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Keywords: Matching theory, market design, stability, college admissions market
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1 Introduction

The field of market design uses microeconomic theory to solve real-life resource allocation problems, which has helped to transport important economic insights from theory to practice. Research, triggered by exchanges between researchers and practitioners, has generated several sucessful mechanisms tailored for real markets — prominent examples include entry-level labor markets, school choice, refugee resettlement, spectrum auctions, organ transplantation, course allocation, and internet advertising.[1] Vital to this field of market design's success has been its fastidious attention to contextual details of allocation problems, details ranging from laws and regulatory constraints to aspects of participants' strategic behavior. In this spirit, this paper presents an analysis of matching markets with participants' choice behavior in focus.

Preferences over potential assignments encode participants' choice behavior in the standard model of matching theory.[2] However, there is plentiful evidence in marketing, psychology, and economics suggesting that participants' choices need not be consistent with the maximization of a preference relation. Among other possibilities, participants may exhibit non-standard choice behavior due to mistakes and behavioral biases. Established phenomenons include choice overload, framing and attraction effects, temptation and self-control, as well as status-quo biases.[3] This paper extends matching theory to the case where participants may exhibit such non-standard choice behavior. The following examples illustrate the significance of such an exercise in the context of matching markets.

(i) (Choice Complexity and Overload) Take the case of kidney exchange programs. The complexity of the choice problem is apparent given the amount of information needed

[1]The initial leading applications of matching were school choice and kidney exchange (Abdulkadiroglu and Sönmez (2003), Abdulkadiroğlu et al. (2005b), Abdulkadiroğlu et al. (2005a), Roth et al. (2005)). Auction applications include radio spectrum, electricity, and internet advertising (see McMillan (1994), Milgrom (2000), Wilson (2002), Edelman et al. (2007) and Milgrom and Segal (2020). Market design has since developed in various directions, for recent surveys see Sönmez and Ünver (2011), Sönmez and Ünver (2017), Kominers et al. (2017), Roth (2018) and Milgrom and Tadelis (2018).

[2]This assumption facilitates the direct use of elegant algorithms in practice, like the deferred Acceptance algorithm (Gale and Shapley (1962)) or Gale's Top Trading Cycles algorithm (Shapley and Scarf (1974)). Moreover, limiting participants' strategic considerations to preference manipulations makes the analysis of several commonly used mechanisms tractable (see, e.g., Roth (1982), Roth (1984) and Sönmez (1997)).

[3]The literature on non-standard choice is too large to summarize here. Instead, we mention a few papers that can help interested readers find the many strands of this literature. Non-standard choice behavior can be seen resulting from status-quo bias (Masatlioglu and Ok (2005), Masatlioglu and Ok (2014)), multiple conflicted selves (Kalai et al. (2002), Xu and Zhou (2007), Ambrus and Rozen (2015)), framing and order effects (Rubinstein and Salant (2006), Rubinstein and Salant (2008), Bernheim and Rangel (2009)), sequential procedures such as shortlisting (Manzini and Mariotti (2007), Horan (2016)), limited attention (Lleras et al. (2017), Manzini and Mariotti (2012), Masatlioglu et al. (2012), Cherepanov et al. (2013)), and lastly temptation and self-control (Lipman et al. (2013)).

to decide whether a kidney is a good match. A practical difficulty with the procedures that match donors to recipients is that doctors hesitate to state preferences over kidneys. However, they do not struggle to select the "best" kidney for a particular patient from a given "menu".[4] Another example is that of the US Army's branching system, where assignments have two attributes — branch assignment and length of service commitment. Ranking both branch assignment and length of service commitment jointly is considered too complex (see Greenberg et al. (2021)). In general, when potential assignments have multiple attributes considerations about the complexity of choice cannot be kept aside.[5] In such cases, eliciting a ranking over all alternatives from participants will likely inaccurately reflect their actual choice behavior.[6] Thus, analysis of such instances falls beyond the scope of standard matching theory.

(ii) (Groups as Participants) Consider the case of school admissions, where parents report a ranking over schools to a centralized authority. Preferences of parents and the various persons they consult to make this decision need not be perfectly aligned. They may therefore reach decisions by aggregating several preferences in some fashion. As seen in social choice theory, such decisions need not be consistent with maximization of a single preference. Thus participants may exhibit non-standard choice behavior even without behavioral biases and mistakes.

(iii) (Hiring with Attraction Effect) Consider a hypothetical labor market choice situation where a manager is choosing among three job candidates: $\{a, b, c\}$.[7] Candidate a and b are similar, but a is better. The manager's choice of candidate maybe influenced by the availability of a similar inferior alternative due to the attraction effect.[8] For example,

[4]See Bade (2016) for a conversation between Sophie Bade and Utku Ünver regarding this hesitancy. Utku Ünver, along with Alvin E. Roth and Tayfun Sönmez, has played a key role in the establishment of kidney exchange programs that use economic and optimization-based principles around the world (Roth et al. (2004), Roth et al. (2005)).

[5]Multi-attribute assignments are commonplace in real-life matching problems. For example, assignments in the US Army's branching system consist of branch assignment and length of service (Sönmez and Switzer (2013), Sönmez (2013), Greenberg et al. (2021)). Assignments in centralized college admissions markets (e.g., that of the University of Delhi) consist of college-course pairs.

[6]There is evidence suggesting that in choice situations involving alternatives with multiple attributes, participants make use of operational procedures and consequently exhibit non-standard choice behavior. See, e.g., Apesteguia and Ballester (2013).

[7]Shapley and Shubik (1971) and Kelso Jr and Crawford (1982) consider application of two-sided matching models to labor markets. They show that the properties of stable matchings are robust to generalizations of the model, which allow both matching and wage determination to be considered together.

[8]Identified by Huber et al. (1982), the attraction effect has been observed in job candidate evaluation (Highhouse (1996), Slaughter (2007), Slaughter et al. (1999)) among various other settings. See footnote 3 in Ok et al. (2015) for other settings and references.

choosing c out of $\{a, c\}$, but choosing a out of $\{a, b, c\}$. Again, such choices are not consistent with the maximization of a single preference relation.

In this paper, we incorporate more general choice behavior into the classical theory of stable matchings (Gale and Shapley (1962)). We show that well-functioning matching markets can be designed even if participants may make choices inconsistent with a single preference relation. We identify two weak conditions on choice behavior that are both necessary and sufficient for the existence of a large class of desirable mechanisms. We contrast the results obtained under such choice behavior with known results from the classical setup. Lastly, we show that a commonly used mechanism for university admissions can be tweaked to accommodate non-standard choice behavior adequately.

Section 2 formally introduces the model and relevant definitions. We consider an admissions problem that consists of individuals and institutions. Institutions are non-strategic agents, equipped with exogenously determined capacities and priority orderings over individuals.[9] In contrast to the standard setup, where individuals have preferences over potential assignments, we equip the individuals in our model with choice functions that determine choice (singleton or empty) from any non-empty subset of assignments.[10] The advantage of having choice functions is that with varying restrictions on choice functions, we can contrast the results we will obtain in this perturbed setup with known results from the classical setup.

A matching is a solution to the admissions problem. It matches individuals and institutions with each other. A matching is *(pairwise) stable* if no individual is assigned an unacceptable institution,[11] no institution is assigned an unacceptable individual, and no individual-institution pair (who are originally not matched with each other) prefer being matched with each other compared to their current assignments.[12] In Proposition 9 we show that in our setup (pairwise) stability is equivalent to the seemingly more involved notion of *group stability*. Group stability rules out coalitions consisting of multiple individuals and institutions that prefer being matched with each other compared to their current assignments. Thus, stability that concentrates on individual-institution pairs will be adequate for

[9]Priority orderings are typically determined by exam scores, neighborhood proximity, interviews, or affirmative action considerations. For example, centralized admissions are based entirely on priority orderings determined by exam scores in Turkey and China. It is a generally held belief that these institutions do not have an incentive to alter or manipulate their priority orderings strategically. In matching theory, this is a defining feature of the student placement model of Balinski and Sönmez (1999) and the school choice model of Abdulkadiroglu and Sönmez (2003).

[10]In many real-life matching mechanisms, individuals report choices, not preferences. See, e.g., the college admissions procedures used in Brazil (Bo and Hakimov (2019)) and Inner Mongolia, China (Gong and Liang (2020)).

[11]An assignment is *unacceptable* for an individual if it is not chosen from the singleton set containing it.

[12]An individual *prefers* an institution over her assignment, if it chooses that institution over her assignment in a *pairwise* comparison.

studying stable matchings in our setup.

Importing the notion of (pairwise) stability into our setup with choice functions, our objective is to analyze the existence of stable matchings when individuals exhibit non-standard choice behavior.[13] Without any sophistication in choice behavior, stable matchings may not exist. Section 3 provides two necessary and sufficient conditions on individuals' choice behavior for the existence of stable matchings (Theorem 1). The first condition, *weak acyclicity*, rules out the presence of strict cycles in choices for any sequence of binary menus. The second condition, *acceptable-consistency*, requires that an unacceptable assignment is not chosen over an acceptable assignment when offered as a pair. In Section 3.2 we illustrate how these conditions differ from the ones that standard choice behavior requires.

In Theorem 2, we present a novel characterization of individuals' ability to strictly order (rank) alternatives in terms of Plott (1973)'s *path independence*.[14] Path independence requires that if a menu of institutions is segmented arbitrarily, choice from the menu consisting of only the chosen assignments from each segment, must be the same as the choice made from the unsegmented menu. The purpose of Theorem 2 is twofold. First, it connects our model to the standard model in matching theory with (strict) preferences over potential assignments. Second, it helps contrast the requirements of standard choice behavior with the two conditions we have identified in Theorem 1. In Proposition 1 we show that path independence demands significantly more sophistication in choice behavior than required for the existence of stable matchings.

In Section 4 we discuss whether there is a way to reconcile our perturbed setup with the standard one. We start by construct an associated market with proxy preferences — which mimic the choice functions of individuals as closely as possible (Lemma 2). We then ask whether the two markets yield the same set of stable matchings? The answer is yes if the choice functions are path independent (Proposition 3). However, if the choice functions are weakly acyclic and acceptable-consistent, even though any stable matching in the proxy admission market is stable in the original market, the converse statement does not hold (Proposition 2). In other words, allowing for more general choice behavior not only affects

[13]There are two motivations for studying stable matchings. First, there is a strong correlation between the success of a matching exchange and its capability of delivering stable matchings (see Roth (2002)). Matching markets in the UK provide field evidence that supports this finding. Moreover, Kagel and Roth (2000) confirm this hypothesis in a controlled lab environment. The second motivation comes from a mathematically equivalent fairness notion introduced for priority-based allocation mechanisms, where failure to respect priorities can have legal implications. This fairness notion, known as *elimination of justified envy*, was introduced by Balinski and Sönmez (1999) in the context of centralized school admissions and since then has appeared in several other real-world matching market proposals to emphasize the requirement of respecting priorities.

[14]Path independent choice rules have been studied in the context of matching markets with contracts in Chambers and Yenmez (2017).

how one finds a stable matching but also the structure of stable matchings — in particular, enlarging the set of stable matchings. The idiosyncrasies of our setup also imply that the lattice structure of stable matchings is absent, and there are no side-optimal matchings.[15] In fact, we show an even stronger result that there are markets where every stable matching is Pareto dominated (for individuals) by another stable matching (Proposition 4).

Section 5 is dedicated to studying the incentives of individuals. A mechanism is said to be *incentive compatible (for individuals)* if for any admission problem with individual i's choice function, denoted C_i, there does not exist another choice function C'_i such that the assignment of individual i under C'_i is better than that under C_i (when analyzed with respect to the original choice function C_i). In Theorem 3, we show that weakly acyclic and acceptable-consistent choice functions are sufficient for stable and incentive compatible mechanisms to exist. Furthermore, these two conditions are necessary for the existence of the larger class of individually rational, weakly non-wasteful, and incentive compatible mechanisms, that contains the class of stable and incentive compatible mechanisms. Individually rational mechanisms require that no individual is assigned an unacceptable institution and vice versa, thus ruling out trivial incentive compatible mechanisms that assign every individual the same institution regardless of choices reported. Weakly non-wasteful mechanisms ensure that no unassigned individual prefers an institution with one or more empty slots where she is acceptable, thus ruling out trivial incentive compatible mechanisms that leave every individual unassigned. In other words, weakly acyclic and acceptable-consistent choice functions are not only necessary and sufficient for the existence of a stable mechanism but also — under two mild requirements — for the existence of an incentive compatible mechanism.

Section 6 presents an application that shows how a commonly used procedure for assigning undergraduates to university programs can be modified to accommodate non-standard choice behavior.[16] Publicly announced cut-offs demarcate each step of the procedure and circumvent the requirement of eliciting a rank-ordered list of programs from students. At each step, the cut-offs reveal the minimum score required for acceptance at each program, thus offering students a menu of programs to choose from (or switch to). Proposition 5 shows that even though the procedure offers choice menus, when programs announce cut-offs simultaneously, it takes path independent choice functions for the procedure to yield stable outcomes. Proposition 6 shows that requiring programs to announce their cut-offs sequentially instead of simultaneously enables the mechanism to accommodate non-standard choice behavior. The intuition underlying these results is rather immediate. Announcing cut-offs sequentially

[15]See Roth and Sotomayor (1990) for the lattice property and side-optimality results of stable matchings in Gale and Shapley (1962)'s college admissions model.

[16]Such procedures are used in many countries, e.g., Brazil and China. See Bó and Hakimov (2020b) for details of the mechanisms used in Brazil and China.

ensures that individuals decide between at most two alternatives at a time, so that no irrelevant options can distort their choices. The message here is that reducing the menu size in matching contexts facilitates choice and therefore benefits individuals, especially in admissions problems where comparing institutions is likely complex and non-standard choice behavior is expected.

Contributions with respect to the Related Literature

The interest in designing mechanisms that accommodate non-standard choice behavior has spurred a rich academic literature. Bounded rationality in strategic play and choice biases have featured in game theory, mechanism design, implementation theory and industrial organization among others (see, e.g., Compte and Postlewaite (2019), Jehiel (2020), De Clippel et al. (2019), De Clippel (2014), Bochet and Tumennasan (2019), Grubb (2015)). In matching theory, departures from standard preferences have been motivated by bounded rationality, mistakes, indifferences, indecisiveness, complementarities, externalities, and peer preferences.[17] Zhang (2021) and Bade (2016) are studies motivated by bounded rationality and therefore are most relevant to our study. Zhang (2021) studies implications of heterogeneous strategic sophistication of individuals under the Boston mechanism and the deferred acceptance mechanism.[18] Bade (2016)'s analysis of boundedly rational individuals focuses on Pareto optimality of matching mechanisms in housing markets when Pápai (2000)'s hierarchical exchange mechanisms are used. By contrast, we are the first to extend matching theory of admissions markets to problems where individuals may exhibit non-standard choice behavior due to choice biases among other possibilities.

Features affecting real-world performance of mechanisms have gained considerable interest in economic theory, and complexity considerations are at the forefront (see, e.g., Oprea (2020)). Today's market designer strives to design cognitively simple mechanisms by primarily easing the complexity of strategic considerations (see, e.g., Li (2017), Börgers and Li (2019), Bochet and Tumennasan (2018)). Another source of complexity concerns choice situations faced by participants when interacting with the mechanism (see Salant and Spenkuch (2021)). These take the form of shortlisting and ranking schools in school choice,[19]

[17]See, e.g., Bade (2016) and Zhang (2021) for bounded rationality, see Echenique et al. (2016) for mistakes incorporated in individuals' behavior, see Erdil and Ergin (2008) and Erdil and Ergin (2017) for indifferences, see Kuvalekar (2020) for indecisiveness, see Hatfield and Kojima (2010), Pycia (2012) and Hatfield and Kominers (2015) for preferences exhibiting complementarity, see Sasaki and Toda (1996) and Pycia and Yenmez (2021) for analysis of matching problems with externalities, see Leshno (2021) and Cox et al. (2021) for peer-dependent preferences.

