proportional to $i^{(-0.5)}$. Other parameters include: $\beta = 0.8$, $\theta = 0.25$, and $\alpha = 0.85$, and the benchmark value of h is 0.3. Note that we require the minimum price to be above 25% of the first best price $1/(\beta + 1)$ so that technically there is some lower bound on capital price.

characterize the constrained-efficient allocation, in which the social planner makes investment decisions, subject to the same constraints that are present in the competitive equilibrium. By doing this, we can analyze the pecuniary externality and the inefficiencies in the competitive equilibrium.

A planner maximizes the present discounted value of aggregate dividends:[18]

$$\max_{\left\{d_{1,t+1},k^n_{t+1},k^o_{t+1},k^l_{t+1}\right\}} \int \left[d_{10}(w) + \sum_{t=0}^{\infty} \beta^{t+1} d_{1,t+1}(w)\right] d\pi\left(w\right)$$

subject to the firms' budget constraints for the current and next period, the collateral constraints, as well as a market clearing condition for old capital, and the rental market condition of $\tau_t = \beta^{-1} - q_{t+1}(1-h)$ (see Appendix B.2 for details on the specifications and derivations).

The first-order condition with respect to the price of old capital q_t is (using λ as the firms' multiplier for the planner; see details in Appendix B.2):

$$\int k^o_{t+1}(w)(1+\lambda_t(w))d\pi(w) = \int k^n_t(w)\left(1+\theta\lambda_t(w)\right)d\pi(w) + \int \left(1-h\right)k^l_t d\pi(w),$$

and we subtract both sides with the market clearing condition (Eq. (B4) in the Appendix), we can have,

$$\int k^o_{t+1}(w)\lambda_t(w)d\pi(w) = \theta \int k^n_t(w)\lambda_t(w)d\pi(w). \tag{15}$$

The left-hand side of Eq. (15) represents the aggregate distributive externality induced by a marginal increase in the price of old capital q_t: firms that purchase old capital at t value the additional expenditure they need to incur as the product of the quantity purchased k^o_{t+1} and firm multipliers. The right-hand side of Eq. (15) represents the aggregate collateral externality induced by the same marginal increase in q_t: firms that purchase new capital at t and face a binding collateral constraint are able to borrow against a fraction θ of the additional collateral value. This is because as the price increases, the collateral value increases. This is the marginal benefit for firms.

[18]Strictly speaking, the planner's objective is to maximize the welfare of the representative household; since firms and households have the same discount factor, it is equivalent to maximizing the present value of dividends, adjusted for some initial condition of the household. See Section C.5 in the Appendix for details of the general dynamic model.

In short, Eq. (15) highlights the offsetting effects of the positive externality and negative externality induced by an increase in old price q for financially constrained firms in the constrained-efficient allocation.

We focus on stationary economy. Denote $\Delta^l = \Delta_{L1} - \Delta_{L2}$, in which $\Delta_{L1} = \int k^o(w)\lambda(w)d\pi(w)$ and $\Delta_{L2} = \theta \int k^n(w)\lambda(w)d\pi(w)$. Then, we have $\Delta^l = 0$ in the constrained efficient allocation. However, in a stationary competitive equilibrium, the aggregate distributive externality is not necessarily equal to the aggregate collateral externalities. We can show that the aggregate distributive externality is larger than the aggregate collateral externality in stationary equilibrium (see Section B.5 in the Appendix for the details). That is, $\Delta^l > 0$. It indicates that in the competitive equilibrium firms are constrained too much, and the equilibrium price of old capital tends to be higher than the constrained-efficient one. Eq. (15) further shows that a marginal reduction in the price of old capital may have a positive effect on aggregate welfare.[19]

2 Inefficiency in C.E. without Leasing Market

Next, for further comparison we consider a counterfactual, special case in which the leasing market is artificially shut down. It is equivalent to assuming that the monitoring cost is sufficiently large in our previous model, with $h > \frac{\beta u^o(w_{min})-1+\beta q}{\beta q}$. This is similar to the economy studied in Lanteri and Rampini (2023).

In this economy, firms can only invest in new or/and old capital. Under stationary equilibrium,

[19] For the constrained efficient problem, typically we also must ensure the price q above some minimal level (i.e., firms can always scrap some fraction of the capital (into output goods)). See Lorenzoni (2008) and Lanteri and Rampini (2023) for similar treatment. Otherwise, if we do not have any constraints for planners' q, it is possible that she would like to induce an extremely low price, even a negative price for old capital (often seen in our numerical examples), to help those financially constrained firms. A simple assumption on the minimal level of q, for example, could be 25% of the first best price. In this case, the equation for optimal q would be slightly modified as:

$$\int k^o_{t+1}(w)\lambda_t(w)d\pi(w) = \theta \int k^n_t(w)\lambda_t(w)d\pi(w) + \chi_{min},$$

in which χ_{min} denotes the multiplier for the additional constraint of $q \geq q_{min}$, with $\chi_{min} \geq 0$. Numerically, we can solve the planner's problem both with and without constraints on q, and the constrained planner always tries to maximize the aggregate welfare for the representative household. Quantitatively, we find that our results are robust to different assumptions on price constraints.

all our previous derivations remain (except those related to leased capital). We summarize the results in Proposition 2 in the Appendix. To measure the inefficiencies in this economy, similarly as before, we can also solve for the constrained-efficient allocation subject to the same constraints that are present in the competitive equilibrium. We can see the same form as that in Eq. (15). That is, positive externality and negative externalities offset each other induced by an increase in old price q for financially constrained firms in the constrained-efficient allocation. We denote $\Delta = \Delta_1 - \Delta_2$, in which $\Delta_1 = \int k^o \lambda d\pi$ and $\Delta_2 = \theta \int k^n \lambda d\pi$. Thus, $\Delta = 0$ in the constrained efficient allocation. Following similar logic as before, we can show that the aggregate distributive externality is larger than the aggregate collateral externality in the market economy, as in Lanteri and Rampini (2023).

3 C.E.: Welfare Improvement due to Leasing

We are now in a position to compare the above two cases. Here we consider an experiment in which h is sufficiently high, $h = 0.8$ as compared to $h = 0.3$ in the benchmark. Setting h to be 0.8 is effectively shutting down the leasing market, since it is very expensive to deploy capital through leasing now. For this extreme comparison, we see that aggregate output drops about 14%, consumption drops about 10%. In addition, for the impacts on q when changing h, there are two main effects. On the one hand, allowing firms to lease capital increases the supply of old capital in the next period (since leased capital turns into old capital after one period); on the other hand, using leased capital could make constrained firms "richer" and demand relatively less for old capital and relatively more for new capital. Both effects lead to a drop of old capital price. We confirm this in our numerical example in Table I.

[Place Table I about here]

With respect to the source of welfare improvements, there are two main channels. First, when the leasing market is available, sufficiently constrained firms are able to lease capital, in addition to purchasing old capital. This directly improves these firms' capital allocation and increases their output. We can see the effects through different indicators: without leasing options, both output and consumption will decrease by more than 10%, the average firm multipliers $\int \lambda(w)d\pi(w)$ increases by more than 60%, the average marginal product of capital (MPK) increases by 13% and the dispersion almost doubled. In short, this is the direct effect of leasing in mitigating financial frictions. This channel is also discussed and highlighted in Hu, Li and Xu (2020). Note that this channel works for any given level of asset price, and thus not related to pecuniary externalities and any inefficient movements in asset prices.

Second, and perhaps more importantly for our purpose, the ability to lease improves efficiency through **"reducing externalities".** The externalities will be largely impacted when there is no leasing market. For the externality differences introduced previously (Δ^l with leasing, and Δ without leasing), we find that it increases from 0.10 to 0.12 when shutting down the leasing market. Since Δ^l can help measure the inefficiencies due to pecuniary externalities from a social planner's perspective (although not perfect, see more discussions later in Section II.E), this suggests that without a leasing market, the externalities and the inefficiencies all increase. This channel is different from the former one, and it is our main focus and new contribution in this paper. To measure the inefficiency more precisely (Δ is just one indicator), we need to compare the planner's allocation with competitive equilibrium in detail. Below, we provide more analysis for the inefficiency from a constrained social planner's perspective.

II.E Constrained Social Planner: Welfare Improvement due to Leasing

Up to now, we have discussed the efficiency improvement with leasing in competitive equilibrium. We now further discuss the role of leasing in constrained efficient allocations. That is, even for social planners, leasing options could also improve their solutions.

First best or not. To begin with, we first note that in the stationary equilibrium of constrained efficient allocation, the optimality condition for the price of old capital indicates that, $\int k^o(w)\lambda(w)d\pi(w) = \theta \int k^n(w)\lambda(w)d\pi(w)$. There are two possible scenarios in which this condition can be satisfied. Clearly, if all firms are unconstrained, that is, $\lambda = 0$ for all w, then it satisfies this condition. In this allocation, the first-best level of welfare can be achieved for the constrained social planner. For example, if we assume w is sufficiently large for all firms, then this is possible. In this case, leasing has no role in improving the overall efficiency in constrained efficient allocation, as leasing is expensive and involves monitoring costs.

First best cannot be achieved by the constrained social planner. In more general cases, the constrained social planner cannot achieve the first best; however, with better leasing options, even for a planner, improvement possibilities still exist. For example, when there are initially a lot of sufficiently small firms, the leasing fee is relatively large. We illustrate these in Figure II.

[Place Figure II about here]

In Figure II, as we increase h, we plot the equilibrium outcomes from both the competitive equilibrium and also the constrained social planner. A few points are clear: (1) as h decreases, both the competitive equilibrium and constrained social planner have higher output and consumption, use more leased capital, and have more old capital in the economy. This is mainly due to the previously mentioned technological improvement, as those constrained firms now have more and better technological options (i.e., leasing). (2) Holding constant h, we can see that the constrained social planner can always improve the market economy. This is precisely related to our previous discussions in that there are inefficiencies in the market economy: we call these pecuniary externalities, and we can decompose the inefficiencies into two types such that the planner can internalize these pecuniary externalities as much as possible (subject to minimal price constraints). (3) Further, when h is sufficiently large (h close to 0.8), both economies will not use leased capital, as it is too expensive, even for those constrained firms. In this case, there is a larger role of the social planner to improve the overall efficiency in the competitive equilibrium. For example, the consumption improvement is about 33% for $h = 0.8$, in contrast to 16% for $h = 0.3$. (4) For the capital price q, we know it is relatively too high in the market economy, as those unconstrained firms tend to invest less than the social optimal, and those constrained firms would benefit with a lower capital price. Based on our analysis of pecuniary externalities, the planner then would like to induce a lower price for old capital as much as possible, since in this way she can help those constrained firms. Qualitatively, in Figure II as h increases, the planner will use more positive multiplier ϕ to do so.[20] Or, equivalently, the planner will reward firms investing in new capital and punish those for investing in the old capital; and leasing partially helps with this direction. Numerically, ϕ increases from about 0.4389 to about 0.4450 as h increases, and the induced price of q varies in a very small range between 0.1392 and 0.1393.[21]

[Place Figure III about here]

Further, to measure how much the **constrained social planner** can improve upon market economies with different h, we can explore more on other indicators (beyond aggregate consumption, aggregate output, optimal planner multiplier).[22] One intuitive method is to compute the average and the dispersion of the marginal product of capital across firms. This method is then very similar to those used in Hsieh and Klenow (2009) and Restuccia and Rogerson (2008), and it is appropriate in our framework, since we have only one period for production essentially and all different types of capital are perfect substitutes (alternatively, we can use the previously defined individual-specific user costs, and define averages and dispersion; nonetheless, our results would be equivalent). In particular, in Figure III, we compute the average for the marginal product of capital, $\beta \int f_k(k^{total}(w))d\pi(w)$, and also the standard deviation for $\beta f_k(k^{total}(w))$. We can see that: (1) For any given h, both the averages and the dispersion are lower in the planner's economy, sug-

[20]Recall that ϕ is the Lagrangian multiplier on the market clearing condition of old capital, and the first-order conditions for the planner in the stationary case are:

$$1 + \lambda = \beta \left[f_k \left(k_{t+1} \right) + q \right] + \beta \theta \lambda q + \beta \phi,$$
$$q \left(1 + \lambda \right) = \beta f_k \left(k_{t+1} \right) - \phi.$$

[21]Technically, when h is sufficiently large, q is sufficiently close to the assumed minimal price (0.1389) in our numerical computations if we simulate using tens of millions of firms.

[22]For social planner's allocations, in an ideal case in which there are no constraints on minimal price at all, we would have $\chi_{min} = 0$, and the differences in externality (as defined previously Δ^l) would be 0 in the constrained efficient allocations. However, numerically, we find that it is often the case that the planner would like to induce very low or negative prices in q if there were no constraints; therefore, $\chi_{min} > 0$ is often seen when we impose realistic constraints on q. In this case, Δ^l is related to $\chi_{min} > 0$, and the interpretation is not so straightforward.

gesting efficiency improvement. (2) Clearly, when h is high (h=0.8), no leased capital will be used, and the improvement upon the market economy is much larger than that with $h = 0.2$. Moreover, the inefficiencies due to pecuniary externality with lower h are much smaller than that when h is higher or when leasing is not available. Quantitatively, as shown in Table II, the numbers are sizable: with $favorable$ leasing conditions, the planner can only improve the average of the marginal product by about 2.9%, but she can improve it by about 11% if h is higher. Similar numbers can also be seen in the dispersion.

[Place Table II about here]

Summary for the Two-period Model. So far we have developed a relatively simple, two-period, overlapping generation model to characterize the important impacts of the leasing market on mitigating financial frictions and improving welfare. In sum, we have learned the following thus far.

First, with better leasing market conditions, both the competitive equilibrium and also the constrained social planner can have higher output and consumption, use more leased capital, and have more old capital in the economy.[23] Second, from the social planner's perspective, even if she is constrained, there are still inefficiencies in the market economy for a given level of h, which we can identify as two distinct externalities: distributive and collateral externalities. We show that there is a positive gap between those two. This inefficiency, as measured by the positive gap between distributive externalities and collateral externalities, increases in economies with higher h. The option for firms to lease capital can alleviate such inefficiency since financially constrained firms can directly use more leased capital; also, with leasing markets it increases the supply of old capital and decreases the demand on old capital eventually in equilibrium.

[23]We note that when using leased capital, there will be some efficiency loss due to the monitoring costs/depreciation costs of leased capital; however, this cost is dominated by the benefits of helping constrained firms.

III A Dynamic Quantitative Model

We now consider a dynamic quantitative general equilibrium model. Compared to the two-period model which focuses on theoretical properties and analytical results, the dynamic model allows us to quantify the importance of leased capital. The dynamic model features a stochastic firm life cycle, persistent idiosyncratic productivity shocks, long-lived capital for production, and capital reallocation across different firms. Output goods can be consumed by the representative household, transformed into new capital by the firm, or converted to leased capital by the lessor (the household). Investment requires one period to build. In the model, firms' total net worth is endogenously determined and affected by stochastic productivity.

III.A The Representative Households and Leased Capital Market

A representative household with linear utility over consumption streams and discount factor β owns all firms in the economy. The representative household also supplies leased capital (hence acts as the lessor) to firms in the economy and collects rental payments. When leased capital is used for individual firms' production, there are monitoring costs incurred in the process, for which firms must pay; this is different from the case in which firms use old or new capital. In a general case, we assume monitoring cost, $H(K_t^l)$, is proportional to the amount of leased capital K_t^l, or, $H(K_t^l) = dK_t^l$ with a positive monitoring cost parameter d.

For the leased capital that firms rent for production in period $t + 1$, households have the technology to transform output goods into leased capital one to one; the total supplied leased capital in period t is denoted as K_{t+1}^l, and households collect the payments of $\tau_t K_{t+1}^l$ in period t. In the next period, the leased capital is returned to the household after all firms' production, and its remaining value is $\delta^l q_{t+1} K_{t+1}^l + (1 - \delta^l) K_{t+1}^l - H(K_{t+1}^l)\cdot$. That is, δ^l fraction of leased capital will become old capital, and the market value for that part is $\delta^l q_{t+1} K_{t+1}^l$, and $(1 - \delta^l) K_{t+1}^l - H(K_{t+1}^l)$ is the remaining undepreciated leased capital, which can be transformed to output one-to-one in period $t + 1$.

The budget constraint for the representative household now can be written as:

$$C_t + (1 + r_t)B_t^H + K_{t+1}^l = B_{t+1}^H + \delta^l q_t K_t^l + (1 - \delta^l)K_t^l - H(K_t^l) + \tau_t K_{t+1}^l + \int d_t^i di - \rho w_0,$$

in which C_t is current consumption, B_t^H is the households' debt the position determined in the previous period (if B_t^H is negative, it means the household has positive savings), and B_{t+1}^H is the new debt raised in period t. d_t^i denotes all the dividend payments from all firms (members of the large household family) and all the transfers to new firms are ρw_0, which we will detail next.

The optimality condition for households to supply leased capital suggests that the user cost for leased capital in terms of a period-t unit of consumption goods is:

$$\tau_t = 1 - \beta \left[\delta^l q_{t+1} + (1 - \delta^l) - d\right]. \tag{16}$$

III.B Firms, Financial Frictions, and Production

In the previous two-period model, firms live, and capital is productive only for two periods. The assumption that firms live for two periods rules out endogenous net worth dynamics. The assumption that old capital can be productive for only one period rules out the possibility of using old capital as collateral. In the general model, firms have a stochastic life cycle, and capital is long-lived. First, at each period, with exogenous probability $\rho \in (0, 1]$ (the death shock is realized at the end of each period/beginning of the next period), firms learn that they could die after this period's production and pay their remaining net worth as a dividend. With probability $1 - \rho$, firms continue their production in the next period. Thus, at each date, a measure ρ of new firms with initial net worth w_0 is born, and firm net worth evolves endogenously. We let $\gamma_a = (1 - \rho)^a$ denote the survival probability of a new firm up to age a. The total mass of firms is always 1.

Second, capital goods depreciate as follows. For each unit of new capital, a fraction $\delta^n \in (0, 1]$ becomes old capital after production, and a firm can pledge a faction of θ of the resale value of capital the next period, $(1-\delta^n(1-q_{t+1}))k^n_{t+1}+q_{t+1}(1-\delta^o)k^o_{t+1}$, as collateral. We note that when $\delta^n=\delta^o=1$, this specification nests the two-period model, and both new and old capital can serve as collateral. In our benchmark case, we let $\delta^n=\delta^l$. We denote age by a and let s^a be a history of realizations of idiosyncratic shocks up to firm age a, with associated exogenous probability $p(s^a)$. The measure of firms of age a that survive and invest to produce in the following period is then $\rho\gamma_a=\rho(1-\rho)^a$.

Specifically, the details for the dynamic model are as follows. Time is discrete and infinite. In every period, a continuum of new firms is born and receives a common initial endowment of output w_0 from the household. At their initial date, firms draw a level of productivity $s_i\in S\equiv\{s_1,s_2,...,s_N\}$. At the production date, firms i produce output with production function, $y_{it}=s_{it}f(k_{i,t})$, $f_k>0$,$f_{kk}<0$, $k_{i,t}\equiv g(k^o_{i,t},k^l_{i,t},k^n_{it})$, in which g is a constant-return-to-scale bundle of the sum of new and leased capital, and old capital. In this model, we specify it as:

$$g(k^n_{it},k^l_{i,t},k^o_{i,t})=\left[(\sigma)^{\frac{1}{\varepsilon}}(k^n_{it}+k^l_{it})^{\frac{\varepsilon-1}{\varepsilon}}+(1-\sigma)^{\frac{1}{\varepsilon}}(k^o_{it})^{\frac{\varepsilon-1}{\varepsilon}}\right]^{\frac{\varepsilon}{\varepsilon-1}},$$

where σ is the share of new and leased capital in the CES bundle. Here new capital and leased capital are perfect substitutes, but there is a plausible degree of imperfect substitutability between new and old capital.[24] Idiosyncratic productivity evolves following a Markov transition matrix.[25] Firms can finance themselves through non-contingent debt, subject to collateral constraints, which specify that promised repayment cannot exceed a fraction θ of the total resale value of new and old capital. For external finance through equities, we assume it is limited and, in turn, dividend distribution cannot be negative.

Firm optimization problem The objective function of a firm, born at time t, is to maximize the expected present discounted value of dividends. The details of the problem are delegated to

[24]This is empirically plausible and facilitates computation by avoiding corner solutions.

[25]In addition, for a technical assumption (as discussed before; also, as in Lorenzoni (2008) and Lanteri and Rampini (2023)), we also assume that there is some scrap value of old capital, say, q_{min}: when the capital price is extremely low, then firms can choose to recover q_{min} fraction of old capital into output goods. We choose q_{min} so that the market economy and first best all feature higher prices.

Section C.1 in the Appendix. We denote by $\beta^{t+1}\lambda_t(s^a)$ the multiplier on the collateral constraint (Eq. (C8) in the Appendix) and $\beta^t\eta_t(s^a)$ the multiplier on non-negativity dividend (Eq. (C9) in the Appendix). The firm optimality conditions for new capital, old capital, leased capital, and debt are:

$$\begin{aligned}1+\eta_t\left(s^a\right) = \beta E_t\left\{\left[s_{a+1}f_k\left(k_{t+1}\left(s^a\right)\right)g_{n,t+1}\left(s^a\right)+\left(1-\delta^n\left(1-q_{t+1}\right)\right)\right]\left(1+(1-\rho)\eta_{t+1}\left(s^{a+1}\right)\right)\right\}\\ +\beta\theta\lambda_t\left(s^a\right)\left(1-\delta^n\left(1-q_{t+1}\right)\right),\end{aligned} \tag{17}$$

$$\begin{aligned}\left(1+\eta_t\left(s^a\right)\right)q_t = \beta E_t\left\{\left[s_{a+1}f_k\left(k_{t+1}\left(s^a\right)\right)\right)g_{o,t+1}\left(s^a\right)+q_{t+1}\left(1-\delta^o\right)\right]\left(1+(1-\rho)\eta_{t+1}\left(s^{a+1}\right)\right)\right\}\\ +\beta\theta\lambda_t\left(s^a\right)q_{t+1}\left(1-\delta^o\right),\end{aligned} \tag{18}$$

$$(1+\eta_t(s^a))\tau_t = E_t\left\{\left[s_{a+1}f_k\left(k_{t+1}\left(s^a\right)\right)g_{L,t+1}\left(s^a\right)\right]\left(1+(1-\rho)\eta_{t+1}\left(s^{a+1}\right)\right)\right\}, \tag{19}$$

$$\eta_t\left(s^a\right) = E_t\left[(1-\rho)\eta_{t+1}\left(s^{a+1}\right)\right]+\lambda_t\left(s^a\right). \tag{20}$$

We can compare the difference in the optimality condition between the previous two-period model and the dynamic model. Because both new and old capital are long-lived and have collateral values, the marginal for the optimal condition of old (new) capital (Eq. (17) and Eq. (18)) depends on the future marginal product, as well as the future resale value and also the effect of old (new) capital on the collateral constraint. Also, productivity is stochastic, implying that future marginal products and future marginal shadow costs for financing are also stochastic. Therefore, all four optimality conditions (17), (18), (19), and (20) involve the conditional-expectation operation E_t.

We can also rewrite the optimization problem with the standard Bellman equation[26]:

$$V_t(s_t,w_t) = \max_{b_{t+1},k^n_{t+1},k^o_{t+1},k^l_{t+1}} d_t+\beta(1-\rho)E_tV_{t+1}(s_{t+1},w_{t+1}(s_{t+1}))+\beta\rho E_tw_{t+1}(s_{t+1}),$$

[26]Our numerical algorithm and numerical exercises subsequently are based on the recursive formulations.

and the firm's constraints are:

$$w_t = s_t f(k_t) + (1 - \delta^n(1 - q_t))k_t^n + q_t(1 - \delta^o)k_t^o - (1 + r_{t-1})b_t,$$

$$d_t = w_t + b_{t+1} - k_{t+1}^n - q_t k_{t+1}^o - \tau_t k_{t+1}^l, d_t \geq 0,$$

$$\beta\theta \left[\omega_{t+1}^n k_{t+1}^n + q_t \omega_{t+1}^o k_{t+1}^o\right] - b_{t+1} \geq 0.$$

More details are delegated to Section C.1 in the Appendix. We then define the competitive equilibrium for the economy. That is, in the equilibrium all firms optimize, and the markets for old capital, leased capital, and output goods all clear. For brevity's sake, we delegate all details to C.2 in the Appendix.

III.C Constrained Efficiency

We first consider first best (FB). As in the two-period model, firms' investment will not depend on net worth but will solely depend on firm productivity (again for the sake of space, the details are delegated to C.3 in the Appendix). We next consider the constrained-efficient allocation for the economy. This is motivated by the fact that the social planner in the first best can hypothetically *eliminate* and overcome *any* market frictions faced by individual firms in the market economy. It is different, and more restricted, in the case of constrained-efficient allocation. The question we ask is whether the planner can improve the market allocation by simply commanding different levels of new, old, and leased capital, as well as debt, all while respecting firms' constraints and internalizing the effects of individuals' choices on the price of the old capital and rental fees. That is, the social planner is constrained to consider allocations with zero net transfers across any firms (no firm receives a positive transfer from other firms, and no firm has net transfers to other firms).[27] With these assumptions, for the social planner to maximize the lifetime welfare for the representative household, it is equivalent to maximizing the present discounted value of aggregate dividends,

[27] See, for example, Joseph E Stiglitz (1982), Lorenzoni (2008), Julio Davila, Jay H Hong, Per Krusell and José-Víctor Ríos-Rull (2012), Eduardo Dávila and Anton Korinek (2018), Lanteri and Rampini (2023), among others.

adjusted for some initial condition of leased capital.[28] For maximizing dividends, the problem is as follows:

$$\sum_{t=0}^{\infty}\beta^t\left[\sum_{t=0}^{\infty}\sum_{s^a}p(s^a)\gamma_a d_t(s^a)+\sum_{a=1}^{\infty}\sum_{s^a}p(s^a)\gamma_{a-1}\rho w_t(s^a),\right]$$

subject to firms' budget constraints, current collateral constraints, with multiplier $\lambda_t(s^a)$, current non-negativity conditions on dividend (Eq. (C9) in the Appendix), with multiplier $\eta_t(s^a)$, and the market clearing condition, with a multiplier $\beta^t\phi_t(s^a)$, and a multiplier χ_{min} for the price cannot fall below the scrap value (i.e., there is some lower bound on the price of old capital that the social planner can induce so that the allocation problem is neither trivial nor realistic). As in the two-period model, the social planner now can recognize the additional marginal benefit to society if there is an additional supply of old capital.[29] The optimality condition for the price of old capital q_t is as follows[30]:

$$\begin{aligned}
&\sum_{a=0}^{\infty}\gamma_a\sum_{s^a}p(s^a)k^o_{t+1}(s^a)\left[1+\eta_t(s^a)\right]\\
=&\sum_{a=0}^{\infty}\gamma_a\sum_{s^{a+1}}p(s^{a+1})\left[\delta^n k^n_t(s^a)+(1-\delta^o)k^o_t(s^a)\right]\left(1+\eta_t(s^{a+1})(1-\rho)+\theta\lambda_{t-1}(s^a)\right)\\
&+\sum_{a=0}^{\infty}\gamma_a\sum_{s^{a+1}}p(s^{a+1})\left[\delta^l k^l_t(s^a)(1+\eta_{t-1}(s^a))\right]+\chi_{min}.
\end{aligned}$$

The left-hand side reports the total marginal cost of increasing the current old capital price q_t. Those firms that must purchase more capital bear higher costs. The right-hand side represents the total marginal benefit and includes three components. First, the total wealth of those firms owning old capital increases (old capital is predetermined in time t). Second, the collateral value of financially constrained firms at period $t-1$ increases. Third, a higher price q_t enables the equilibrium rental fee to decrease at period $t-1$, which benefits those sufficiently financially constrained firms. Thus, through this optimality condition, we can distinguish total distributive externality, total collateral

[28] See the notes in Section C.5 in the Appendix. In numerical practice, we utilize the first-order conditions, budget constraints, and value functions for firms, household optimality conditions, and market clearing conditions to maximize the lifetime welfare for the representative household.

[29] See the details of the firm's optimality conditions and discussions in Section C.5 in the Appendix.

[30] Also see Section C.5 for another format using notations of firm distributions and firm policy functions.

externality, and the externality related to leased capital. The social planner optimally balances these different types of externalities as much as possible, and the additional multiplier χ_{min} is possibly binding since the price must be higher than the minimum possible level.[31]

IV Quantitative Results

IV.A Calibration and Estimation

We calibrate the model in two steps. For several parameters that are relatively standard in the literature, we directly set those values. Another set of parameters in the model, they are generally hard to observe and equally hard to measure with realistic data; thus, we resort to the method of moments matching and use closely related empirical moments to help us discipline these parameter values. We delegate all details to Section C.8 in the Appendix.