[18]First identified by Pathak and Sönmez (2008), the Boston mechanism has been shown to favor strategically sophisticated parents (Dur et al. (2018)).

[19]See Calsamiglia et al. (2010) and Haeringer and Klijn (2009) for complications surrounding this research.

or choosing an assignment after higher priority individuals have made their pick in a serial dictatorship procedure. There is growing evidence on preference-reporting errors and their detrimental effects on mechanism's performance (see, e.g., Rees-Jones (2018), Rees-Jones and Skowronek (2018), Hassidim et al. (2021)). Choice complexity leading to non-standard choice behavior analyzed in this paper poses a possible explanation for these occurrences.

There are multiple reasons to believe that choice complexity could be a cause for real-world underperformance of matching mechanisms. For instance, in school admissions, parents report challenges in navigating choice.[20] In the US Army's branching system, ranking both branch assignment and length of service commitment jointly is considered complex (see Greenberg et al. (2021)). Moreover, in practice, individuals have often been found to make mistakes that strategic considerations cannot explain (see, e.g., Narita (2018) and Shorrer and Sóvágó (2018)).

One approach to mitigating choice complexity is simply reducing the number of alternatives in the various choice situations that may arise. Recent laboratory experiments suggest that this approach could be advantageous. Sequential (step-by-step) implementation of the Deferred Acceptance algorithm, in which participants choose from relevant menus that occur at each step of the algorithm, is shown to outperform its static counterpart in Bó and Hakimov (2020a), Klijn et al. (2019) and Grenet et al. (2019). Bó and Hakimov (2020b) and Mackenzie and Zhou (2020) have theoretically investigated the advantages of sequential implementation, yet only strategic considerations have been analyzed. The analysis presented in our paper hints that sequential mechanisms may also be better at accommodating non-standard choice behavior.

Another approach to mitigate choice complexity is designing better preference-reporting language (as discussed in Milgrom (2009) and Milgrom (2011)). Experimental findings of Budish and Kessler (2021) show that this is a promising direction to explore. Budish and Kessler (2021) show that Budish (2011)'s mechanisms for combinatorial assignments can be successfully implemented with a limited set of preference data on binary choices. Therefore, if eliciting entire choice functions seems impractical, one way to accommodate non-standard choice would be tailoring messages. We take this approach in Section 6 when discussing a particular application of university admissions.

Finally, we contribute to the literature on the existence of stable matchings in many-to-one matching markets. The limits of generalizing institutional preferences over individuals (encoded via choice correspondences) in the universe of stable matching mechanisms are

[20] Jochim et al. (2014) reports "Parents with less education, minority parents, and parents of children with special needs are more likely to report challenges navigating choice," and recommends investing heavily in information systems as "Parents in high-choice cities are seeking information on their options, but sorting through it all can be overwhelming."

known (see Hatfield and Milgrom (2005), Hatfield and Kojima (2010), Hatfield et al. (2020)). In a similar vein, our study pins down the limits of generalizing individual preferences over institutions (encoded via choice functions). In particular, Theorem 1 provides a complete characterization of conditions on individual choice functions under which a stable matching is guaranteed to exist.

2 Model

We start by introducing a model for two-sided matching markets that consists of institutions and individuals. Examples include assigning students to schools, children to daycare centers, asylum seekers to member states, refugees to localities, or undergraduates to university programs. Institutions in our model are not strategic agents, while individuals potentially are. Institutions can accept only a limited number of individuals represented by their capacities. Moreover, institutions have priority orderings over individuals that, depending on the context, are based on exam scores, interviews, or other criteria such as geographic proximity to the institution and affirmative action considerations. We deviate from the standard matching models in the way we model individuals' preferences. In order to allow for more general choice behavior, we equip the individuals in our model with choice functions instead of preference relations. Let us formally define the model — referred to as an admissions problem.

An **admissions problem** $\gamma \in \Gamma$ is a five-tuple $\langle I, S, q, C, \pi \rangle$ that consists of:

(i) a non-empty finite set of individuals I,

(ii) a non-empty finite set of institutions S,

(iii) a list of capacities of institutions $q = (q_s)_{s \in S}$,

(iv) a list of priority orders of institutions $\pi = (\pi_s)_{s \in S}$ over $I \cup \{\varnothing\}$, and

(v) a list of choice functions of individuals $C = (C_i)_{i \in I}$ over 2^S.

Each institution s has a capacity of q_s seats that represents the maximum number of individuals it can accept. Priority order π_s represents the way institution s ranks individuals. Formally, a **priority order** π_s is a strict simple order over $I \cup \{\varnothing\}$. Let Π denote the of all possible lists of priority orders. We assume that, from an institutional viewpoint there are no complementarities between individuals, so the priority order π_s and capacity

q_s of an institution s translate into a (partial order) preference over sets of individuals in a straightforward way.[21]

Each individual i is equipped with a choice function C_i that represents her choice from any menu of institutions. Formally, a (unit demand) **choice function** C_i is a mapping $C_i : 2^S \to 2^S$ such that for every $S' \subseteq S$ we have $C_i(S') \subseteq S'$ and $|C_i(S')| \leq 1$.

Let us define a few basic terms. An institution s is **acceptable to individual** i if $C_i(\{s\}) = \{s\}$ and unacceptable if $C_i(\{s\}) = \varnothing$. Similarly, an individual i is **acceptable to institution** s if $i \; \pi_s \; \varnothing$ and unacceptable otherwise.

We are seeking matchings such that each individual is assigned a seat at only one institution and no institution exceeds its capacity. Formally, a (feasible) **matching** is a correspondence $\mu : I \cup S \mapsto I \cup S \cup \{\varnothing\}$ that satisfies:

(i) $\mu(i) \subseteq S$ such that $|\mu(i)| \leq 1$ for all $i \in I$,

(ii) $\mu(s) \subseteq I$ such that $|\mu(s)| \leq q_s$ for all $s \in S$, and

(iii) $i \in \mu(s)$ if and only if $s \in \mu(i)$ for all $i \in I$ and $s \in S$.

Let $\mathcal{M}$ denote the set of all (feasible) matchings.

We next define an analog of the standard notion of (pairwise) stability in our setup. A matching μ is individually rational if no individual is assigned an unacceptable institution and no institution is assigned an unacceptable individual. A matching μ has no blocking pair if no individual-institution pair (who are originally not matched with each other) prefer being matched with each other, possibly instead of some of their current assignments. A matching that is individually rational and has no blocking pair is said to be stable. Formally, a matching μ is **(pairwise) stable** if

(i) it is **individually rational**, that is, there is no individual i such that $C_i(\mu(i)) = \varnothing$ and no institution s such that $\varnothing \; \pi_s \; i$ for some $i \in \mu(s)$, and

(ii) there is no **blocking pair**, that is, there is no pair $(i, s) \in I \times S$ such that

[21] In a nutshell, an institution chooses the q_s highest priority individuals from any set of acceptable individuals. Formally, let $\succ_s$ be an partial order over 2^I. We assume that $\succ_s$ is responsive (Roth (1985)), that is,

(i) for any $I' \subset I$ with $|I'| < q_s$ and any $i \in I \setminus I'$,

$$(I' \cup \{i\}) \succ_s I' \iff \{i\} \; \pi_s \; \varnothing,$$

(ii) for any $I' \subset I$ with $|I'| < q_s$ and any $i, i' \in I \setminus I'$,

$$(I' \cup \{i\}) \succ_s (I' \cup \{i'\}) \iff \{i\} \; \pi_s \; \{i'\}.$$

(a) $\mu(i) \neq s$,

(b) $C_i(\mu(i) \cup \{s\}) = \{s\}$, and

(c) (1) either $i \; \pi_s \; i'$ for some $i' \in \mu(s)$, or

(2) $|\mu(s)| < q_s$ and $i \; \pi_s \; \varnothing$.

Notice that a blocking pair requires that both parties are strictly better off from getting together. This is in line with the standard setup of stable matching theory, where it is not plausible that a party would be part of a blocking pair unless it is worthwhile. Secondly, we will show that pairwise stability is equivalent to another prominent stability concept, namely *group stability* (see Konishi and Ünver (2006), and Pycia (2012)). Group stability rules out coalitions consisting of multiple individuals and institutions that can benefit from blocking a matching. The equivalence of pairwise and group stability, presented in Proposition 9, asserts that concentrating on individual-institution pairs will be adequate for studying stable matchings in our setup.[22]

Finally, a **mechanism** is a function $\psi : \Gamma \to \mathcal{M}$ that assigns a matching $\psi[\gamma] \in \mathcal{M}$ to each admission problem $\gamma \in \Gamma$. A **mechanism is stable** if $\psi[\gamma]$ is stable for any admission problem $\gamma \in \Gamma$.

3 Stable Matchings

In our setup, the existence of stable matchings is not guaranteed. Therefore, in the first step, in Section 3.1 we establish necessary and sufficient conditions — weak acyclicity and acceptable-consistency — on individuals' choice functions for the existence of stable matchings. In Section 3.2 we show that choice behavior consistent with the maximization of a single preference relation is equivalent to having a (unit demand) choice function that satisfy a well-known condition called path independence. Lastly, in Proposition 1 and Example 1 we illustrate the contrast between our necessary and sufficient conditions with path independence (that the standard setup assumes), which is a stronger condition. Analysis of the structure of the set of stable matchings is presented in Section 4.

3.1 Stable Matchings under Non-Standard Choice

Without any sophistication in choice behavior, stable matchings may not exist. For instance, a choice function that selects unacceptable alternatives over acceptable ones would

[22]The same result also holds in the standard setup of many-to-one matching markets (see Roth and Sotomayor (1990)).

certainly lead to violations of individual rationality. This subsection describes the weakest requirements from individual choice behavior for the existence of stable matchings.

The first condition, weak acyclicity, rules out the possibility that an individual, regardless of the institution assigned to it, can always find another institution to block with. Formally, choice function C_i is **weakly acyclic (over acceptable institutions)** if for all positive integer $t \geq 3$ and t distinct and acceptable institutions $s^1, s^2, \dots, s^t \in S$,[23]

$$C_i(\{s^1, s^2\}) = \{s^1\}, \dots, C_i(\{s^{t-1}, s^t\}) = \{s^{t-1}\} \text{ implies } C_i(\{s^1, s^t\}) \neq \{s^t\}.$$

The second condition, acceptable-consistency, ensures that an individual does not choose an unacceptable institution over an acceptable one in pairwise comparisons. Formally, choice function C_i is **acceptable-consistent** if for all distinct institutions $s, s' \in S$,

$$C_i(\{s\}) = \{s\} \text{ and } C_i(\{s'\}) = \varnothing \text{ implies } C_i(\{s, s'\}) \neq \{s'\}.$$

The following result shows that, for an individual with a weakly acyclic and acceptable-consistent choice function, any set of institutions with at least one acceptable institution, contains at least one such institution that no other institution in the set is chosen over in pairwise comparisons. Formally, let the set of **C-maximal institutions** for individual i in subset $S' \subseteq S$ be denoted by $U_i(S') \equiv \Big\{ s \in S' : \big\{ C_i(\{s, s'\}) = \{s'\} \text{ for some } s' \in S \setminus \{s\} \big\} = \varnothing \Big\}$. The following lemma holds.

Lemma 1. *For a weakly acyclic and acceptable-consistent choice function C_i and a subset of institutions $S' \subseteq S$ containing at least one acceptable institution, the set of C-maximal institutions $U_i(S')$ is non-empty.*

An analogous but much stronger version of this result trivially holds for the case of strict preferences. That is, every set of institutions, with at least one acceptable institution, must contain an institution preferred to any other institution in the set. Our next result shows that the weaker version presented in Lemma 1 is enough to construct a mechanism that always leads to a stable outcome. Moreover, if either weak acyclicity or acceptable-consistency are violated, the existence of a stable matching is no longer guaranteed.

Theorem 1. *Fix I, S, q, C. There exists a stable matching for every priority order profile $\pi \in \Pi$ if and only if the choice functions are weakly acyclic and acceptable-consistent.*

[23]Weak acyclicity resembles *Strong Axiom of Revealed Preference (SARP)*, however there are important distinctions. Unlike SARP, weak acylicty does not imply *independence of irrelevance alternatives (IIA)* because it places no restriction on choices from menus of size larger than two. This is shown in Example 1. For definitions and an excellent exposition of SARP and IIA, see Bossert et al. (2010).

The proof of Theorem 1, given in Appendix A, describes an algorithm that always yields a stable matching for weakly acyclic and acceptable-consistent choice functions. This result highlights that matching markets can be designed to accommodate a plethora of choice behaviors that are not allowed under the standard setup consisting of individuals with preference relations. However, the exact connection between preference relations and weak acyclic and acceptable-consistent choice functions remains to be established. This is the point of the next subsection.

3.2 Standard Assumptions on Choice Behavior

The two identified conditions are new to the literature on stable matching theory. The standard assumption in the literature is that individuals can rank the institutions (together with the option of remaining unassigned) in a single order. We next show that the ability to rank institutions corresponds to a well-known condition on choice functions in our setup called path independence.

Let us first define path independence formally. A choice function is **path independent** if for all $S', S'' \subseteq S$ we have

$$C_i(S' \cup S'') = C_i(C_i(S') \cup S'').$$

Path independence requires that if a set is segmented arbitrarily, choice from the menu consisting of only the chosen assignments from each segment, must be the same as the choice made from the unsegmented set. We next show that a path independent choice function reflects choice behavior that a strict order over institutions can rationalize. For a path independent choice function, there is a unique strict order over (acceptable) institutions such that the institution chosen from each menu of institutions is simply the best in that menu with respect to the strict order. This benchmark result will facilitate a direct comparison of the perturbed setup consisting of the two conditions identified in Theorem 1 with the standard setup of stable matching theory.

Let us proceed to make the idea of choice behavior consistent with the ability to rank institutions formal. Let R_i be a binary relation over $S \cup \{\varnothing\}$. A binary relation R_i over $S \cup \{\varnothing\}$ is (strongly) **complete over acceptable alternatives** if

(i) for all $s \in S$ either $s \; R_i \; \varnothing$ or $\varnothing \; R_i \; s$, and

(ii) for all $s, s' \in \{s \in S : s \; R_i \; \varnothing\}$ either $s \; R_i \; s'$ or $s' \; R_i \; s$.

A binary relation is **transitive over acceptable choices** if for all $s, s', s'' \in \{s \in S : s \; R_i \; \varnothing\}$ we have that $s \; R_i \; s'$ and $s' \; R_i \; s''$ implies $s \; R_i \; s''$. Finally, a binary relation is **anti-symmetric over acceptable choices** if for all $s, s' \in \{s \in S : s \; R_i \; \varnothing\}$, $s \; R_i \; s'$ and $s' \; R_i \; s$

implies $s = s'$. We say R_i is a **simple order over acceptable choices** in $S \cup \{\varnothing\}$ if it is (strongly) complete, transitive, and anti-symmetric over acceptable choices.

A choice function C_i can be **rationalized by a simple order over acceptable choices** R_i if and only if for all subsets $S' \subseteq S$, we have

(i) $C_i(S') = \varnothing$ if $\varnothing \; R_i \; s$ for all $s \in S'$, and

(ii) $C_i(S') = \big\{s \in S' : s \; R_i \; s' \text{ for all } s' \in \{s \in S : s \; R_i \; \varnothing\}\big\}$ otherwise.

Notice that our choice functions allow for the possibility of choosing nothing (the empty set). This possibility is not present in Plott (1973)'s original analysis of path independent choice functions where individuals' choice from a non-empty menu is not allowed to be empty. The possibility of empty choices from non-empty menus requires some additional careful considerations that lead to the following result.[24]

Theorem 2. *A (unit demand) choice function C_i can be rationalized by a simple order over acceptable choices if and only if it is path independent.*

Theorem 2 shows that the choice behavior of an individual consistent with maximization of a single preference relation over the set of acceptable institutions can be represented with a path independent choice function. It is instructive to understand that path independence is a stronger requirement on choice sophistication than weak acyclicity and acceptable-consistency combined. In Proposition 1 we present a simple observation that path independence implies weak acyclicity and acceptable-consistency.

Proposition 1. *If a choice function is path independent, then it is weakly acyclic and acceptable-consistent. The converse statement may not hold.*

Corollary 1. *Fix I, S, q, C. There exists a stable matching for every priority order profile $\pi \in \Pi$ if the choice functions are path independent.*

Therefore, our identified conditions allow for more general choice behavior than the standard setup that assumes path independent choice behavior. Let us revisit the attraction effect (Huber et al. (1982)) example from the introduction to illustrate choice behavior that satisfies weak acyclicity and acceptable-consistency but not path independence.

Example 1 (Hiring with Attraction Effect)**.** Consider a hypothetical choice situation where a manager is choosing among three job candidates: $\{a, b, c\}$. Candidate a and b are similar, but a is better. The manager's choice of candidate may be influenced by the availability

[24] It is worth noting that a similar result has been proved in Plott (1973) but for non-empty choices from non-empty menus.

of a similar inferior alternative due to the attraction effect. For example, choosing c out of $\{a, c\}$, but choosing a out of $\{a, b, c\}$. Thus exhibiting choices that a single preference relation cannot rationalize.

Consider an admissions problem $\gamma = \langle I, S, q, C, \pi \rangle$ where the manager takes the role of an individual looking to match with a job candidate, and the job candidates take the role of institutions, each with capacity one (assuming that a job candidate cannot work in two firms).

(i) $I = \{i\}$,

(ii) $S = \{a, b, c\}$,

(iii) $q_s = 1$ for all $s \in S$,

(iv) $C_i(\{s\}) = \{s\}$ for all $s \in S$,
$C_i(\{a, b\}) = \{a\}$,
$C_i(\{a, c\}) = \{c\}$,
$C_i(\{b, c\}) = \{c\}$,
$C_i(\{a, b, c\}) = \{a\}$, and

(v) $i \; \pi_s \; \varnothing$ for all $s \in S$.

Three simple observations follow. First, C_i is not path independent because $C_i(\{a, b, c\}) = \{a\} \neq C_i\big(C_i(\{a, c\}) \cup \{b\}\big) = \{c\}$, that is, the choices cannot be rationalized by a single preference relation. Second, C_i is weakly acyclic and acceptable-consistent. Third, there exists a stable matching for this market, in particular, $\mu(i) = \{c\}$ and $\mu(a) = \mu(b) = \varnothing$.

Menu effects that accompany non-standard choice make choices from binary menus more meaningful than those from larger menus, in that they are not influenced by the presence of not chosen (sometimes called *irrelevant*) alternatives. For our example, consider assigning job candidate a to the manager instead. To illustrate possible issue arising from menu effects we can further modify the problem to include four job candidates, where a new candidate d influences choices such that c is now again chosen out of the menu $\{a, b, c, d\}$ and $\{a, c, d\}$ while a is still chosen out of $\{a, b, c\}$. Here, deciding on an outcome based on larger menus seems arbitrary in the face of menu effects.

Similar examples can be constructed for other choice behaviors that exhibits context effects related to a variety of psychological, social, or environmental factors such as status-quo bias (Masatlioglu and Ok (2005), Masatlioglu and Ok (2014)), framing and order effects (Rubinstein and Salant (2006), Rubinstein and Salant (2008), Bernheim and Rangel (2009)) and limited attention (Lleras et al. (2017), Manzini and Mariotti (2012), Masatlioglu et al. (2012), Cherepanov et al. (2013)). It is worth noting that although weak acyclicity and acceptable-consistency allow for more general choice behavior than path independence, there are well-known choice biases that violate even these (weaker) conditions. For example, consider the behavior that involves sequential shortlisting of alternatives or behavior of an individual who is maximizing a preference relation but may overlook some alternatives when making choices.[25] For instance, parents may restrict attention to schools in a five mile radius from their place of residence when there are numerous options but would drop that restriction if only a few schools are capable of nurturing their child's unique talents. Such behavior can lead to a pairwise cycle of choices and thus violate weak-acyclicity (see, e.g., the choice behavior presented in Manzini and Mariotti (2007)).

4 Richness

Shedding light on implications of having weakly acyclic and acceptable-consistent choice functions rather than standard preferences (represented by path independent choice functions) requires more than stating the relationship between the two. One must also understand what they imply for stable matchings. This section shows that the set of stable matchings under weak acyclic and acceptable-consistent choice functions is richer than the set of stable matchings in the standard setup. Moreover, well-known results regarding the lattice structure and side-optimality of stable matchings under the standard setup do not hold in our setup.

We start by constructing an associated proxy admission problem that differs only in that individuals have strict preferences as opposed to choice functions, and the preferences of individuals are in line with their choices from binary menus. Lemma 2 shows that for any admissions problem with weakly acyclic and acceptable-consistent choice functions, there always exists at least one associated proxy admissions problem. Interestingly, each stable matching in the proxy admissions problem is also stable in the original admissions problem. However, there are stable matchings of some admissions problems that are not stable in any associated admissions problem (Proposition 2). On the other hand, for admissions markets with path independent choice functions, the set of stable matchings for any admission problem is identical to the set of stable matchings for its associated proxy admission problems (Proposition 3).

[25]Such behavior was first highlighted in marketing literature, where the set of alternatives describes the set of all options available to choose from, while the consideration set is a subset of those options that the individual actively consider when making a choice (see, e.g., Masatlioglu et al. (2012) for a recent discussion of this idea in economics).

Let us proceed to the definitions. Let P_i be a binary relation over $S \cup \{\varnothing\}$. A binary relation P_i over $S \cup \{\varnothing\}$ is **complete over acceptable alternatives** if

(i) for all $s \in S$ either $s \; P_i \; \varnothing$ or $\varnothing \; P_i \; s$, and

(ii) for all $s, s' \in \{s \in S : s \; P_i \; \varnothing\}$ such that $s \neq s'$ either $s \; P_i \; s'$ or $s' \; P_i \; s$.

A binary relation is **transitive over acceptable choices** if for all $s, s', s'' \in \{s \in S : s \; P_i \; \varnothing\}$ we have that $s \; P_i \; s'$ and $s' \; P_i \; s''$ implies $s \; P_i \; s''$. Finally, a binary relation is **asymmetric over acceptable choices** if for all $s, s' \in \{s \in S : s \; P_i \; \varnothing\}$, $s \; P_i \; s'$ implies $\neg(s' \; P_i \; s)$. We say P_i is a **strict simple order over acceptable choices** in $S \cup \{\varnothing\}$ if it is complete, transitive, and asymmetric over acceptable choices.

Let $\gamma_P = \langle I, S, q, P, \pi \rangle$ be a **proxy admissions problem** for admission problem $\gamma = \langle I, S, q, C, \pi \rangle$, where P is a profile of strict simple orders over acceptable choices satisfying the following conditions:

(i) If $C_i(\{s, s'\}) = \{s\}$ then $s \; P_i \; s'$,

(ii) If $C_i(\{s\}) = \{s\}$ then $s \; P_i \; \varnothing$, and

(iii) If $C_i(\{s\}) = \varnothing$ then $\varnothing \; P_i \; s$.

Let Γ_γ denote the set of all proxy admissions problems for admission problem γ. In order to make sure that an associated market constructed in such a way always exists, we need to check that Γ_γ is non-empty. For admissions problems with weakly acyclic and acceptable-consistent choice functions, the answer is affirmative.

Lemma 2. *For an admissions problem γ with choice functions that are weakly acyclic and acceptable-consistent, the set of proxy admissions problems Γ_γ is non-empty.*

Our following result shows that weakly acyclicity and acceptable-consistency are not only weaker than path independence but that they also yield a larger set of stable matchings. Let us first define stability for proxy problems.

A matching μ is **stable for proxy admissions problem γ_P** if

(1) it is **individually rational**, that is, there is no individual i such that $\varnothing \; P_i \; \mu(i)$ and no institution s such that $\varnothing \; \pi_s \; i$ for some $i \in \mu(s)$, and

(2) there is no **blocking pair**, that is, there is no pair $(i, s) \in I \times S$ such that

  (a) $s \; P_i \; \mu(i)$, and

  (b) (i) either $i \; \pi_s \; i'$ for some $i' \in \mu(s)$, or

(ii) $|\mu(s)| < q_s$ and $i \; \pi_s \; \varnothing$.

Proposition 2. *Fix an admissions problem γ with choice functions that are weakly acyclic and acceptable-consistent.*

1. *If a matching is stable for some associated proxy admissions problem, then it is also stable for the admissions problem.*

2. *The converse statement may not hold.*

We wish to emphasize that this result stems from the possibility that individuals may exhibit more general (non-standard) choice behavior than previously studied. The following example illustrates one such possibility.

Example 2. Consider an admissions problem $\gamma = \langle I, S, q, C, \pi \rangle$ where

(i) $I = \{i\}$,

(ii) $S = \{s_1, s_2, s_3\}$,

(iii) $q_s = 1$ for all $s \in S$,

(iv) $C_i(\{s\}) = \{s\}$ for all $s \in S$,
$C_i(\{s_1, s_2\}) = \{s_1\}$,
$C_i(\{s_2, s_3\}) = \{s_2\}$,
$C_i(\{s_1, s_3\}) = \varnothing$,

(v) $i \; \pi_{s_1} \; \varnothing$,
$\varnothing \; \pi_{s_2} \; i$, and
$i \; \pi_{s_3} \; \varnothing$.

A few simple observations follow. First, under the original admissions problem both the matchings $\mu(i) = s_1$ and $\mu(i) = s_3$ are stable. Second, notice that there is a unique associated proxy admissions problem for this problem. The only preference consistent with these choices is P_i such that $s_1 \; P_i \; s_2 \; P_i \; s_3$. Finally, for the associated proxy problem $\mu(i) = s_1$ is the only stable matching. $\mu(i) = s_3$ is not stable as it is blocked by s_1 and i in the proxy problem.

This is not the case for path independent choice behavior. Our next result shows that for problems with path independent choice functions, each stable matching of the admissions problem is also stable for some associated proxy admissions problem, and each stable matching of proxy admissions problem is also stable in the corresponding admissions problem.

Proposition 3. *Fix an admissions problem γ with choice functions that are path independent.*

1. *If a matching is stable for some associated proxy admissions problem, then it is also stable for the admissions problem.*

2. *The converse statement holds.*

In the standard setup consisting of individuals with preference relations without indifferences, just like in our proxy admissions problem, there exists a unique *individual-optimal stable matching* (Gale and Shapley (1962)) that every individual (weakly) prefers to any other stable matching. For matching markets consisting of individuals that are allowed to have indifferences, a weakening of the notion of individual-optimality, called *constrained efficiency*, is seen in Erdil and Ergin (2008). A constrained efficient stable matching corresponds to a matching that is not Pareto dominated by any other stable matching. We next define this property formally and then show that for some problems in our setup, even a constrained efficient matching might not exist (Proposition 4).

A stable matching μ is **constrained efficient (for individuals)** if it is not Pareto dominated by any other stable matching μ'. That is, μ' **Pareto dominates (for individuals)** μ if $C_i(\mu'(i) \cup \mu(i)) \neq \mu(i)$ for every $i \in I$ with $\mu'(i) \neq \mu(i)$, and $C_j(\mu'(j) \cup \mu(j)) = \mu'(j)$ for some $j \in I$ with $\mu'(j) \neq \mu(j)$. The following result shows that even a constrained efficient stable matching is not guaranteed in our setup.

Proposition 4. *There exists an admissions problem $\gamma \in \Gamma$ with choice functions that are weakly acyclic and acceptable-consistent, that does not have a constrained efficient stable matching.*

The result again stems from the possibility that individuals may exhibit seemingly non-standard choices. The case is rather easy to understand by means of an example where individual choices are weakly acyclic and acceptable-consistent but not path independent.

Example 3. Consider an admissions problem $\gamma = \langle I, S, q, C, \pi \rangle$ where

(i) $I = \{i_1, i_2, i_3\}$,

(ii) $S = \{s_1, s_2, s_3\}$,

(iii) $q_s = 1$ for all $s \in S$,

(iv) $C_i(\{s\}) = \{s\}$ for all $i \in I$ and $s \in S$. Moreover,

		Choices		
		C_{i_1}	C_{i_2}	C_{i_3}
Choice Menu	$\{s_1, s_2\}$	s_1	$\varnothing$	$\varnothing$
	$\{s_2, s_3\}$	s_2	$\varnothing$	s_2
	$\{s_1, s_3\}$	$\varnothing$	$\varnothing$	s_3

(v) $i_2 \; \pi_s \; i_3 \; \pi_s \; i_1$ for all $s \in S$.

Note that i_2 would never block a matching where she is assigned some institution. Similarly, i_1 having the lowest priority, would never contest another individual's assignment. Moreover, i_3 can only block assignments of i_1. It follows that there are exactly three stable matchings for problem γ.

(i) $\mu(i_1) = \{s_2\}$, $\mu(i_2) = \{s_3\}$ and $\mu(i_3) = \{s_1\}$;

(ii) $\nu(i_1) = \{s_1\}$, $\nu(i_2) = \{s_2\}$ and $\nu(i_3) = \{s_3\}$; and

(iii) $\eta(i_1) = \{s_3\}$, $\eta(i_2) = \{s_1\}$ and $\eta(i_3) = \{s_2\}$.

Recall the definition of the Pareto domination relation – μ' Pareto dominates (for individuals) μ if $C_i(\mu'(i) \cup \mu(i)) \neq \mu(i)$ for every $i \in I$ and $C_j(\mu'(j) \cup \mu(j)) = \mu'(j)$ for some $j \in I$ with $\mu'(j) \neq \mu(j)$. In this case, ν Pareto dominates μ, μ Pareto dominates η and η Pareto dominates ν. Since every stable matching has a Pareto improvement which is still stable, a constrained efficient stable matching does not exist in this case.

In conclusion, the results presented in this section imply that more general choice behavior, taking the form of weakly acyclic and acceptable-consistent choice functions, leads to a richer set of stable matchings. The set is shown to be different from the stable set of Gale and Shapley (1962) that has a lattice structure and stable matchings that are preferred over other stable matchings by all the participants on one side of the market.

5 Incentives

We next turn to analyze the strategic incentives of individuals when choices are non-standard. We show that weakly acyclic and acceptable-consistent choice functions are both necessary and sufficient for a large class of incentive compatible mechanisms.

Let us now define incentive compatibility for our setup but first recall the definition of a mechanism. A **mechanism** is a function $\psi : \Gamma \rightarrow \mathcal{M}$ that assigns a matching $\psi[\gamma] \in \mathcal{M}$ to each admission problem $\gamma \in \Gamma$. A **mechanism is stable** if $\psi[\gamma]$ is stable for any admission problem $\gamma \in \Gamma$. A mechanism is said to be incentive compatible (for individuals) if for any admission problem with individual i's choice function denoted C_i, there does not exist another choice function C_i' such that the assignment of individual i under C_i' is better than that under C_i. Formally, a mechanism ψ is said to be **incentive compatible (for individuals)** if for any admissions problem $\gamma = \langle I, S, q, C, \pi \rangle$ there does not exist $\hat{\gamma} = \langle I, S, q, (\hat{C}_i, C_{-i}), \pi \rangle$ such that

$$\psi[\gamma](i) \neq \psi[\hat{\gamma}](i) \quad \text{and} \quad C_i(\psi[\hat{\gamma}](i) \cup \psi[\gamma](i)) = \psi[\hat{\gamma}](i).$$

Theorem 3 shows that weakly acyclic and acceptable-consistent choice functions are sufficient for the existence of stable and incentive compatible mechanisms. However, the two conditions are necessary for a bigger class of mechanisms, a class that contains stable and incentive compatible mechanisms. To define this class, we need two additional definitions that pin down meaningful incentive compatible mechanisms.