[Place Table III about here]

The results for all parameter values are reported in Table A.7 in the Appendix and the comparison between model moments and data moments is reported in Table III. Overall, the model fit is reasonably good, especially since we deliberately keep the model structure relatively simple and transparent. The values for productivity persistence is consistent with some other estimates (Cooper and Haltiwanger (2006) and Clementi and Palazzo (2016) for example); for standard deviations, we note that our model has a fairly close match for several different measures. The results on leased capital, in terms of average shares, persistence, and volatility are also close between our model and our data. Admittedly, for firm leverage (and consequently the capital structure for firms), the model moment is not able to generate very high levels of debt as in the data, as the literature points out that it generally needs a more complicated model structure and/or some other elements such as tax benefits for debt (e.g., Hennessy and Whited (2007)); since our paper's main focus is

[31] For the dynamic model, we find that numerically the social planner's capital price never hits the lower bound (e.g., 25% of first best price); this is slightly different from that in the two-period model.

on leased capital, we leave it to future studies that could incorporate more features from corporate finance and capital structure.

In Table III, we also examined the cross-sectional distribution for the stationary economy. In particular, both in the data and in the model, based on firms' total assets (Compustat item "at" in the data, and $(k^n + qk^o)$ in the model), we can divide all firms into four quartiles with an equal number of firms within each quartile.[32] We find that the model economy also matches well for several moments across the distribution, including the leased capital ratios across different firm size groups.

IV.B Inspecting Firm Optimization

To understand the role of leased capital in the economy, we first inspect firms' optimal choices. The details of the first-order conditions are discussed in Section C.1 in the Appendix. We can further look at the numerical solutions for firms' optimal choices. Figure IV plots a typical firm's optimal choices on different types of capital as functions of firm net worth; in this figure, firms' current productivity is fixed, and firms' optimal choices with different levels of current productivity have very similar patterns. To highlight leased capital and the role of financial constraints, net worth is in logs so that firms with small net worth are clearly represented.

[Place Figure IV about here]

Intuitively, since firms have decreasing returns to scale production technology, firms will always choose a positive k^o_{t+1} and a positive summation in $k^n_{t+1} + k^l_{t+1}$. When the firm's net worth is sufficiently large, it will choose some optimal level of capital so that the effective rates of return from investing in different types of capital are exactly the same as investing in the financial market.

[32] In the following exercise, we also consider dividing firms in the model into different groups based on one of the individual state variables, firm net worth w, and we find that the resulting statistics are very close.

Thus, firms will have:[33]

$$1 = \beta E_t \left[\left(s_{t+1} f_k(k_{t+1}) g_{n,t+1} + \omega^n_{t+1} \right) \right],$$
$$1 = \beta E_t \left[\left(s_{t+1} f_k(k_{t+1}) \right) g_{o,t+1}/q_t + \omega^o_{t+1} \right) \right].$$

At the same time, it is also easy to see that firms will not use any leased capital, since the user cost of leased capital is larger than that of new capital, $\tau_t > \left(1 - \beta\omega^n_{t+1}\right)$, and these two types (new and leased) are perfect substitutes for production; if firms had used leased capital, we would have:

$$\tau_t = \beta E_t \left[s_{t+1} f_k(k_{t+1}) g_{l,t+1} \right] = \beta E_t \left[s_{t+1} f_k(k_{t+1}) g_{n,t+1} \right] = 1 - \beta\omega^n_{t+1},$$

which cannot be true (also see more discussions in Section C.1 of the Appendix). Therefore, we see in Figure IV that, firms' choices on new and old capital are flat after some thresholds in net worth, and the the optimal choice of leased capital is exactly zero.[34]

On the other extreme, when a firm's net worth is very small, the firm will use leased capital instead of new capital, even if the user cost is higher ($\tau_t > 1 - \beta\omega^n_{t+1}$). To see this, intuitively, the firm must have some down payment for using new capital, and when its net worth is very small, the shadow costs of paying these down payments is actually very high. Formally, we can see this from the re-arranged first-order conditions on new capital and leased capital:

$$(1 + \eta_t)\left(1 - \omega^n_{t+1}\beta\right) + \lambda_t \omega^n_{t+1}\beta(1 - \theta) = \beta\left(EV_k\right),$$
$$(1 + \eta_t)\tau_t = \beta\left(EV_k\right),$$

[33]To simplify notations, we had denoted $(1 - \delta^n(1 - q_{t+1}))$ as ω^n_{t+1}, and $q_{t+1}(1 - \delta^o)/q_t$ as ω^o_{t+1}.

[34]Also note that, in Figure IV, there are "jumps" for the policy functions of leasing, new, and old capital when net worth changes. This is a feature of the modeling framework, since firms do not face any adjustment costs in changing capital stocks (making it quite different from standard investment models with adjustment costs); rather, in our model, firms only have state variables in s and w, and none of k^o, k^n, k^l in isolation is a state variable. In addition, firms do not have any other frictions (e.g., fixed lump-sum investment). By doing this, our model is particularly convenient for studying capital allocations and reallocations (including purchases, sales, and renting decisions, among others). These features are also consistent with Eisfeldt and Rampini (2009), Rampini and Viswanathan (2010, 2013), and also Lanteri and Rampini (2023).

in which we used EV_k as a shorthand to denote the marginal product of capital. For a given level of old capital, if firms choose the same level of k^n and k^l, then the marginal products of capital for k^n and k^l are exactly the same since they are perfect substitutes (i.e., $E_t\left[s_{t+1}f_k(k_{t+1})g_{n,t+1}\tilde{\eta}_{t+1}\right] = E_t\left[s_{t+1}f_k(k_{t+1})g_{l,t+1}\tilde{\eta}_{t+1}\right] \equiv EV_k$). In this first-order condition equation for k^n_{t+1}, the left-hand side represents the firm's valuation of the new capital's user cost: the first component is $(1+\eta_t)(1-\omega^n_{t+1}\beta)$, the valuation of those paid costs, and the second component is the shadow value of paying these down payments (θ is the fraction of capital that can be borrowed against in the next period). From Figure A.5 in the Appendix, we can also see numerically that firms have very high values in multipliers when net worth is very small.

As the firm's net worth increases, it uses more leased capital and old capital. In the figure, we can also see intuitively that leased capital increases even faster than old capital, mainly since firms still must pay the down payment when using old capital and this is costly, and firms only have to pay the user cost each period when using leased capital. As net worth is relatively large, the size of leasing capital is almost the same as that of old capital. After that, there is a transition stage when firms start to use new capital and old capital, and do not use any leased capital anymore. In Figure IV in the Appendix, the policy function for leased capital has a discontinuous part and jumps to zero when net worth is larger than some threshold point. From the first-order conditions that we have described, we know in this region firms are still constrained and still have positive multipliers in η_t and λ_t but that their values are already quite close to 0 (i.e., firms are close to being unconstrained). These can also be verified through Figure A.5.

Across different levels of firm productivity, Figure A.4 in the Appendix shows that the pattern for capital choice is very similar for firms with the highest and lowest levels of productivity (recall that we have a finite, discrete Markov space for s): higher productivity is associated with higher levels of capital in general. More interestingly, in the first panel we see that, when productivity is very high, there are some regions for which high-productivity firms will still choose to use leasing capital, no new capital, while low-productivity firms with the same level of net worth will choose some amount of new capital. Intuitively, when current productivity is very high, firms would like to make the best of capital in this period's production since productivity is mean reverting; for the the alternative choice of investing in new capital and using collateral borrowing to finance new capital, it is dominated in this case by using leased capital only.

For firms' optimization, we provide more information about the multipliers of η_t and λ_t in Figure A.5 in the Appendix, and about firm values and dividend distributions in Figure A.6 in the Appendix. The patterns for η, λ, v and d are relatively standard and consistent with other firm dynamics models with financial frictions. Intuitively, when w is small, firms are more likely to be constrained, and firms evaluate net worth much higher than other firms. Thus, the value function is concave in w for a given z. The dividend multiplier η is decreasing and convex in w, the collateral constraint multiplier λ follows similar patterns as η, and it is lower than η since the action of dividend distribution in the current period will take into account the future possibility of being constrained, and the collateral constraint is only for the current period's status. As this figure shows, many firms will be constrained in the next period so that η is strictly higher than λ; in this case, firms have precautionary saving incentives. When firms have relatively large net worth, firms are almost not constrained and firm value function is almost linear in w, and the dividend increases linearly with w. Figure A.6 also shows that, for firms with higher levels of productivity, they may have lower dividend payments since productivity is persistent, and firms will invest more in capital for the next period.

IV.C The Role of Leased Capital in Partial Equilibrium

To further illustrate the role of leased capital in mitigating firms' financial frictions in the credit market, here we consider a simple illustration: we assume the monitoring cost increases (so the per unit rental fee τ increases) but q does not change in a partial equilibrium, and these two cases are labeled as "Low Rental Fee" with a solid black line, versus "High Rental Fee" with a dashed red line. Figure V and Figure A.7 in the Appendix plot the firm's optimal choices on the three types of capital and the associated firm value and multiplier η function.

[Place Figure V about here]

In Figure V, when the firm's net worth is small, higher rental fees will force the firm to use much less leased capital (see the middle panel), and only when firms have an extremely low net the worth will they purchase a small amount of leased capital; at the same time, firms must invest more in new capital with higher rental fees. With higher rental fees, firms basically do not use leased capital, and the amounts of new and old capital are both increasing continuously in the firm net worth. This case is then very similar to the results in Lanteri and Rampini (2023). When net worth is relatively large, firms will start to use new and old capital in both cases, and the figure shows that their optimal policies are exactly the same, regardless of whether rental fees are higher or lower.

Figure A.7 shows that with higher rental fees, firms with low net worth tend to have higher multipliers in η and have lower valuations for the same productivity and net worth; however, this is not the case for firms with relatively higher levels of net worth: as firms gradually become unconstrained, they will use new and old capital in either case. Therefore, based on the differences in the multipliers of η, we observe that leased capital can mitigate firms' financial frictions in the credit market due to the requirement of collateral constraints.

IV.D The Distribution of Firms in General Equilibrium

Previously, we only analyzed firms' optimal decisions and comparisons in partial equilibrium; in general equilibrium, in addition to the fact that firms will endogenously accumulate net worth so that the distribution over different firms is endogenous, the prices for old capital and rental fees are also endogenously determined. Moreover, in the case of pecuniary externality, it is defined in general equilibrium. Figure A.3 in the Appendix reports the distribution of firms from various different perspectives. In short, the economy features a significant share of firms being constrained, and a large portion of these firms lease capital extensively (for the sake of space, see more discussions and details in Section C.9 in the Appendix).

IV.E Quantifying the Efficiency Improvement from a Social Planner

To further understand the role of leased capital in the economy, we compare the three different allocations for the first best case, the constrained social planner, and for the competitive market economy. As introduced in Sections C.3 in the Appendix and also in Section III.C, first best and constrained efficient allocation are two different perspectives that can offer possible improvements over the market economy with financial frictions. In Figure VI we first compare firms' optimal choices under different allocations. We report the choices on leased capital, new capital, and old capital, as well as the multipliers η for each case. For illustrative purposes, we focus on firms with the highest level of productivity in each allocation.

[Place Figure VI about here]

In Panel (a) of Figure VI, we observe that the first best social planner will not use any leased capital, simply because without any financial frictions, the user cost of leased capital is higher than that for new capital, and it is more efficient to invest only in new capital (and old capital). Comparing the constrained efficient economy and the market economy, almost all firms across different levels of net worth will use higher levels of leased capital in the market economy; when the net worth is relatively large, there are some regions at which constrained social planners stop using leased capital while firms in the competitive equilibrium still choose to use leased capital since the capital price is much lower in the constrained efficient economy, and also since there is enough supply of old capital due to the social planner's improvement. This is also consistent with the policy functions for new and old capital in Panels (b) and (c): a first best economy features a constant level of the new and old capital investment (since we fixed productivity in the figure but the firm net worth varies), or net worth does not matter for investment, and there is no role for financial frictions to affect firms' choices.

For the other two economies, however, we see a few points worth noting: (1) the overall pattern for all capital choices is very similar, as the social planner is constrained and must respect all frictions and budgets just like individual firms in the market economy; (2) the constrained social planner tends to use more new capital whenever they are able to, and in the most unconstrained case with a relatively large firm net worth, the social planner also invests more in new capital. (3) For old capital, the social planner tends to use more for production when the firm net worth is relatively small, as capital prices are cheaper; when the firm's net worth is relatively large, the planner actually uses less old capital and more new capital, since by doing so the marginal benefit to the whole society is larger and is reflected in the social planners' multiplier on the market-clearing condition of the old capital (ϕ, as in Section III.C); and also, ϕ is positive in the quantitative model (i.e., there is some reward from the social planner for investing more in new capital and penalties for using more old capital). Therefore, for the constrained efficient economy, the total amount of new and old capital will be higher, and thus welfare improves.

[Place Table IV about here]

In Table IV we report the detailed quantitative results on a series of aggregate variables. We note that the decrease in capital price from the competitive equilibrium to constrained social planner is large (about 38%); this is mainly because the social planner has a very large increase of almost 74% investment in new capital. Even though the leasing fee increases, from 10% of one unit output goods to about 13%, and total leased capital is decreased by 30%, the overall change in the supply of old capital (from the depreciation of both new and leased capital) is not that large (about 2%). We can also now compare other economies to the first best; clearly first best features much higher output and consumption, almost by design such that the social planner in this case can overcome any financial frictions.[35]

In addition, we see both distributive externality and rental externality decrease, mainly due to the decrease in capital price and leased capital;[36] since the price is now lower, the collateral

[35] Also, for asset prices, we can compute user costs for different types of assets, which are more relevant and allow for fair comparisons across different types of economies: (current) user costs for new and old capital for first best are 0.0906 and 0.0934, respectively; they are 0.0945, 0.0884, and 0.1011 for new, old, and leased capital in competitive equilibrium, respectively (all these numbers are computed from a financially unconstrained firm's perspective). Clearly, the first best has lower user costs for new capital, and both new and old capital costs are lower than the rental cost from the perspective of competitive equilibrium.

[36] Recall that in the stationary equilibrium, the distributive externality, $\int k^o(s,w)\,(1+\eta(s,w))\,d\pi(s,w)$, has three

externality increases but the overall magnitude is still relatively small. Thus, both the constrained efficient economy and the market economy suggest that the externality related to capital price due to collateral constraint is relatively small; this is more clearly illustrated in Figure VII for different types of externalities, and we note that the distributive externality across different firms has a larger magnitude. These results from our dynamic model are also consistent with the two-period model.

[Place Figure VII about here]

Lastly, across the firm distribution, we see leased capital ratios are decreasing for all groups with different levels of firm net worth; the individual firms' multipliers η change somewhat monotonically, and it seems that small firms are on average more likely to be financially constrained since leased capital now is more expensive; and for firms with relatively large net worth, they benefit more from capital price decreases.

[**Place Table V about here**]

Overall, the social planner balances marginal benefits and costs for different groups. Aggregate output and consumption increase for about 1.7% and 2.4%, respectively. Intuitively, using leased capital can help financially constrained firms mitigate frictions; for the constrained social planner, the room for further improvement becomes smaller. In Table V we provide more comparisons: holding constant for all other parameters, we experiment with higher or lower levels of agency cost d in the model and then compare aggregate consumption across different economies.[37] We normalize the benchmark competitive equilibrium level to 100%. Higher agency costs are associated with

major terms (see Section C.5); the first term is:

$$\int \sum_{s_{t+1},w_{t+1}} [(1-\delta^o)k^o(s,w)+\delta^n k^n(s,w)]\left[(1+\eta(s_{t+1},w_{t+1}))(1-\rho)+\rho\right] P(s_{t+1}|s) I_{\{w_{t+1}=g(s,w,s_{t+1})\}} d\pi(s,w),$$

and the second term (collateral externality) is:

$$\int [(1-\delta^o)k^o(s,w)+\delta^n k^n(s,w)]\,\theta\lambda(s,w)d\pi(s,w),$$

also, the third term, $\int \delta^l k^l(s,w)\,(1+\eta(s,w))\,d\pi(s,w)$, is the rental externality.

[37]The values for d are: (1) benchmark value d plus 40% of rental fees in the benchmark model; (2) benchmark d plus 20% of rental fees in the benchmark model; (3) 50% of benchmark value d; and (4) $d = 10^{-6}$.

lower welfare; however, for a given higher d, the social planner can improve more. For example, the planner could improve about 13.5% with a relatively higher d (leasing is effectively shutting down in this economy), but has almost no room for improvements if d is already very low. Thus, our findings in the quantitative model are consistent with what we learned from the two-period model. Lastly, compared to the model without leasing as in Lanteri and Rampini (2023), the social planner's improvement in our benchmark economy is also smaller than the 5% reported in their paper, even though we have different model structures and baseline calibrations; this further provides a robustness check on our results.

Implied Taxation. To implement the constrained social planner's problem (e.g., the standard Ramsey optimal taxation problem with commitment), we can implement with firm-specific tax/subsidies separately on different types of capital. In particular, we can define tax/subsidy functions as follows, such that for given state variables on (s, w), a firm's budget constraint is modified as:

$$d_t = w_t + b_{t+1} - \left[1 - \Gamma^N(s,w)\right] k^n_{t+1} - q_t \left[1 - \Gamma^O(s,w)\right] k^o_{t+1} - \tau_t \left[1 - \Gamma^L(s,w)\right] k^l_{t+1} - TR(s,w),$$

in which $\Gamma^N(s,w), \Gamma^O(s,w), \Gamma^L(s,w)$ are the tax/subsidy rate for the three different types of capital purchases, and $TR(s,w)$ is any lump-sum transfer or subsidy that is individual-specific and is used to make sure that there are zero transfers across firms. By inspecting the optimality conditions for the constrained social planner, we can define tax rates as follows such that in a competitive equilibrium with tax/subsidy, the constrained efficient allocation can be implementable and has the same market allocations:

$$\begin{aligned}
\Gamma^N(s,w) &= \frac{\beta\phi\delta^n}{1+\eta^{SP}(s,w)}, \Gamma^O(s,w) = \frac{\phi\left[\beta(1-\delta^o)-1\right]}{1+\eta^{SP}(s,w)}, \Gamma^L(s,w) = \frac{\beta\phi\delta^l/\tau}{1+\eta^{SP}(s,w)}, \\
TR(s,w) &= \Gamma^N(s,w)k^n_{t+1} + q_t\Gamma^O(s,w)k^o_{t+1} + \tau_t\Gamma^L(s,w)k^l_{t+1},
\end{aligned}$$

in which ϕ is the social planner's multiplier, η^{SP} is the firm's multipliers in the constrained efficient allocation, and all the capital choices for $TR(s,w)$ are evaluated exactly as the social planner's choices. Intuitively, this form of taxation can provide tax benefits to firms for investing in new capital ($\Gamma^N(s,w) > 0$ if $\phi > 0$), and the benefit is larger if the firm is more likely to be financially constrained; hence, its investment is more elastic for additional marginal benefits on price, which is consistent with the basic principles of optimal taxation. On the other hand, for firms close to being financially unconstrained, the effective tax rate is relatively large, as these firms typically are large and not so elastic in changing their marginal investment behavior. Quantitatively, in Figure VIII we report the implied tax rates for firms with the highest and the lowest levels of productivity, and also note that the domain for firm net worth is restricted for which we have active firms in the allocation. We can see that, on average, the subsidy rates are around 2-3% for new capital investment, and about 15-25% for capital leasing;[38] higher productivity is associated with lower subsidies. Lastly, the subsidy rate increases with firm net worth for any given productivity, which is mainly driven by the planner's motive to induce these firms to invest and provide more new capital to the entire economy. Across different economies, when the monitoring cost is doubled comparing to the benchmark model, the equilibrium rental fee increases by about 6%, and the optimal subsidy rate on leased capital will increase on average by about 4.5 percentage points. That is, in this case, the planner has stronger incentives to induce firms to rent more (typically for those constrained firms).

Welfare Loss If Ignoring Leasing Markets. Lastly, we consider another counterfactual experiment and ask what would happen if the social planner ignores the leasing market. In that case, the planner makes a mistake in the model specification and she would place a different multiplier ϕ in our benchmark model; or, she would have a different incentive for stimulating investment on new capital.

In particular, we first compute the social planner's best multiplier ϕ by assuming the agent cost d for leasing is prohibitively high such that the leasing market is effectively shut down. We then apply this ϕ to our benchmark model where leasing cost is realistic. We report the results in Table VI in

[38] For the orders of magnitude, we note that the price of new capital investment is 1, and the price of leased capital is τ_t, which is roughly about 10% of that for new capital.

the Appendix. The planner's best multiplier ϕ is 0.46 when d is very high as 0.1081, much higher than benchmark value d of 0.007. In this experiment, compared to the benchmark competitive equilibrium or the benchmark social planner, we find that the misspecified social planner would induce a too high multiplier ϕ, a too low capital price q.

This planner would also completely shut down the leasing market (see the rows for leasing capital, or leasing capital ratio), but at the same time, she would have too strong incentives for investing in new capital. For example, her new capital would be almost doubled compared to the benchmark CE. Intuitively, this is because she completely ignored the role of leasing market in helping those financially constrained firms. In the experiment, the resulting aggregate output would be lower by about 4.5%, consumption lower by 4.6% compared to the benchmark CE; or, the cost of ignoring leasing market would be 4.6% of steady state consumption, which is economically significant. Therefore, a lesson is that ignoring leasing market may lead to incomplete and biased welfare implications; and in turn, the implied optimal tax/subsidy policies should take into account the developments of both the leasing market and financial market.

[Place Figure VIII about here]

IV.F Policy Implications: Regulatory Change For The Leasing Market

Based on our previous analysis, we see that leased capital is important in the cross-section, especially for those firms more likely to be financially constrained. Here we suppose there are some changes in terms of regulations or institutional arrangements for the leasing market; in such a case, what would happen to individual firms and to the aggregate economy? This is motivated by several recent policy changes. For example, in the U.S. there are some changes in state laws around the 2000s which make it easier for lenders to repossess collateral in the case of firm bankruptcy. Different states implemented these law changes in different years (for example, Texas and Louisiana in 1997, followed by Alabama in 2001, Delaware in 2002, South Dakota in 2003, Virginia in 2004, and lastly Nevada in 2005). Therefore, several papers have used these regulatory changes (such as

Li, Whited and Wu (2016) and Chu (2020)) - typically through a difference in difference method and argued that these law changes will affect firms' collateral asset valuations and secured lending. Indirectly, this may affect firms' leasing decisions (e.g., Chu (2020)). With respect to our model, these changes could be captured parsimoniously by changes in the collateral parameter θ or by decreases in monitoring cost d.

We, therefore, assess the aggregate and distributional impacts of regulation changes in our model. Specifically, we experiment with alternative parameter values and then compare them with the benchmark model. For the aggregate variables, we report the percent deviations relative to the benchmark economy. In Table A.8 columns (1) and (2), we consider different monitoring costs d, 50% higher or 50% lower than the benchmark value. We focus on a lower d. As suggested by $\tau_t = 1-\beta[\delta^l q_{t+1} + (1-\delta^l) - d]$, we can see that when d decreases, on impact it directly decreases rental fees, and indirectly it affects other aggregate variables and the price of old capital q in particular. As rental fees decrease, financially constrained firms use more leased capital (as shown in the previous partial equilibrium analysis) and produce more. As a result, the total demand for leasing is higher, and the unconstrained lessor can meet the higher demand. In turn, the aggregate leased capital is higher, and since part of it will depreciate into old capital in the next period, the total supply of old capital increases, and the price q decreases. Quantitatively, when the monitoring cost exogenously drops by 50%, the equilibrium rental fee decreases by 2.4%, aggregate output increases by about 1.9% relative to the benchmark economy, aggregate consumption by 2.1%, dividend increases by 2.4%. Thus, the elasticity of output to rental price is roughly -0.9.

We also inspect several other macro variables related to externalities that we introduced for the constrained social planner. The distributive externality, the net marginal effects to the society involving capital purchases due to capital price change, increase slightly, mainly due to price decreases. The externality involving rental capital (the term $\int \delta^l k^l(s,w)\,(1+\eta(s,w))\,d\pi(s,w)$) increases, since the total amount of leased capital increases and the multipliers η do not change as much (see the rows below). Lastly, collateral externality, generally with a relatively small magnitude, increases slightly as the total amount of new and old capital that can be used as collateral now increases. In addition, we also report several statistics from different groups of firms by their net worth in the cross-section. We find for firms with relatively small net worth, they tend to use more leased capital when d decreases; however, as the capital price now is cheaper, firms with relatively large net worth tend to use more other types of capital, and in turn, the leased capital ratio decreases slightly. Overall, with different monitoring costs d in these experiments, we find similar patterns for the firms in the cross-section, and the quantitative differences are not that large.

Closely related, we also consider changes in δ^l, with 50% higher or 50% lower than the benchmark value (0.12). The results are in Table A.8 columns (3) and (4). As we can see, the effect on rental fees is quite large at 20% (column (4) for example). Intuitively, if δ^l decreases, the user cost charged by the lessor will also decrease, and the resulting increased demand on leased capital will induce firms to produce more. We see the elasticity for aggregate output to changes in rental fees is again close to about -0.8 in this case; thus, both experiments show that, with changes of different underlying sources in the leasing market, output and consumption appear to increase as rental fees decrease. With respect to the impacts on other firm variables in the cross-section (see Figure A.8 in the Appendix), as more leased capital becomes available in the economy, it clearly decreases the value of η, simply because leased capital is relatively cheaper now; even firms with a net worth in the highest quantile also tend to use more leased capital on average and have smaller firm leverages.

Lastly, we consider different values in the collateral parameter θ. Our results are reported in columns (5) and (6) of Table A.8, as in the example of law changes mentioned above. Intuitively, when firms can borrow more against a given value of new and old capital, they will borrow more, invest relatively more in new capital, and produce more. Also, their financial frictions can be mitigated, and as a result, the demand for leased capital will decrease. This is exactly what we observe in column (5). It is worth noting that since the amount of aggregate new capital will increase but aggregate leased capital tends to fall, the impacts on capital price reflect these two forces and tend to increase in the equilibrium. Consistent with those empirical estimates (such as Chu (2020)), we find leased capital ratio also decreases when θ increases. Firms on average borrow more and have smaller shadow values for additional external finance (the value of η). Overall, improved regulations for both the leasing and collateral market can mitigate firms' financial frictions and increase aggregate output and consumption.

IV.G Alternative Model Specifications, Robustness, and Aggregate Implications

Lastly, we also check our model with alternative specifications, and we confirm that our aggregate and distributional variables are robust to several alternative assumptions and specifications. We delegate all details to Section C.10 in the Appendix.

V Conclusion

Empirical evidence shows firms extensively use leased capital in their production, and more so for those firms more likely to be financially constrained. However, leased capital typically is not studied in the literature that focus on firm heterogeneity and financial frictions. We investigate the impacts of leased capital on aggregate inefficiency due to externalities in the credit markets. We do so both theoretically and quantitatively. In our model, firms have heterogeneous productivity and endogenously accumulated net worth, and firms can produce with new, old, or leased capital, but firms' borrowing capacity is subject to collateral constraints against the market value of owned capital assets. The dynamic model can fit data moments reasonably well, and is suitable to study capital reallocation and pecuniary externalities that arise since individual firms do not internalize their actions on market prices.

Theoretically, in our two-period model, we show that for competitive equilibrium with leased capital, inefficiencies still exist due to pecuniary externalities from the constrained social planner's perspective. We further analyze in detail two particular types of pecuniary externalities: the distributive externality (net benefits across capital buyers and sellers) and the collateral externality (impacts through collateral values), and we find that the former is larger than the latter; however, the gap between them can be mitigated by using leasing capital. The constrained social planner can further improve the market economy and reduce the overall pecuniary externalities as much as possible. Numerically, we find that the room for improvement decreases in leasing costs. Intuitively, those very constrained firms already have several good options from leasing, and the planner/policy maker cannot improve much further.

Quantitatively, in the fully-developed dynamic model, we find that when there are favorable regulation changes or law changes in the leasing market (e.g., the implementation of anti-recharacterization laws), the aggregate output and consumption can be further improved. Quantitatively, for example, the elasticity of aggregate output with respect to rental fees is about -1 across steady states, and the ratio of aggregate consumption changes relative to leasing market expenditure changes could be as large as 4.

We also compare three different allocations to study welfare improvement: the first best allocation for the totally unconstrained social planner, the constrained efficient allocation, and the market economy with financial frictions. We find that the constrained social planner can induce firms to invest more in new capital and use less leased capital, and the overall benefit for aggregate output or consumption are both around about 2%, which is economically sizable. In addition, we also find that when leasing markets have fewer frictions, the extent to which a planner can further improve the market economy becomes more limited; As a result, missing leasing capital market in the previous literature may have biased welfare implications.

We believe that it would be interesting for future research to further explore the leased capital market, both empirically and quantitatively. For example, a more detailed empirical analysis with detailed micro-level data could help explain the role that leased capital plays for different firms in different situations. Also, it would be interesting to know more about the dynamics of using leased capital in response to changes in macroeconomic and financial regulations and policies, such as those related to collateral constraints, asset prices, and firm risks, among others. Quantitatively, it would be interesting to explore more broadly the role of leased capital interacting with different model environments or elements; for example, one could extend our relatively transparent model with several features that are possibly important for future studies, such as the capital structure of firms, arrangement in the equity finance markets, and different specifications on investment and capital adjustment costs, among other features.

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Tables and Figures

Table I: Welfare Improvement Due to Leasing in Competitive Equilibrium

Variable	EQ with leasing	EQ without leasing	Change (%)
Output	1.71	1.47	-14.16
Consumption	0.71	0.64	-10.15
Old capital price q	0.558	0.560	0.36
Leasing fee τ	0.86	1.36	58.59
Average Firm multipliers	0.14	0.23	63.15
Average MPK	0.61	0.69	13.11
Dispersion in MPK	0.04	0.11	189
Externality: (Δ^l vs. Δ)	0.10	0.12	22.88

Table II: Reducing Pecuniary Externality with the Social Planner

	$h = 0.3$ (With leasing)	$h = 0.8$ (Without leasing in EQ)
Average MPK		
Competitive equilibrium	0.61	0.69
Constrained social planner	0.60	0.61
Improvement	-2.9%	-10.76%
Dispersion in MPK		
Competitive equilibrium	0.04	0.11
Constrained social planner	0.03	0.05
Improvement	-30.6%	-57.6%

Table III: Moments: Model and Data

Moments	Model	Data
Agg. leased capital/all agg. productive capital	0.302	0.387
Leverage: agg. debt/agg. assets	0.078	0.224
Std: firm productivity	0.206	0.373
Std: firm leased capital/productive capital	0.237	0.161
Std: firm leverage	0.255	0.159
Corr: firm output/assets with firm leased capital ratio	0.452	0.279
Corr: firm output/assets with firm leverage	0.232	-0.095
Corr: firm productivity with firm leased capital ratio	0.439	0.246
Corr: firm productivity with firm leverage	0.263	-0.072
Auto corr: firm output/assets	0.851	0.869
Auto corr: firm leased capital/assets	0.860	0.873
Auto corr: firm leasing/productive capital	0.741	0.924
Auto corr: firm leverage	0.427	0.828
Agg. leased capital/agg. productive capital: Q1	0.776	0.506
Agg. leased capital/agg. productive capital: Q2	0.584	0.450
Agg. leased capital/agg. productive capital: Q3	0.290	0.391
Agg. leased capital/agg. productive capital: Q4	0.018	0.297
Agg. Debt/agg. assets: Q1	0.214	0.141

Table IV: First Best, Constrained Social Planner, and Market Economy

	First best	Competitive market	Constrained SP	% change
q	0.64	0.61	0.38	-38.0%
τ		0.10	0.13	26.1%
Output	15.88	12.15	12.36	**1.7%**
Consumption	10.52	8.39	8.59	**2.4%**
Investment	5.36	3.75	3.77	**0.5%**
Dividend	10.52	10.93	10.24	-6.3%
Leased capital		19.96	13.02	-34.8%
New capital	44.66	10.14	17.62	73.8%
Old capital	53.59	36.03	36.74	2.0%
Distributive externality		3.64	2.31	-36.5%
Rental externality		2.95	1.95	-33.8%
Collateral externality		0.26	0.36	40.1%
Leased capital ratio		0.30	0.19	
Leased capital ratios: Q1		0.77	0.60	
Leased capital ratios: Q2		0.57	0.42	
Leased capital ratios: Q3		0.29	0.10	
Leased capital ratios: Q4		0.01	0.00	
Avg. η: Q1		0.41	0.43	
Avg. η: Q2		0.17	0.15	
Avg. η: Q3		0.05	0.04	
Avg. η: Q4		0.01	0.00	
Avg. leverage: Q1		0.21	0.21	
Avg. leverage: Q2		0.21	0.21	
Avg. leverage: Q3		0.21	0.18	
Avg. leverage: Q4		-0.13	-0.19	

Table V: Constrained Social Planner Improving Market Economy

	CE	Planner	Planner improvement
Higher agency cost	82.0%	95.5%	13.5%
High agency cost	89.0%	97.4%	8.3%
Benchmark	100.0%	102.4%	2.4%
Low agency cost	102.6%	103.9%	1.3%
Lower agency cost	105.0%	105.2%	0.2%

Figure I: Firm Capital Choices in Competitive Equilibrium

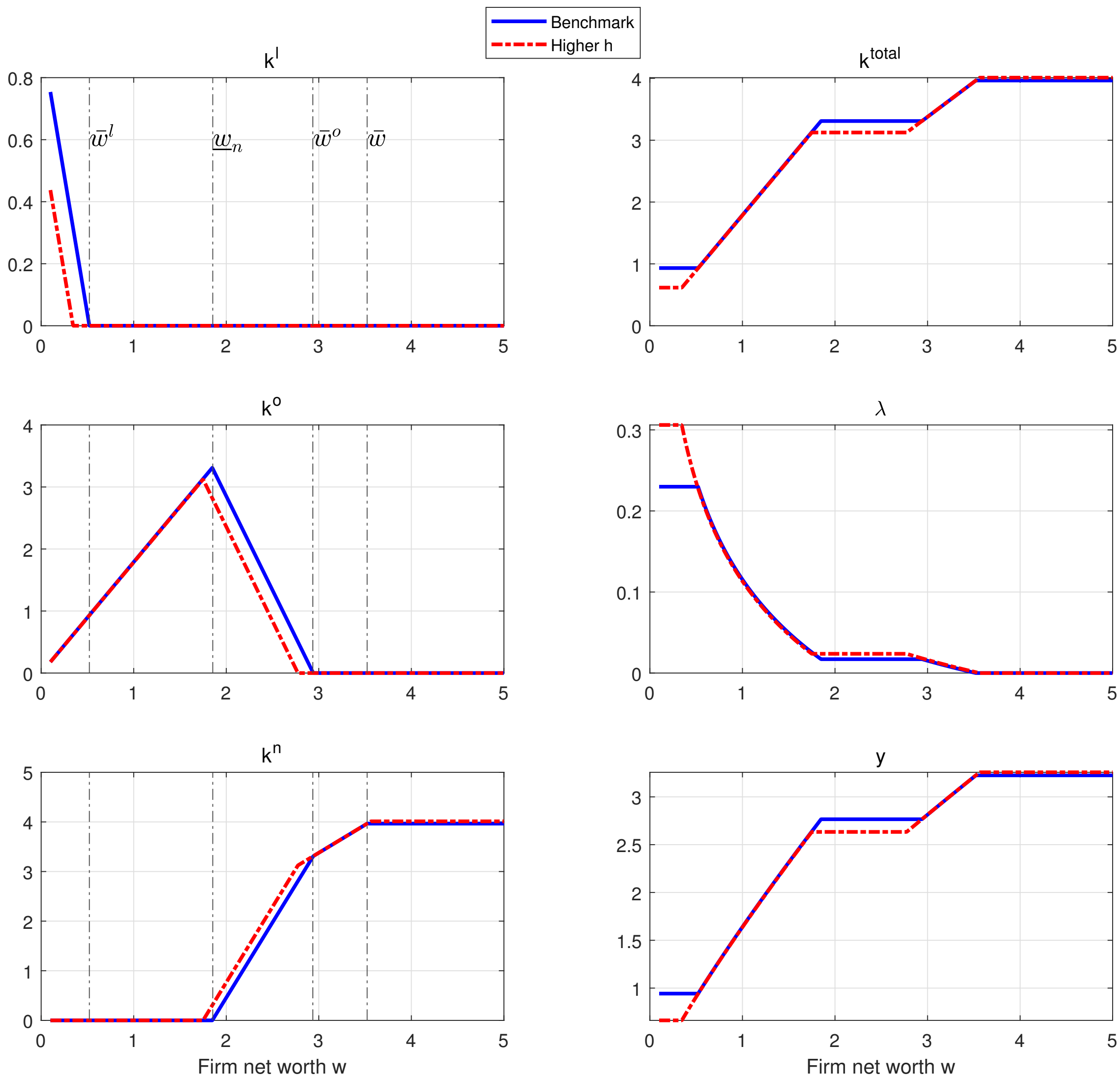


Figure II: Competitive Equilibrium vs. Constrained Social Planner

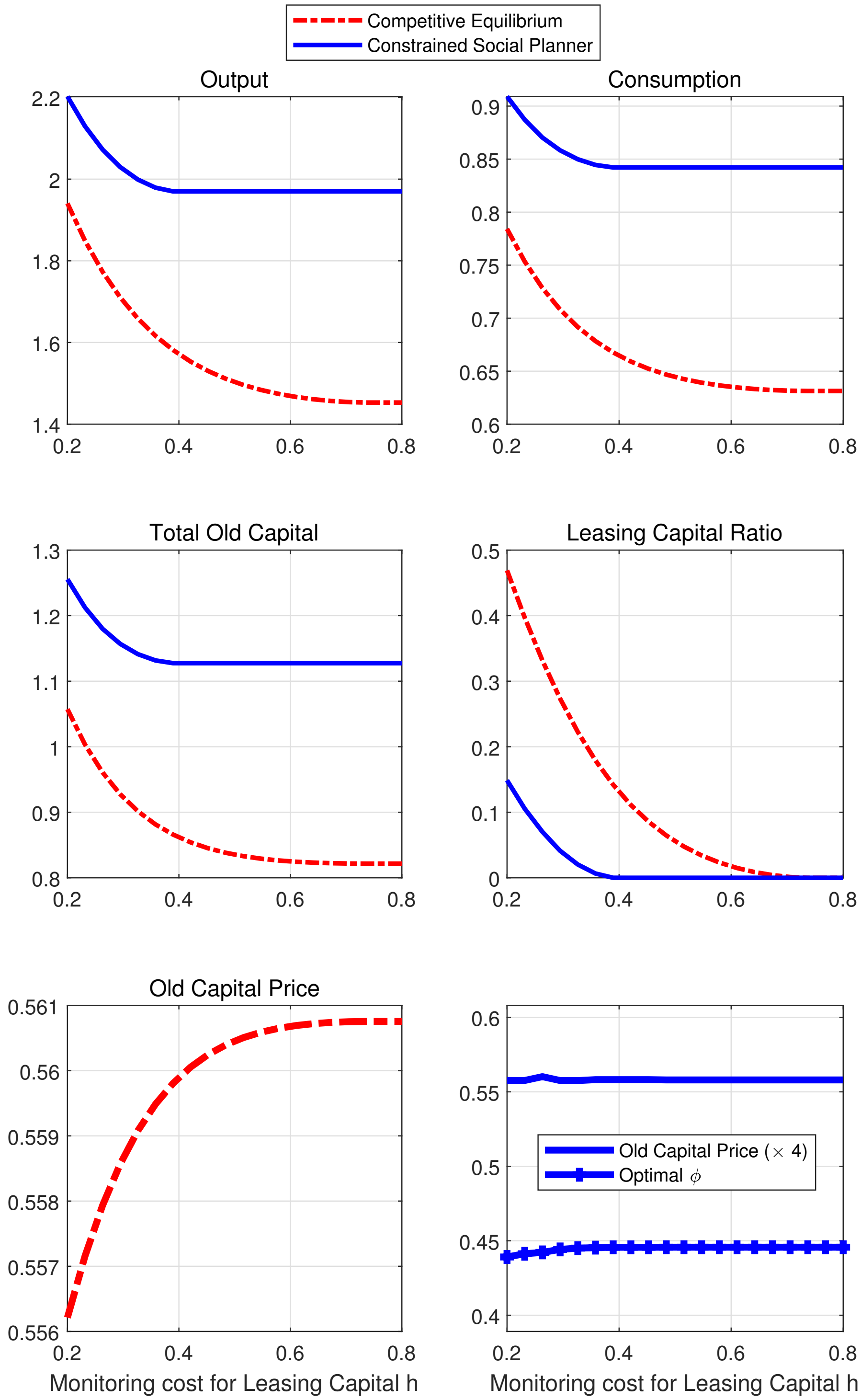


Figure III: Competitive Equilibrium vs. Constrained Social Planner: Marginal Product Of Capital

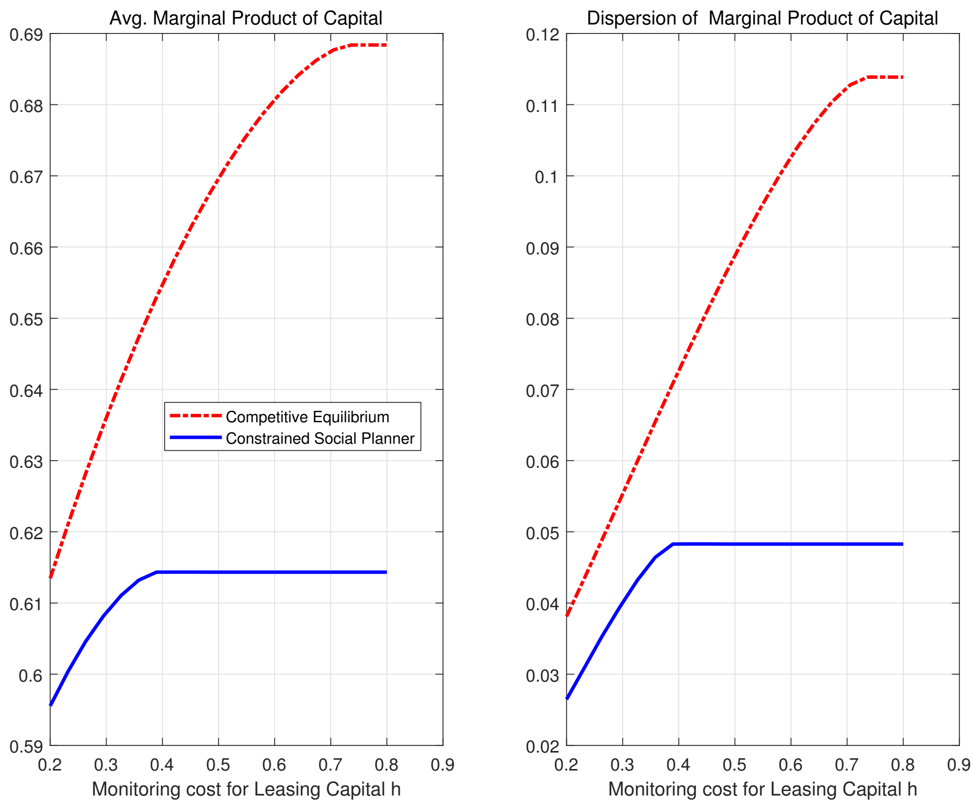


Figure IV: Optimal choices on different types of capital

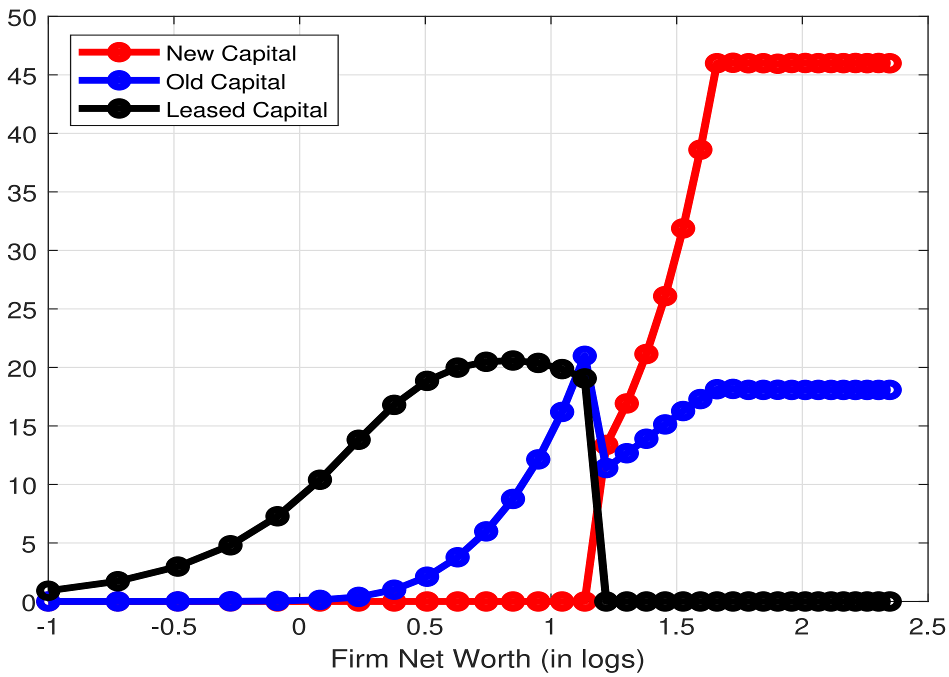


Figure V: Capital Choice With Low Vs. High Rental Fees

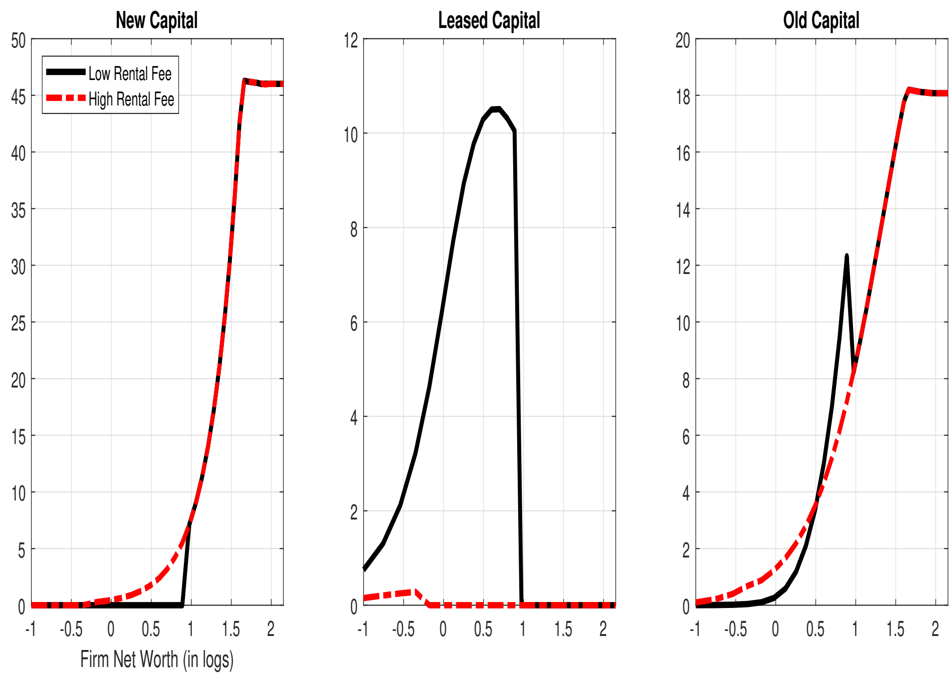


Figure VI: First Best, Constrained Social Planner, and Market Economy

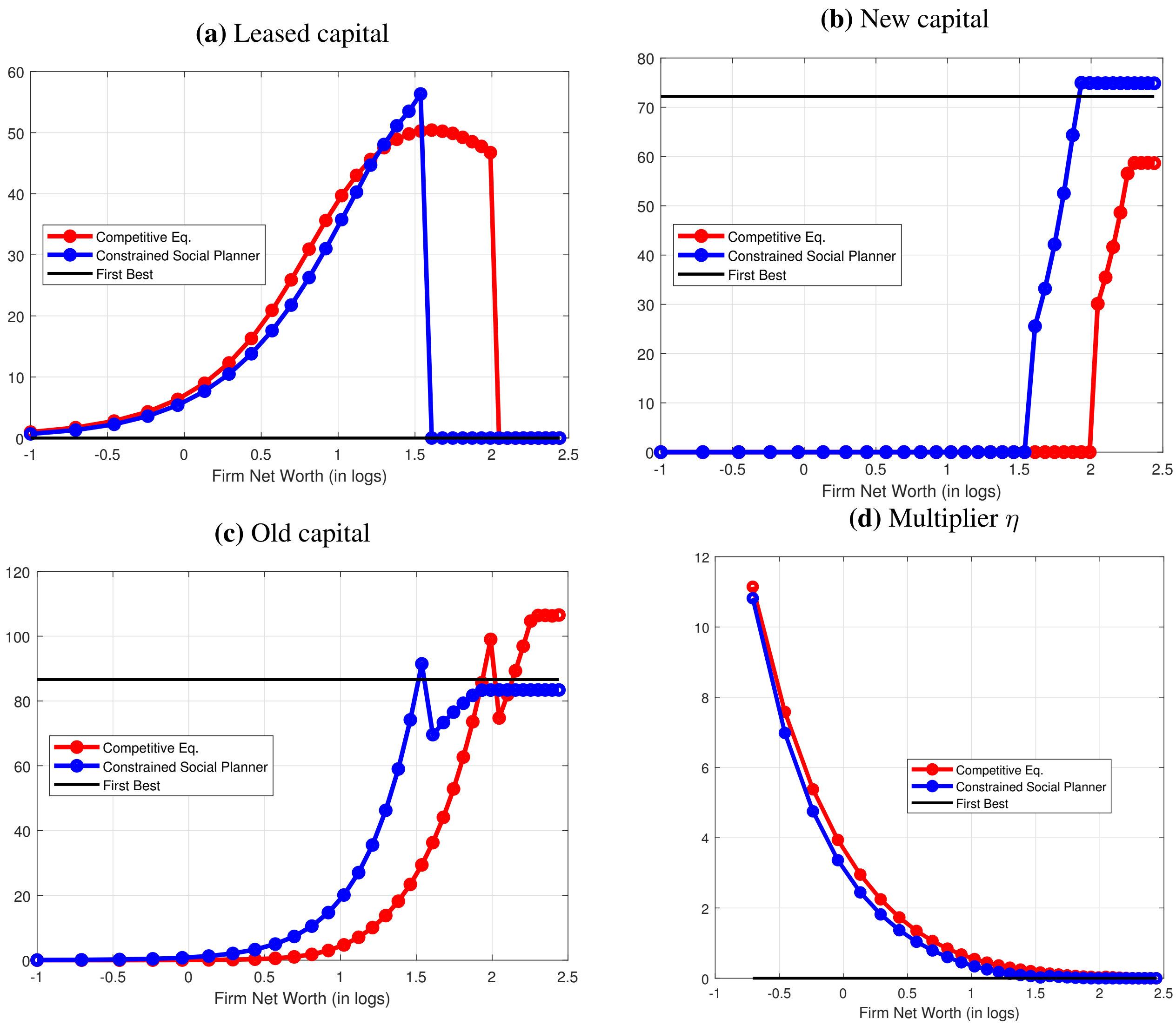
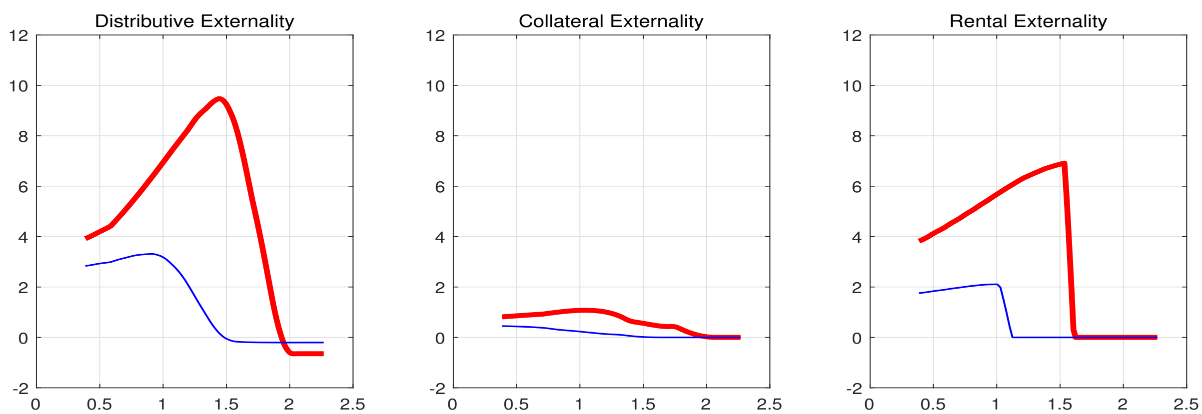
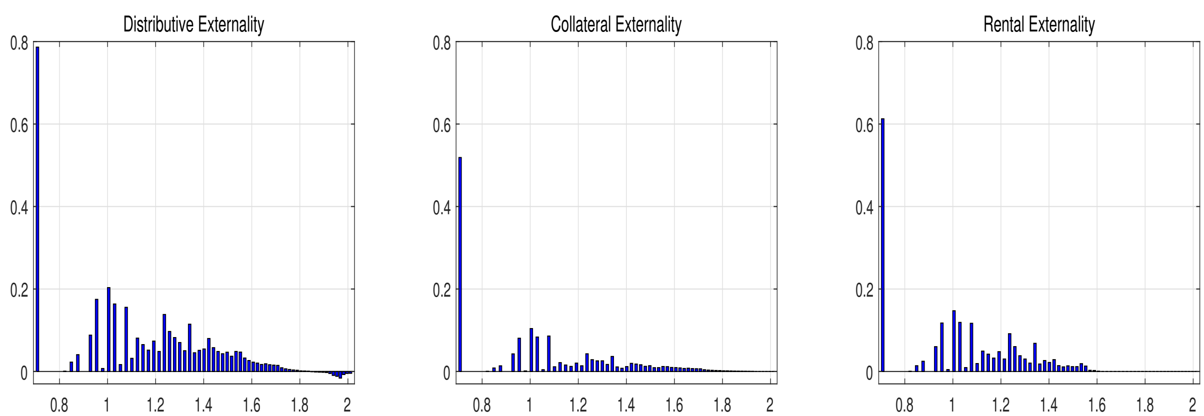


Figure VII: Different Types of Externality in the Constrained Efficient Economy

(a) Based on policy functions



(b) Based on equilibrium distributions



NOTE: Based on firm policy functions and the associated multiplier functions in the constrained social planner problem, the upper panel figure plots the three different types of externalities across firm net worth; the thick line is for highest productivity firms, and the thin line is for lowest productivity firms. See the text for the definitions of externalities. In the lower panel, for a given level of firm net worth, we sum up different types of externalities in the stationary economy (across different levels of firm productivity).

Figure VIII: Tax/Subsidy Rate for Leasing and New Capital in Constrained Efficient Allocation

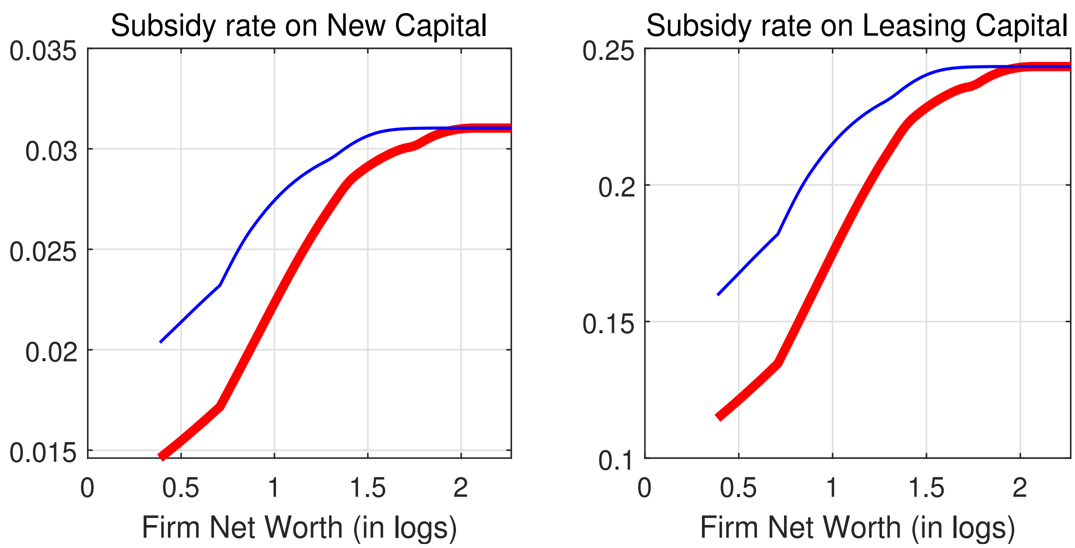


Table VI: Welfare Loss If Ignoring Leasing Market

	CE	SP	Misspecified SP	Changes w.r.t. CE
d	0.007	0.007	0.007 (0.1081)	
ϕ	0.00	0.27	0.46	
q	0.61	0.38	0.25	
τ	0.10	0.13	0.14	
Leasing Capital	19.96	13.02	0.00	
New Capital	10.14	17.62	28.88	184.9%
Old Capital	36.03	36.74	33.97	-5.7%
Composite Capital	66.13	67.38	62.84	-5.0%
Leasing Capital ratio	0.30	0.19	0.00	
Output	12.15	12.36	11.60	-4.5%
Consumption	8.39	8.59	8.01	-4.6%
Investment	3.75	3.77	3.59	-4.2%
Dividend	10.93	10.24	8.13	-25.6%

Appendix for Online Publication

A Motivational Facts with More Details

We present motivational facts with more details here. Our sample, which we obtain from Compustat, consists of firms with positive rental expenditure data and non-missing SIC codes. The sample period ranges from 1977 to 2017. We focus on firms trading on NYSE, AMEX, and NASDAQ, except for utility firms that have four-digit standard industrial classification (SIC) codes between 4900 and 4999, finance firms that have SIC codes between 6000 and 6999 (finance, insurance, trusts, and real estate sectors), and public administrative firms that have SIC codes between 9000 and 9999. We also explicitly exclude industries that serve as lessors (i.e., industries with SIC 3 digits of 735 and 751, and with SIC 4 digits of 7377).