First, we need individual rationality to make sure the mechanism assigns an acceptable institution to every individual. This rules out trivial incentive compatible mechanisms that assign every individual the same institution regardless of choices reported. Formally, a matching μ is **individually rational** if there is no individual i such that $C_i(\mu(i)) = \varnothing$ and no institution s such that $\varnothing \pi_s i$ for some $i \in \mu(s)$. A **mechanism is individually rational** if $\psi[\gamma]$ is individually rational for any admission problem $\gamma \in \Gamma$. It is worth noting that a stable matching is always individual rational. Therefore, stability implies individual rationality.

Second, we need to ensure that no unassigned individual prefers an institution with one or more empty slots and where she is acceptable. Thus rules out trivial incentive compatible and individual rational mechanisms that leave all individuals unassigned. Formally, a matching μ is **weakly non-wasteful** if there exists no individual i with $\mu(i) = \varnothing$ and s with $|\mu(s)| < q_s$ such that $C_i(\{s\}) = \{s\}$ and $s \pi_s \varnothing$. A **mechanism is weakly non-wasteful** if $\psi[\gamma]$ is weakly non-wasteful for any admission problem $\gamma \in \Gamma$. Again it is worth noting that a stable matching is always weakly non-wasteful because weak non-wastefulness is a weaker requirement than having no blocking pairs. Under weak non-wastefulness, only unassigned individuals block with only those institutions that have empty seats available. Therefore, stability implies weak non-wastefulness.

We next show that weakly acyclic and acceptable-consistent choice functions are both necessary and sufficient for existence of individually rational, weakly non-wasteful, and incentive compatible mechanisms (Corollary 2). Moreover, they are sufficient for the existence of stable and incentive compatible mechanisms (Theorem 3).

Theorem 3. *Fix I, S, q.*

1. *If the choice functions are weakly acyclic and acceptable-consistent, then a stable and incentive compatible mechanism exists.*

2. *There exists an individually rational, weakly non-wasteful, and incentive compatible mechanism for every priority order profile $\pi \in \Pi$ only if the choice functions are weakly acyclic and acceptable-consistent.*

Corollary 2. *Fix I, S, q. There exists an individually rational, weakly non-wasteful, and incentive compatible mechanism for every priority order profile $\pi \in \Pi$ if and only if the choice functions are weakly acyclic and acceptable-consistent.*

6 Application

Many admissions procedures use mechanisms where individuals get multiple opportunities to report their choices. For instance, college admissions in Brazil allow students to revise choices over four consecutive days after knowing the cut-off score at each university (Bo and Hakimov (2019)). In Inner Mongolia, Chinese students choose one college at a time and are allowed to change their choices at any time before a pre-announced deadline (Gong and Liang (2020)).[26] Such dynamic (implementation of static) mechanisms have been shown to outperform their static counterparts in lab and field experiments, with existing explanations encompassing simplicity of strategic considerations, transparency, and credibility (Bó and Hakimov (2020a), Klijn et al. (2019) and Grenet et al. (2019)).[27]

Our analysis suggests dynamic mechanisms are a promising avenue for accommodating non-standard choice behavior. Intuitively, dynamic mechanisms can serve as compelling alternatives to direct mechanisms by limiting the number of options under consideration at each step, thereby mitigating the problem of choice overload (Grenet et al. (2019), Hakimov et al. (2021)).[28] This section shows that dynamic mechanisms can be tailored further to adequately accommodate non-standard choice behavior by reducing the size of the encountered choice sets.

[26]Market designers have studied many such procedures. See Dur et al. (2018) for Wake County, North Carolina; Luflade (2018) for Tunisia; Grenet et al. (2019) for Germany; and Haeringer and Iehlé (2019) for France.

[27]There is no set term for such mechanisms in the market design literature. Some researchers have referred to them as sequential mechanisms (Bó and Hakimov (2020b)), and others have termed them as dynamic mechanisms (Mackenzie and Zhou (2020)). We use the latter.

[28]Thus explaining the wide-spread use of such mechanisms in nationwide college admissions, that come with a very large number of options.

We consider a dynamic college admissions procedure, motivated by the University of Delhi's procedure, which resembles the steps of college-proposing Deferred Acceptance algorithm by Gale and Shapley (1962).[29] The admissions procedure begins with high-school graduates applying to college programs and reporting their national high school exam scores to the university. The university uses these scores to release a public list of cut-off marks for college programs that represent the lowest score necessary to be accepted at a college program. All college programs are required to admit all applicants who meet the announced cut-off criteria. On declaration of the list of cut-offs, applicants are required to choose a college program from the list of college programs that they are eligible for, that is, the programs where their score is higher than the cut-off. The applicants then take (provisional) admission in their choice of program by submitting required documents within the prescribed duration of the cut-off round. For college programs that could not fill all their seats, a new list of (weakly) lower cut-offs is released. If in the updated list, applicants find themselves eligible for admission to another college program that they prefer over their current assignment, they can cancel their previous admission and take provisional admission at this preferred program. Once an applicant cancels a provisional admission, the applicant cannot be re-admitted to the same program.

Notice that whenever a new list of cut-offs is released, an applicant faces a choice situation where only the relevant programs are under consideration. That is, the programs where the applicant satisfies the cut-off criteria. We will show that even though only relevant choice menus are offered during each step of the procedure, the mechanism demands great sophistication in choice behavior. So much so that one can claim that the outcomes are stable if and only if the applicants exhibit path independent choice behavior (Proposition 5).

Let us adopt the University of Delhi's admissions procedure to our model and give a schematic algorithm to compute the matching. The only difference between our model and the problem that the University of Delhi faces is that merit scores determine the priority orderings of the programs over applicants.[30] Let us establish this connection before moving on to the algorithm.

Let the **merit score** of individual i at institution s be defined as

$$m_s(i) = \begin{cases} |\{i' \in I : i \pi_s i'\}| + 1, & \text{if } i \; \pi_s \; \varnothing \\ 0, & \text{otherwise} \end{cases}$$

[29] See University of Delhi's undergraduate admissions procedure on page 18 of the following file: https://www.du.ac.in/adm2019/pdf/BulletinForUpload30May2019.pdf

[30] Since priorities are strict, our model corresponds to situations where there is a tie-break rule that determines how to rank applicants that got the same exam score.

Let the lowest non-zero merit score at institution s be m_s.

Admissions using Simultaneous Cut-offs Algorithm

- **Step 0:** Each individual i applies to all acceptable institutions s such that $C_i(\{s\}) = \{s\}$. Let $\psi[\gamma]^0(i) \equiv \varnothing$, $\psi[\gamma]^0(s) \equiv \varnothing$ and $c_s^0 \equiv n+1$.

- **Step t (t $\geq$ 1) with Simultaneous Cut-offs:** Each institution s announces a cut-off $c_s^t \in \{m_s, \ldots, n+1\}$ such that $|\{i \in I : C_i(\{s\}) = \{s\}$ and $c_s^t \leq m_s(i) < c_s^{t-1}\}| + |\psi[\gamma]^{t-1}(s)| \leq q_s$ and $c_s^t < c_s^{t-1}$. If there is no such cut-off, the institution announces $c_s^t = c_s^{t-1}$.

  Let $B_i^t \equiv \{s \in S : C_i(\{s\}) = \{s\}$ and $c_s^t \leq m_s(i) < c_s^{t-1}\} \cup \psi[\gamma]^{t-1}(i)$ denote the set of institutions that individual i can choose from. Individual i's assignment is updated as follows:

$$\psi[\gamma]^t(i) = \begin{cases} C_i(B_i^t), & \text{if } C_i(B_i^t) \neq \varnothing \\ \psi[\gamma]^{t-1}(i), & \text{otherwise} \end{cases}$$

  Institution s's assignment is $\psi[\gamma]^t(s) = \{i \in I : \psi[\gamma]^t(i) = \{s\}\}$. If each institution s has either filled its capacity or has announced the lowest cut-off possible ($c_s = m_s$), then stop and return $\psi[\gamma]^t$ as the outcome.

The algorithm terminates in a finite number of steps because there is a new rejection in every step that is not terminal, and there are only a finite number of students (and therefore merit scores). Therefore, the outcome of the algorithm is well-defined.

Let ψ^{sim} be the mechanism based on the simultaneous cut-offs algorithm. The following result formalizes the requirements of this mechanism in terms of the choice sophistication of the applicants.

Proposition 5. *The simultaneous cut-offs mechanism ψ^{sim} leads to a stable outcome $\psi^{sim}[\gamma]$ for every admissions problem $\gamma \in \Gamma$ if and only if the choice functions are path independent.*

This result shows that not all dynamic/sequential mechanisms are proficient at handling choice complexity. This particular mechanism requires choices consistent with a ranking of programs. We next check whether there is a way to tailor this mechanism so that it is less demanding in terms of choice sophistication. The answer is affirmative.

Next, we modify the mechanism such that the cut-offs for programs are released not simultaneously but sequentially. The modified mechanism identifies and presents the individuals with a pair of programs at each step with the presumption that applicants have no difficulty discarding the program that they dislike when choosing from two options. Thus, the individual reaches the final choice by discarding programs in binary comparisons in a some order. Let us formally define this mechanism.

Admissions with Sequential Cut-offs Algorithm

- **Step 0:** Each individual i applies to all acceptable institutions s such that $C_i(\{s\}) = \{s\}$. Let $\psi[\gamma]^0(i) \equiv \varnothing$, $\psi[\gamma]^0(s) \equiv \varnothing$ and $c_s^0 \equiv n+1$.

- **Step t (t $\geq$ 1) with Sequential Cut-offs:** A **single** institution s announces a cut-off $c_s^t \in \{1, \ldots, n+1\}$ such that $|\{i \in I : c_s^t \leq \pi_s(i) < c_s^{t-1}\}| + |\psi[\gamma]^{t-1}(s)| \leq q_s$ and $c_s^t < c_s^{t-1}$. Other institutions $s' \neq s$ announce $c_{s'}^t = c_{s'}^{t-1}$.

  Let $B_i^t \equiv \{s \in S : C_i(\{s\}) = \{s\} \text{ and } c_s^t \leq m_s(i) < c_s^{t-1}\} \cup \psi[\gamma]^{t-1}(i)$ denote the set of institutions that individual i can choose from. Individual i's assignment is updated as follows:

$$\psi[\gamma]^t(i) = \begin{cases} C_i(B_i^t), & \text{if } C_i(B_i^t) \neq \varnothing \\ \psi[\gamma]^{t-1}(i), & \text{otherwise} \end{cases}$$

  Institution s's assignment is $\psi[\gamma]^t(s) = \{i \in I : \psi[\gamma]^t(i) = \{s\}\}$. If each institution s has filled its capacity or has announced the lowest cut-off possible ($c_s = m_s$), then stop and return $\psi[\gamma]^t$ as the outcome.

The algorithm terminates in a finite number of steps for the same reason as the simultaneous cut-offs algorithm. Therefore, the outcome of the algorithm is well-defined.

Let ψ^{seq} be the mechanism based on the sequential cut-offs algorithm. Notice that the outcomes of this mechanism are order dependent. That is, the outcome depends on the order of cut-offs announcements from institutions (which is not fixed). Therefore, the mechanism is said to lead to stable outcomes if they lead to stable outcomes for any order of cut-offs announcements from institutions.

Restricting choice situations to binary menus allows the sequential cut-offs mechanism to accommodate more choice behaviors than the simultaneous cut-offs mechanism. Even so, the weakly acyclic choices are not accommodated. In addition to acceptable-consistency, this mechanism requires acyclic choices, that is, choices without any weak and strict cycles in binary menus. This requirement is stronger than weak acyclicity, which rules out only strict cycles.

Formally, a choice function C_i is **acyclic (over acceptable institutions)** if for all sequences of acceptable institutions $s^1, s^2, \ldots, s^t \in S$,

$$C_i(\{s^1, s^2\}) = \{s^1\}, \ldots, C_i(\{s^{t-1}, s^t\}) = \{s^{t-1}\} \text{ implies } C_i(\{s^1, s^t\}) = \{s^1\}.$$

Proposition 6. *The sequential cut-offs mechanism ψ^{seq} leads to a stable outcome $\psi^{seq}[\gamma]$ for every admissions problem $\gamma \in \Gamma$ if and only if the choice functions are acyclic and acceptable-consistent.*

Therefore, in line with the intuition that making consistent choices from binary sets is easier than choosing from potentially larger sets, the sequential cut-offs mechanism requires much less choice sophistication than its simultaneous counterpart. Note that both ψ^{sim} and the proposed modification ψ^{seq} are not incentive compatible.[31] That is, the two mechanisms yield stable outcomes for the observed choices, given that the choices satisfy the corresponding conditions.

7 Conclusion

We extend matching theory to problems where individuals may exhibit a plethora of non-standard choice behaviors. We show that weak acyclic and acceptable-consistent choice functions are both necessary and sufficient for the existence of stable matchings and a large class of incentive compatible mechanisms. Compared to the standard choice behavior, characterized by path independent choice functions, our identified conditions allow for more general choice behavior of individuals and lead to a larger set of stable matchings. In our setup, classical results, such as the existence of an individual-optimal matching or a lattice structure, cease to exist. We find a stronger implication of non-standard choice behavior on the set of stable matchings. We show that for some problems, with weak acyclic and acceptable-consistent choice functions, even a Pareto undominated stable matching may not exist. In other words, allowing for more general choice behavior not only affects how one finds a stable matching but also the structure of the set of stable matchings.

We investigate an application in the context of centralized university admissions. Building on insights from the literature on preference-reporting language (see, e.g., Milgrom (2009), Milgrom (2011), Budish and Kessler (2021)), we tweak a commonly used mechanism to

[31] In rare situations, misrepresenting choices by rejecting an acceptable institution (that is preferred over the current tentative assignment) can lead to better outcomes as follows. The rejected institution lowers its cut-off, attracting an individual from another institution, which in turn lowers its cut-off — opening up a slot for the original manipulator. Notice that the problem disappears if the relevant market is sufficiently large as a single individual can no longer affect an institution's cut-off by rejecting that institution.

reduce the complexity of choice situations that individuals face when interacting with the mechanism to mere binary comparisons. In contrast to the original mechanism, the modified mechanism is shown to accommodate non-standard choice behavior adequately. Focusing on mechanisms where choices are made directly — as opposed to elicited prior to the match by a centralized authority — circumvent the need to choose a preference-reporting language that accurately reflects individuals' choice behaviors. An important dimension is identifying the minimum amount of choice information required to arrive at stable outcomes, knowledge of which could greatly facilitate the use of such mechanisms in practice. This will be the subject of future work.

A commonality in our approach in defining concepts is that while in strategic situations, such as participating in a block (in both the definitions of stability) or misreporting preferences (in the definition of incentive compatibility), the individual is assumed to care about only two outcomes. The first outcome is the one that precedes the strategic action, a matching that is to be blocked, or a matching lead by truthfully revealing preferences. The second outcome succeeds the strategic action, a matching resulting from a successful block (or coalition), or a matching lead by untruthful revelation of preferences. Thus, from the individual's point of view benefit from strategic actions entails choices from singleton or binary menus. If we are to broaden the reach of institution design to settings in which agents have un-orthodox but realistic patters of preferences, such approaches to adapting and redefining key concepts are worth scrutinizing further.