Two salient observations can be represented. First, leased capital accounts for a significant fraction of total assets or total productive capital on average. As shown in Table A.1, the leased capital ratio is slightly higher than leased capital/assets. Overall, the ratio is in the range of 0.20 and 0.38, which is sizable. In Table A.2, we further show that the leased capital ratio is large across different industries. Generally, firms in the service sector use more leased capital (30-60% in retail trade and services compared to around 20% in manufacturing); these facts not only reflect sector-level heterogeneity (such as technological differences), but also reflect different levels of financial constraints in different industries.

[**Place Table A.1 about here**]

[**Place Table A.2 about here**]

Second, leased capital is particularly important for those young and financially constrained firms on the cross-section. In Table A.3, we further present summary statistics for different groups of firms with different total assets, and we can confirm that leased capital is still important in firms' capital structure and real production, especially for smaller and/or constrained firms. We then follow Eisfeldt and Rampini (2009) and study the relation between the leasing ratio and measures of financial constraints in Table A.4. We use firm size (defined as log assets) and the Whited-Wu index (WW-index) as proxies for financial constraints. We also consider a number of controls capturing additional measures of firm characteristics, as is typical in the empirical corporate finance literature. Across all specifications, we find negative coefficients on size and positive coefficients on the WW-index. Indeed, variables that indicate that a firm is financially constrained exhibit a positive correlation with the decision to lease, consistent with Eisfeldt and Rampini (2009). Moreover, such a pattern is more salient for young firms and among service industries, since these firms and industries tend to lease more. It is worth mentioning that since Compustat firms are relatively large, one might expect the relationship between financial variables and the leased capital ratio to be even stronger for private firms for which financial characteristics are not observed.

[**Place Table A.3 about here**]

[**Place Table A.4 about here**]

B Additional Proofs and Results for the Two-period Model

[Place Figure A.1 about here]

B.1 First Best, and the Competitive Equilibrium

First best (FB) allocation and its solution.

The first best (FB) allocation is to choose aggregate consumption C_t, and an allocation of the new and old capital, $k^n_{t+1}(w)$ and $k^o_{t+1}(w)$, so that the utility of the representative household is maximized:

$$\max_{\{C_t, k^n_{t+1}(w), k^o_{t+1}(w)\}} \sum_{t=0}^{\infty} \beta^t C_t$$
$$\int f(k^n_t(w) + k^o_t(w)) d\pi(w) = C_t + \int k^n_{t+1}(w) d\pi(w),$$
$$\int k^n_t(w) d\pi(w) = \int k^o_{t+1}(w) d\pi(w).$$

The first-order conditions for firms suggest that we should then have:

$$1 = \beta(f_k(k^{FB}_t) + q^{FB}_{t+1}),$$
$$q^{FB}_t = \beta f_k(k^{FB}_t).$$

In stationary equilibrium, we can pin down the capital price as $q^{FB} = \frac{1}{1+\beta}$, which is equal to the discounted marginal product of capital, and the optimal scale of production for all firms is $k^{FB} = f_k^{-1}\left(\frac{1}{\beta(1+\beta)}\right)$.

Proofs for Proposition 1:

Proof. To begin with, we first note that the old capital price q is the same economy-wide, such that $u^l(w)$ is a constant. $u^n(w)$ and $u^o(w)$ varies along different w and linear functions of λ. We start by comparing the user cost of new and old capital:

$$u^n(w) - u^o(w) = 1 - \beta q - q + \lambda(w)(1 - \beta\theta q - q) = \underline{\nu}^n(w) - \underline{\nu}^o(w).$$

If the user cost of new capital exceeds the user cost of the old capital, which depends on the firm's net worth w, $\underline{\nu}^n(w) - \underline{\nu}^o(w) > 0$, or, $\underline{\nu}^n(w) > \underline{\nu}^o(w) \geq 0$, and the firm strictly prefers old capital to new capital. If the reverse is true, $\underline{\nu}^o(w) > 0$, and old capital is dominated. When $\underline{\nu}^n(w) = \underline{\nu}^o(w) = 0$, the firm is indifferent between new and old capital at the margin.

Similarly, we next compare the user cost of new capital and leased capital:

$$u^n(w) - u^l(w) = \lambda(w)(1 - \beta\theta q) - \beta h q = \underline{\nu}^n(w) - \underline{\nu}^l(w).$$

If the user cost of new capital exceeds the user cost of leased capital, $\underline{\nu}^n(w) > 0$ and the firm prefers leased capital to new capital. If the reverse is true, $\underline{\nu}^l(w) > 0$, and leased capital is dominated. Similarly, we can compare the user costs between leased capital and old capital $u^l(w) - u^o(w) = 1 - \beta q + \beta h q - q - \lambda(w) q = \underline{\nu}^l(w) - \underline{\nu}^o(w)$.

We first provide the following lemmas so that we can better understand the capital choices across firms.

Lemma 1. *No firm only invests in leased capital.*

Proof. Since the monitoring cost is positive ($h > 0$), it is obvious that financially unconstrained firms ($\lambda = 0$) will always prefer new capital to leased capital, since $u^n(w) - u^l(w) = -\beta h q$. For the financially constrained firms ($\lambda > 0$), suppose there exists one firm that only invests in leased capital, then we know debt $b = 0$ and also dividends $d = 0$. The budget constraint Eq. (3) will then reduce to $w_0 + 0 = 0$, which contradicts our assumption that $w_{min} > 0$. Therefore, no firm only invests in leased capital. Intuitively, this is due to the feature that the leasing fee is paid after production in the second period, and it requires no down payment and thus does not take up resources in the first period; for constrained firms, they will still use other types of capital to the maximum extent possible. □

Intuitively, even for the most constrained firm, it still uses its initial wealth to purchase capital. The market clearing condition also outlines boundaries for the equilibrium price q for old capital as follows.

Lemma 2. $q < \frac{1}{1+\beta\theta}$.

Proof. In stationary equilibrium, (8) and (9) imply that:

$$\varphi_n - \varphi_o = \beta\frac{(1-\theta)q}{1+\lambda} + \frac{\underline{\nu}^n - \underline{\nu}^o}{1+\lambda}.$$

If $\varphi_n - \varphi_o \leq 0$, then this equation implies that $\underline{\nu}^o > 0$. Consequently, no firm would buy old capital, which obviously violates the market clearing condition. Hence, in a stationary equilibrium, we have $\varphi_n - \varphi_o > 0$, equivalently, $q < \frac{1}{1+\beta\theta}$, so that the market clearing condition of old capital is satisfied. Intuitively, the capital price cannot be too high; otherwise, no firm would buy old capital.

$\square$

Intuitively, the capital price cannot be too high; otherwise, no firm would buy old capital. The following assumption ensures that the equilibrium price q is such that new capital is never strictly dominated; this rules out the uninteresting special cases in which firms never use new capital.

Assumption 1. $q \geq \frac{1}{1+\beta}$.

Assumption 1 ensures that new capital is never dominated. To see this, suppose instead, $q < \frac{1}{1+\beta}$, equivalently $1 - \beta q - q > 0$, along with $q < \frac{1}{1+\beta\theta}$, we will have $u^n(w) - u^o(w) > 0$ for any w, i.e., there is no usage of new capital in this economy. Hence, even financially unconstrained firms only use old capital, which is unrealistic.

Assumption 2. *The monitoring cost h is not too high or too low, such that we have the following: $u^l \leq min\{u^o(w_{min}), u^n(w_{min})\}$; or, $h \in (\frac{(1-\beta\theta q)[q(1+\beta)-1]}{\beta q(1-q-\beta\theta q)}, \frac{\beta u^o(w_{min})-1+\beta q}{\beta q})$. Alternatively, we must make sure w_{min} is sufficiently small and also ensures that h is reasonably sizable.*

Assumption 2 says that the monitoring cost h must satisfy a certain range: it is not too high, so that leased capital is not dominated by old capital for the most constrained firm; also, we assume $h > 0$, but h should not be too low (not too close to 0), so that for firms that are relatively large and use new capital, they do not use leased capital (as leasing capital still charges a premium); therefore, we need to have $u^n(w) > u^l$, $u^n(w) > u^o(w)$, for these firms. As we shall see below, these conditions can be satisfied with our assumptions on the parameters.

In particular, in this economy, to ensure that leased capital is used by some firms, firms with the lowest net worth w_{min} (thus sufficiently constrained) must find it beneficial to use leased capital. That is, $u^l \leq min\{u^o(w_{min}), u^n(w_{min})\}$. Note that the value of $u^o(w_{min})$ itself is endogenous, and we can further assume w_{min} is sufficiently small such that $u^l < \beta f_k(\frac{w_{min}}{q})$, i.e., for these firms w_{min} is too small and then using leased capital can improve the marginal product of capital. Further, assuming that leased capital doesn't coexist with new capital, we have $u^l < u^n(w_{min})$. Along with Lemma 1, we know that leased capital must be coupled with old capital. Hence, we have, at least, $u^l\left(w_{min}\right) = u^o\left(w_{min}\right) < u^n\left(w_{min}\right)$. These together indicate that:

$$h \in (\frac{(1-\beta\theta q)[q(1+\beta)-1]}{\beta q(1-q-\beta\theta q)}, \frac{\beta u^o(w_{min}) - 1 + \beta q}{\beta q}).$$

Assumption 3. *w_{max} is sufficiently large, and there must exist unconstrained firms in the economy.*

An unconstrained firm chooses only new capital, and at the level which achieves the optimal marginal product of capital. This gives a threshold of $\bar{w}$: when firms have a net worth larger than $\bar{w}$, they are unconstrained and invest in new capital only.

With respect to our solutions, we first discuss the general case in which $q > \frac{1}{1+\beta}$ (recall that $\frac{1}{1+\beta}$ is the capital price for the first best, which typically cannot be achieved). Since the firm's optimal solution is crucial for understanding the two-period model, as well as instructive for the dynamic model, we therefore lay out all the details next.

Old and leased capital. We start from the region with the smallest net worth, $w \in [w_{min}, \bar{w}^l)$, in which firms invest in positive amounts of old and leased capital. Combined with Eqs. (3), (9) and (10) and using the notions of user costs, we are able to solve capital choices within this range:

$$u^l = 1 - \beta q(1-h) = \beta f_k(k^l + k^o),$$

$$k^o = \frac{w_0}{q}, k^l \geq 0,$$

and when firm net worth is at the margin of $\bar{w}^l$,$k^l = 0$, such that $\bar{w}^l$ is simply defined by:

$$u^l = 1 - \beta q(1-h) = \beta f_k(\frac{\bar{w}^l}{q}).$$

Old capital only. As the firm's net worth w further increases, firms are still constrained and they use old capital only. In this region, we should have the following conditions satisfied:

$$k^o = \frac{w}{q}, u^o\left(w\right) = \beta f_k(\frac{w}{q}),$$
$$u^l(w) > u^o(w), u^n\left(w\right) > u^o\left(w\right),$$

and for range of $(\bar{w}^l, \underline{w}_n)$, the bound $\underline{w}_n$ is defined as the level of net worth such that the user costs of u^n and u^o are just equal, and firms are starting to use new capital:

$$u^n\left(w\right) = u^o\left(w\right) = \beta f_k(\frac{\underline{w}_n}{q}).$$

Through the expressions for $u^n\left(w\right)$ and $u^o\left(w\right)$, we can find the unique $\bar{\lambda}^n$, at which $u^n\left(\bar{\lambda}^n\right) = u^o\left(\bar{\lambda}^n\right)$:

$$u^n\left(\bar{\lambda}^n\right) = 1 - \beta q + \bar{\lambda}^n\left(1 - \beta\theta q\right) = \beta f_k(\frac{\underline{w}_n}{q}),$$
$$u^o\left(\bar{\lambda}^n\right) = q + \bar{\lambda}^n q,$$
$$\bar{\lambda}^n = \frac{\left(\beta+1\right)q - 1}{1 - \beta\theta q - q} > 0,$$

for which we have used the previous lemma on $q < \frac{1}{1+\beta\theta}$ and the assumption of $q > \frac{1}{1+\beta}$.

Old and new capital. As a firm's net worth w further increases, firms start to use new capital, but at the same time, they also use old capital (at the interaction point of the two lines for k^o and

k^n). These firms have the following conditions satisfied:

$$u^n\left(\bar{\lambda}^n\right) = u^o\left(\bar{\lambda}^n\right) = \beta f_k(k^o + k^n),$$
$$w = qk^o + (1-\beta\theta q)k^n.$$

That is, the total amount of capital is constant but the composition shifts to more new capital. We can also define the upper bound of wealth for these firms as $\bar{w}^o$ (beyond which firms do not use old capital anymore):

$$u^n\left(\bar{\lambda}^n\right) = u^o\left(\bar{\lambda}^n\right) = \beta f_k(\frac{\bar{w}^o}{1-\beta\theta q}),$$

and at this point, firms only use new capital but borrow to the maximum possible. From $\beta f_k(\frac{\underline{w}_n}{q}) = u^o\left(\bar{\lambda}^n\right) = \beta f_k(\frac{\bar{w}^o}{1-\beta\theta q})$, it is clear that $\bar{w}^o > \underline{w}_n$, since $1-\beta\theta q > q$. Thus, the relevant region for these firms is $(\underline{w}_n, \bar{w}^o)$.

New capital only. As firm net worth w further increases, they only use new capital, $u^n(w) = 1-\beta q+\lambda(1-\beta\theta q) = \beta f_k(k^n)$. Also, the smallest required net worth for firms to be unconstrained, $\bar{w}$, can be defined from: $u^n(\bar{w}) = 1-\beta q = \beta f_k(\frac{\bar{w}}{1-\beta\theta q})$. We can see clearly $\bar{w} > \bar{w}^o$. Beyond $\bar{w}$ we know that firms are totally unconstrained.

We next discuss the case when $q = \frac{1}{1+\beta}$. The cases when firms become constrained are the same as those when $q > \frac{1}{1+\beta}$. For unconstrained firms, we must have $u^o = u^n$. Therefore, firms are indifferent between new and old capital, and they invest in both.

In summary, the detailed algebras in Proposition 1 can be listed as follows for convenience.

(1). $w_0 \in [w_{min}, \bar{w}^l]$, the solution is given by: $k^n = 0, k^o = \frac{w_0}{q}, k^l = \left(\frac{1+\beta qh-\beta q}{\alpha\beta}\right)^{\frac{1}{\alpha-1}} - \frac{w_0}{q}$.

(2). $w_0 \in (\bar{w}^l, \underline{w}_n]$, the solution is: $k^n = 0, k^o = \frac{w_0}{q}, k^l = 0$.

(3). $w_0 \in (\underline{w}_n, \bar{w}_o)$, the solution is: $k^l = 0$, and also:

$$k^n = \frac{q}{\beta q\theta + q - 1}\left(\frac{q^2(\theta-1)}{\alpha(\beta q^o\theta + q^o - 1)}\right)^{\frac{1}{\alpha-1}} - \frac{w_0}{\beta q\theta + q - 1},$$
$$k^o = \frac{\beta q\theta - 1}{\beta q\theta + q - 1}\left(\frac{q^2(\theta-1)}{\alpha(\beta q\theta + q - 1)}\right)^{\frac{1}{\alpha-1}} + \frac{w_0}{\beta q\theta + q - 1}.$$

(4). $w_0 \in [\bar{w}_o, \bar{w})$, the solution is: $k^n = \frac{w_0}{1-\beta\theta q}, k^o = 0, k^l = 0$.

(5). $w_0 \in [\bar{w}, w_{max}]$, the solution is: $k^n = \left(\frac{1-\beta q}{\alpha\beta}\right)^{\frac{1}{\alpha-1}}, k^o = 0, k^l = 0$. The thresholds are given by:

$$\bar{w}^l = q\left(\frac{1+\beta q h - \beta q}{\alpha\beta}\right)^{\frac{1}{\alpha-1}}$$
$$\underline{w}_n = (1-\beta\theta q)\left(\frac{q^2(\theta-1)}{\alpha\beta q\theta + \alpha q - \alpha}\right)^{\frac{1}{\alpha-1}}$$
$$\bar{w}_o = (1-\beta\theta q)\left(\frac{q^2(\theta-1)}{\alpha\beta q\theta + \alpha q - \alpha}\right)^{\frac{1}{\alpha-1}}$$
$$\bar{w} = q\left(\frac{1-\beta q}{\alpha\beta}\right)^{\frac{1}{\alpha-1}}.$$

Figure A.2 provides an intuitive illustration for the user costs of different types of capital.

□

[Place Figure A.2 about here]

Proposition 2. *Shutting down the leasing market, we can characterize stationary competitive equilibrium as follows:*

1. *If $q > \frac{1}{1+\beta}$, there exists threshold $\underline{w}_n < \bar{w}_o < \bar{w}$ such that: firms with $w < \underline{w}_n$ invest only in old capital; firms with $w \in (\underline{w}_n, \bar{w}_o)$ invest in new and old capital; firms with $w \geq \bar{w}_o$ invest only in new capital; and $w \geq \bar{w}$, firms are unconstrained, invest only in new capital, and achieve the optimal marginal product of capital.*

2. *If $q = \frac{1}{1+\beta}$, then $\bar{w}_o = \bar{w}$, so firms with $w \leq \underline{w}_n$ invest only in old capital; firms with $w > \underline{w}_n$ invest in the new and old capital. In particular, firms with $w \geq \bar{w}_o = \bar{w}$ invest in new and old capital so that they achieve the optimal marginal product of capital.*

B.2 Lagrangian, Inefficiency and Externality

A planner maximizes the present discounted value of aggregate dividends:[39]

$$\max_{\left\{d_{1,t+1},k_{t+1}^{n},k_{t+1}^{o},k_{t+1}^{l}\right\}} \int \left[d_{10}(w) + \sum_{t=0}^{\infty} \beta^{t+1} d_{1,t+1}(w)\right] d\pi\left(w\right)$$

subject to the firms' budget constraints for the current and next period, as well as a market clearing condition for old capital, and the rental market condition of $\tau_t = \beta^{-1} - q_{t+1}(1-h)$,

$$\begin{aligned}
w_{0t}\left(w\right) + b_{t+1}\left(w\right) &= d_{0,t} + k_{t+1}^{n}\left(w\right) + q_t k_{t+1}^{o}\left(w\right), \forall w &\text{(B1)}\\
f\left(k_{t+1}^{n}\left(w\right) + k_{t+1}^{o}\left(w\right) + k_{t+1}^{l}\left(w\right)\right) + q_{t+1}k_{t+1}^{n}\left(w\right) &= d_{1,t+1}\left(w\right) + \tau_t k_{t+1}^{l}\left(w\right) + \beta^{-1}b_{t+1}\left(w\right), \forall w &\text{(B2)}\\
\theta q_{t+1}k_{t+1}^{n}\left(w\right) &\geq \beta^{-1}b_{t+1}\left(w\right), \forall w &\text{(B3)}\\
\int k_t^{n}(w)d\pi(w) + \int \left(1-h\right)k_t^{l}(w)d\pi(w) &= \int k_{t+1}^{o}(w)d\pi(w). &\text{(B4)}
\end{aligned}$$

We denote with multipliers $\beta^t\mu_{0,t}\left(w\right)$ and $\beta^{t+1}\mu_{1,t+1}\left(w\right)$ for individual budget constraints when a firm is young and old, and $\beta^{t+1}\lambda_t\left(w\right)$ for the collateral constraints. Further, we denote the multipliers on non-negativity constraint for $k_{t+1}^{n}\left(w\right)$, $k_{t+1}^{o}\left(w\right)$, $k_{t+1}^{l}\left(w\right)$, and $d_{1,t+1}\left(w\right)$ by $\underline{\nu}_t^{n}\left(w\right)$, $\underline{\nu}_t^{o}\left(w\right)$, $\underline{\nu}_t^{l}\left(w\right)$, and $\eta_t\left(w\right)$, respectively. Importantly, we have an additional multiplier $\beta^t\phi_t$ for the market clearing condition Eq. (B4).

We set up the Lagrangian, drop the dependence of w to simplify notations and get the first-order

[39]Strictly speaking, the planner's objective is to maximize the welfare of the representative household; since firms and households have the same discount factor, it is equivalent to maximizing the present value of dividends, adjusted for some initial condition of the household. See Section C.5 in the Online Appendix for details of the general dynamic model.

conditions with respect to new, old and leased capital:

$$1+\lambda_t = \beta\left[f_k\left(k_{t+1}\right)+q_{t+1}\right]+\beta\theta\lambda_t q_{t+1}+\beta\phi_{t+1}+\underline{\nu}^n_{t+1},$$

$$q_t\left(1+\lambda_t\right) = \beta f_k\left(k_{t+1}\right)-\phi_t+\underline{\nu}^o_{t+1},$$

$$\beta\tau_{t+1} = \beta f_k\left(k_{t+1}\right)+\beta\phi_{t+1}(1-h)+\underline{\nu}^l_{t+1}.$$

These conditions indicate that, when firms make investment decisions at time t, firms ignore the additional marginal benefit of new capital $\left(\beta\phi_{t+1}\right)$ and leased capital $\left(\beta\phi_{t+1}(1-h)\right)$, while overestimating the additional benefit of old capital by ϕ_t, when typically ϕ_t is positive. This may result in a higher price of the old capital in the competitive equilibrium, as compared to the allocation decisions made by the benevolent social planner.

B.3 Constrained Efficient Problem for Social Planner in the Two-period Model

Recall that the constrained social planner's problem is given by:

$$\max_{\left\{d_{1,t+1},k^n_{t+1},k^o_{t+1},k^l_{t+1}\right\}} \int\left[d_{1,t=0}(w)+\sum_{t=0}^{\infty}\beta^{t+1}d_{1,t+1}(w)\right]d\pi\left(w\right)$$

subject to firms' budget constraints for the current and next period, as well as a market clearing condition:

$$\begin{aligned}
w_{0t}\left(w\right)+b_{t+1}\left(w\right) &= d_{0,t}\left(w\right)+k^n_{t+1}\left(w\right)+q_t k^o_{t+1}\left(w\right),\forall w\\
f\left(k^n_{t+1}\left(w\right)+k^o_{t+1}\left(w\right)+k^l_{t+1}\left(w\right)\right)+q_{t+1}k^n_{t+1}\left(w\right) &= d_{1,t+1}\left(w\right)+\tau_t k^l_{t+1}\left(w\right)+\beta^{-1}b_{t+1}\left(w\right),\forall w\\
\beta\theta q_{t+1}k^n_{t+1}\left(w\right) &\geq b_{t+1}\left(w\right),\forall w\\
d_{0,t}\left(w\right) &\geq 0,\forall w\\
\int k^n_t(w)d\pi(w)+\int\left(1-h\right)k^l_t(w)d\pi(w) &= \int k^o_{t+1}(w)d\pi(w).
\end{aligned}$$

We denote with current multipliers $\eta_t\left(w\right)$ and $\eta_{t+1}\left(w\right)$ for individual constraints on dividends when a firm is young and old, respectively, and $\lambda_t\left(w\right)$ for the collateral constraints. Further, we denote the multipliers on non-negativity constraints for $k_{t+1}^n(w)$, $k_{t+1}^o(w)$, $k_{t+1}^l(w)$ by $\underline{\nu}_t^n(w)$, $\underline{\nu}_t^o(w)$, $\underline{\nu}_t^l(w)$, respectively. Importantly, we have an additional multiplier $\beta^t\phi_t$ for the old capital market clearing condition. Also, we note that the constrained social planner must use the equilibrium condition for a rental market:

$$\tau_t = \beta^{-1} - q_{t+1}(1-h).$$

Similarly, as before, we note that since firms can always choose trivial choices, $d_{0,t}(w) = w_{0t}(w) > 0$ and $d_{1,t+1}(w) = 0$, or $d_{0,t}(w) = 0, b_{t+1}(w) = -w_{0t}(w), d_{1,t+1}(w) = \beta^{-1}w_{0t}(w)$, and no production at all, the optimal solution for firms must be the case in which firms can do better; without a loss of generality, we can assume $d_{1,t+1}(w) > 0$ is always true. Otherwise, we always slightly reduce $d_{0,t}(w)$, which saves more and yields $d_{1,t+1}(w) > 0$.

We consider a stationary economy so that q and ϕ are both constant. The first-order conditions with respect to k_{t+1}^n, k_{t+1}^o,k_{t+1}^l and b_{t+1} for the constrained social planner includes the following:

$$\begin{aligned}
&1+\eta_t = \beta\left[f_k(k_{t+1}) + q\right] + \lambda_t\beta\theta q + \beta\phi + \underline{\nu}_{t+1}^n,\\
&(1+\eta_t)\,q = \beta f_k(k_{t+1}) - \phi + \underline{\nu}_{t+1}^o,\\
&\beta\tau = \beta f_k(k_{t+1}) + \beta\phi(1-h) + \underline{\nu}_{t+1}^l,\\
&1+\eta_t - \lambda_t - \beta\beta^{-1} = 0.
\end{aligned}$$

Simplifying these conditions, we have:

$$\begin{aligned}
&(1+\lambda(w)) = \beta f_k(k(w)) + \beta q + \lambda(w)\beta\theta q + \beta\phi + \underline{\nu}_{t+1}^n,\\
&(1+\lambda(w))\,q = \beta f_k(k(w)) - \phi + \underline{\nu}_{t+1}^o,\\
&1 = \beta f_k(k(w)) + \beta\phi(1-h) + \beta q(1-h) + \underline{\nu}_{t+1}^l,
\end{aligned}$$

with $\lambda(w), k(w)$ depending on the individual wealth of w, q and ϕ are economy-wide variables.

From these conditions, we obtain:

$$\begin{aligned}
& 1-q-\beta\theta q+\lambda(1-\beta\theta q-q) \\
= \;& (1-\beta\theta q-q)(1+\lambda) \\
= \;& \beta\left(q+\phi\right)-\beta\theta q+\phi+\underline{\nu}_{t+1}^{n}-\underline{\nu}_{t+1}^{o} \\
= \;& \beta q(1-\theta)+\phi+\beta\phi+\underline{\nu}_{t+1}^{n}-\underline{\nu}_{t+1}^{o},
\end{aligned}$$

when $\phi \geq 0$, we must have $1-\beta\theta q-q>0$: this is because there must be some firms that demand old capital ($\underline{\nu}_{t+1}^{o}=0$) in the equilibrium, and for these firms, we have $\beta q(1-\theta)+\phi+\beta\phi+\underline{\nu}_{t+1}^{n}>0$, which implies that the aggregate variables should satisfy $1-\beta\theta q-q>0$. In stationary equilibrium, we can use simple notations and define the user cost for new, old, and leased capital: that is, in the optimal solution, if a given type of capital is used, then the current value of the marginal cost for that capital is:

$$\begin{aligned}
& u^{n}=1-\beta\left(q+\phi\right)+\lambda(1-\beta\theta q), \\
& u^{o}=\left(q+\phi\right)+\lambda q, \\
& u^{l}=1-\beta\left(q+\phi\right)\left(1-h\right).
\end{aligned}$$

Since the optimization problem for a given firm is well-defined, we can then use extensively these optimality conditions, just as in the main text for the problem of decentralized competitive equilibrium. Also, the problem has some special features: since capital is a perfect substitute in production, we must compare user costs for the solution.

For unconstrained firms, $\lambda=0$. Since $h>0$, we know that these firms do not use leased capital. Since it is realistic to assume that these unconstrained firms use new capital, we can find the condition that should be satisfied is: $u^{n}<u^{o}$, or:

$$\frac{1}{1+\beta}<q+\phi.$$

We then choose the economy's fundamentals so that:

$$\begin{aligned}\frac{1}{1+\beta} &< q+\phi, \\ 1-\beta\theta q - q &> 0, \text{ or } q < \frac{1}{1+\beta\theta}\end{aligned}$$

are all satisfied.

We can divide the firms' solutions into a few types:

(1). Unconstrained firms, for which $\lambda = 0$. As discussed earlier, these firms only use new capital k^n, and satisfy the following conditions:

$$1-\beta(q+\phi) = \beta f_k\left(\bar{k}^n\right),$$
$$d = w + b - k^n \geq 0,$$
$$\beta\theta q k^n \geq b,$$

such that we can determine a threshold for wealth w, and $\bar{w} = \bar{k}^n(1-\beta\theta q)$; if firms have $w \geq \bar{w}$, firms could either save more or pay more dividends in the amount of $w - \bar{w}$, and they are indifferent in doing so.

(2). Constrained firms, $\lambda > 0$. First, since firms are constrained, by definition, the borrowing constraint must be binding. Second, firms face a portfolio choice problem over (k^n, k^o) in the first period, and since k^n, k^o are perfect substitutes for production, we simply need to compare u^n and u^o for their first-period choices.

Since both u^n and u^o are linear in λ, and the slope for u^n is larger than that for u^o, i.e., $1-\beta\theta q - q > 0$. We can find, first, for relatively low λ, that firms prefer to use k^n, and for firms that are more constrained, they prefer to use k^o. The cutoff is given by $\bar{\lambda}^n > 0$, using the relation in which the user cost of new capital is equal to that of the old capital, i.e.,$u^n = u^o$, this equation can be further rewritten as:

$$\bar{\lambda}^n(1-\beta\theta q-q)=(\beta+1)(q+\phi)-1,$$

and the cutoff user costs are denoted as $u^n(\bar{\lambda}^n)=u^o(\bar{\lambda}^n)$. Now, we discuss three sub-cases and also discuss the impact on k^l.

(2.1). $\lambda>0,\lambda\leq\bar{\lambda}^n,k^n>0,k^o=0$. In this sub-case, $k^n=w/(1-\beta\theta q)$, and the firm's optimal condition for k^n is then:

$$u^n=\beta f_k\left(k^{Total}\right).$$

when this firm uses leased capital, $k^l>0$, we then have the following conditions satisfied simultaneously:

$$\begin{aligned}
&u^n=1-\beta(q+\phi)+\lambda(1-\beta\theta q)=\beta f_k\left(k^{Total}\right),\\
&u^l=1-\beta(q+\phi)(1-h)=\beta f_k\left(k^{Total}\right),\\
&k^l>0,\\
&w/(1-\beta\theta q)+k^l=k^{Total},\\
&\lambda>0,\lambda\leq\bar{\lambda}^n,k^n>0,k^o=0
\end{aligned}$$

If these conditions cannot be satisfied simultaneously, then the optimal set of conditions must be:

$$
\begin{aligned}
&u^n = \beta f_k\left(k^{Total}\right), \\
&k^l = 0, \\
&u^n < u^l, \\
&w/\left(1-\beta\theta q\right) = k^{Total}, \\
&\lambda > 0, \lambda \leq \bar{\lambda}^n, k^n > 0, k^o = 0.
\end{aligned}
$$

Intuitively, these firms have almost enough w for the unconstrained solution, such that they still use only new capital, and do not use leasing or old capital as such capital is still too expensive.

(2.2). $\lambda > 0, \lambda > \bar{\lambda}^n, k^n = 0, k^o > 0$. In this sub-case, $k^o = w/q$, and the firm's optimal condition for k^o is:

$$u^o = (q+\phi) + \lambda q = \beta f_k\left(k^{Total}\right).$$

When this firm uses leased capital, $k^l > 0$, we should have the following conditions satisfied simultaneously:

$$
\begin{aligned}
&u^o = (q+\phi) + \lambda q = \beta f_k\left(k^{Total}\right), \\
&u^l = 1 - \beta\left(q+\phi\right)\left(1-h\right) = \beta f_k\left(k^{Total}\right), \\
&k^l > 0, \\
&w/q + k^l = k^{Total}, \\
&\lambda > 0, \lambda > \bar{\lambda}^n, k^n = 0, k^o > 0.
\end{aligned}
$$

If these conditions cannot be satisfied simultaneously, then the optimal set of conditions must be:

$$u^o = (q+\phi) + \lambda q = \beta f_k\left(k^{Total}\right),$$
$$k^l = 0,$$
$$u^o < u^l,$$
$$w/q = k^{Total},$$
$$\lambda > 0, \lambda > \bar{\lambda}^n, k^n = 0, k^o > 0.$$

(2.3). $\lambda > 0, \lambda = \bar{\lambda}^n, k^n > 0, k^o > 0$. In this case, firms use both k^n, k^o, and the relative composition changes as firm w changes (also, the debt level changes); also, in this case, we see that firms do not use k^l (unless in very special cases in which $u^n(\bar{\lambda}^n) = u^o(\bar{\lambda}^n) = u^l$)

$$u^n = 1 - \beta\left(q+\phi\right) + \lambda(1-\beta\theta q) = \beta f_k\left(k^{Total}\right),$$
$$u^o = (q+\phi) + \lambda q = \beta f_k\left(k^{Total}\right),$$
$$u^n = u^o < u^l, k^l = 0,$$
$$w + \beta\theta q k^n = k^n + q k^o,$$
$$k^n + k^o = k^{Total}, \text{ or } k^n = \frac{w - qk^{Total}}{1-\beta\theta q - q}.$$

B.4 Equilibrium and Aggregate Variables in the Two-period Model

We can derive all the aggregate variables in the model. First, we focus on time $t+1$, when the household budget constraint is:

$$B^H_{t+2} + q_{t+1}(1-h)K^l_{t+1} + \tau_{t+1}K^l_{t+1} + \int d_{1,t+1}\left(w\right)d\pi(w) = C_{t+1} + \beta^{-1}B^H_{t+1} + K^l_{t+2}.$$

In time $t+1$, all production is from firms created in time t, and their budget constraints at time $t+1$ (the second period from these firms' perspective) is:

$$f\left(k_{t+1}^{n}(w)+k_{t+1}^{o}(w)+k_{t+1}^{l}(w)\right)+q_{t+1}k_{t+1}^{n}(w)=d_{1,t+1}(w)+\tau_{t+1}k_{t+1}^{l}(w)+\beta^{-1}b_{t+1}(w).$$

Also, in time $t+1$, the new generation of firms are created, and their budget constraints at time $t+1$ are (the first period from these firms' perspective, and also $d_{0,t+1}(w)=0$):

$$w+b_{t+2}(w)=d_{0,t+1}(w)+k_{t+2}^{n}(w)+q_{t+1}k_{t+2}^{o}(w).$$

Market clearing conditions: the market clearing condition for old capital in time $t+1$ is:

$$\int k_{t+1}^{n}(w)\,d\pi(w)+(1-h)\,K_{t+1}^{l}=\int k_{t+2}^{o}(w)d\pi(w).$$

The market clearing condition for leased capital in time $t+1$ is:

$$K_{t+1}^{l}=\int k_{t+1}^{l}(w)\,d\pi(w).$$

The market clearing condition for bonds across all firms and households in time $t+1$ is:

$$\int b_{t+1}(w)\,d\pi(w)+B_{t+1}^{H}=0.$$

To derive the output goods, clearing condition (also, the resource constraint for the economy) (denote $k^{Total}(w)=k_{t+1}^{n}(w)+k_{t+1}^{o}(w)+k_{t+1}^{l}(w)$):

Combing the consumer and the firms' budget constraints, we have:

$$B^H_{t+2} + q_{t+1}(1-h)K^l_{t+1} + \tau_{t+1}K^l_{t+1} + \int d_{1,t+1}(w)\, d\pi(w) = C_{t+1} + \beta^{-1}B^H_{t+1} + K^l_{t+2},$$
$$\int \left[f\left(k^{Total}(w)\right) + q_{t+1}k^n_{t+1}(w)\right] d\pi(w) = \int \left[d_{1,t+1}(w) + \tau_{t+1}k^l_{t+1}(w) + \beta^{-1}b_{t+1}(w)\right] d\pi(w),$$
$$\int \left[w + b_{t+2}(w)\right] d\pi(w) = \int \left[d_{0,t+1}(w) + k^n_{t+2}(w) + q_{t+1}k^o_{t+2}(w)\right] d\pi(w).$$

Adding up these equations, and using the market clearing conditions, we have:

$$\int \left[f\left(k^{Total}(w)\right)\right] d\pi(w) + \int w d\pi(w) = C_{t+1} + K^l_{t+2} + \int k^n_{t+2}(w)\, d\pi(w).$$

In stationary equilibrium, we have:

$$\int f\left(k^{Total}(w)\right) d\pi(w) + \int w d\pi(w) = C + K^l + \underbrace{\int k^n(w)\, d\pi(w)}_{\text{Investment in New Capital}},$$

or, we can define national income accounting as:

$$Y + \int w d\pi(w) = C + (K^l + k^n).$$

In stationary equilibrium, for different economies (for example, when we have different fundamental parameters), the present value for the representative HH is just $C/(1-\beta)$. Therefore, comparing different economies' welfare is equivalent to comparing $Y - (K^l + k^n)$, which are obtained from firms' optimal solutions.

B.5 Details for Comparing Two Types of Externalities

In particular, when $q > q^{FB}$, we know that $k^n = 0$ for $w < \underline{w}_n$, $k^o = 0$ for $w > \bar{w}^o$, and $\lambda(w) = 0$ for $w > \bar{w}$, in which $\underline{w}_n < \bar{w}^o < \bar{w}$. Firms with $w \in (\underline{w}_n, \bar{w}^o)$, have the same positive collateral multiplier. We denote it as $\bar{\lambda}$. As $\lambda(w)$ is weakly decreasing in w, we have $\lambda(w) \geq \bar{\lambda}$ for $w \leq \bar{w}^o$, and $\lambda(w) \leq \bar{\lambda}$ for $w \geq \underline{w}_n$. That is, firms purchasing old capital must have a marginal value of net worth larger than or equal to $1 + \bar{\lambda}$, and firms purchasing new capital have a marginal value of net worth no larger than $1 + \bar{\lambda}$. Hence:

$$\int k^o(w)\lambda(w)d\pi(w) = \int^{\bar{w}^o} k^o(w)\lambda(w)d\pi(w) \geq \bar{\lambda} \int^{\bar{w}^o} k^o(w)d\pi(w),$$

and

$$\int k^n(w)\lambda(w)d\pi(w) \leq \bar{\lambda} \int k^n(w)d\pi(w) = \bar{\lambda} \int_{\underline{w}_n}^{w_{max}} k^n(w)d\pi(w).$$

Furthermore, the market-clearing condition for the old capital, coupled with the characterization of equilibrium ($k^n = 0$ for $w < \underline{w}_n$, $k^o = 0$ for $w > \bar{w}^o$, and the fact that leased capital amount is positive), implies:

$$\int_{\underline{w}_n}^{w_{max}} k^n(w)d\pi(w) < \int^{\bar{w}_o} k^o(w)d\pi(w).$$

As a result, we have:

$$\int k^n(w)\lambda(w)d\pi(w) \leq \bar{\lambda} \int_{\underline{w}_n}^{w_{max}} k^n(w)d\pi(w) < \bar{\lambda} \int^{\bar{w}^o} k^o(w)d\pi(w) \leq \int k^o(w)\lambda(w)d\pi(w).$$

Thus, we have the inequality:

$$\int k^o(w)\lambda(w)d\pi(w) > \int k^n(w)\lambda(w)d\pi(w).$$

Finally, as $\theta < 1$, we can obtain that in a stationary competitive equilibrium, the aggregate distributive externality is larger than aggregate collateral externalities.

Let us now consider the special case $q = q^{FB}$. First, $\int k^n(w)\lambda(w)d\pi(w) = 0$ because firms who invest in new capital have a net worth larger than $\underline{w}_n$, and they are unconstrained with a zero collateral multiplier. On the other hand, $\int k^o(w)\lambda(w)d\pi(w)$ is strictly positive. Therefore, we have $\int k^o(w)\lambda(w)d\pi(w) > \theta \int k^n(w)\lambda(w)d\pi(w)$.

B.6 Proof for $\Delta > 0$ When Leasing Market Is Shut Down

When $q > q^{FB}$, we know that $k^n = 0$ for $w < \underline{w}_n$, $k^o = 0$ for $w > \bar{w}_o$, and $\lambda(w) = 0$ for $w > \bar{w}$, in which $\underline{w}_n < \bar{w}_o < \bar{w}$. Firms with $w \in (\underline{w}_n, \bar{w}_o)$, have the same positive collateral multiplier. We denote it as $\bar{\lambda}$. As $\lambda(w)$ is weakly decreasing in w, we have $\lambda(w) \geq \bar{\lambda}$ for $w \leq \bar{w}_O$, and $\lambda(w) \leq \bar{\lambda}$ for $w \geq \underline{w}_N$. That is, firms purchasing old capital must have a marginal value of net worth larger than or equal to $1 + \bar{\lambda}$, and firms purchasing new capital has a marginal value of net worth no larger than $1 + \bar{\lambda}$. Hence:

$$\int k^o(w)\lambda(w)d\pi(w) = \int^{\bar{w}_o} k^o(w)\lambda(w)d\pi(w) \geq \bar{\lambda} \int^{\bar{w}_o} k^o(w)d\pi(w)$$

and

$$\int k^n(w)\lambda(w)d\pi(w) \leq \bar{\lambda} \int k^n(w)d\pi(w) = \bar{\lambda} \int_{\underline{w}_n}^{\bar{w}} k^n(w)d\pi(w).$$

Furthermore, the market-clearing condition for old capital, coupled with the characterization of equilibrium, implies:

$$\int_{\underline{w}_n}^{\bar{w}} k^n(w)d\pi(w) < \int^{\bar{w}_o} k^o(w)d\pi(w)$$

due to the fact that the leased capital amount is nonzero. As a result, we have:

$$\int k^n(w)\lambda(w)d\pi(w) \leq \bar{\lambda} \int_{\underline{w}_n}^{\bar{w}} k^n(w)d\pi(w) < \bar{\lambda} \int^{\bar{w}_o} k^o(w)d\pi(w) \leq \int k^o(w)\lambda(w)d\pi(w).$$

Therefore:

$$\int k^o(w)\lambda(w)d\pi(w) > \int k^n(w)\lambda(w)d\pi(w).$$

Finally, as $\theta < 1$, we can obtain that in a stationary competitive equilibrium, the aggregate distributive externality is larger than aggregate collateral externalities.

Let us now consider the case $q = q^{FB}$. In this case, $\int k^n(w)\lambda(w)d\pi(w) = 0$ because firms that invest in new capital have a net worth larger than $\underline{w}_n$, and they are unconstrained with a zero collateral multiplier. On the other hand, $\int k^o(w)\lambda(w)d\pi(w)$ is strictly positive. Therefore, we have $\theta \int k^n(w)\lambda(w)d\pi(w) < \int k^o(w)\lambda(w)d\pi(w)$.

C Additional Proofs and Results for the Dynamic Model

C.1 Firm Optimization Problem for the Dynamic Model

Firm optimization problem The objective function of a firm, born at time t, is to maximize the expected present discounted value of dividends as:

$$\sum_{a=0}^{\infty}\beta^a\gamma_a\sum_{s^a}p(s^a)d_{t+a}(s^a) + \sum_{a=1}^{\infty}\beta^a\gamma_{a-1}\rho\sum_{s^a}p(s^a)w_{t+a}(s^a)$$

The budget constraint, collateral constraint, and dividend constraint follow:

$$d_t(s^a) = w_t(s^a) + b_{t+1}(s^a) - k^n_{t+1}(s^a) - q_t k^o_{t+1}(s^a) - \tau_t k^l_{t+1}(s^a), \tag{C5}$$

$$\begin{aligned} w_t(s^a) =& s_a f(k_t(s^{a-1})) + (1-\delta^n(1-q_t))k^n_t(s^{a-1}) \\ &+ q_t(1-\delta^o)k^o_t(s^{a-1}) - \beta^{-1}b_t(s^{a-1}), \end{aligned} \tag{C6}$$

$$k_t(s^{a-1}) = g(k^n_t(s^{a-1}), k^o_t(s^{a-1}), k^l_t(s^{a-1})), \tag{C7}$$

$$b_{t+1}(s^a) \le \beta\theta\left[(1-\delta^n(1-q_{t+1}))k^n_{t+1}(s^a) + q_{t+1}(1-\delta^o)k^o_{t+1}(s^a)\right], \tag{C8}$$

$$d_t(s^a) \geq 0. \tag{C9}$$

Here, $d_t(s^a)$ are dividends of continuing firms and $w_t(s^a)$ is net worth (paid as a dividend by existing firms). The dividend of a continuing firm satisfies constraint (C5). q_t is the price of the old capital, and τ_t is the rental fee, which is determined at time t as in Eq. (16). The gross interest is β^{-1} for non-contingent debt $b_t(s^{a-1})$. The firms' worth evolves according to Eq. (C6). Firms face a collateral constraint, which is shown in (C8), the collateral value includes undepreciated new capital, depreciated new capital that is transformed into old capital, and undepreciated old capital.[40]

We consider the first-order conditions. Denote $\left[1+(1-\rho)\eta_{t+1}(s_{t+1}, w_{t+1}(s_{t+1}))\right]$ as $\tilde{\eta}_{t+1}$, and $\tilde{\eta}_{t+1} \geq 1$. The first-order condition for b_{t+1} is then:

$$1+\eta_t = \lambda_t + \beta(1+r_t)E_t\tilde{\eta}_{t+1}.$$

If a firm has a large enough net worth and is not constrained in period t and $t+1$, the first-order condition is reduced to $1 = \beta(1+r_t)$, or, the firms is indifferent in borrowing one more unit of debt or one unit less. Otherwise, firms with limited net worth will typically have positive λ_t and positive η_t.

For choices on new, old, and leased capital with interior values, similarly, we have:

$$\begin{aligned}
&1+\eta_t = \beta E_t\left[\left(s' f_k(k_{t+1})g_{n,t+1} + \omega^n_{t+1}\right)\tilde{\eta}_{t+1}\right] + \lambda_t\beta\theta\omega^n_{t+1},\\
&(1+\eta_t)q_t = \beta E_t\left[\left(s_{t+1} f_k(k_{t+1})\right)g_{o,t+1} + q_{t+1}(1-\delta^o)\right)\tilde{\eta}_{t+1}\right] + \lambda_t\beta\theta q_{t+1}(1-\delta^o),\\
&(1+\eta_t)\tau_t = \beta E_t\left[s_{t+1} f_k(k_{t+1})g_{l,t+1}\tilde{\eta}_{t+1}\right],
\end{aligned}$$

and the first-order condition for old capital can be further simplified into:

$$1+\eta_t = \beta E_t\left[\left(s' f_k(k_{t+1})\right)g_{o,t+1}/q_t + \omega^o_{t+1}\right)\tilde{\eta}_{t+1}\right] + \lambda_t\beta\theta\omega^o_{t+1}.$$

Intuitively, these first-order conditions can be easily understood by analyzing the corresponding

[40]Note that, firms do not face any capital adjustment costs in this setting: for example, when k_{t+1} is different from k_t, or k^o_{t+1} is different from k^o_t and so on, there is no additional adjustment cost. We do this mainly to keep the model simple and transparent, and also so we can exclusively focus on the role of leasing in an environment with financial frictions. Admittedly, this is not to say that adjustment cost is not important per se. In Li and You (2023), capital adjustment and flexibility of leasing are explicitly considered.

marginal cost of investing one more unit and the associated marginal benefit of doing so. For example, investing one more unit in new capital costs 1 unit of output goods in period t, and the firm evaluates it with $1 + \eta_t$; the expected discounted return from production and resale is $\beta E_t \left[\left(s_{t+1} f_k(k_{t+1}) g_{n,t+1} + \omega^n_{t+1} \right) \tilde{\eta}_{t+1} \right]$; in addition, one more unit of new capital allows firms to borrow more $\beta\theta\omega^n$ debt in the credit market, and this generates shadow value at $\lambda_t \beta\theta\omega^n_{t+1}$. In optimal choices, marginal benefits exactly equal marginal costs.

We can also rewrite the optimization problem with the standard Bellman equation as follows (our numerical algorithm and numerical exercises subsequently are based on the recursive formulations)[41]:

$$V_t(s_t, w_t) = \max_{\{b_{t+1}, k^n_{t+1}, k^o_{t+1}, k^l_{t+1}\}} d_t + \beta(1-\rho) E_t V_{t+1}(s_{t+1}, w_{t+1}(s_{t+1})) + \beta\rho E_t w_{t+1}(s_{t+1}),$$

in which an individual firm's state variable is denoted as (s, w), where s is for firms' individual productivity and w is for net worth (or cash on hand). w is measured at the end of each period t, and w_t is defined as:

$$w_t = s_t f(k_t) + (1 - \delta^n(1 - q_t))k^n_t + q_t(1 - \delta^o)k^o_t - (1 + r_{t-1})b_t.$$

That is, for all active firms after the production stage is finished, firms sell any old capital on the market and the resale value is $q_t(1-\delta^o)k^o_t$, and firms repay all the debt $(1 + r_{t-1})b_t$; lastly, firms can transform all the remaining new capital back into output goods one to one, and the value is $(1 - \delta^n(1 - q_t))k^n_t$.

[41] Here we have used the time index for value functions, intended for more general cases (e.g., transition analysis, or other non-stationary equilibrium). Also, we assume that firms have perfect foresight about any future aggregate prices (q_{t+1}, τ_{t+1}).

The constraints for the optimization problem are:

$$
\begin{aligned}
& d_t = w_t + b_{t+1} - k^n_{t+1} - q_t k^o_{t+1} - \tau_t k^l_{t+1}, \\
& \beta\theta \left[\omega^n_{t+1} k^n_{t+1} + q_t \omega^o_{t+1} k^o_{t+1}\right] - b_{t+1} \geq 0, \\
& d_t \geq 0, \\
& k_t = g(k^n_t, k^o_t, k^l_t).
\end{aligned}
$$

in which we have used η_t as the multiplier for the dividend inequality constraint, and λ_t as the multiplier for the collateral inequality constraint. As noted before, k_t is a CES composite capital goods with $g(k^n_t, k^o_t, k^l_t)$. In period t, with cash on hand w_t, firms must optimally choose different types of capital for the next period's production with $(k^n_{t+1}, k^o_{t+1}, k^l_{t+1})$, and dividend distribution of d_t in the current period, as well as the debt position b_{t+1} for the next period; for exiting firms, they simply transfer w_t back to households and do not need to make any other choices. To simplify notations, we denote $(1 - \delta^n(1 - q_{t+1}))$ as ω^n_{t+1}, and $q_{t+1}(1 - \delta^o)/q_t$ as ω^o_{t+1}; these stand for the rates of return for the two types of capital that are not related to production, respectively. Going to the next period, after the productivity shocks are realized, $w_{t+1}(s_{t+1})$ can be defined consistently as:

$$
w_{t+1}(s_{t+1}) = s_{t+1} f(k_{t+1}) + \omega^n_{t+1} k^n_{t+1} + q_t \omega^o_{t+1} k^o_{t+1} - (1 + r_t) b_{t+1}.
$$

C.2 Defining Stationary Competitive Equilibrium

To define a stationary competitive equilibrium, we first let $\pi_t(s, w)$ denote the CDF function that measures the distribution of all firms over (s, w) at the very end of time t. For the sake of convenience, the timing for measurement is after all time-t actions, including all production stages, all capital resales, and all debt repayments, as well as after all firms' possible exiting and new firms' possible entry, but before the next period's productivity shocks and death shocks are realized. Also, for those firms receiving death shocks at the beginning of time t, they re-distribute all the outputs in time t and capital resales as dividends back to the household; they are replaced with exiting firms, and new firms are born at the end of each period and begin with an initial net worth w_0, which is transferred from the household, such that they have the same productivity as those exiting firms for simplicity's sake.

The market for leased capital in period t clears, $K^l_{t+1} = \int k^l_{t+1}(s, w)d\pi_t(s, w)$, and the relevant market price is τ_t. The market for old capital clears with the relevant price q_t:

$$\int k^o_{t+1}(s, w)d\pi_t(s, w) = \int (1 - \delta^o)k^o_t(s_{t-1}, w_{t-1})d\pi_{t-1}(s_{t-1}, w_{t-1})$$
$$+ \int \delta^n k^n_t(s_{t-1}, w_{t-1})d\pi_{t-1}(s_{t-1}, w_{t-1}) + \int \delta^l k^l_t(s_{t-1}, w_{t-1})d\pi_{t-1}(s_{t-1}, w_{t-1}).$$

That is, all individual firms' demand for old capital in period t (the left-hand side) equals the total supply of old capital available at time t, which includes undepreciated old capital from the previous period, and the depreciated fraction of new capital and leased capital from the previous period. We have used the notations (s_{t-1} and w_{t-1}) to specifically highlight the individual states that are in the previous period.[42]

Lastly, the output goods market also clears. For the output goods market, by combining the

[42]We can also write down the supply and demand for old capital with sequential formulation, as it may be used later for further analysis:

$$\sum_{a=0}^{\infty}\gamma_a \sum_{s^a} p(s^a)\left[\delta^n k^n_t(s^a) + (1 - \delta^o)k^o_t(s^a) + \delta^l k^l_t(s^a)\right] = \sum_{a=0}^{\infty}\gamma_a \sum_{s^a} p(s^a)k^o_{t+1}(s^a).$$

The left-hand side is the sum of depreciated new capital, depreciated leased capital, and undepreciated old capital from the previous period (viewed from the perspective of individual state s^a); that is, the the aggregate supply of old capital at the end of period t with all possible individual states in s^a, and the right-hand side is the aggregate demand for old capital at the end of period t. The price of old capital q_t should adjust such that the market-clearing condition is satisfied.

household's budget constraint with all firms' budget constraints, we have:

$$
\begin{aligned}
&C_t + \underbrace{\int k^n_{t+1} d\pi_t(s,w) - \int (1-\delta^n) k^n_t d\pi_{t-1}(s,w)}_{\text{Investment in New Capital}} + \underbrace{\left[K^l_{t+1} - (1-\delta^l)K^l_t + H(K^l_t)\right]}_{\text{Investment in Leased Capital}} \\
&= \underbrace{\int \sum_{s'} P(s_{t+1}|s) s_{t+1} f(k_t) d\pi_{t-1}(s,w)}_{\text{Total Output}} + NFA
\end{aligned}
$$

in which NFA refers to the net foreign financial assets position for the whole economy (equal to $B^H_{t+1} - (1+r)B^H_t + \int b_{t+1}(w) - (1+r)b_t(w) d\pi_t(s,w)$). For output, since the next period's productivity shocks are realized after this period's capital choices of (k^n_t, k^o_t, k^l_t) being made, we must take into account the transition probabilities $P(s_{t+1}|s)$ in computing output.

For a stationary equilibrium, the time index drops out, and all distributions are stationary. All aggregate variables are constants. For the resource constraint, we should have:

$$
C + \int \delta^n k^n d\pi(s,w) + \left[\delta^l K^l + H(K^l)\right] = \int \sum_{s_{t+1}} P(s_{t+1}|s) s_{t+1} f(k) d\pi(s,w) + NFA_{ss}.
$$

For the evolution of the distribution, its details are as follows. For any given admissible $\hat{w}$,

$$
\int_{w \le \hat{w}} d\pi_t(s,w) = \int_{w \le \hat{w}} (1-\rho) \sum_{s_{t-1}} P(s|s_{t-1}) I_{\{w = g(s_{t-1}, w_{t-1}, s)\}} d\pi_{t-1}(s_{t-1}, w_{t-1}) + \rho I_{\{w_0 \le \hat{w}\}}
$$

in which $P(s|s_{t-1})$ is the exogenous transition probability from state s_{t-1} to s, and $I_{\{w=g(s_{t-1},w_{t-1},s)\}}$ is the indicator function that a firm's state variable is transitioned from (s_{t-1}, w_{t-1}) to (s, w), and g is shorthand for the transition process that uses the firm's optimal policy function.

C.3 First Best

If the economy is frictionless in the credit market for borrowing and for external equity issuances, then firms are not incentivized to use leased capital; rather, they will use new capital and old capital for production. To see this, we note that in a frictionless economy, we have the multipliers $\eta_t = \eta_{t+1} = \lambda_t$=0. As shown in Eq. (17), the optimal choice for new capital follows:

$$\beta E_t s_{a+1} f_{k+1}\left(k_{t+1}\left(s^a\right)\right) g_{n,t+1}\left(s^a\right) = 1 - \beta\left(1 - \delta^n\left(1 - q_{t+1}\right)\right).$$

From the optimal F.o.c for leased capital Eq. (19), and Eq. (16) for equilibrium rental fees, and the fact that leasing capital and new capital are perfect substitutes in production (i.e., $g_{n,t+1} = g_{l,t+1}$), we can conclude that firms never use leased capital in this economy. Intuitively, this is simply because the user cost for leasing is higher than that for new capital, due to monitoring costs.

To further analyze the first best economy, we consider the first-order conditions in the stationary economy, which can be arranged as follows:

$$\begin{aligned} 1 - \beta(1 - \delta^n(1 - q^{FB})) &= \beta E_a\left[s_{a+1}\right] f_k(k^{FB}(s^a)) g_n(s^a), \\ q^{FB}\left[1 - \beta(1 - \delta^o)\right] &= \beta E_a\left[s_{a+1}\right] f_k(k^{FB}(s^a)) g_o(s^a). \end{aligned}$$

We note that the left-hand sides are exactly the user costs for new and old capital, respectively. These equations hold for any history at s^a. By using some algebra (see Section C.4 in Appendix), we can show that for capital choice on $k^{FB}(s^a)$, it should satisfy:

$$f_k(k^{FB}(s^a)) = \frac{q^{FB}\left[1 - \beta(1 - \delta^o)\right]}{\beta E_a\left[s_{a+1}\right] g_o(s^a)},$$

in which g_o is a constant in stationary equilibrium.

In sum, the level of capital choices with respect to first best only depend on firms' current productivity at s^a, and do not depend on firms' net worth at all. This is different from competitive equilibrium, for which net worth will matter due to financial frictions. We also see that $k^n(s^a)$ is always proportional to $k^o(s^a)$ for any s^a. With a finite Markovian space for productivity s, we can see that across all firms, there are only a finite number of heterogenous firms in terms of capital choice and output. We will compare other allocations to this first best later in this paper.