References

Abdulkadiroglu, Atila and Tayfun Sönmez, "School choice: A mechanism design approach," *The American Economic Review*, 2003, *93* (3), 729–747.

Abdulkadiroğlu, Atila, Parag A Pathak, Alvin E Roth, and Tayfun Sönmez, "The Boston public school match," *American Economic Review*, 2005, *95* (2), 368–371.

_ , _ , and _ , "The new york city high school match," *American Economic Review*, 2005, *95* (2), 364–367.

Ambrus, Attila and Kareen Rozen, "Rationalising Choice with Multi-self Models," *The Economic Journal*, 2015, *125* (585), 1136–1156.

Apesteguia, Jose and Miguel A. Ballester, "Choice by sequential procedures," *Games and Economic Behavior*, 2013, *77* (1), 90 – 99.

Bade, Sophie, "Pareto-optimal matching allocation mechanisms for boundedly rational agents," *Social Choice and Welfare*, 2016, *47* (3), 501–510.

Balinski, Michel and Tayfun Sönmez, "A tale of two mechanisms: student placement," *Journal of Economic theory*, 1999, *84* (1), 73–94.

Bernheim, B Douglas and Antonio Rangel, "Beyond revealed preference: choice-theoretic foundations for behavioral welfare economics," *The Quarterly Journal of Economics*, 2009, *124* (1), 51–104.

Bo, Inacio and Rustamdjan Hakimov, "The iterative deferred acceptance mechanism," *Available at SSRN 2881880*, 2019.

Bó, Inácio and Rustamdjan Hakimov, "Iterative versus standard deferred acceptance: Experimental evidence," *The Economic Journal*, 2020, *130* (626), 356–392.

_ and _ , "Pick-an-object mechanisms," *Available at SSRN*, 2020.

Bochet, Olivier and Norovsambuu Tumennasan, "Prevalence of truthtelling and implementation," 2018.

_ and _ , "One truth and a thousand lies: Defaults and benchmarks in mechanism design," 2019.

Börgers, Tilman and Jiangtao Li, "Strategically simple mechanisms," *Econometrica*, 2019, *87* (6), 2003–2035.

Bossert, Walter, Kotaro Suzumura, Kōtarō Suzumura et al., *Consistency, choice, and rationality*, Harvard University Press, 2010.

Budish, Eric, "The combinatorial assignment problem: Approximate competitive equilibrium from equal incomes," *Journal of Political Economy*, 2011, *119* (6), 1061–1103.

_ **and Judd B Kessler**, "Can Market Participants Report Their Preferences Accurately (Enough)?," *Management Science*, 2021.

Calsamiglia, Caterina, Guillaume Haeringer, and Flip Klijn, "Constrained school choice: An experimental study," *American Economic Review*, 2010, *100* (4), 1860–74.

Chambers, Christopher P and M Bumin Yenmez, "Choice and matching," *American Economic Journal: Microeconomics*, 2017, *9* (3), 126–47.

Cherepanov, Vadim, Timothy Feddersen, and Alvaro Sandroni, "Rationalization," *Theoretical Economics*, 2013, *8* (3), 775–800.

Clippel, Geoffroy De, "Behavioral implementation," *American Economic Review*, 2014, *104* (10), 2975–3002.

_ **, Rene Saran, and Roberto Serrano**, "Level-k mechanism design," *The Review of Economic Studies*, 2019, *86* (3), 1207–1227.

Compte, Olivier and Andrew Postlewaite, *Ignorance and Uncertainty* number 61, Cambridge University Press, 2019.

Cox, Natalie, Bobby Pakzad-Hurson, and Ricardo Fonseca, "Do Peer Preferences Matter in School Choice Market Design? Theory and Evidence," 2021.

Dubins, Lester E and David A Freedman, "Machiavelli and the Gale-Shapley algorithm," *The American Mathematical Monthly*, 1981, *88* (7), 485–494.

Dur, Umut, Robert G Hammond, and Thayer Morrill, "Identifying the harm of manipulable school-choice mechanisms," *American Economic Journal: Economic Policy*, 2018, *10* (1), 187–213.

Echenique, Federico, Alistair J Wilson, and Leeat Yariv, "Clearinghouses for two-sided matching: An experimental study," *Quantitative Economics*, 2016, *7* (2), 449–482.

Edelman, Benjamin, Michael Ostrovsky, and Michael Schwarz, "Internet advertising and the generalized second-price auction: Selling billions of dollars worth of keywords," *American economic review*, 2007, *97* (1), 242–259.

Erdil, Aytek and Haluk Ergin, "What's the matter with tie-breaking? Improving efficiency in school choice," *American Economic Review*, 2008, *98* (3), 669–89.

_ **and** _ , "Two-sided matching with indifferences," *Journal of Economic Theory*, 2017, *171*, 268–292.

Gale, David and Lloyd S Shapley, "College admissions and the stability of marriage," *The American Mathematical Monthly*, 1962, *69* (1), 9–15.

Gong, Binglin and Yingzhi Liang, "A dynamic college admission mechanism in inner mongolia: Theory and experiment," Technical Report, Working paper 2020.

Greenberg, Kyle, Parag A Pathak, and Tayfun Sönmez, "Mechanism Design meets Priority Design: Redesigning the US Army's Branching Process," Technical Report, National Bureau of Economic Research 2021.

Grenet, Julien, Yinghua He, and Dorothea Kübler, "Decentralizing Centralized Matching Markets: Implications from Early Offers in University Admissions," 2019.

Grubb, Michael D, "Consumer inattention and bill-shock regulation," *The Review of Economic Studies*, 2015, *82* (1), 219–257.

Haeringer, Guillaume and Flip Klijn, "Constrained school choice," *Journal of Economic Theory*, 2009, *144* (5), 1921 – 1947.

_ **and Vincent Iehlé**, "Gradual college admission," *Université Paris-Dauphine Research Paper*, 2019, (3488038).

Hakimov, Rustamdjan, Dorothea Kübler, and Siqi Pan, "Costly Information Acquisition in Centralized Matching Markets," 2021.

Hassidim, Avinatan, Assaf Romm, and Ran I Shorrer, "The limits of incentives in economic matching procedures," *Management Science*, 2021, *67* (2), 951–963.

Hatfield, John William and Fuhito Kojima, "Substitutes and stability for matching with contracts," *Journal of Economic theory*, 2010, *145* (5), 1704–1723.

_ **and Paul R Milgrom**, "Matching with contracts," *American Economic Review*, 2005, *95* (4), 913–935.

_ **and Scott Duke Kominers**, "Hidden Substitutes.," in "EC" 2015, p. 37.

_ , _ , and Alexander Westkamp, "Stability, Strategy-Proofness, and Cumulative Offer Mechanisms," *The Review of Economic Studies*, 09 2020, *88* (3), 1457–1502.

Highhouse, Scott, "Context-dependent selection: The effects of decoy and phantom job candidates," *Organizational Behavior and Human Decision Processes*, 1996, *65* (1), 68–76.

Horan, Sean, "A simple model of two-stage choice," *Journal of Economic Theory*, 2016, *162*, 372–406.

Huber, Joel, John W Payne, and Christopher Puto, "Adding asymmetrically dominated alternatives: Violations of regularity and the similarity hypothesis," *Journal of consumer research*, 1982, *9* (1), 90–98.

Jehiel, Philippe, "Analogy-Based Expectation Equilibrium and Related Concepts: Theory, Applications, and Beyond," in "World Congress of the Econometric Society" 2020.

Jochim, Ashley, Michael DeArmond, Betheny Gross, and Robin Lake, "How parents experience public school choice," *Center for Reinventing Public Education. Retrieved from http://www. crpe. org/publications/how-parents-experience-public-school-choice*, 2014.

Jr, Alexander S Kelso and Vincent P Crawford, "Job matching, coalition formation, and gross substitutes," *Econometrica: Journal of the Econometric Society*, 1982, pp. 1483–1504.

Kagel, John H and Alvin E Roth, "The dynamics of reorganization in matching markets: A laboratory experiment motivated by a natural experiment," *The Quarterly Journal of Economics*, 2000, *115* (1), 201–235.

Kalai, Gil, Ariel Rubinstein, and Ran Spiegler, "Rationalizing choice functions by multiple rationales," *Econometrica*, 2002, *70* (6), 2481–2488.

Klijn, Flip, Joana Pais, and Marc Vorsatz, "Static versus dynamic deferred acceptance in school choice: Theory and experiment," *Games and Economic Behavior*, 2019, *113*, 147 – 163.

Kominers, Scott Duke, Alexander Teytelboym, and Vincent P Crawford, "An invitation to market design," *Oxford Review of Economic Policy*, 2017, *33* (4), 541–571.

Konishi, Hideo and M Utku Ünver, "Credible group stability in many-to-many matching problems," *Journal of Economic Theory*, 2006, *129* (1), 57–80.

Kuvalekar, Aditya, "Matching with Incomplete Preferences," *Available at SSRN 3560889*, 2020.

Leshno, Jacob, "Stable Matching with Peer-Dependent Preferences in Large Markets: Existence and Cutoff Characterization," *Available at SSRN 3822060*, 2021.

Li, Shengwu, "Obviously strategy-proof mechanisms," *American Economic Review*, 2017, *107* (11), 3257–87.

Lipman, Barton L, Wolfgang Pesendorfer et al., "Temptation," in "Advances in economics and econometrics: Tenth World Congress," Vol. 1 Citeseer 2013, pp. 243–288.

Lleras, Juan Sebastian, Yusufcan Masatlioglu, Daisuke Nakajima, and Erkut Y Ozbay, "When more is less: Limited consideration," *Journal of Economic Theory*, 2017, *170*, 70–85.

Luflade, Margaux, "The value of information in centralized school choice systems," 2018.

Mackenzie, Andrew and Yu Zhou, "Menu mechanisms," *Available at SSRN 3521257*, 2020.

Manzini, Paola and Marco Mariotti, "Sequentially rationalizable choice," *American Economic Review*, 2007, *97* (5), 1824–1839.

_ **and** _ , "Categorize then choose: Boundedly rational choice and welfare," *Journal of the European Economic Association*, 2012, *10* (5), 1141–1165.

Masatlioglu, Yusufcan and Efe A Ok, "Rational choice with status quo bias," *Journal of economic theory*, 2005, *121* (1), 1–29.

_ **and** _ , "A canonical model of choice with initial endowments," *Review of Economic Studies*, 2014, *81* (2), 851–883.

_ **, Daisuke Nakajima, and Erkut Y Ozbay**, "Revealed attention," *American Economic Review*, 2012, *102* (5), 2183–2205.

McMillan, John, "Selling spectrum rights," *Journal of Economic Perspectives*, 1994, *8* (3), 145–162.

Milgrom, Paul, "Putting auction theory to work: The simultaneous ascending auction," *Journal of political economy*, 2000, *108* (2), 245–272.

— , "Assignment messages and exchanges," *American Economic Journal: Microeconomics*, 2009, *1* (2), 95–113.

— , "Critical issues in the practice of market design," *Economic Inquiry*, 2011, *49* (2), 311–320.

— **and Ilya Segal**, "Clock auctions and radio spectrum reallocation," *Journal of Political Economy*, 2020, *128* (1), 1–31.

— **and Steven Tadelis**, "How Artificial Intelligence and Machine Learning Can Impact Market Design," in "The Economics of Artificial Intelligence: An Agenda," National Bureau of Economic Research, Inc, 2018, pp. 567–585.

Narita, Yusuke, "Match or mismatch? Learning and inertia in school choice," 2018.

Ok, Efe A, Pietro Ortoleva, and Gil Riella, "Revealed (p) reference theory," *American Economic Review*, 2015, *105* (1), 299–321.

Oprea, Ryan, "What Makes a Rule Complex?," *American Economic Review*, 2020, *110* (12), 3913–51.

Pápai, Szilvia, "Strategyproof assignment by hierarchical exchange," *Econometrica*, 2000, *68* (6), 1403–1433.

Pathak, Parag A and Tayfun Sönmez, "Leveling the playing field: Sincere and sophisticated players in the Boston mechanism," *American Economic Review*, 2008, *98* (4), 1636–52.

Plott, Charles R, "Path independence, rationality, and social choice," *Econometrica: Journal of the Econometric Society*, 1973, pp. 1075–1091.

Pycia, Marek, "Stability and preference alignment in matching and coalition formation," *Econometrica*, 2012, *80* (1), 323–362.

— **and M Bumin Yenmez**, "Matching with externalities," *University of Zurich, Department of Economics, Working Paper*, 2021, (392).

Rees-Jones, Alex, "Suboptimal behavior in strategy-proof mechanisms: Evidence from the residency match," *Games and Economic Behavior*, 2018, *108*, 317–330.

— **and Samuel Skowronek**, "An experimental investigation of preference misrepresentation in the residency match," *Proceedings of the National Academy of Sciences*, 2018, *115* (45), 11471–11476.

Roth, Alvin E, "The economics of matching: Stability and incentives," *Mathematics of operations research*, 1982, *7* (4), 617–628.

— , "Misrepresentation and stability in the marriage problem," *Journal of Economic theory*, 1984, *34* (2), 383–387.

— , "The college admissions problem is not equivalent to the marriage problem," *Journal of economic Theory*, 1985, *36* (2), 277–288.

— , "The economist as engineer: Game theory, experimentation, and computation as tools for design economics," *Econometrica*, 2002, *70* (4), 1341–1378.

— , "Marketplaces, markets, and market design," *American Economic Review*, 2018, *108* (7), 1609–58.

Roth, Alvin E. and Marilda A. Oliveira Sotomayor, *Two-Sided Matching: A Study in Game-Theoretic Modeling and Analysis* Econometric Society Monographs, Cambridge University Press, 1990.

Roth, Alvin E, Tayfun Sönmez, and M Utku Ünver, "Kidney exchange," *The Quarterly journal of economics*, 2004, *119* (2), 457–488.

— , — , and — , "Pairwise kidney exchange," *Journal of Economic theory*, 2005, *125* (2), 151–188.

Rubinstein, Ariel and Yuval Salant, "A model of choice from lists," *Theoretical Economics*, 2006, *1* (1), 3–17.

— and — , "(A, f): choice with frames," *The Review of Economic Studies*, 2008, *75* (4), 1287–1296.

Salant, Yuval and Jörg L Spenkuch, "Complexity and Choice," 2021.

Sasaki, Hiroo and Manabu Toda, "Two-sided matching problems with externalities," *Journal of Economic Theory*, 1996, *70* (1), 93–108.

Shapley, Lloyd and Herbert Scarf, "On cores and indivisibility," *Journal of mathematical economics*, 1974, *1* (1), 23–37.

Shapley, Lloyd S and Martin Shubik, "The assignment game I: The core," *International Journal of game theory*, 1971, *1* (1), 111–130.

Shorrer, Ran I and Sándor Sóvágó, "Obvious mistakes in a strategically simple college admissions environment: Causes and consequences," *Available at SSRN 2993538*, 2018.

Slaughter, Jerel E, "Effects of Two Selection Batteries on Decoy Effects in Job-Finalist Choice 1," *Journal of applied social psychology*, 2007, *37* (1), 76–90.

_ **, Evan F Sinar, and Scott Highhouse**, "Decoy effects and attribute-level inferences.," *Journal of applied psychology*, 1999, *84* (5), 823.

Sönmez, Tayfun, "Games of manipulation in marriage problems," *Games and Economic Behavior*, 1997, *20* (2), 169–176.

_ , "Bidding for army career specialties: Improving the ROTC branching mechanism," *Journal of Political Economy*, 2013, *121* (1), 186–219.

_ **and M Utku Ünver**, "Matching, allocation, and exchange of discrete resources," in "Handbook of social Economics," Vol. 1, Elsevier, 2011, pp. 781–852.

_ **and** _ , "Market design for living-donor organ exchanges: An economic policy perspective," *Oxford Review of Economic Policy*, 2017, *33* (4), 676–704.

_ **and Tobias B Switzer**, "Matching with (branch-of-choice) contracts at the United States military academy," *Econometrica*, 2013, *81* (2), 451–488.

Wilson, Robert, "Architecture of power markets," *Econometrica*, 2002, *70* (4), 1299–1340.

Xu, Yongsheng and Lin Zhou, "Rationalizability of choice functions by game trees," *Journal of Economic theory*, 2007, *134* (1), 548–556.

Zhang, Jun, "Level-k reasoning in school choice," *Games and Economic Behavior*, 2021, *128*, 1–17.

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A Mathematical Appendix. Proofs

Lemma 1

Proof. Consider a subset $S' \subseteq S$ and individual $i \in I$ with at least one acceptable institution, that is, $s \in S'$ such that $C_i(\{s\}) = \{s\}$. Moreover, suppose that $U_i(S') = \varnothing$. Define the set of acceptable institutions as $\overline{S'} = \{s \in S' : C_i(\{s\}) = \{s\}\} \neq \varnothing$ and the set of unacceptable institutions as $\underline{S'} = \{s \in S' : C_i(\{s\}) = \{s\}\}$.

First, suppose there exists an unacceptable institution $s' \in \underline{S'}$ that is chosen over an acceptable institution $s \in \overline{S'}$, i.e., $C_i(\{s, s'\}) = \{s'\}$. Since choice functions are acceptable-consistent we have that $C_i(\{s\}) = \{s\}$ and $C_i(\{s'\}) = \varnothing$ implies $C_i(\{s, s'\}) \neq \{s'\}$ — a contradiction. Therefore, only an acceptable institution can be chosen over another acceptable institution.

Second, suppose that $|\overline{S'}| \leq 2$. Then there trivially exists a C-maximal institution, as only an acceptable institution can be chosen over another acceptable institution.

Third, suppose that for every acceptable institution $s \in \overline{S'}$, there is another institution $s' \in S'$ that is chosen in pairwise comparison. By the first part, only an acceptable institution can be chosen over another acceptable institution. That is, for all $s \in \overline{S'}$ there exists $s' \in \overline{S'} \setminus \{s\}$ such that $C_i(\{s, s'\}) = \{s'\}$. By the second part, $|\overline{S'}| \geq 3$. But this implies that there exists a positive integer $t \geq 3$ and t distinct and acceptable alternatives $s^1, s^2, \ldots, s^t$, such that $C_i(\{s^1, s^2\}) = \{s^2\}, \ldots C_i(\{s^{t-1}, s^t\}) = \{s^t\},$ and $C_i(\{s^t, s^1\}) = \{s^1\}$ — a contradiction to weak acyclicity. $\square$

Theorem 1

We construct an outcome for every admission problem $\gamma \in \Gamma$ using the algorithm described next.

Non-Block Algorithm

Step 0: For each $i \in I$ consider the set of mutually acceptable institutions $S_i^0 \equiv \{s \in S : C_i(s) = \{s\}$ and $s \pi_s \varnothing\}$. Let the set of individuals proposing to institution s be denoted by $I_s^0 \equiv \varnothing$. Finally, let $A_s(I') \equiv \{i \in I' : |\{i' \in I' : i' \pi_s i\}| < q_s\}$ denote the set of individuals in $I' \subseteq I$ tentatively assigned to institution s.

Step $k \geq 1$: If there are any individual currently not tentatively admitted, i.e., $i \notin \bigcup_{s \in S} A_s(I_s^{k-1})$, and that still has a mutually acceptable institution left to propose, i.e., $S_i^{k-1} \neq \varnothing$. Then let each such individual i propose to an C-maximal institution $s_i^k \in S_i^{k-1}$ (if there are multiple, take the lowest-subscript institution) — by Lemma 1 such an institution exists since $S_i^{k-1} \neq \varnothing$ and contains only acceptable institutions. Let P_s^k denote the set of individuals proposing to institution s at step k. Set $S_i^k = S_i^{k-1} \setminus \{s_i^k\}$ and $I_s^k = I_s^{k-1} \cup P_s^k$. Go to step $k+1$.

Otherwise, if no such individual exists, the algorithm stops. Then each institution is assigned $A_s(I_s^{k-1})$ while each individual i is assigned $s \in S$ such that $i \in A_s(I_s^{k-1})$ and $\varnothing$ otherwise.

Let the **non-block mechanism** ψ^{nb} be described by the function that associates outcome of the non-block algorithm to any admission problem $\gamma \in \Gamma$. We now prove Theorem 1.

Proof. **The "if" part:** If the choice functions $(C_i)_{i\in I}$ are weakly acyclic and acceptable-consistent, then the non-block mechanism ψ^{nb} yields a stable matching $\psi^{nb}[\gamma]$ for every admission problem $\gamma \in \Gamma$.

For any admission problem $\pi \in \Pi$, consider the outcome of the non-block mechanism $\psi^{nb}[\pi]$. Since individuals propose to mutually acceptable institutions only, the outcome $\psi^{nb}[\pi]$ is trivially individually rational for both institutions and individuals.

Next, suppose that there exists $s \in S \setminus \psi^{nb}[\pi](i)$ with $C_i(\{s\} \cup \psi[\pi](i)) = \{s\}$ and $s \pi_s \varnothing$. By acceptable-consistency, s must be acceptable. Together with Lemma 1, this implies that i must have proposed to s at some step k and subsequently been rejected. Therefore, for institution s it must be the case that $|\psi^{nb}[\pi](s)| = q_s$ and $i' \pi_s i$ for all $i' \in \mu[\pi](s)$.

The "only if" part: A stable matching μ exists for every admission problems $\gamma \in \Gamma$ only if the choice functions $(C_i)_{i\in I}$ are weakly acyclic and acceptable-consistent. The contrapositive statement is, if the choice functions $(C_i)_{i\in I}$ are not weakly acyclic or acceptable-consistent, then for some admissions problem $\gamma \in \Gamma$ a stable matching μ does not exist.

Part 1. Weak Acyclicity is necessary.
Suppose that for some C_i we have a cycle for $t \geq 3$ and distinct acceptable alternatives $s^1, s^2, \ldots, s^t$ in S such that $C_i(\{s^1, s^2\}) = \{s^2\}, \ldots C_i(\{s^{t-1}, s^t\}) = \{s^t\}$, and $C_i(\{s^t, s^1\}) = \{s^1\}$.

Consider an admissions problem $\tilde{\gamma} = \langle I, S, q, C, \tilde{\pi}\rangle$ such that $i\ \tilde{\pi}_s\ i'$ for all $i' \in I \setminus \{i\}$ and $s \in \{s^1, s^2, \ldots, s^t\}$ and $\varnothing\ \tilde{\pi}_s\ i$ for all $s \in S \setminus \{s^1, s^2, \ldots, s^t\}$. Consider a matching μ for problem $\tilde{\gamma}$. If $\mu(i) = \{s\}$ for some $s \in \{s^1, s^2, \ldots, s^t\}$ or $\mu(i) = \varnothing$, i forms a blocking pair with some $s' \in \{s^1, s^2, \ldots, s^t\} \setminus \mu(i)$. While if $\mu(i) = \{s\}$ for $s \in S \setminus \{s^1, s^2, \ldots, s^t\}$ we have a violation of individual rationality for the appropriate institution. Therefore, there is no stable matching for problem $\tilde{\gamma}$.

Part 2. Acceptable-consistency is necessary.
Suppose that for some C_i we have $C_i(\{s^1\}) = \{s^1\}$ and $C_i(\{s^2\}) = \varnothing$ but $C_i(\{s^1, s^2\}) = \{s^2\}$.

Consider an admission problem $\tilde{\gamma} \in \Gamma$ such that $i \ \tilde{\pi}_s \ i'$ for all $i' \in I \setminus \{i\}$ and $s \in \{s^1, s^2\}$ and $\varnothing \ \tilde{\pi}_s \ i$ for all $s \in S \setminus \{s^1, s^2\}$. Consider a matching μ for problem $\tilde{\gamma}$. There are four possibilities. (i) If $\mu(i) = \{s^1\}$, i forms a blocking pair with s^2. (ii) If $\mu(i) = \{s^2\}$ we have a violation of individual rationality for individual i. (iii) If $\mu(i) = \varnothing$, i forms a blocking pair with s^1. (iv) While if $\mu(i) = \{s\}$ for $s \in S \setminus \{s^1, s^2\}$ we have a violation of individual rationality for the appropriate institution. Therefore, there is no stable matching for problem $\tilde{\gamma}$. □

Theorem 2

We start by proving the following lemma.

Lemma 3. *Consider a path independent choice function C_i, then*

(i) an unacceptable alternative cannot be chosen, that is,

$$C_i(\{s\}) = \varnothing \implies C_i(\{s\} \cup S') \neq \{s\} \quad \textit{for any} \quad S' \subseteq S \setminus \{s\}.$$

(ii) choice from a set containing at least one acceptable alternative is non-empty, that is,

$$C_i(\{s\}) = \{s\} \implies C_i(\{s\} \cup S') \neq \varnothing \quad \textit{for any} \quad S' \subseteq S \setminus \{s\}.$$

(iii) choice from a set containing unacceptable alternatives is empty, that is,

$$C_i(\{s\}) = \varnothing \quad \textit{for all} \quad s \in S' \subseteq S \implies C_i(S') = \varnothing.$$

Proof. (i) Consider $s \in S$ such that $C_i(\{s\}) = \varnothing$ and any $S' \subseteq S \setminus \{s\}$. By path independence we have

$$\begin{aligned} C_i(\{s\} \cup S') &= C_i(C_i(\{s\}) \cup S') \\ &= C_i(S') \subseteq S' \\ &\neq \{s\}. \end{aligned}$$

(ii) Consider $s \in S$ such that $C_i(\{s\}) = \{s\}$ and any $S' \subseteq S \setminus \{s\}$. By path independence we have

$$
\begin{aligned}
C_i(\{s\} \cup S') &= C_i(\{s\} \cup (\{s\} \cup S')) \\
&= C_i(\{s\} \cup C_i(S' \cup \{s\})) \\
&= C_i(\{s\}) \\
&= \{s\} \\
&\neq \varnothing.
\end{aligned}
$$

(iii) Consider any $S' \subseteq S$ with $C_i(\{s\}) = \varnothing$ for all $s \in S'$. Let $S' = \{s_1, \dots , s_k\}$. By path independence we can remove alternatives one by one to reach the desired conclusion, that is,

$$
\begin{aligned}
C_i(S') &= C_i(C_i(\{s_1\}) \cup (S' \setminus \{s_1\})) \\
&= C_i(S' \setminus \{s_1\}) \\
&= C_i(C_i(\{s_2\}) \cup (S' \setminus \{s_1, s_2\})) \\
&= \dots \\
&= C_i(\{s_k\}) \\
&= \varnothing.
\end{aligned}
$$

□

We now prove Theorem 2.

Proof. **The "if" part:** If a unit demand choice function C_i is path independent then it is rationalizable by a simple order R_i over acceptable choices.

Consider C_i such that $C_i(\{s, s'\}) = \{s\}$ and $C_i(\{s', s''\}) = \{s'\}$. Note that path independence implies that

$$
\begin{aligned}
\{s\} &= C_i(\{s\} \cup \{s'\}) \\
&= C_i(\{s\} \cup C_i(\{s', s''\})) \\
&= C_i(\{s\} \cup \{s', s''\}) && \text{by path independence} \\
&= C_i(C_i(\{s, s'\}) \cup \{s''\}) && \text{by path independence} \\
&= C_i(\{s, s''\}).
\end{aligned}
$$

Therefore, C_i is transitive over binary menus.

Next, define binary relation P_i such that $s \; P_i \; s'$ for $s, s' \in S$ with $s \neq s'$ if $C_i(\{s, s'\}) = \{s\}$, $C_i(\{s\}) = \{s\}$, and $C_i(\{s'\}) = \{s'\}$. Moreover, $s \; P_i \; \varnothing$ if $C_i(\{s\}) = \{s\}$, and $\varnothing \; P_i \; s$ if

$C_i(\{s\}) = \varnothing$. Similarly, define R_i such that sR_is' if and only if $[s\ P_i\ s'$ or $s = s']$, $sR_i\varnothing$ if and only if $s\ P_i\ \varnothing$, and $\varnothing R_i s$ if and only if $\varnothing\ P_i\ s$. Note that, R_i is anti-symmetric, and (strongly) complete over acceptable choices by construction. Moreover, R_i is transitive over acceptable choices as C_i is transitive over binary choices. For the constructed order R_i we next show that

(i) $C_i(S') = \varnothing$ if $\varnothing R_i s$ for all $s \in S'$, and

(ii) $C_i(S') = \{s \in S' : sR_is' \text{ for all } s' \in S'\}$ otherwise.

(i) Consider $S' \subseteq S$ such that $\varnothing R_i s$ for all $s \in S'$. For any $s \in S'$, by construction $\varnothing R_i s$ implies $\varnothing\ P_i\ s$ which implies $C_i(\{s\}) = \varnothing$. By Lemma 3, if $C_i(\{s\}) = \varnothing$ for all $s \in S'$ then $C_i(S') = \varnothing$.

(ii) Consider $S' \subseteq S$ with at least one $s \in S'$ such that $sR_i\varnothing$. By construction $sR_i\varnothing$ implies $s\ P_i\ \varnothing$ which implies $C_i(\{s\}) = \{s\}$. That is, S' contains at least one acceptable alternative. Suppose by contradiction that $C_i(S') \neq \{s \in S' : sR_is' \text{ for all } s' \in S'\}$. By Lemma 3, $C_i(S') \neq \varnothing$. Hence, let $C_i(S') = \{s'\}$ and $\{s \in S' : sR_is' \text{ for all } s' \in S'\} = \{s\}$. Since $s \neq s'$ and sR_is' we have $s\ P_i\ s'$ respectively $\{s\} = C_i(\{s, s'\})$. From this observation we reach a contradiction since

$$\begin{aligned}
\{s'\} &= C_i(\{s, s'\} \cup (S' \setminus \{s, s'\})) \\
&= C_i(\{s\} \cup (S' \setminus \{s, s'\})) && \text{by path independence} \\
&= C_i(S' \setminus \{s'\}) \\
&\neq \{s'\}.
\end{aligned}$$

The "only if" part: If a unit demand choice function C_i is rationalizable by a simple order R_i then it is path independent.
There are three cases to consider.

(i) Consider $S', S'' \subseteq S$ both containing at least one acceptable alternative, that is, s such that $sR_i\varnothing$. Let $\{s^*\} = \{s \in S' \cup S'' : sR_is' \text{ for all } s' \in S' \cup S''\}$, and $\{s^{*\prime}\} = \{s \in S' : sR_is' \text{ for all } s' \in S'\}$. Since s^*R_is' for all $s' \in S' \cup S''$ and $s^{*\prime} \in S'$ we have $\{s^*\} = \{s \in \{s^{*\prime}\} \cup S'' : sR_is' \text{ for all } s' \in \{s^{*\prime}\} \cup S''\}$. Rewriting this in terms of choice functions leads

$$\begin{aligned}
C_i(S' \cup S'') &= C_i(\{s^{*\prime}\} \cup S'') \\
&= C_i(C_i(S') \cup S'')
\end{aligned}$$

(ii) Consider $S', S'' \subseteq S$ both containing no acceptable alternatives. We have $C(S') = \varnothing$, $C(S'') = \varnothing$ and $C(S' \cup S'') = \varnothing$. It directly follows that $C_i(S' \cup S'') = C_i(S'') = C_i(C_i(S') \cup S'')$.