C.4 First Best for the Dynamic Model: Further Details

To analyze more on the first best economy, we consider the first-order conditions in the stationary economy, which can be arranged as follows:

$$\begin{aligned} 1-\beta(1-\delta^n(1-q^{FB})) =& \beta E_a\left[s_{a+1} f_k(k^{FB}(s^a)) g_n(s^a)\right], \\ q^{FB}\left[1-\beta(1-\delta^o)\right] =& \beta E_a\left[s_{a+1} f_k(k^{FB}(s^a)) g_o(s^a)\right]. \end{aligned}$$

Furthermore, from the above first-order conditions, we can compare the ratio of two marginal products for g_n and g_o,

$$\frac{1-\beta(1-\delta^n(1-q^{FB}))}{q^{FB}\left[1-\beta(1-\delta^o)\right]} = \frac{\beta E_a\left[s_{a+1}\right] f_k(k^{FB}(s^a)) g_n(s^a)}{\beta E_a\left[s_{a+1}\right] f_k(k^{FB}(s^a)) g_o(s^a)} = \frac{g_n(s^a)}{g_o(s^a)}.$$

in which we have defined notations before:

$$\begin{aligned} k \equiv g(k^n, k^o) &= \left[(\sigma)^{\frac{1}{\varepsilon}}(k^n)^{\frac{\varepsilon-1}{\varepsilon}} + (1-\sigma)^{\frac{1}{\varepsilon}}(k^o)^{\frac{\varepsilon-1}{\varepsilon}}\right]^{\frac{\varepsilon}{\varepsilon-1}}, \\ g_n &\equiv \left[(\sigma)^{\frac{1}{\varepsilon}}(k^n)^{\frac{\varepsilon-1}{\varepsilon}} + (1-\sigma)^{\frac{1}{\varepsilon}}(k^o)^{\frac{\varepsilon-1}{\varepsilon}}\right]^{\frac{\varepsilon}{\varepsilon-1}-1}(\sigma)^{\frac{1}{\varepsilon}}(k^n)^{\frac{\varepsilon-1}{\varepsilon}-1}, \\ g_o &\equiv \left[(\sigma)^{\frac{1}{\varepsilon}}(k^n)^{\frac{\varepsilon-1}{\varepsilon}} + (1-\sigma)^{\frac{1}{\varepsilon}}(k^o)^{\frac{\varepsilon-1}{\varepsilon}}\right]^{\frac{\varepsilon}{\varepsilon-1}-1}(1-\sigma)^{\frac{1}{\varepsilon}}(k^o)^{\frac{\varepsilon-1}{\varepsilon}-1}. \end{aligned}$$

To further characterize the first best economy, we note that the aggregate resource constraint of the economy is:

$$\sum_{a=0}^{\infty}\gamma_a\sum_{s^{a+1}}p(s^{a+1})\left[s_{a+1}f(g(k_t^n(s^a),k_t^o(s^a))) + (1-\delta^n)k_t^n(s^a)\right] = C_t + \sum_{a=0}^{\infty}\gamma_a\sum_{s^a}p(s^a)k_{t+1}^n(s^a),$$

in which the left-hand side is aggregate output and undepreciated new capital, and the right-hand side is the consumption of the representative household and aggregate new capital. The stock of old capital follows:

$$\sum_{a=0}^{\infty}\gamma_a\sum_{s^a}p(s^{a+1})\left[\delta^n k_t^n(s^a)+\delta^l k_t^l(s^a)+(1-\delta^o)k_t^o(s^a)\right]=\sum_{a=0}^{\infty}\gamma_a\sum_{s^a}p(s^a)k_{t+1}^o(s^a).$$

The left-hand side is the sum of depreciated new capital, depreciated leased capital (zero), and undepreciated old capital from the previous period (viewed from the perspective of individual state s^a), that is, the aggregate supply of old capital at the end of period t with all possible individual states in s^a, and the right-hand side is the aggregate demand for old capital at the end of period t. The price of old capital q_t should adjust so that the market-clearing condition is satisfied.

Further simplifying the first-order conditions, we observe that:

$$\begin{aligned}
\frac{g_n(s^a)}{g_o(s^a)} &= \frac{\left[(\sigma)^{\frac{1}{\varepsilon}}(k^n)^{\frac{\varepsilon-1}{\varepsilon}}+(1-\sigma)^{\frac{1}{\varepsilon}}(k^o)^{\frac{\varepsilon-1}{\varepsilon}}\right]^{\frac{\varepsilon}{\varepsilon-1}-1}(\sigma)^{\frac{1}{\varepsilon}}(k^n)^{\frac{\varepsilon-1}{\varepsilon}-1}}{\left[(\sigma)^{\frac{1}{\varepsilon}}(k^n)^{\frac{\varepsilon-1}{\varepsilon}}+(1-\sigma)^{\frac{1}{\varepsilon}}(k^o)^{\frac{\varepsilon-1}{\varepsilon}}\right]^{\frac{\varepsilon}{\varepsilon-1}-1}(1-\sigma)^{\frac{1}{\varepsilon}},(k^o)^{\frac{\varepsilon-1}{\varepsilon}-1}} \\
&= \frac{1-\beta(1-\delta^n(1-q^{FB}))}{q^{FB}\left[1-\beta(1-\delta^o)\right]}.
\end{aligned}$$

That is, across different individual states of s^a, $k^n(s^a)/k^o(s^a)$ is a constant, and it does not depend on individual firms' productivity. Alternatively, $k^n(s^a)$ is always proportional to $k^o(s^a)$ for any s^a in the first best, simply because without any financial frictions, the relative user cost between the two types of capital are constant; however, as we show later, the level of capital is different across different productivity levels. Therefore, we can denote now $k^n = C(q^{FB})k^o$ for some constant $C(q^{FB})(q^{FB}$ remains to be solved) and $C(q^{FB})$ satisfies:

$$\frac{(\sigma)^{\frac{1}{\varepsilon}}}{(1-\sigma)^{\frac{1}{\varepsilon}}}C(q^{FB})^{\frac{-1}{\varepsilon}} \equiv \frac{1-\beta(1-\delta^n(1-q^{FB}))}{q^{FB}\left[1-\beta(1-\delta^o)\right]}.$$

Furthermore, using the market clearing condition for the old capital, and $k^n = C(q^{FB})k^o$ for any individual state, we can now compare aggregate demand and aggregate supply:

$$
\begin{aligned}
& \int (1-\delta^o) k_t^o(s,w) d\pi_{t-1}(s,w) + \int \delta^n k_t^n(s,w) d\pi_{t-1}(s,w) \\
= \; & \int \left[(1-\delta^o) + \delta^n C(q^{FB})\right] k_t^o(s,w) d\pi_{t-1}(s,w) \\
= \; & \left[(1-\delta^o) + \delta^n C(q^{FB})\right] \int k_t^o(s,w) d\pi_{t-1}(s,w) \\
= \; & \left[(1-\delta^o) + \delta^n C(q^{FB})\right] \int k_{t+1}^o(s,w) d\pi_t(s,w) \\
= \; & \int k_{t+1}^o(s,w) d\pi_t(s,w) \text{ (By market clearing condition)},
\end{aligned}
$$

which results in $\left[(1-\delta^o) + \delta^n C(q^{FB})\right] = 1$ for the first best. In a few special cases, if we had $\delta^o = \delta^n$, it would give $C(q^{FB}) = 1$. If we further used $\sigma = 1-\sigma = 1/2$, then we can simplify the final solution for q^{FB} as:

$$
\frac{(\sigma)^{\frac{1}{\varepsilon}}}{(1-\sigma)^{\frac{1}{\varepsilon}}} C(q^{FB})^{\frac{-1}{\varepsilon}} = 1 = \frac{1-\beta(1-\delta^n(1-q^{FB}))}{q^{FB}\left[1-\beta(1-\delta^o)\right]}.
$$

For more general cases, $C(q^{FB}) = \delta^o/\delta^n$, and clearly q^{FB} depends on parameters σ and ε. Using the fact that $g(k^n, k^o)$ is a homogeneous degree of 1, we then have the composite capital goods k is linear in k^o,

$$
\begin{aligned}
k \equiv & g(k^n, k^o) = \left[(\sigma)^{\frac{1}{\varepsilon}} (k^n)^{\frac{\varepsilon-1}{\varepsilon}} + (1-\sigma)^{\frac{1}{\varepsilon}} (k^o)^{\frac{\varepsilon-1}{\varepsilon}}\right]^{\frac{\varepsilon}{\varepsilon-1}} \\
& = \left[(\sigma)^{\frac{1}{\varepsilon}} C(q^{FB})^{\frac{\varepsilon-1}{\varepsilon}} + (1-\sigma)^{\frac{1}{\varepsilon}}\right]^{\frac{\varepsilon}{\varepsilon-1}} k^o.
\end{aligned}
$$

Lastly, the solution for capital choice on $k^{FB}(s^a)$, should satisfy:

$$
f_k(k^{FB}(s^a)) = \frac{q^{FB}\left[1-\beta(1-\delta^o)\right]}{\beta E_a\left[s_{a+1}\right] g_o(s^a)},
$$

which depends on current productivity at s^a, and g_n and g_o follow:

$$
\begin{aligned}
g_o &\equiv \left[(\sigma)^{\frac{1}{\varepsilon}}(k^n)^{\frac{\varepsilon-1}{\varepsilon}} + (1-\sigma)^{\frac{1}{\varepsilon}}(k^o)^{\frac{\varepsilon-1}{\varepsilon}}\right]^{\frac{\varepsilon}{\varepsilon-1}-1}(1-\sigma)^{\frac{1}{\varepsilon}}(k^o)^{\frac{\varepsilon-1}{\varepsilon}-1} \\
&= \left[(\sigma)^{\frac{1}{\varepsilon}}(C(q^{FB}))^{\frac{\varepsilon-1}{\varepsilon}} + (1-\sigma)^{\frac{1}{\varepsilon}}\right]^{\frac{\varepsilon}{\varepsilon-1}-1}(1-\sigma)^{\frac{1}{\varepsilon}}, \\
g_n &= \left[(\sigma)^{\frac{1}{\varepsilon}}(k^n)^{\frac{\varepsilon-1}{\varepsilon}} + (1-\sigma)^{\frac{1}{\varepsilon}}(k^o)^{\frac{\varepsilon-1}{\varepsilon}}\right]^{\frac{\varepsilon}{\varepsilon-1}-1}(\sigma)^{\frac{1}{\varepsilon}}(k^n)^{\frac{\varepsilon-1}{\varepsilon}-1} \\
&= \left[(\sigma)^{\frac{1}{\varepsilon}}(C(q^{FB}))^{\frac{\varepsilon-1}{\varepsilon}} + (1-\sigma)^{\frac{1}{\varepsilon}}\right]^{\frac{\varepsilon}{\varepsilon-1}-1}(\sigma)^{\frac{1}{\varepsilon}}(C(q^{FB}))^{\frac{-1}{\varepsilon}}.
\end{aligned}
$$

We note that g_n and g_o are both constants in the stationary economy.

C.5 On Constrained Efficient Economy

Objective function. First, we note that the present value of consumption can be re-written as:

$$
\begin{aligned}
& \sum_{t=0}^{\infty}\beta^t C_t \\
= \ & \sum_{t=0}^{\infty}\beta^t\left[B_{t+1}^H + D_t - (1+r_t)B_t^H + \delta^l q_{t+1}K_t^l + (1-\delta_l)K_t^l - H(K_t^l) + \tau_t K_{t+1}^l - K_{t+1}^l - \rho w_0\right] \\
= \ & \sum_{t=0}^{\infty}\beta^t D_t + \sum_{t=0}^{\infty}\beta^t B_{t+1}^H - \sum_{t=0}^{\infty}\beta^t[\frac{1}{\beta}B_t^H] + \sum_{t=0}^{\infty}\beta^t\left[\delta_l q_t K_t^l + (1-\delta^l)K_t^l - H(K_t^l) + \tau_t K_{t+1}^l - K_{t+1}^l - \rho w_0\right] \\
= \ & \sum_{t=0}^{\infty}\beta^t D_t - B_0^H + \sum_{t=0}^{\infty}\beta^t\left[\delta^l q_t K_t^l + (1-\delta_l)K_t^l - hK_t^l - K_{t+1}^l(1-\tau_t) - \rho w_0\right] \\
= \ & \sum_{t=0}^{\infty}\beta^t D_t - B_0^H + \sum_{t=0}^{\infty}\beta^t\left[\delta^l q_t + (1-\delta^l) - h\right]K_t^l - \sum_{t=0}^{\infty}\beta^t K_{t+1}^l(1-\tau_t) + \sum_{t=0}^{\infty}\beta^t\left[-\rho w_0\right] \\
= \ & \sum_{t=0}^{\infty}\beta^t D_t - B_0^H + \sum_{t=0}^{\infty}\beta^t\frac{1}{\beta}(1-\tau_{t-1})K_t^l - \sum_{t=0}^{\infty}\beta^t K_{t+1}^l(1-\tau_t) + \sum_{t=0}^{\infty}\beta^t\left[-\rho w_0\right] \\
= \ & \sum_{t=0}^{\infty}\beta^t D_t - B_0^H + \frac{1}{\beta}(1-\tau_{-1})K_0^l + \sum_{t=0}^{\infty}\beta^t\left[-\rho w_0\right],
\end{aligned}
$$

for which we have used $1-\tau_t = \beta\left[\delta^l q_{t+1} + (1-\delta_l) - h\right]$. We can see that, with assumptions on interest rates and the optimality condition for households to supply leased capital, for given initial levels of bond, leased capital, and constant level of w_0, maximizing the present value of consumption is equivalent to maximizing the present value of all firms' dividends. In a stationary economy, we let all the aggregate variables remain constant, such that:

$$\sum_{t=0}^{\infty}\beta^t C_t = \frac{1}{1-\beta}C_{ss} = \frac{1}{1-\beta}D_{ss} - B_0^H + \frac{1}{\beta}(1-\tau_{ss})K_{ss}^l + \frac{-\rho w_0}{1-\beta},$$

or in this case, if we maximize C_{ss} then we maximize $D_{ss}+\frac{1-\beta}{\beta}(1-\tau_{ss})K_{ss}^l$.

Externality. We can also write the equation that the optimal level of the old capital price should satisfy, using the notations for distributions and the state variable of (s, w) as follows:

$$\begin{aligned}
&\beta \int k_{t+1}^o(s,w)\left(1+\eta_t(s,w)\right)d\pi_t(s,w) \\
=&\beta \int \sum_{s,w} \left[(1-\delta^o)k_t^o(s_{t-1},w_{t-1}) + \delta^n k_t^n(s_{t-1},w_{t-1})\right]\left[\left(1+\eta_t(s,w)\right)(1-\rho)+\rho\right]\hat{P}(s|s_{t-1})d\pi_{t-1}(s_{t-1},w_{t-1}) \\
+&\int \left[(1-\delta^o)k_t^o(s_{t-1},w_{t-1}) + \delta^n k_t^n(s_{t-1},w_{t-1})\right]\left(\beta\theta\lambda_{t-1}(s_{t-1},w_{t-1})\right)d\pi_{t-1}(s_{t-1},w_{t-1}) \\
+&\int \left[\beta\delta^l k_t^l(s_{t-1},w_{t-1})\right]\left(1+\eta_{t-1}(s_{t-1},w_{t-1})\right)d\pi_{t-1}(s_{t-1},w_{t-1}).
\end{aligned}$$

in which $\hat{P}(s|s_{t-1}) = P(s|s_{t-1})I_{\{w=g(s_{t-1},w_{t-1},s)\}}$. $P(s|s_{t-1})$ is the transition probability from state s_{t-1} to s, and $I_{\{w=g(s_{t-1},w_{t-1},s)\}}$ is the indicator function that a firm's state variable is transitioned from (s_{t-1}, w_{t-1}) to (s, w).

The left-hand side is the marginal costs of capital purchases, if q_t increases with one unit, discounted to t-1 value. The first term of the right-hand side is the benefits of capital resale values in t; the second term of the right-hand side is the benefits of collateral in t-1, and the last term of the right-hand side is the benefits of decreases in rental fees in t-1. For a stationary equilibrium, this can be further simplified into:

$$
\begin{aligned}
&\int k^{o}(s,w)\left(1+\eta(s,w)\right)d\pi(s,w)\\
&=\int\sum_{s_{t+1},w_{t+1}}\left[(1-\delta^{o})k^{o}(s,w)+\delta^{n}k^{n}(s,w)\right]\left[(1+\eta(s',w_{t+1}))\left(1-\rho\right)+\rho\right]P(s_{t+1}|s)I_{\{w'=g(s,w,s_{t+1})\}}d\pi(s,w)\\
&+\int\left[(1-\delta^{o})k^{o}(s,w)+\delta^{n}k^{n}(s,w)\right]\theta\lambda(s,w)d\pi(s,w)\\
&+\int\delta^{l}k^{l}(s,w)\left(1+\eta(s,w)\right)d\pi(s,w).
\end{aligned}
$$

If there are no financial constraints at all, $\eta = 0$ and $\lambda = 0$, then we should have the reduced condition, which is exactly coincidental with the market clearing condition; that is, without any financial frictions, the competitive equilibrium is efficient:

$$
\begin{aligned}
&\int k^{o}(s,w)d\pi(s,w)\\
&=\int\left[(1-\delta^{o})k^{o}(s,w)+\delta^{n}k^{n}(s,w)\right]d\pi(s,w)\\
&+\int\delta^{l}k^{l}(s,w)d\pi(s,w).
\end{aligned}
$$

Optimal Conditions. Following the main text, the firm's optimality conditions for new capital, the old capital, and leased capital are as follows:

$$
\begin{aligned}
1+\eta_{t}\left(s^{a}\right)=&\ \beta E_{t}\left\{\left[s_{a+1}f_{k}\left(k_{t+1}\left(s^{a}\right)\right)g_{n,t+1}\left(s^{a}\right)+\left(1-\delta^{n}\left(1-q_{t+1}\right)\right)\right]\left(1+(1-\rho)\eta_{t+1}\left(s^{a+1}\right)\right)\right\}\\
&+\beta\theta\lambda_{t}\left(s^{a}\right)\left(1-\delta^{n}\left(1-q_{t+1}\right)\right)+\beta\delta^{n}\phi_{t+1}, \qquad \text{(C10)}\\
\left(1+\eta_{t}\left(s^{a}\right)\right)q_{t}=&\ \beta E_{t}\left\{\left[s_{a+1}f_{k}\left(k_{t+1}\left(s^{a}\right)\right)\right)g_{o,t+1}\left(s^{a}\right)+q_{t+1}\left(1-\delta^{o}\right)\right]\left(1+(1-\rho)\eta_{t+1}\left(s^{a+1}\right)\right)\right\}\\
&+\beta\theta\lambda_{t}\left(s^{a}\right)q_{t+1}\left(1-\delta^{o}\right)-\phi_{t}+\beta\left(1-\delta^{o}\right)\phi_{t+1}, \qquad \text{(C11)}\\
(1+\eta_{t}(s^{a}))\tau_{t}=&\ \beta E_{t}\left\{\left[s_{a+1}f_{k}\left(k_{t+1}\left(s^{a}\right)\right)g_{l,t+1}\left(s^{a}\right)\right]\left(1+(1-\rho)\eta_{t+1}\left(s^{a+1}\right)\right)\right\}+\beta\delta^{l}\phi_{t+1}. \qquad \text{(C12)}
\end{aligned}
$$

Intuitively, the social planner can recognize the additional marginal benefit to society if there is an additional supply of old capital. As shown in Eq. (C10), new capital invested on time t could bring additional marginal benefit to the society: one unit of new capital generates δ^n more old capital next period, which is why we see $+\beta\delta^n\phi_{t+1}$ in the optimality equation. Similarly, as shown in Eq. (C12), we have an additional marginal benefit $\beta\delta^l\phi_{t+1}$. This component represents the externality of the usage of leased capital on the supply of old capital in the next period, and it is positive if $\phi_{t+1} > 0$. Intuitively, financially constrained firms tend to use leased capital, and some fraction of leased capital becomes old capital in the next period, such that the supply of old capital increases and the corresponding price of old capital decreases. As a result, this could mitigate capital misallocation. However, this benefit is ignored by each individual firm. On the other hand, leased capital is supplied with monitoring costs; for social planners, if the effective user cost for leased capital is too high or higher than the corresponding user cost for new capital, then even if there are some potential benefits of providing leased capital, the cost also needs to be considered. For example, in Eq. (C10), there is a component of $\beta\theta\lambda_t(s^a)(1-\delta^n(1-q_{t+1}))$, which contributes to the marginal benefit of new capital. As in the case with the two-period model, if this benefit - due to collateral constraint - is relatively large, then the social planner still prefers to use new capital instead of leased capital in the constrained efficient case. Lastly, for the old capital, current demand can have both marginal benefit and marginal cost to society: current increases in demand drive up time t price, but also provides more old capital through equilibrium for the next period. On net, we have a term $-\phi_t + \beta(1-\delta^o)\phi_{t+1}$ in Eq. (C11).

To further derive the optimal solutions for the social planer in the constrained efficient economy, as we did with the competitive equilibrium, we first obtain the market clearing condition for old capital at time t as follows, with a multiplier of ϕ_t:

$$\int (1-\delta^o)k_t^o(s_{t-1}, w_{t-1}) + \delta^n k_t^n(s_{t-1}, w_{t-1}) + \delta^l k_t^l(s_{t-1}, w_{t-1}) d\pi_{t-1}(s_{t-1}, w_{t-1}) = \int k_{t+1}^o(s, w) d\pi_t(s, w).$$

First-order conditions with respect to k_{t+1}^n, k_{t+1}^o, k_{t+1}^l, and b_{t+1} can be simplified using shorthand notations:

$$
\begin{aligned}
1+\eta_t =& \beta E_t\left[\left(s' f_k(k_{t+1})g_{n,t+1}+\omega^n_{t+1}\right)\tilde{\eta}_{t+1}\right] \\
&+\lambda_t\beta\theta\omega^n_{t+1}+\beta\phi_{t+1}\delta^n, \\
(1+\eta_t)q_t =& \beta E_t\left[\left(s_{t+1}f_k(k_{t+1})g_{o,t+1}+q_{t+1}(1-\delta^o)\right)\tilde{\eta}_{t+1}\right] \\
&+\lambda_t\beta\theta q_{t+1}(1-\delta^o)+\beta\phi_{t+1}(1-\delta^o)-\phi_t, \\
(1+\eta_t)\tau_t =& \beta E_t\left[s_{t+1}f_k(k_{t+1})g_{l,t+1}\tilde{\eta}_{t+1}\right]+\beta\phi_{t+1}\delta^l, \\
1+\eta_t =& \lambda_t+E_t\tilde{\eta}_{t+1}.
\end{aligned}
$$

Stationary Equilibrium. In stationary equilibrium, we can further simplify these conditions using $\omega^o = 1-\delta^o, \omega^n = 1-\delta^n(1-q)$:

$$
\begin{aligned}
1+\eta_t &= \beta E_t\left[\left(s' f_k(k_{t+1})g_{n,t+1}+\omega^n\right)\tilde{\eta}_{t+1}\right]+\lambda_t\beta\theta\omega^n+\beta\phi\delta^n, \\
1+\eta_t &= \beta E_t\left[\left(s' f_k(k_{t+1})g_{o,t+1}/q+\omega^o\right)\tilde{\eta}_{t+1}\right]+\lambda_t\beta\theta(1-\delta^o)+\phi\left[\beta(1-\delta^o)-1\right]/q, \\
1+\eta_t &= \beta E_t\left[s' f_k(k_{t+1})g_{l,t+1}\tilde{\eta}_{t+1}/\tau\right]+\beta\phi\delta_l/\tau, \\
1+\eta_t &= \lambda_t+E_t\tilde{\eta}_{t+1}.
\end{aligned}
$$

Numerical Algorithms for SP. For numerical algorithms, as we did with the competitive equilibrium, we also have different cases to discuss as follows, and eventually, we must also ensure that all value functions and policy functions converge.

(1). $\eta_t = 0$ and $\lambda_t = 0$, and the conditions on k^n_{t+1} and k^o_{t+1} now are:

$$
\begin{aligned}
1 &= \beta\omega^n+\beta E_t\left[\left(s_{t+1}f_k(k_{t+1})g_{n,t+1}\right)\right]+\beta\phi\delta^n, \\
1 &= \beta\omega^o+\beta/qE_t\left[\left(s_{t+1}f_k(k_{t+1})g_{o,t+1}\right)\right]+\phi\left[\beta(1-\delta^o)-1\right]/q.
\end{aligned}
$$

(2). If there is a solution such that $\eta_t > 0$ and $\lambda_t > 0$, we can separately discuss:

(2.1). $k^n_{t+1} > 0, k^o_{t+1} > 0, k^l_{t+1} = 0$; the first-order conditions with respect to k^n_{t+1} and k^o_{t+1} follow:

$$\begin{aligned}
0 =& \beta E_t \left[\left(s' f_k(k_{t+1}) g_{n,t+1} + \omega^n \right) \tilde{\eta}_{t+1} \right] + \lambda_t \beta \theta \omega^n + \beta \phi \delta^n \\
& - \lambda_t - \beta(1+r_t) E_t \tilde{\eta}_{t+1}, \\
0 =& \beta E_t \left[\left(s' f_k(k_{t+1}) g_{o,t+1}/q_t + \omega^o \right) \tilde{\eta}_{t+1} \right] + \lambda_t \beta \theta \omega^o \\
& + \phi \left[\beta(1-\delta^o) - 1 \right]/q - \lambda_t - \beta(1+r_t) E_t \tilde{\eta}_{t+1}.
\end{aligned}$$

and we can further simplify it and eliminate λ_t :

$$\begin{aligned}
& \beta E_t \left[\left(s_{t+1} f_k(k_{t+1}) g_{n,t+1} + \omega^n \right) \tilde{\eta}_{t+1} \right]/(1-\beta\theta\omega^n) - \beta(1+r_t) E_t \tilde{\eta}_{t+1}/(1-\beta\theta\omega^n) \\
& + \beta \phi \delta^n/(1-\beta\theta\omega^n) \\
& = \beta E_t \left[\left(s' f_k(k_{t+1}) g_{o,t+1}/q_t + \omega^o \right) \tilde{\eta}_{t+1} \right]/(1-\beta\theta\omega^o) - \beta(1+r_t) E_t \tilde{\eta}_{t+1}/(1-\beta\theta\omega^o) \\
& + \phi \left[\beta(1-\delta^o) - 1 \right]/q/(1-\beta\theta\omega^o).
\end{aligned}$$

(2.2). Very similarly, $k^n_{t+1} = 0, k^o_{t+1} = \frac{w_t - \tau_t k^l_{t+1}}{q_t(1-\beta\theta\omega^o)}, b_{t+1} = \beta\theta \left[\omega^n k^n_{t+1} + q_t \omega^o k^o_{t+1} \right]$, and $k^l_{t+1} > 0$ and $k^l_{t+1} < w_t/\tau_t$.

First-order conditions with respect to k^o_{t+1} and k^l_{t+1} can be simplified to:

$$\begin{aligned}
0 =& \beta E_t \left[\left(s' f_k(k_{t+1}) g_{o,t+1}/q_t + \omega^o \right) \tilde{\eta}_{t+1} \right] + \lambda_t \beta \theta \omega^o - \lambda_t \\
& - \beta(1+r_t) E_t \tilde{\eta}_{t+1} \ + \phi \left[\beta(1-\delta^o) - 1 \right]/q, \\
0 =& \beta E_t \left[s' f_k(k_{t+1}) g_{l,t+1} \tilde{\eta}_{t+1} \right]/\tau_t - \lambda_t - \beta(1+r_t) E_t \tilde{\eta}_{t+1} + \beta \phi \delta^l/\tau,
\end{aligned}$$

and can be further simplified and we eliminate λ_t :

$$\begin{aligned}
&\beta E_t\left[s_{t+1}f_k(k_{t+1})g_{l,t+1}\tilde{\eta}_{t+1}\right]/\tau_t-\beta(1+r_t)E_t\tilde{\eta}_{t+1}+\beta\phi\delta^l/\tau\\
=&\beta E_t\left[\left(s'f_k(k_{t+1})g_{o,t+1}/q_t+\omega^o\right)\tilde{\eta}_{t+1}\right]/(1-\beta\theta\omega^o)\\
&-\beta(1+r_t)E_t\tilde{\eta}_{t+1}/(1-\beta\theta\omega^o)+\phi\left[\beta(1-\delta^o)-1\right]/q/(1-\beta\theta\omega^o).
\end{aligned}$$

(3). When $\eta_t>0$ and $\lambda_t=0$, the conditions with respect to k^n_{t+1} and k^o_{t+1} now are:

$$\begin{aligned}
0&=\beta E_t\left[\left(s'f_k(k_{t+1})g_{n,t+1}+\omega^n\right)\tilde{\eta}_{t+1}\right]-\beta(1+r_t)E_t\tilde{\eta}_{t+1}+\beta\phi\delta^n,\\
0&=\beta E_t\left[\left(s'f_k(k_{t+1})g_{o,t+1}/q_t+\omega^o\right)\tilde{\eta}_{t+1}\right]-\beta(1+r_t)E_t\tilde{\eta}_{t+1}+\phi\left[\beta(1-\delta^o)-1\right]/q.
\end{aligned}$$

C.6 Numerical Algorithm for Value Function and Policy Function

(1). We denote the multiplier functions as $\eta(s,w),\lambda(s,w),V(s,w)$ in general (we also use variables with prime $'$ to denote values in the next period).