(iii) Consider $S', S'' \subseteq S$ where only S' contains at least one acceptable alternative. Let $\{s^*\} = \{s \in S' \cup S'' : sR_is' \text{ for all } s' \in S' \cup S''\}$. It follows that $\{s^*\} = \{s \in S' : sR_is' \text{ for all } s' \in S'\}$ as well as $\{s^*\} = \{s \in S'' \cup \{s^*\} : sR_is' \text{ for all } s' \in S'' \cup \{s^*\}\}$. Again, we get that $C_i(S' \cup S'') = C_i(C_i(S') \cup S'')$.

□

Proposition 1

Part 1. Path independent choice functions are weakly acyclic.
Consider a path independent choice function C_i, an integer $t \geq 3$ and t distinct and acceptable institutions $s^1, s^2, \ldots, s^t \in S$, $C_i(\{s^1, s^2\}) = \{s^1\}, \ldots, C_i(\{s^{t-1}, s^t\}) = \{s^{t-1}\}$.

First, notice that path independence implies that

$$\begin{aligned} C_i(\{s^1, s^2, \ldots, s^t\}) &= C_i(\{s^1, s^2, \ldots, s^{t-2}\} \cup C_i\{s^{t-1}, s^t\}) \\ &= C_i(\{s^1, s^2, \ldots, s^{t-1}\}) \\ &\neq \{s^t\}. \end{aligned}$$

Second, path independence implies that

$$\begin{aligned} C_i(\{s^1, s^2, \ldots, s^t\}) &= C_i(\{s^1, s^2, \ldots, s^{t-3}\} \cup C_i\{s^{t-2}, s^{t-1}\} \cup \{s^t\}) \\ &= C_i(\{s^1, s^2, \ldots, s^{t-4}\} \cup C_i\{s^{t-3}, s^{t-2}\} \cup \{s^t\}) \\ &= \ldots \\ &= C_i(\{s^1, s^t\}. \end{aligned}$$

Therefore, $C_i(\{s^1, s^t\} \neq \{s^t\}$, that is, C_i is weakly acyclic.

Part 2. Path independent choice functions are acceptable-consistent.
Consider a path independent choice function C_i and distinct institutions $s, s' \in S$ such that $C_i(\{s\}) = \{s\}$ and $C_i(\{s'\}) = \varnothing$. Path independence implies that

$$\begin{aligned} C_i(\{s, s'\}) &= C_i(\{s\} \cup C_i\{s'\}) \\ &= C_i(\{s\}) \\ &\neq \{s'\}. \end{aligned}$$

Therefore, C_i is acceptable-consistent.

Lemma 2

Proof. Consider the following construction for each individual $i \in I$:

Step 0: First, consider the unacceptable assignments, that is, $\{s \in S : C_i(\{s\}) = \varnothing\}$ and order them as $\varnothing P_i s$. Second, consider the acceptable assignments $S^0 = S' \setminus \{s \in S : C_i(\{s\}) = \varnothing\}$. If $S^0 = \varnothing$ the construction stops. Otherwise consider the set of C-maximal institutions $U(S^0) = \{s \in S^0 : \{C_i(\{s, s'\}) = \{s'\} \text{ for some } s' \in S \setminus \{s\}\} = \varnothing\}$ — which is non-empty by Lemma 1. Let

(i) sPs' for all $s \in U(S^0)$ and $s' \in S^0$,

(ii) order $s, s' \in U(S^0)$ in any order, that is, sPs' or $s'Ps$.

Step $k \geq 1$: Consider $S^k = S^{k-1} \setminus U(S^{k-1})$. If $S^k = \varnothing$ the construction stops. Otherwise consider the set of C-maximal institutions $U(S^k)$ —- which is non-empty by Lemma 1. Let

(i) sPs' for all $s \in U(S^k)$ and $s' \in S^k$,

(ii) order $s, s' \in U(S^k)$ in any order, that is, sPs' or $s'Ps$.

This construction yields a simple order over acceptable choices satisfying the conditions that (1) if $C_i(\{s, s'\}) = \{s\}$ then $s\ P_i\ s'$; (2) if $C_i(\{s\}) = \{s\}$ then $s\ P_i\ \varnothing$; and, (3) if $C_i(\{s\}) = \varnothing$ then $\varnothing\ P_i\ s$.

□

Proposition 2

Proof. Fix an admissions problem Γ with choice functions that are weakly acyclic and acceptable-consistent.

The "if" part: If a matching is stable for some associated proxy admissions problem then it is also stable for the admissions problem.

Consider a one-to-one mapping $f : \Gamma \mapsto \Gamma_{\mathcal{P}}$ such that $f(\gamma) \in \Gamma_\gamma$ — which is non-empty by Lemma 2. Let $\hat{\psi}^s$ denote a stable mechanism for proxy admissions problems. Now consider the mechanism ψ^s for admissions problems such that $\psi^s[\gamma] \equiv \hat{\psi}^s[f(\gamma)]$ for all $\gamma \in \Gamma$.

(i) Suppose $\psi^s[\gamma]$ is not individually rational for some $\gamma \in \Gamma$, then there exists $i \in I$ with $\psi^s[\gamma](i) \neq \varnothing$ such that $C_i(\psi^s[\gamma](i)) = \varnothing$. This implies that in the associated proxy admissions problem $\varnothing \; P_i \; \hat{\psi}^s[f(\gamma)](i)$. Thus contradicting that $\hat{\psi}^s$ is stable for proxy admissions problems.

(ii) Suppose there exist $\gamma \in \Gamma$ such that $\psi^s[\gamma]$ has a blocking pair (i, s), i.e., $C_i(\psi^s[\gamma](i) \cup s) = s$ and $i \pi_s i'$ for some $i' \in \psi^s[\gamma](s)$, or $|\psi^s[\gamma](s)| < q_s$ with $i \pi_s \varnothing$. Note that, $C_i(\psi^s[\gamma](i) \cup s) = s$ implies that in the associated proxy admissions problem $s \; P_i \; \hat{\psi}^s[f(\gamma)]$. Thus contradicting that $\hat{\psi}^s$ is stable for proxy admissions problems.

The converse may not hold: A stable matching for the admissions problem may not be stable for any associated proxy admissions problem in Γ_γ.

See Example 2. □

Proposition 3

Proof. Fix an admissions problem Γ with choice functions that are path independent.

The "if" part: If a matching is stable for some associated proxy admissions problem then it is also stable for the admissions problem.

Since path independent choice functions are weakly acyclic and acceptable-consistent (Proposition 1), this part is a corollary to Proposition 2.

The converse holds: A stable matching for the admissions problem is also stable for some associated proxy admissions problem in Γ_γ.

Notice that with path independent choice functions there is a unique proxy admissions problem for every admissions problem with the preferences constructed in the same way as in Theorem 2.

Consider a one-to-one mapping $f : \Gamma \mapsto \Gamma_{\mathcal{P}}$ such that $f(\gamma) \in \Gamma_\gamma$ — where P the simple order over acceptable choices is constructed in the same way as in Theorem 2. Let ψ^s denote a stable mechanism for admissions problems. Now consider the mechanism $\hat{\psi}^s$ for the proxy admissions problems such that $\hat{\psi}^s[f(\gamma)] \equiv \psi^s[\gamma]$ for all $\gamma \in \Gamma$.

(i) Suppose $\hat{\psi}^s[f(\gamma)]$ is not individually rational for some $\gamma \in \Gamma$, then there exists $i \in I$ with $\hat{\psi}^s[f(\gamma)](i) \neq \varnothing$ such that $\varnothing \; P_i \; \hat{\psi}^s[f(\gamma)](i)$. This implies that in the associated admissions problem $C_i(\psi^s[\gamma](i)) = \varnothing$. Thus contradicting that ψ^s is stable for proxy admissions problems.

(ii) Suppose there exist $\gamma \in \Gamma$ such that $\hat{\psi}^s[f(\gamma)]$ has a blocking pair (i, s), that is, $s\ P_i\ \hat{\psi}^s[f(\gamma)]$ and $i\pi_s i'$ for some $i' \in \hat{\psi}^s[f(\gamma)](s)$, or $|\hat{\psi}^s[f(\gamma)](s)| < q_s$ with $i\pi_s\varnothing$. Note that, $s\ P_i\ \hat{\psi}^s[f(\gamma)]$ implies that in the associated admissions problem $C_i(\psi^s[\gamma](i) \cup s) = s$. Thus contradicting that ψ^s is stable for proxy admissions problems.

□

Theorem 3

We start by defining a related proxy admission problem for any assignment problem, as well as the stability and strategy-proofness in the proxy admission problem. In a second step, we will use the connection between the two problems to prove our result.

Recall the **proxy admissions problem** $\gamma_P = \langle I, S, q, P, \pi\rangle$ for admissions problem $\gamma = \langle I, S, q, C, \pi\rangle$, where P is a profile of strict simple orders over acceptable choices satisfying the following conditions for each choice function:

(1) If $C_i(\{s, s'\}) = \{s\}$ then $s\ P_i\ s'$;

(2) If $C_i(\{s\}) = \{s\}$ then $s\ P_i\ \varnothing$; and,

(3) If $C_i(\{s\}) = \varnothing$ then $\varnothing\ P_i\ s$.

We denote the set of all proxy admission problems by $\Gamma_{\mathcal{P}}$.

A **mechanism for proxy admissions problems** is a function $\hat{\psi} : \Gamma_{\mathcal{P}} \to \mathcal{M}$ that assigns a matching $\hat{\psi}[\gamma_P] \in \mathcal{M}$ to each proxy admission problem $\gamma_P \in \Gamma_{\mathcal{P}}$. A mechanism $\hat{\psi}$ is said to be **incentive compatible (for individuals) for proxy admissions problems** if for any $\gamma_P = \langle I, S, q, P, \pi\rangle$ there does not exist $\gamma_{\hat{P}} = \langle I, S, q, (\hat{P}_i, P_{-i}), \pi\rangle$ such that

$$\hat{\psi}[\gamma_{\hat{P}}](i)\ P_i\ \hat{\psi}[\gamma_P](i).$$

A matching μ is **stable for proxy admissions problems** γ_P if

(1) it is **individually rational**, that is, there is no individual i such that $\varnothing\ P_i\ \mu(i)$ and no institution s such that $\varnothing\ \pi_s\ i$ for some $i \in \mu(s)$, and

(2) there is no **blocking pair**, that is, there is pair $(i, s) \in I \times S$ such that

   (a) $s\ P_i\ \mu(i)$, and

   (b) (i) either $i\ \pi_s\ i'$ for some $i' \in \mu(s)$, or

   (ii) $|\mu(s)| < q_s$ and $i\ \pi_s\ \varnothing$.

A mechanism $\hat{\psi}$ for proxy admissions problems is said to be **stable for proxy admissions problems** if it assigns a stable matching $\hat{\psi}[\gamma_P]$ to each proxy admission problem $\gamma_P \in \Gamma_{\mathcal{P}}$. Let $\hat{\psi}^{GS}$ denote the individual-proposing deferred acceptance algorithm defined in Gale and Shapley (1962). Recall that this mechanism is both stable and incentive compatible.

Proposition 7 (Gale and Shapley (1962))**.** *The individual-proposing deferred acceptance mechanism $\hat{\psi}^{GS}$ is stable for proxy admissions problems.*

Proposition 8 (Dubins and Freedman (1981), Roth (1982))**.** *The individual-proposing deferred acceptance mechanism $\hat{\psi}^{GS}$ is incentive compatible (for individuals) for proxy admissions problems.*

We now prove Theorem 3.

Proof. **The "if" part.** Consider a one-to-one mapping $f : \Gamma \mapsto \Gamma_{\mathcal{P}}$ such that $f(\gamma) \in \Gamma_\gamma$ — which is non-empty by Lemma 2. Moreover let f be such that for any $\gamma = \langle I, S, q, C, \pi\rangle$ and $\hat{\gamma} = \langle I, S, q, (\hat{C}_i, C_{-i}), \pi\rangle$ the proxy admissions problem only differ by the preference relation of individual i. Such a requirement can be easily accommodated following a construction akin to the one in Lemma 2.

Consider the mechanism ψ^{GS} such that $\psi^{GS}[\gamma] \equiv \hat{\psi}^{GS}[f(\gamma)]$ for all $\gamma \in \Gamma$.

(i) Suppose $\psi^{GS}[\gamma]$ is not individually rational for some $\gamma \in \Gamma$, then there exists $i \in I$ with $\psi^{GS}[\gamma](i) \neq \varnothing$ such that $C_i(\psi^{GS}[\gamma](i)) = \varnothing$. This implies that in the associated proxy admissions problem $\varnothing\ P_i\ \hat{\psi}^{GS}[f(\gamma)](i)$. Thus contradicting that $\hat{\psi^{GS}}$ is stable for every proxy admissions problem (Proposition 7).

(ii) Suppose there exist $\gamma \in \Gamma$ such that $\psi^{GS}[\gamma]$ has a blocking pair (i, s), i.e., $C_i(\psi^{GS}[\gamma](i) \cup s) = s$ and $i\pi_s i'$ for some $i' \in \psi^{GS}[\gamma](s)$, or $|\psi^{GS}[\gamma](s)| < q_s$ with $i\pi_s\varnothing$. Note that, $C_i(\psi^{GS}[\gamma](i) \cup s) = s$ implies that in the associated proxy admissions problem $s\ P_i\ \hat{\psi}^{GS}[f(\gamma)]$. Thus contradicting that $\hat{\psi}^{GS}$ is stable for every proxy admissions problem (Proposition 7).

(iii) Suppose ψ^{GS} is not incentive compatible for some $\gamma \in \Gamma$. That is, there exist $\gamma = \langle I, S, q, C, \pi\rangle$ and $\hat{\gamma} = \langle I, S, q, (\hat{C}_i, C_{-i}), \pi\rangle$ such that $\psi^{GS}[\gamma](i) \neq \psi^{GS}[\hat{\gamma}](i)$ and $C_i(\psi^{GS}[\hat{\gamma}](i) \cup \psi^{GS}[\gamma](i)) = \psi^{GS}[\hat{\gamma}](i)$. By construction, in the proxy admissions problem we have that $\hat{\psi}^{GS}[f(\hat{\gamma})](i)\ P_i\ \hat{\psi}^{GS}[f(\gamma)](i)$, thus contradicting that $\hat{\psi}^{GS}$ is incentive compatible (Proposition 8).

The "only if" part.

Part 1. Weak Acyclicity is necessary.

Suppose that for some C_i we have a cycle for $t \geq 3$ and distinct acceptable alternatives $s^1, s^2, \ldots, s^t$ in S such that $C_i(\{s^1, s^2\}) = \{s^2\}, \ldots C_i(\{s^{t-1}, s^t\}) = \{s^t\}$, and $C_i(\{s^t, s^1\}) = \{s^1\}$.

Consider an admissions problem $\tilde{\gamma} = \langle I, S, q, C, \tilde{\pi} \rangle$ such that $i \; \tilde{\pi}_s \; i'$ for all $i' \in I \setminus \{i\}$ and $s \in \{s^1, s^2, \ldots, s^t\}$ and $\varnothing \; \tilde{\pi}_s \; i$ for all $s \in S \setminus \{s^1, s^2, \ldots, s^t\}$. Moreover, $\tilde{\pi}$ is such that $|\{i' \in I \setminus \{i\} : i' \; \tilde{\pi}_s \; \varnothing\}| < q_s$ for all $s \in \{s^1, s^2, \ldots, s^t\}$ — which there is at least one such priority order as $q_s \geq 1$.