(2). We can make initial guesses for these functions in the next period, denoted as $\eta^{(0)}_{t+1}(s,w)$, $\lambda^{(0)}_{t+1}(s,w)$,$V^{(0)}_{t+1}(s,w)$, and (0) is used to track the number of iterations.

(3). For each (s,w), we must find the best solution on $\left(b_{t+1},k^n_{t+1},k^o_{t+1},k^l_{t+1}\right)$ using case discussions and the first-order conditions. One important note is that the next period's state variable $w_{t+1}(s_{t+1})$ is also a function of $\left(b_{t+1},k^n_{t+1},k^o_{t+1},k^l_{t+1}\right)$:

$$w_{t+1}(s_{t+1})=s_{t+1}f(k_{t+1})+(\omega^n)k^n_{t+1}+\left(\omega^oq_t\right)k^o_{t+1}-(1+r_t)b_{t+1},$$

and so is $\tilde{\eta}_{t+1}$.

The details are as follows:

(3.1). **Case 1:** If there is a solution such that $\eta_t=0$, we should have $\lambda_t=0$ and $\tilde{\eta}_{t+1}(s_{t+1},w_{t+1})=1$ for any (s_{t+1},w_{t+1}) and $w_{t+1}(s_{t+1})$ is not depending on $\left(b_{t+1},k^n_{t+1},k^o_{t+1},k^l_{t+1}\right)$.

In this case, we should have $d_t\geq 0$, and $d_t=w_t+b_{t+1}-k^n_{t+1}-q_tk^o_{t+1}-\tau_tk^l_{t+1}$.

For more details, as $\tau_t \geq 1 - \beta\omega^n$, then we should have $k^n_{t+1} > 0, k^o_{t+1} > 0, k^l_{t+1} = 0$ (also, see the notes in the end for more detailed analysis).

Using the following two optimal conditions, first we need to solve for k^n_{t+1} and k^o_{t+1}:

$$1 - \beta\omega^n = \beta E_t \left[s_{t+1} f_k(k_{t+1}) g_{n,t+1} \right],$$
$$1 - \beta\omega^o = \beta E_t \left[s_{t+1} f_k(k_{t+1}) g_{o,t+1}/q_t \right].$$

After we find possible solutions, we must check for consistency: $d_t \geq 0$, and $b_{t+1} <= \beta\theta \left[\omega^n k^n_{t+1} + q_t \omega^O k^o_{t+1} \right]$, and the implied $\eta_{t+1}(s_{t+1}, w_{t+1}) = 0$ for any (s_{t+1}, w_{t+1}). For a given (s, w), there will be some minimal possible level of positive saving so that even for the worse productivity shock in the next period, the firm can still be unconstrained under the guessed $\eta(s, w)$ function. When w is sufficiently large and the firm can support that level of saving, d_t is strictly positive and increasing linearly in w.

(3.2). **Case 2:** If there is a solution such that: $\eta_t > 0$ and $\lambda_t > 0$, we should have

$$d_t = 0 = w_t + b_{t+1} - k^n_{t+1} - q_t k^o_{t+1} - \tau_t k^l_{t+1},$$

and

$$b_{t+1} = \beta\theta \left[\omega^n k^n_{t+1} + q_t \omega^o k^o_{t+1} \right].$$

We can separately discuss the following sub-cases:

(3.2.1). $k^n_{t+1} > 0, k^o_{t+1} > 0, k^l_{t+1} = 0$; in this case, we have

$$k^o_{t+1} = \frac{w_t + k^n_{t+1} \left(\beta\theta\omega^n - 1 \right)}{q_t \left(1 - \beta\theta\omega^o \right)},$$

and if $\beta\theta\omega^n - 1 < 0$, we should bound k^n_{t+1}. We have two first-order conditions for new and old capital:

$$0 = \beta E_t \left[\left(s' f_k(k_{t+1}) g_{n,t+1} + \omega^n \right) \tilde{\eta}_{t+1} \right] + \lambda_t \beta \theta \omega^n - \lambda_t - \beta(1+r_t) E_t \tilde{\eta}_{t+1},$$

$$0 = \beta E_t \left[\left(s' f_k(k_{t+1}) g_{o,t+1}/q_t + \omega^o \right) \tilde{\eta}_{t+1} \right] + \lambda_t \beta \theta \omega^o - \lambda_t - \beta(1+r_t) E_t \tilde{\eta}_{t+1}.$$

and we can further simplify it and eliminate λ_t :

$$\beta E_t \left[\left(s' f_k(k_{t+1}) g_{n,t+1} + \omega^n \right) \tilde{\eta}_{t+1} \right] / (1 - \beta\theta\omega^n) - \beta(1+r_t) E_t \tilde{\eta}_{t+1} / (1 - \beta\theta\omega^n)$$
$$= \beta E_t \left[\left(s' f_k(k_{t+1}) g_{o,t+1}/q_t + \omega^o \right) \tilde{\eta}_{t+1} \right] / (1 - \beta\theta\omega^o) - \beta(1+r_t) E_t \tilde{\eta}_{t+1} / (1 - \beta\theta\omega^o).$$

so we simply need to solve for k^n_{t+1}; If there is a solution, we then check the implied $\eta_t > 0$, $\lambda_t > 0$ and then check for the consistency.

(3.2.2). $k^n_{t+1} = 0, k^o_{t+1} = \frac{w_t - \tau_t k^l_{t+1}}{q_t(1-\beta\theta\omega^o)}, b_{t+1} = \beta\theta \left[\omega^n k^n_{t+1} + q_t \omega^o k^o_{t+1} \right]$, and $k^l_{t+1} > 0$ and $k^l_{t+1} < w_t/\tau_t$. First-order conditions on owned capital and leased capital can be simplified to:

$$0 = \beta E_t \left[\left(s' f_k(k_{t+1}) g_{o,t+1}/q_t + \omega^o \right) \tilde{\eta}_{t+1} \right] + \lambda_t \beta \theta \omega^o - \lambda_t - \beta(1+r_t) E_t \tilde{\eta}_{t+1},$$

$$0 = \beta E_t \left[s' f_k(k_{t+1}) g_{l,t+1} \tilde{\eta}_{t+1} \right] / \tau_t - \lambda_t - \beta(1+r_t) E_t \tilde{\eta}_{t+1}.$$

and can be further simplified, so we may eliminate λ_t :

$$\beta E_t \left[s' f_k(k_{t+1}) g_{l,t+1} \tilde{\eta}_{t+1} \right] / \tau_t - \beta(1+r_t) E_t \tilde{\eta}_{t+1}$$
$$= \beta E_t \left[\left(s' f_k(k_{t+1}) g_{o,t+1}/q_t + \omega^o \right) \tilde{\eta}_{t+1} \right] / (1 - \beta\theta\omega^o) - \beta(1+r_t) E_t \tilde{\eta}_{t+1} / (1 - \beta\theta\omega^o),$$

so we must solve for k^l_{t+1}; If there is a solution, we check the implied $\eta_t > 0$, $\lambda_t > 0$, and then check for the consistency.

(3.3). **Case 3:** If there is a solution such that: $\eta_t > 0$ and $\lambda_t = 0$. In this case, similarly, we have $d_t = 0 = w_t + b_{t+1} - k^n_{t+1} - q_t k^o_{t+1} - \tau_t k^l_{t+1}$,

and $b_{t+1} < \beta\theta \left[\omega^n k^n_{t+1} + q_t \omega^O k^o_{t+1}\right]$. As $\tau_t \geq 1 - \beta\omega^n$, we should then have $k^n_{t+1} > 0, k^o_{t+1} > 0, k^l_{t+1} = 0$. The first-order conditions on new capital and old capital follow:

$$0 = \beta E_t \left[\left(s' f_k(k_{t+1}) g_{n,t+1} + \omega^n\right) \tilde{\eta}_{t+1}\right] - \beta(1 + r_t) E_t \tilde{\eta}_{t+1},$$
$$0 = \beta E_t \left[\left(s' f_k(k_{t+1}) g_{o,t+1}/q_t + \omega^o\right) \tilde{\eta}_{t+1}\right] - \beta(1 + r_t) E_t \tilde{\eta}_{t+1}.$$

so we must solve for k^n_{t+1} and k^o_{t+1}. If there is a solution, we check the implied $\eta_t > 0$ from $(1 + \eta_t) = E_t \tilde{\eta}_{t+1}$, and then check the consistency for b_{t+1}.

(3.4). After all these three cases (some may have solutions and some may not), we must compare the implied value function $V_t(s, w)$ with different solutions:

$$V_t(s, w) = d_t + \beta(1 - \rho) E_t V^{(0)}_{t+1}(s_{t+1}, w_{t+1}(s_{t+1})) + \beta\rho E_t w_{t+1}(s_{t+1}).$$

and we select the one with the highest value in $V_t(s, w)$. In rare cases for which we do not find solutions in the equation system (since the solutions rely on the guessed functions), we can always choose a feasible solution (similar to Case 2): dividend is constrained and collateral constraint is binding, and when we set $k^n_{t+1} = k^o_{t+1} > 0$, we find consistent and feasible values.

(4). After all these grid points (s, w), we update the initial guesses on $\eta^{(1)}_t(s, w)$, $\lambda^{(1)}_t(s, w)$, $V^{(1)}_t(s, w)$. (in stationary equilibrium, the time index drops out; in transition analysis, this is related to backward induction). We should then have final convergence on these functions.

Discussion for corner solutions. When firms must decide between new capital or leased capital, i.e., $k^n = 0$ or $k^l = 0$, we observe the following. We first note that for a given level of the old capital, if firms choose the same level of k^n and k^l, then we know the marginal product of capital for k^n and k^l are exactly the same since they are perfect substitutes, $E_t \left[s_{t+1} f_k(k_{t+1}) g_{n,t+1} \tilde{\eta}_{t+1}\right] = E_t \left[s_{t+1} f_k(k_{t+1}) g_{l,t+1} \tilde{\eta}_{t+1}\right] \equiv EV_k$. By re-arranging the first-order conditions for interior solutions, we can simplify the first-order conditions on new capital and leased capital:

$$
\begin{aligned}
&(1+\eta_t)\left(1-\omega^n\beta\right)+\lambda_t\omega^n\beta(1-\theta)=\beta\left(EV_k\right), \\
&(1+\eta_t)\tau_t=\beta\left(EV_k\right).
\end{aligned}
$$

τ_t is the user cost of leased capital at time t, and $1-\omega^n\beta$ is the user cost of new capital at time t. We note that τ_t and ω^n are aggregate variables, and individual firms take those as given. Typically, we have $(1-\omega^n\beta)<\tau_t$. For cases with $\lambda_t=0$ (unconstrained firms in collateral), if $(1-\omega^n\beta)<\tau_t$, then the marginal cost of investing in k^l_{t+1} is too high, and firms choose $k^l_{t+1}=0$. In contrast, for cases with $\lambda_t>0$, when λ_t is sufficiently large (firms with very limited net worth and being constrained), the effective user cost for using new capital is too large, so these firms never use new capital. For $\lambda_t>0$ but λ_t is sufficiently close to 0, then the firms' collateral constraint is binding but these firms start to use new capital. We note that λ_t depends on individual states, so different firms in the cross section will have different λ_t.

C.7 Numerical Algorithm for Finding Equilibrium or Efficient Allocations

The numerical algorithm that we used for solving the competitive equilibrium (stationary distribution and stationary equilibrium), or for the constrained efficient economy with a (constrained) social planner, in general includes the following steps:

(1). Guess the initial price q and the rental price τ;

- (1.1). Solve for individual firms' optimization problem and obtain optimal policy functions on $k^n_{t+1}(s,w),k^o_{t+1}(s,w),k^l_{t+1}(s,w),b_{t+1}(s,w)$.
- (1.2). Simulate the economy by starting from some initial wealth for firms until the distribution does not change.
- (1.3). Compute the implied aggregate demand and supply for the old capital, and the implied aggregate demand and supply for leased capital.

(2). Adjust the initial guesses (slowly) until convergence.

(3). Compute statistics for the economy, including aggregate moments and also distributional moments.

(4). For the constrained social planner problem, we also must find the best multiplier (or the society's shadow price for old capital) so that the aggregate consumption (adjusted) is maximized; that is, for each guessed value on the multiplier, we solve for the social planner's problem (see our modified numerical algorithms on value function and policy function iterations in this case in the previous section).

C.8 Details for Calibration and Estimation

We calibrate the model in two steps. For several parameters that are relatively standard in the literature, we directly set those values. For example, for the subject discount factor of the representative household, we assume it is 0.95 so that the implied annual risk-free interest rate is about 5%. For α since we only have capital inputs in the production function, it is close to the value of capital share used in the literature. For depreciation parameters, we follow most of the business cycle literature (e.g., Cooley, Prescott et al. (1995) and Bernanke, Gertler and Gilchrist (1999)) and set the rate for old capital δ^o equal to 0.10. For leased capital and new capital, the rates δ^l and δ^n are assumed to be the same, 0.12, which is slightly higher than δ^o; also, recall that leasing capital has some additional agency/monitoring costs in the production process. Later on, we will also confirm that our results are not sensitive to these arrangements. In addition, we set the collateral constraint parameter θ as 0.25, in line with several seminal papers (such as Quadrini (2000), Buera and Shin (2013), Moll (2014) and Midrigan and Xu (2014), among others).

Another set of parameters in the model, they are generally hard to observe and equally hard to measure with realistic data; thus, we resort to the method of moments matching and use closely related empirical moments to help us discipline these parameter values. These parameters include: ρ, the probability for firms' death shock; ε, the elasticity of substitution between new (and leasing) and old capital in the production function; σ, the parameter related to the share of new (and leasing) capital relative to old capital; d, the monitoring costs for leasing capital; and lastly, the persistence ρ_s and the standard deviation σ_s for the idiosyncratic productivity process. We choose a set of moments to discipline these parameters: for ρ, it is closely related to the level of borrowing given other parameters (similar to finite life cycles in the financial friction literature, as in Bernanke, Gertler and Gilchrist (1999) and Arellano, Bai and Kehoe (2019)). For ρ_s and σ_s, they are closely related to the persistence, standard deviation of productivity and output/assets ratios in the model. For ε and σ, they are mostly related to the share, persistence, and standard deviation of leasing capital ratios. It is worth noting that the model moments mentioned here depend on these parameters jointly, and thus we use the Euclidean distance between model moments and data moments; also, we use identical weights for simplicity's sake.

C.9 The Distribution of Firms in the Cross Section

Following the main text, to assess the importance of leased capital in the stationary equilibrium, after the calibration we can simulate the economy for the stationary equilibrium in which firm distribution over productivity and net worth is endogenous and does not change over time. We can then investigate the distribution of firms in the cross-section to better understand leased capital. Panel (a) of Figure A.3 first reports the density function for all the firms in stationary equilibrium by their net worth. Interestingly, we see that there are two modes for the distribution with relatively small and large net worth, respectively: intuitively, since new firms begin with relatively small net worth and it takes time to accumulate, and also when firms have relatively large net worth for investment and production, they do not have strong incentives to further accumulate since returns then would be quite close to the risk-free return.

In Figure A.3 Panel (b), we divide firms into 4 different quantiles based on their net worth so that within each group we have the same number of firms in the stationary equilibrium. We then plot the ratio of total leased capital to total productive capital that firms use within each group. That is, we use $\frac{k^l}{k^n+k^o+k^l}$, to best reflect the importance of leased capital in production. The ratio monotonically decreases with the firm net worth. For the first quantile, firms use leased capital for up to almost 60% of their total productive capital, and this number is about 30% for the second quantile. Also, for firms with relatively large net worth, the ratio for leased capital is essentially zero (i.e., these firms do not need to use leased capital to mitigate financial frictions at all). Recall that all firms begin with the same level of initial (small) wealth and may endogenously accumulate net worth to overcome these financial frictions (from the credit market and also limited opportunities to raise external equity finance).

[Place Figure A.3 about here]

We can also investigate the distribution of financial constraints across different firms in the stationary equilibrium (Panel (c) of Figure A.3). As before, we divide firms into 4 quantiles and then plot the averages of firm-level multiplier η. Intuitively, the average η decreases monotonically with firm net worth. For one more dollar of net worth, firms in the first quantile would have an additional valuation at about 0.4 dollars; for firms with large net worth, the additional valuation is essentially zero. We can also assess firms' possible financial constraints from another perspective by looking at firm leverage, which is typically used in practice and in the financial friction literature. Overall, we see average firm leverage ratios decrease with firm net worth. In particular, firms in the first quantile actually use a significant portion of leased capital and only need to borrow to finance old capital. Since firms can at most borrow $\beta\theta q_t\omega^o k^o_{t+1}$ for a given level of k^o_{t+1}, the leverage ratio is measured as $\beta\theta\omega^o$. When firms start to use new capital, and ω^n is slightly higher than ω^o with our calibrated parameters, thus we see the average leverage ratio increase slightly. When firms have larger net worth, they tend to remain financially unconstrained and save positive financial assets; in turn, they may have quite low leverage ratios or even hold many liquid assets such that leverage ratio is negative.

In addition, from another perspective, if we sort firms by their productivity in the stationary equilibrium, Panel (d) of Figure A.3 reports the average capital ratio and the average firm leverage ratio within each productivity group in the stationary economy. The mean productivity of the economy is 1,[43] and we consider firms with higher or lower productivity in the figure. It is evident that high-productivity firms will use more leased capital and less new capital; they also have higher values in multipliers of η and higher firm leverage ratios on average. Thus, we obtain a more complete picture now: these high-productivity firms would like to invest more for the next period's production since productivity is persistent, and are more likely to be financially constrained. With the option of leased capital, they can rent more and produce more.

C.10 Alternative Model Specifications, Robustness, and Aggregate Implications

We also check our model with alternative specifications, and we confirm that our aggregate and distributional variables are robust to several alternative assumptions and specifications.

In particular, in Table A.5, we consider different values for ε, the elasticity of substitution between new and old capital, by higher or lower by 50% relative to the benchmark case in columns (1) and (2), and in columns (3) and (4) for 50% higher or lower values in σ, the share of new capital in the production function, and in columns (5) and (6) for higher or lower values in ρ_s (0.90 vs. 0.35), the persistence for idiosyncratic productivity, and in columns (7) and (8) for 50% higher or lower values in σ_s, the standard deviation for idiosyncratic productivity. In short, we find the basic pattern regarding aggregate variables and distributional moments on leasing ratios, multipliers, and firm leverages do not change too much. In particular, when the elasticity of substitution is higher, intuitively firms can more easily substitute between new (also leasing) and old capital, and firms invest more on new capital and also demand more leased capital relative to the benchmark case. Therefore, we see that leasing ratios and, rental fees also increase, and output increases as well. In another experiment, with higher σ, the desired share of new capital in the production function is to be higher; thus, firms invest more on new capital as much as possible, and the aggregate amount of

[43] This is due to normalization in the productivity space and also the assumption that new firms inherit old exiting firms' productivity.

new capital is higher. Also, the leased capital ratio is reduced since old capital is cheaper and the rental price is higher now. We also find, with more persistent and more volatile productivity, that there could be more firms with very high productivity now. Output, consumption, and investment all increase for these two cases, although the changes in capital price and leased capital rental fees are very small and could have different directions.

In Table A.6, we check the implications for different values in β, the death probability ρ, and the depreciation rate for new capital δ^n. Similarly, as before, we consider higher and lower values in these parameters (0.98 vs. 0.80 for β, and other parameters have 50% higher or 50% lower values as in previous experiments). We find output and consumption increase with higher β or lower death probability. These can be understood intuitively, since if firms are more patient and live longer, they can accumulate more net worth to overcome financial frictions; also, they tend to invest more in new capital since they discount future less, and at the same time they use less leasing capital when rental fees decrease. In equilibrium, aggregate variables' changes are relatively large (consistent with the literature that investment and capital are quite sensitive to changes in the discount factor). In particular, when β is very low (such as 0.80), we see almost no firms are financially constrained simply because they do not value the future very much and do not need to invest much otherwise. Lastly, for the experiment of increasing the depreciation rate for new capital δ^n, intuitively, this will make investing in new capital even less attractive than the benchmark case, and the capital price will tend to increase. At the same time, households will invest more in leased capital since the return of providing more leased capital is higher. Therefore, we see leased capital ratios increase for most firms, although aggregate output decreases.

[**Place Table A.5 about here**]

[**Place Table A.6 about here**]

C.11 Additional Numerical Results for the Dynamic Model

[Place Table A.7 about here]

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[Place Table A.8 about here]

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Appendix A Tables and Figures for Online Appendix Publication

Table A.1: Summary Statistics for Leased Capital

	p10	p25	p50	p75	p90	Mean	No. of firms	Std
Output/Assets	0.558	0.809	1.140	1.577	2.133	1.225	2330	0.431
Leased Capital/Assets	0.047	0.085	0.149	0.277	0.526	0.224	2330	0.176
Leased Capital Ratio	0.102	0.206	0.384	0.620	0.777	0.387	2330	0.161
Leverage	0.003	0.032	0.155	0.331	0.513	0.224	2330	0.159
Productivity (log)	2.980	3.393	3.792	4.182	4.572	3.803	2330	0.373

The table reports the time-series averages of the cross-sectional statistics. We measure output using SALE and measure total assets using AT. The owned physical capital is measured using PPENT. We measure market leverage as the ratio of long-term debt (DLTT) over the sum of market capitalization and long-term debt. The leased capital stock is estimated by 10 times the rental expenses, as in Rampini and Viswanathan (2013), Li and Tsou (2019), as well as in common industry practice. The leased capital ratio is defined as the fraction of leased capital over the sum of leased capital and owned capital. We calculate firm-level productivity using the control function approach. The sample selections are in Appendix A. Note that p10, p25, p50, p75, p90 represent the corresponding quantile. The mean is weighted by firm-level total assets. For standard deviations, we focus on the within industry concept. Hence, we remove the industry-year fixed effect from each raw data series and then calculate year-by-year standard deviations of the residuals.

Table A.2: Leased Capital Ratio at Industry Level

	Median	Mean
Agriculture	0.162	0.230
Mining	0.052	0.097
Construction	0.136	0.200
Manufacturing	0.120	0.163
Transportation	0.138	0.267
Wholesale Trade	0.184	0.234
Retail Trade	0.563	0.678
Services	0.228	0.312

The table presents the leased capital ratio at the industry level. We report pooled means by two-digital SIC codes on the sample of Compustat firms. The detailed data description is in Appendix A.

Table A.3: Summary Statistics for Leased Capital by Groups

G1	p10	p25	p50	p75	p90	Mean	No. of obs	Std
Output/Assets	0.681	0.949	1.305	1.749	2.261	1.395	583	0.484
Leased Capital/Assets	0.055	0.106	0.194	0.346	0.617	0.276	583	0.198
Leased Capital Ratio	0.146	0.301	0.542	0.735	0.837	0.506	583	0.186
Leverage	0.000	0.008	0.055	0.206	0.406	0.141	583	0.147
Productivity (log)	2.934	3.341	3.745	4.105	4.477	3.733	583	0.401
G2	p10	p25	p50	p75	p90	Mean	No. of obs	Std
Output/Assets	0.615	0.862	1.202	1.640	2.177	1.321	582	0.429
Leased Capital/Assets	0.050	0.090	0.160	0.302	0.591	0.259	582	0.184
Leased Capital Ratio	0.125	0.246	0.444	0.656	0.788	0.450	582	0.163
Leverage	0.003	0.026	0.127	0.321	0.516	0.199	582	0.164
Productivity (log)	2.989	3.386	3.791	4.178	4.576	3.789	582	0.370
G3	p10	p25	p50	p75	p90	Mean	No. of obs	Std
Output/Assets	0.544	0.819	1.135	1.553	2.135	1.267	583	0.398
Leased Capital/Assets	0.045	0.084	0.145	0.264	0.534	0.238	583	0.174
Leased Capital Ratio	0.101	0.201	0.365	0.569	0.736	0.391	583	0.148
Leverage	0.011	0.067	0.202	0.378	0.557	0.246	583	0.165
Productivity (log)	2.981	3.407	3.816	4.209	4.599	3.810	583	0.343
G4	p10	p25	p50	p75	p90	Mean	No. of obs	Std
Output/Assets	0.456	0.674	0.943	1.292	1.822	1.067	582	0.364
Leased Capital/Assets	0.043	0.072	0.115	0.195	0.371	0.173	582	0.137
Leased Capital Ratio	0.074	0.146	0.258	0.421	0.618	0.297	582	0.130
Leverage	0.037	0.109	0.222	0.370	0.534	0.258	582	0.150
Productivity (log)	3.018	3.439	3.819	4.237	4.624	3.834	582	0.339

The table reports the time-series averages of the cross-sectional statistics by groups. G1 - G4 denotes firm groups sorted by total assets, in which G1 refers to the smallest size group, while G4 refers to the largest size group. All other details remain the same as in Table A.1.

Table A.4: Regression Results: Leased Capital Ratio and Firm Characteristics

Regression	All	All	All	All	Manu-facturing	Services	Young	Old
Size	-0.0150		-0.0059		-0.0041	-0.0067	-0.0079	-0.0031
	(-13.65)		(-5.16)		(-3.41)	(-2.29)	(-4.98)	(-2.05)
WW-index		0.2886		0.1512				
		(13.81)		(6.86)				
Other controls: (at t-1)								
Dividend-to-asset			-0.4092	-0.2515	-0.3305	-0.0099	-0.2581	-0.4665
			(-4.61)	(-2.70)	(-3.54)	(-0.05)	(-1.92)	(-4.21)
Cash-to-asset			-0.0756	-0.0787	-0.0552	-0.0940	-0.0790	-0.0573
			(-6.47)	(-6.72)	(-4.57)	(-3.79)	(-5.22)	(-3.54)
Leverage			-0.1616	-0.1638	-0.0861	-0.2534	-0.1866	-0.1365
			(-13.10)	(-13.4)	(-7.32)	(-7.36)	(-11.3)	(-8.21)
q			-0.0060	-0.0062	-0.0038	-0.0113	-0.0092	-0.0009
			(-3.66)	(-3.81)	(-2.52)	(-3.53)	(-4.94)	(-0.35)
Profitability			-0.1565	-0.1532	-0.1425	-0.1420	-0.1780	-0.1401
			(-8.31)	(-8.15)	(-5.75)	(-4.49)	(-8.3)	(-4.74)
B/M ratio			-0.0272	-0.0278	-0.0245	-0.0410	-0.0242	-0.0290
			(-6.76)	(-6.86)	(-5.94)	(-4.32)	(-4.2)	(-5.05)
Firm age			0.0000	0.0001	0.0001	-0.0009	0.0001	0.0000
			(-0.05)	(0.64)	(0.40)	(-1.88)	(0.15)	(-0.07)
Sale-to-asset			0.0983	0.0971	0.0830	0.1521	0.1128	0.0913
			(18.99)	(18.82)	(14.83)	(12.07)	(16.31)	(11.78)
Tangibility			0.0260	0.0262	0.0167	0.0519	0.0289	0.0268
			(7.87)	(7.91)	(5.39)	(7.00)	(5.53)	(6.05)
Year FE	YES	YES	YES	YES	YES	YES	YES	YES
Industry FE	YES	YES	YES	YES	YES	YES	YES	YES
Adj-R^2	0.6005	0.6637	0.6388	0.6394	0.3571	0.5101	0.6310	0.6341

This table presents the regression results of the leasing ratio and firm characteristics. The leasing ratio is defined as leased capital over total assets. Dividends are Dividends—Common plus (when available) Dividends Preferred; Cash is Cash and Short-Term Investments; and Tobin's q is market equity minus Common Equity Total minus Deferred Taxes in Balance Sheet, all divided by total assets. Leverage is long-term debt over total assets. B/M ratio is book equity over market equity. Profitability is net income plus depreciation, divided by total assets. Tangibility is physical capital over book equity. The sample selections are in Appendix A. Standard errors are two-way clustered by firm and industry-year. We report t-statistics in parentheses.

Table A.5: Aggregate Implications with Different Parameters for Productivity and Production

	Benchmark	Higher ε	Lower ε	Higher σ	Lower σ	Higher ρ_s	Lower ρ_s	Higher σ_s	Lower σ_s
		(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
q	0.610	-0.21%	1.68%	-11.02%	13.66%	0.05%	0.11%	-0.39%	0.13%
τ	0.101	0.15%	-1.15%	7.59%	-9.39%	-0.03%	-0.07%	0.27%	-0.08%
Output	12.145	3.48%	-5.32%	2.86%	-9.39%	2.08%	-1.58%	1.97%	-1.51%
Consumption	8.395	2.54%	-3.94%	1.57%	-8.25%	2.08%	-1.63%	2.24%	-1.50%
Investment	3.750	5.59%	-8.42%	5.72%	-11.94%	2.09%	-1.46%	1.35%	-1.56%
Dividend	10.929	5.25%	-7.69%	1.64%	-8.41%	2.07%	-2.06%	2.36%	-2.00%
Agg. leased capital	19.959	14.18%	-20.16%	1.82%	-8.96%	1.99%	-3.51%	2.70%	-3.72%
Agg. new capital	10.137	-12.37%	15.97%	13.78%	-18.21%	2.21%	2.73%	-1.56%	2.87%
Agg. old capital	36.032	5.19%	-7.87%	5.82%	-12.03%	2.12%	-1.24%	1.01%	-1.04%
Distributive externality	3.637	2.86%	-3.51%	1.02%	-8.95%	1.16%	-2.96%	4.46%	-3.08%
Rental externality	2.949	12.56%	-19.39%	1.04%	-9.27%	2.87%	-3.76%	4.16%	-4.06%
Collateral externality	0.259	-18.87%	28.64%	-0.32%	-9.22%	-0.32%	-2.42%	8.58%	-3.06%
Leased capital ratio	0.302	0.328	0.262	0.290	0.312	0.302	0.295	0.307	0.294
Leased capital ratios: Q1	0.774	0.836	0.654	0.773	0.762	0.765	0.785	0.768	0.780
Leased capital ratios: Q2	0.569	0.597	0.525	0.554	0.575	0.566	0.569	0.577	0.565
Leased capital ratios: Q3	0.286	0.283	0.280	0.253	0.311	0.304	0.256	0.316	0.249
Leased capital ratios: Q4	0.012	0.019	0.016	0.010	0.021	0.031	0.000	0.035	0.000
Avg. η: Q1	0.412	0.359	0.491	0.393	0.403	0.401	0.417	0.432	0.406
Avg. η: Q2	0.167	0.143	0.202	0.151	0.172	0.165	0.162	0.185	0.162
Avg. η: Q3	0.051	0.043	0.065	0.049	0.049	0.055	0.048	0.065	0.047
Avg. η: Q4	0.005	0.004	0.007	0.004	0.005	0.007	0.003	0.009	0.003
Avg. leverage: Q1	0.214	0.214	0.214	0.214	0.214	0.214	0.214	0.214	0.214
Avg. leverage: Q2	0.214	0.214	0.214	0.214	0.214	0.214	0.214	0.214	0.214
Avg. leverage: Q3	0.206	0.205	0.209	0.207	0.207	0.190	0.215	0.179	0.215
Avg. leverage: Q4	-0.129	-0.137	-0.107	-0.147	-0.099	-0.176	0.033	-0.330	0.044

Table A.6: Aggregate Implications with Different β, ρ and δ^n

	Benchmark	Higher β	Lower β	Higher ρ	Lower ρ	Higher δ^n	Lower δ^n
		(1)	(2)	(3)	(4)	(5)	(6)
q	0.610	-10.53%	32.22%	-4.30%	3.48%	3.00%	-2.97%
τ	0.101	-20.60%	121.72%	2.97%	-2.38%	-2.05%	2.05%
Output	12.145	40.52%	-73.34%	-9.74%	12.26%	-1.88%	1.13%
Consumption	8.395	21.20%	-65.95%	-8.10%	10.22%	-3.63%	6.88%
Investment	3.750	83.76%	-89.88%	-13.39%	16.81%	2.06%	-11.75%
Dividend	10.929	44.42%	-73.72%	-4.16%	0.20%	-1.20%	1.37%
Agg. Leased capital	19.959	121.27%	-99.49%	8.86%	-33.03%	0.068	-0.169
Agg. New capital	10.137	5.47%	-69.85%	-59.81%	120.56%	-0.386	0.979
Agg. Old capital	36.032	83.07%	-89.49%	-14.47%	18.88%	0.019	-0.115
Distributive externality	3.637	125.09%	-99.48%	18.72%	-35.83%	-2.34%	4.25%
Rental externality	2.949	139.07%	-99.58%	10.57%	-33.42%	4.70%	-10.13%
Collateral externality	0.259	124.98%	-99.36%	40.08%	-42.25%	-18.71%	40.43%
Leased capital ratio	0.302	0.366	0.015	0.384	0.170	0.332	0.242
Leased capital ratios: Q1	0.774	0.864	0.064	0.801	0.692	0.799	0.746
Leased capital ratios: Q2	0.569	0.672	0.000	0.617	0.287	0.602	0.533
Leased capital ratios: Q3	0.286	0.412	0.000	0.426	0.008	0.289	0.211
Leased capital ratios: Q4	0.012	0.092	0.000	0.128	0.000	0.016	0.001
Avg. η: Q1	0.412	0.707	0.006	0.463	0.315	0.377	0.497
Avg. η: Q2	0.167	0.306	0.000	0.231	0.061	0.139	0.249
Avg. η: Q3	0.051	0.109	0.000	0.104	0.004	0.032	0.125
Avg. η: Q4	0.005	0.019	0.000	0.023	0.000	0.002	0.020
Avg. leverage: Q1	0.214	0.221	-0.157	0.214	0.214	0.214	0.214
Avg. leverage: Q2	0.214	0.221	-0.540	0.214	0.205	0.214	0.214
Avg. leverage: Q3	0.206	0.221	-0.143	0.214	-0.339	0.195	0.219
Avg. leverage: Q4	-0.129	0.087	0.092	0.097	-0.055	-0.210	0.024

Table A.7: Parameters Calibration and Estimation

Parameter	Description	Value	Relevant Data Moments
β	Discount factor	0.95	Risk free interest rates
α	Decreasing return to scale	0.60	Standard values
δ^o	Depreciation rate for old capital	0.10	See text.
δ^n	Depreciation rate for new capital	0.12	See text.
δ^l	Depreciation rate for leased capital	0.12	See text.
θ	Collateral constraint parameter	0.25	See text.
ρ	Firm death shocks	0.1845	Avg. and Std. of Firm Leverage
ε	Elasticity of substitution between new (and leasing) and old capital	4.2803	Avg., persistence, and Std. of leasing ratio
σ	Share of new (and leasing) capital	0.4218	Avg., persistence, and Std. of leasing ratio
d	Monitoring costs for leased capital	0.0070	Avg. leased capital ratio
ρ_s	Persistence for idiosyncratic productivity	0.6639	Persistence of output/assets
σ_s	Std. for idiosyncratic productivity	0.1976	Std. of Output/Assets

Table A.8: Agency Costs, Capital Depreciation, and Implications

	Benchmark	Higher d	Lower d	Higher δ^l	Lower δ^l	Higher θ	Lower θ
		(1)	(2)	(3)	(4)	(5)	(6)
q	0.610	1.20%	-1.21%	2.23%	-0.79%	1.34%	-1.39%
τ	0.101	2.47%	-2.45%	19.70%	-21.71%	-0.91%	0.96%
Output	12.145	-1.89%	1.90%	-12.34%	15.92%	2.27%	-1.83%
Consumption	8.395	-2.08%	2.14%	-12.13%	21.14%	2.05%	-1.64%
Investment	3.750	-1.47%	1.38%	-12.80%	4.22%	2.78%	-2.27%
Dividend	10.929	-2.34%	2.42%	-15.30%	15.99%	0.06%	0.22%
Agg. leased Capital	19.959	-5.85%	6.24%	-49.61%	87.44%	-6.54%	6.33%
Agg. new Capital	10.137	2.17%	-2.71%	14.23%	15.25%	22.12%	-20.27%
Agg. old Capital	36.032	-3.13%	3.22%	-11.39%	1.03%	3.42%	-2.48%
Leased capital ratio	0.302	0.293	0.311	0.188	0.438	0.273	0.329
Distributive externality	3.637	-4.22%	4.46%	-5.78%	-14.06%	-3.10%	2.99%
Rental externality	2.949	-6.00%	6.40%	-19.13%	-7.81%	-6.83%	6.17%
Collateral externality	0.259	-1.23%	1.23%	28.74%	-28.13%	55.49%	-52.33%

Figure A.1: Timeline for the Two-period Model

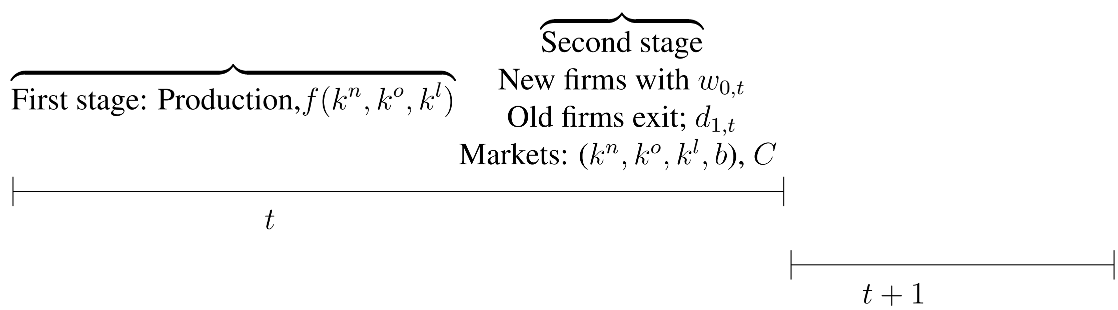


Figure A.2: Illustrations for different user costs and optimal solutions

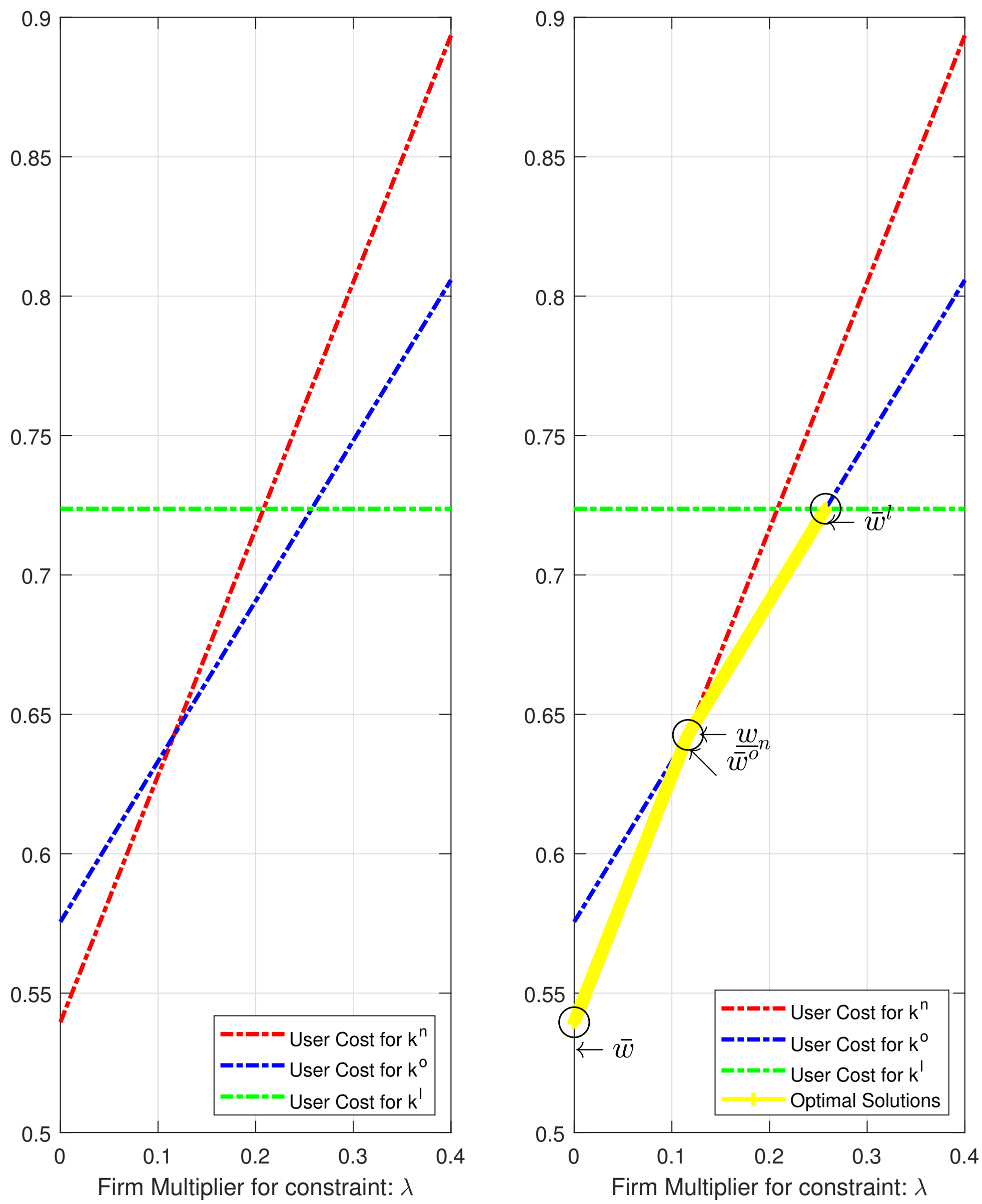


Figure A.3: Firm Distributions And Characteristics

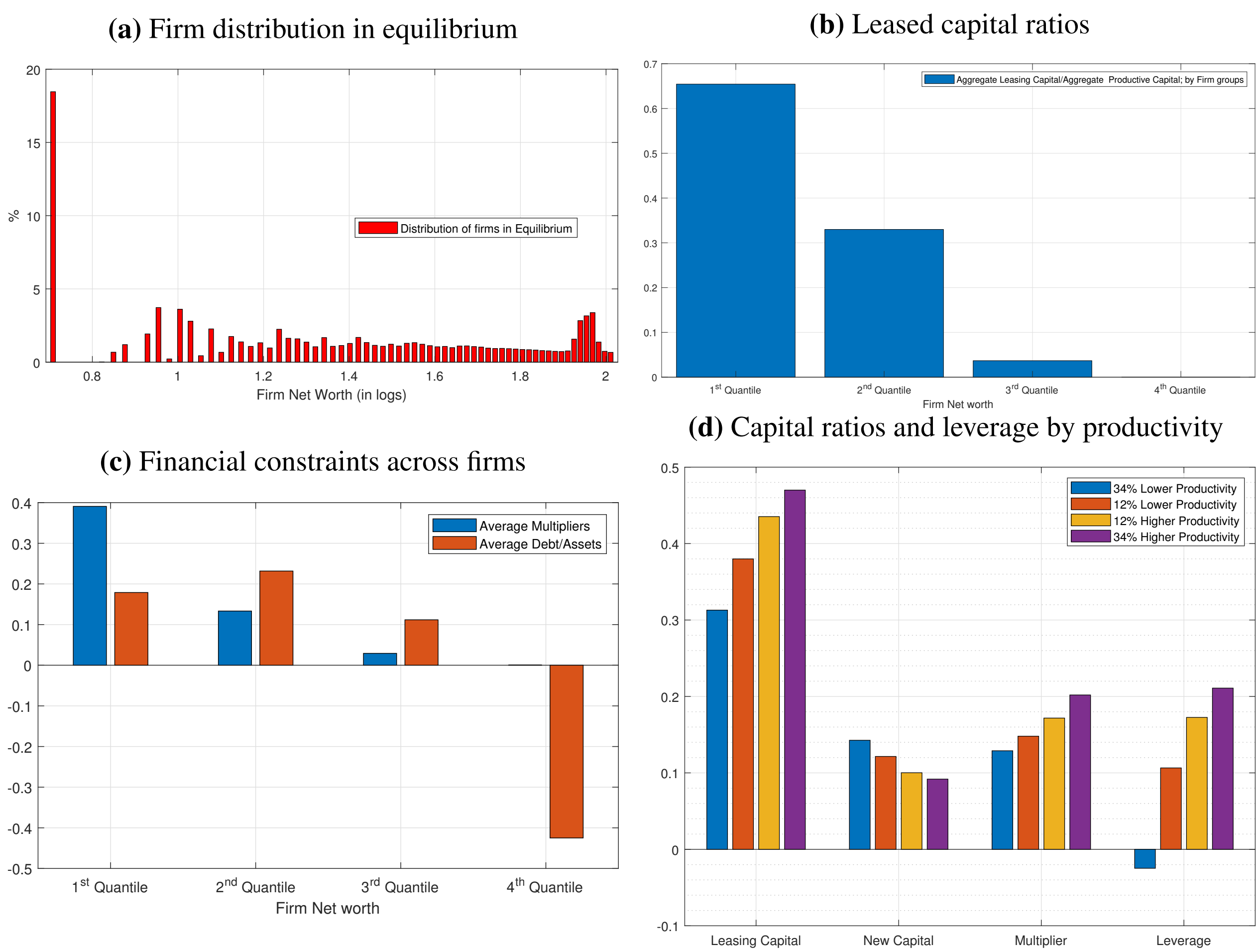


Figure A.4: Productivity and Capital Choices

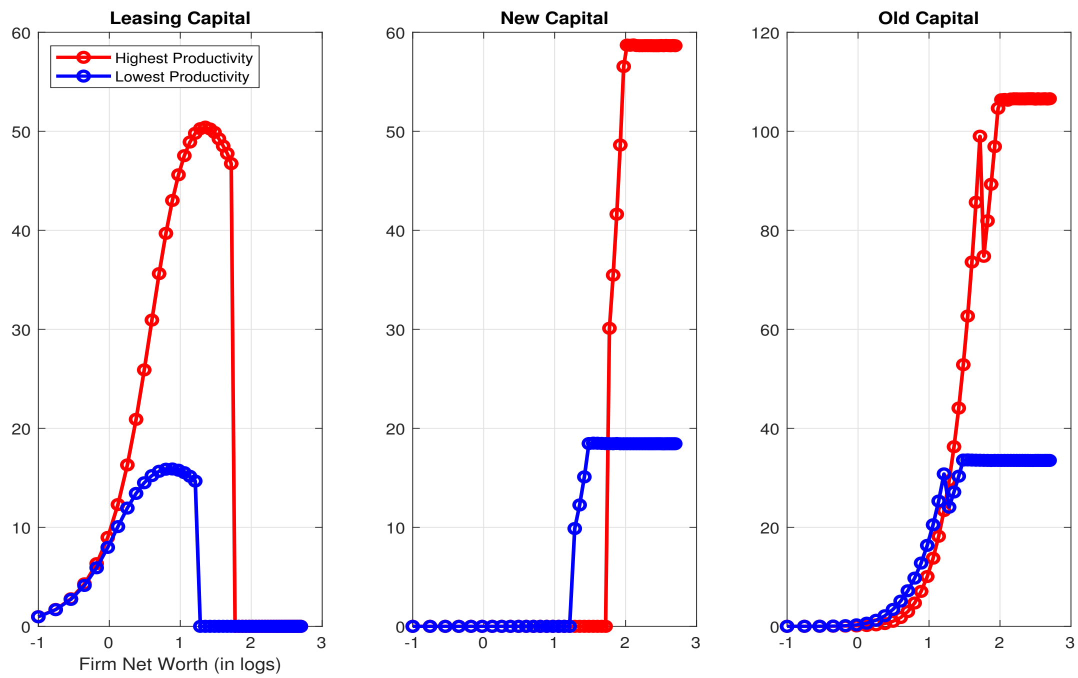


Figure A.5: Firm Multipliers

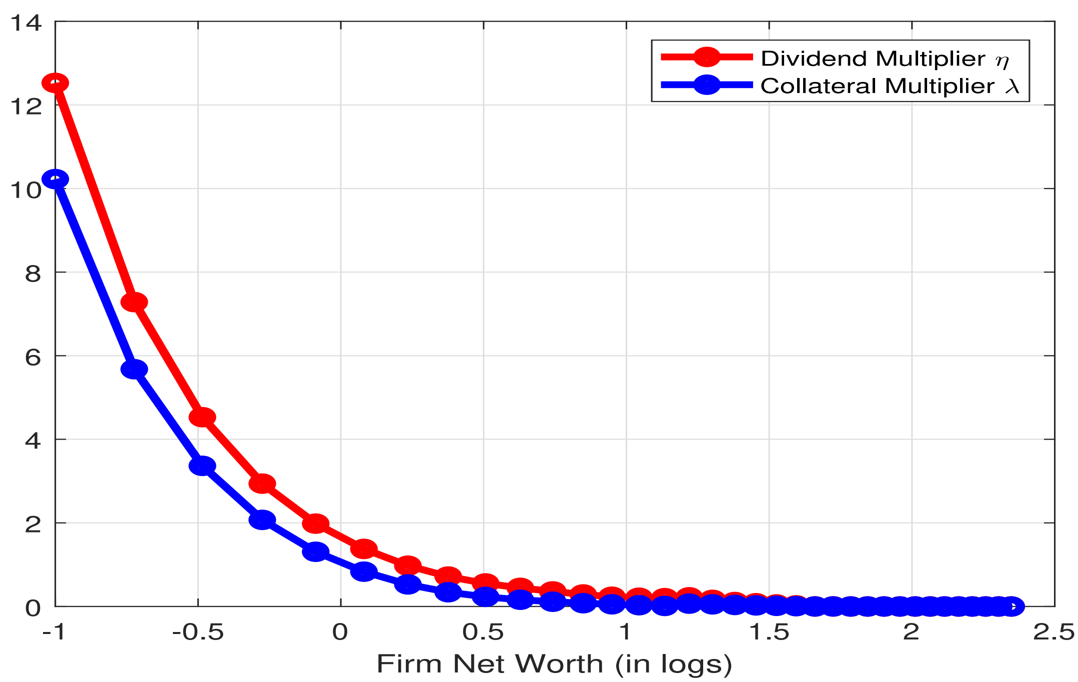


Figure A.6: Firm Value and Dividend Distribution

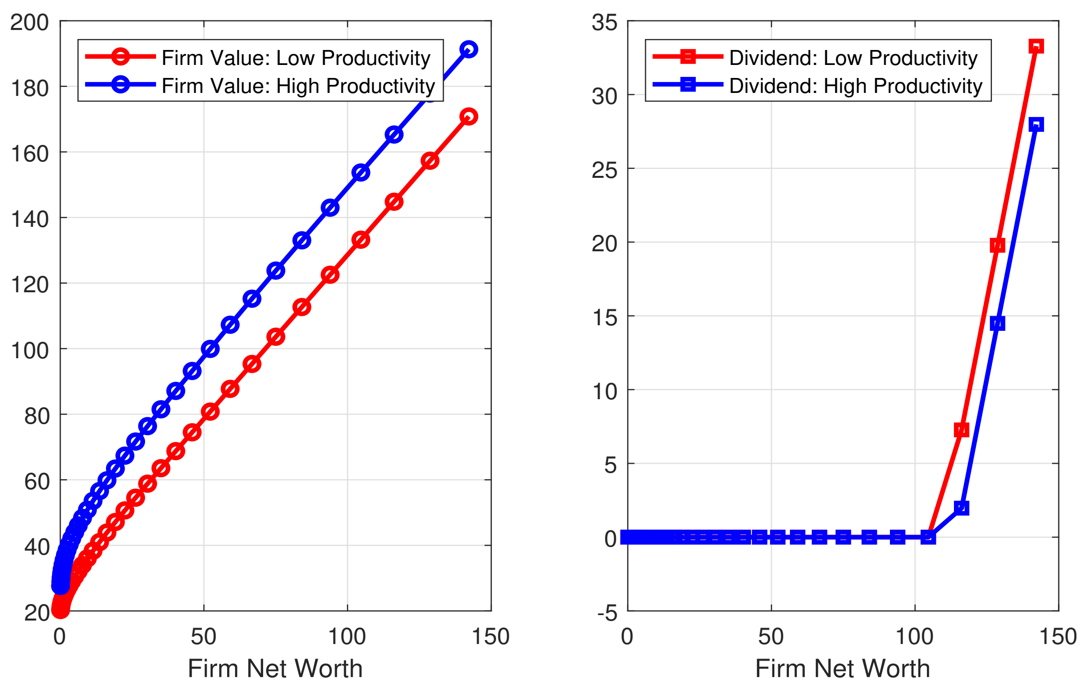


Figure A.7: Firm Value and Multipliers with Low vs. High Rental Fees

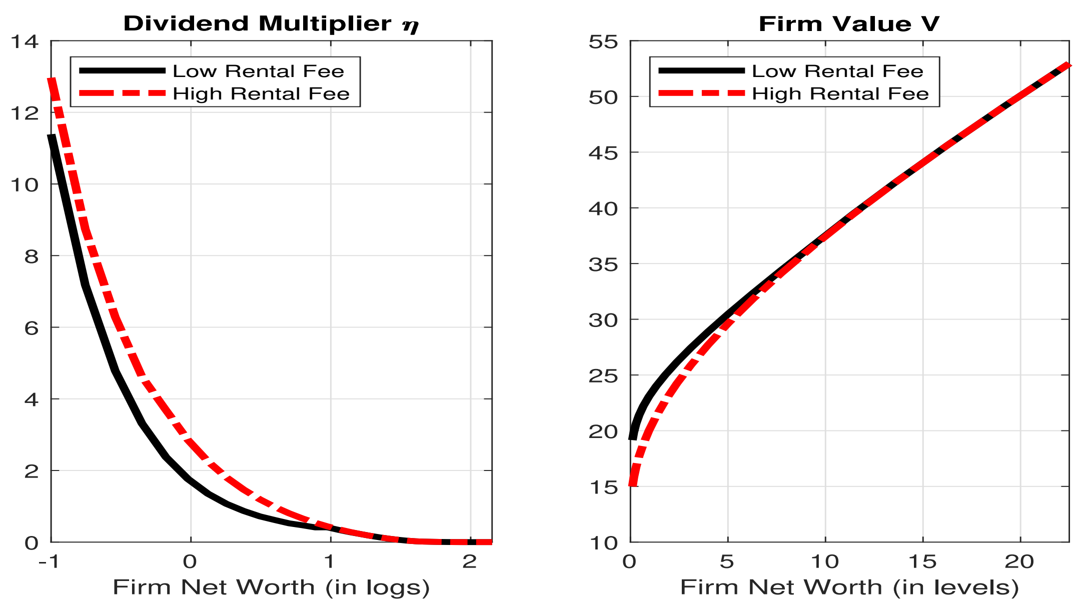
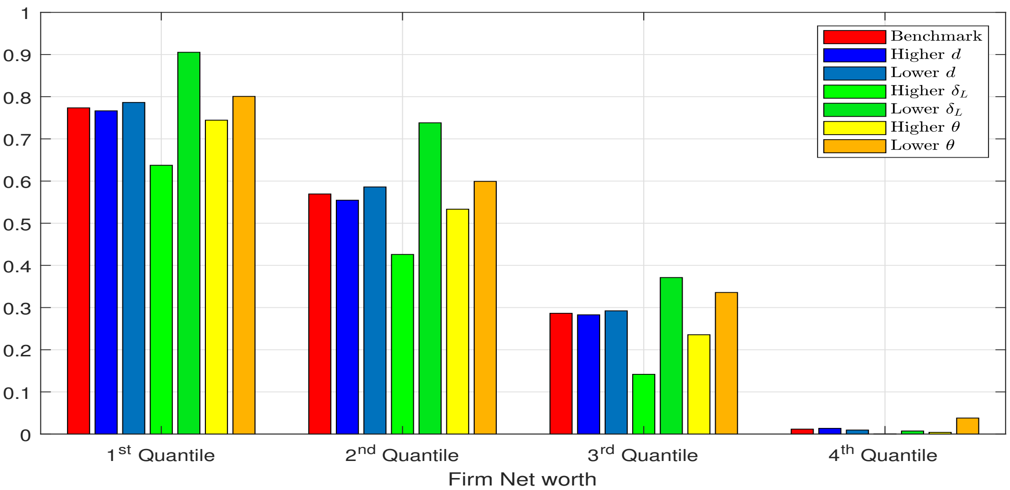
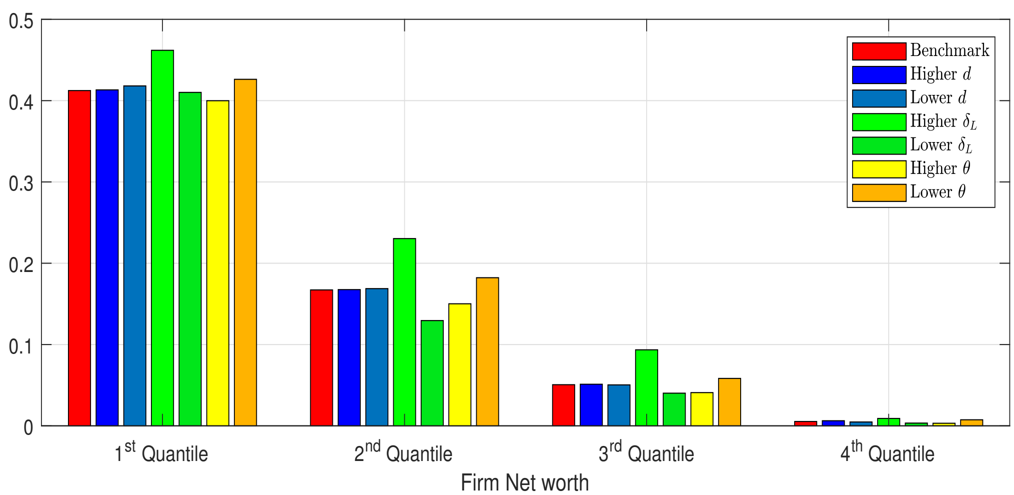


Figure A.8: Agency Costs, Capital Depreciation, Collateral, and Implications Across Firms

(a) Leased capital ratios



(b) Multiplier η



(c) Firm leverages