If $\psi[\gamma](i) = \{s\}$ for $s \in S \setminus \{s^1, s^2, \ldots, s^t\}$ we have a violation of individual rationality for the appropriate institution. Suppose then that the outcome of a mechanism is $\psi[\gamma](i) = \{s\}$ for some $s \in \{s^1, s^2, \ldots, s^t\}$ or $\psi[\gamma](i) = \varnothing$. Consider the following problem $\hat{\gamma} = \langle I, S, q, (\hat{C}_i, C_{-i}), \tilde{\pi} \rangle$ with $\hat{C}_i$ as follows:

(i) $\hat{C}_i(\{s\}) = \{s\}$ for some $s \in \{s^1, s^2, \ldots, s^t\} \setminus \{\psi[\gamma](i)\}$ such that $C_i(\psi[\gamma](i) \cup \{s\}) = \{s\}$, and

(ii) $\hat{C}_i(\{s'\}) = \varnothing$ for all $s' \in \{s^1, s^2, \ldots, s^t\} \setminus \{s\}$.

By individual rationality and weak non-wastefulness, for any mechanism ψ we have $\psi[\hat{\gamma}](i) = \{s\}$. With $\psi[\gamma](i) \neq \psi[\hat{\gamma}](i)$ and $C_i(\psi[\hat{\gamma}](i) \cup \psi[\gamma](i)) = \psi[\hat{\gamma}](i)$, ψ therefore violates incentive compatibility.

Part 2. Acceptable-consistency is necessary.

Suppose that for some C_i we have $C_i(\{s^1\}) = \{s^1\}$ and $C_i(\{s^2\}) = \varnothing$ but $C_i(\{s^1, s^2\}) = \{s^2\}$.

Consider an admissions problem $\tilde{\gamma} = \langle I, S, q, C, \tilde{\pi} \rangle$ such that $i \; \tilde{\pi}_s \; i'$ for all $i' \in I \setminus \{i\}$ and $s \in \{s^1, s^2\}$ and $\varnothing \; \tilde{\pi}_s \; i$ for all $s \in S \setminus \{s^1, s^2\}$. Moreover, $\tilde{\pi}$ is such that $|\{i' \in I \setminus \{i\} : i' \; \tilde{\pi}_s \; \varnothing\}| < q_s$ for all $s \in \{s^1, s^2\}$.

If $\psi[\gamma](i) = \{s\}$ for $s \in S \setminus \{s^1, s^2\}$ we have a violation of individual rationality for the appropriate institution. By weak non-wastefulness we have that for any mechanism $\psi[\gamma](i) \neq \varnothing$ and by individual rationality we have $\psi[\gamma](i) \neq s^2$. Therefore we have $\psi[\gamma](i) = \{s^1\}$.

Consider the following problem $\hat{\gamma} = \langle I, S, q, (\hat{C}_i, C_{-i}), \tilde{\pi} \rangle$ with $\hat{C}_i$ as follows:

(i) $\hat{C}_i(\{s^2\}) = \{s^2\}$, and

(ii) $\hat{C}_i(\{s\}) = \varnothing$ for all $s \in S \setminus \{s^2\}$

By individual rationality and weak non-wastefulness, for any mechanism ψ we have $\psi[\hat{\gamma}](i) = \{s^2\}$. With $\psi[\gamma](i) \neq \psi[\hat{\gamma}](i)$ and $C_i(\psi[\hat{\gamma}](i) \cup \psi[\gamma](i)) = \psi[\hat{\gamma}](i)$, ψ therefore violates incentive compatibility. $\square$

Proposition 5

Lemma 4. *Consider a path independent choice function C_i, then choice from a set is pairwise preferred to alternatives in the set, that is,*

$$C_i(S) = \{s\} \implies C_i(\{s, s'\}) = \{s\} \quad \textit{for all} \quad s' \in S \setminus \{s\}.$$

Proof. Suppose $C_i(S) = \{s\}$ and $C_i(\{s, s'\}) \neq \{s\}$ for some $s' \in S \setminus \{s\}$. Then by path independence we have that, $C_i(S) = C_i\big(C_i(\{s, s'\}) \cup (S \setminus \{s, s'\})\big) \neq \{s\}$ since $C_i\{s, s'\} \neq \{s\}$, a contradiction.

$\square$

We now prove Proposition 5.

Proof. **The "if" part:** If choice functions are path independent, the simultaneous cut-offs mechanism ψ^{sim} yields a stable matching for every admissions problem.

We start with individual rationality. By definition, a institution s never announces cut-off c_s lower than m_s the lowest merit scores at institution s. Hence, for all $\gamma \in \Gamma$, $s \in S$, we have that $i \in \psi^{sim}[\gamma](s)$ implies $i \ \pi_s \ \varnothing$. Similarly for individuals, due to step 0 where individuals apply to only acceptable institutions, individual i is assigned only acceptable institution s. Hence, for all for all $\gamma \in \Gamma$, $i \in I$, we have that $\psi^{sim}[\gamma](i) \neq \varnothing$ implies $C_i(\{\psi^{sim}[\gamma](i)\}) = \{\psi^{sim}[\gamma](i)\}$.

Moving on to the blocking pairs. First, an individual i can never block with a institution $s \in S$ that never proposed to her during the simultaneous cut-offs algorithm, as in that case either the institution has filled its capacity with individuals that have a higher merit score than i and/or individual i is unacceptable. Second, due to path independence, individual i will never block an acceptable institution with an unacceptable one. Combining both observations, it suffices to consider the sequence of sets of (acceptable) institutions proposing to individual i during the simultaneous cut-offs algorithm. For some admission problem $\gamma \in \Gamma$ and individual i let $(S^1, \ldots, S^K)[\gamma](i)$ denote a sequence of institutions proposing to i during the sequential cut-offs mechanism. That is, S^1 proposes first and so on and so forth until S^K, which proposes last.

If the sequence is empty, then there is no institution willing to block with i. Otherwise by Lemma 4, individual i holds a C-maximal institution at every step of the simultaneous cut-offs algorithm. Therefore, the outcome $\psi^{sim}[\gamma](i) \in (S^1 \cup \cdots \cup S^K)$ is such that $C_i(\{\psi^{sim}[\gamma](i), s'\}) = \{\psi^{sim}[\gamma](i)\}$ for all $s' \in (S^1 \cup \cdots \cup S^K) \setminus \{\psi^{sim}[\gamma](i)\}$.

The "only if" part: The simultaneous cut-offs mechanism ψ^{sim} is stable for every admissions problem only if the choice functions are path independent. The contrapositive is, if some choice functions are not path independent then the simultaneous cut-offs mechanism ψ^{sim} is not stable, that is, there exists at least one admission problem $\gamma \in \Gamma$ for which the outcome $\psi^{sim}[\gamma] \in \mathcal{M}$ is not stable.

Consider a violation of path independence $C_i(S' \cup S'') \neq C_i(C_i(S') \cup S'')$ for individual i. Let $C_i(S' \cup S'') \equiv s^1 \in S \cup \{\varnothing\}$ and $C_i(C_i(S') \cup S'') \equiv s^2 \in S \cup \{\varnothing\}$, with $s^1 \neq s^2$. Moreover, consider the following admission problems:

(i) admissions problem $\tilde{\gamma}^1 \in \Gamma$ such that $i\ \tilde{\pi}^1{}_s\ i'$ for all $i' \in I \setminus \{i\}$ and $s \in S' \cup S''$ and $\varnothing\ \tilde{\pi}^1{}_s\ i$ for all $s \in S \setminus (S' \cup S'')$;

(ii) admissions problem $\tilde{\gamma}^2 \in \Gamma$ such that $i\ \tilde{\pi}^2{}_s\ i'$ for all $i' \in I \setminus \{i\}$ and $s \in C_i(S') \cup S''$ and $\varnothing\ \tilde{\pi}^2{}_s\ i$ for all $s \in S \setminus (C_i(S') \cup S'')$; and

(iii) admissions problem $\tilde{\gamma}^3 \in \Gamma$ such that $i\ \tilde{\pi}^3{}_s\ i'$ for all $i' \in I \setminus \{i\}$ and $s \in S'$ and $\varnothing\ \tilde{\pi}^3{}_s\ i$ for all $s \in S \setminus S'$.

Case 1. Suppose $C_i(\{s^1, s^2\}) = \{s^2\}$.
We have $\psi^{sim}[\tilde{\gamma}^1](i) = \{s^1\}$ as all institutions in $S' \cup S''$ propose to i in the first round and no other institution proposes. Moreover, as $s^2 \in S' \cup S''$ individual i has the highest priority at s^2. Slightly abusing notation, note that there are three possibilities:

(a) $s^1, s^2 \in S$, or

(b) $s^1 = \varnothing$ and $s^2 \in S$, or

(c) $s^2 = \varnothing$ and $s^1 \in S$.

Under case (a) and (b) we have a blocking pair (i, s^2), while under (c) we have a violation of individual rationality as s^1 is unacceptable.

This distinction holds for the remaining cases — instead, we will simply write that some (i, s) constitutes a blocking pair.

Case 2. Suppose $C_i(\{s^1, s^2\}) = \{s^1\}$ and $s^1 \in C_i(S') \cup S''$.
We have $\psi^{sim}[\tilde{\gamma}^2](i) = \{s^2\}$ as all institutions in $C_i(S') \cup S''$ propose to i in the first round and no other institution proposes. Moreover, as $s^1 \in C_i(S') \cup S''$ — and therefore i has the highest priority at s^1 — we have a blocking pair (i, s^1).

Case 3. Suppose $C_i(\{s^1, s^2\}) = \{s^1\}$ and $s^1 \in S' \setminus C_i(S')$ and $s^2 = C_i(S')$.
We have $\psi^{sim}[\tilde{\gamma}^3](i) = \{s^2\}$ as all institutions in S' propose to i in the first round and no other institution proposes. Moreover, as $s^1 \in S'$ - we have a blocking pair (i, s^1).

Case 4. Suppose $C_i(\{s^1, s^2\}) = \{s^1\}$ and $s^1 \in S'$ and $s^2 \in S''$.
Let $C_i(S') \equiv s^3$.

Case 4.1. Suppose $C_i(\{s^1, s^3\}) = \{s^1\}$.
We have $\psi^{sim}[\tilde{\gamma}^3](i) = \{s^3\}$ as all institutions in S' propose to i in the first round and no other institution proposes. Moreover, as $s^1 \in S'$ we have a blocking pair (i, s^1).

Case 4.2. Suppose $C_i(\{s^1, s^3\}) = \{s^3\}$.
Recall that $\psi^{sim}[\tilde{\gamma}^1](i) = \{s^1\}$ as all institutions in $S' \cup S''$ propose to i in the first round and no other institution proposes. Similarly, as $s^3 \in S' \cup S''$ we have a blocking pair (i, s^3). □

Proposition 6

Lemma 5. *Consider an acyclic and acceptable-consistent choice function C_i and a finite sequence of acceptable alternatives $s^1, \dots s^K$ that propose to individual i (one at a time) under the sequential cut-offs algorithm. That is, s^1 proposes first and so on and so forth until s^K, which proposes last. If the individual's final choice is s^*, then $C_i(\{s^k, s^*\}) \neq \{s^k\}$ for any $s^k \in \{s^1, \dots s^K\} \setminus \{s^*\}$.*

Proof. Let $s^{(j)}$ denote the tentative assignment of individual i when it receives proposal from institution s^j. Therefore, when institution s^j proposes individual i's choice menu is $\{s^j, s^{(j)}\}$.

Let individual's final choice be s^*, that is, $C_i(\{s^K, s^{(K)}\}) = s^*$. Since $s^1, \dots s^K$ are acceptable alternatives, by acceptable-consistency $s^* \neq \varnothing$. Suppose there exists $s^k \in \{s^1, \dots s^K\} \setminus \{s^*\}$ such that $C_i(\{s^k, s^*\}) = \{s^k\}$. There are two possibilities to consider.

(i) Suppose $C_i(\{s^k, s^{(k)}\}) = s^k$. Since s^k is not the final choice there must exist $s^{k'} \in \{s^{k+1}, \dots s^K\}$ such that $C_i(\{s^k, s^{k'}\}) = \{s^{k'}\}$. If $s^{k'} = s^*$ we have a contradiction. Otherwise, if $s^{k'}$ is not the final choice there must exists $s^{k''} \in \{s^{k'+1}, \dots s^K\}$ such that $C_i(\{s^{k'}, s^{k''}\}) = \{s^{k''}\}$. By acyclicity, $C_i(\{s^k, s^{k'}\}) = \{s^{k'}\}$ and $C_i(\{s^{k'}, s^{k''}\}) = \{s^{k''}\}$ imply that $C_i(\{s^k, s^{k''}\}) = \{s^{k''}\}$. If $s^{k''} = s^*$ we have a contradiction. Otherwise, if $s^{k''}$ is not the final choice we can finitely repeat the same steps until we arrive at the final choice, which due to acyclicity will lead to a contradiction.

(ii) Suppose $C_i(\{s^k, s^{(k)}\}) \neq s^k$. If $s^{(k)} = s^*$ we have a contradiction. Otherwise, if $s^{(k)}$ is not the final choice there must exist $s^{k'} \in \{s^{k+1}, \dots s^K\}$ such that $C_i(\{s^{(k)}, s^{k'}\}) = \{s^{k'}\}$. If $s^{k'} = s^*$, then by acyclicity $C_i(\{s^k, s^*\}) = \{s^k\}$ and $C_i(\{s^{(k)}, s^{k'}\}) = \{s^{k'}\}$ imply that $C_i(\{s^k, s^{(k)}\}) = s^k$, therefore we have a contradiction. Otherwise, if $s^{k'}$ is not the final choice there must exists $s^{k''} \in \{s^{k'+1}, \dots s^K\}$ such that $C_i(\{s^{k'}, s^{k''}\}) = \{s^{k''}\}$. By acyclicity, $C_i(\{s^{(k)}, s^{k'}\}) = \{s^{k'}\}$ and $C_i(\{s^{k'}, s^{k''}\}) = \{s^{k''}\}$ imply that

$C_i(\{s^{(k)}, s^{k''}\}) = \{s^{k''}\}$. If $s^{k''} = s^*$, then by acyclicity $C_i(\{s^k, s^*\}) = \{s^k\}$ and $C_i(\{s^{(k)}, s^{k''}\}) = \{s^{k''}\}$ imply that $C_i(\{s^k, s^{(k)}\}) = s^k$, therefore we have a contradiction. Otherwise, if $s^{k''}$ is not the final choice we can finitely repeat the same steps until we arrive at the final choice, which due to acyclicity will lead to a contradiction.

□

We now prove Proposition 6.

Proof. **The "if" part:** If choice functions are weakly acylic and acceptable-consistent, the sequential cut-offs mechanism ψ^{seq} yields a stable matching for every admissions problem.

We start with individual rationality. By definition, a institution s never announces cut-off c_s lower than m_s the lowest merit scores at institution s. Hence, for all $\gamma \in \Gamma$, $s \in S$, we have that $i \in \psi^{seq}[\gamma](s)$ implies $i \; \pi_s \; \varnothing$. Similarly for individuals, due to step 0 where individuals apply to only acceptable institutions, individual i is assigned only acceptable institution s. Hence, for all for all $\gamma \in \Gamma$, $i \in I$, we have that $\psi^{seq}[\gamma](i) \neq \varnothing$ implies $C_i(\{\psi^{seq}[\gamma](i)\}) = \{\psi^{seq}[\gamma](i)\}$.

Moving on to the blocking pairs. First, an individual i can never block with a institution $s \in S$ that never proposed to her during the sequential cut-offs algorithm, as in that case either the institution has filled its capacity with individuals that have a higher merit score than i and/or individual i is unacceptable. Second, due to acceptable-consistency, individual i will never block an acceptable institution with an unacceptable one. Combining both observations, it suffices to consider the sequence of (acceptable) institutions proposing to individual i during the sequential cut-offs algorithm. For some admission problem $\gamma \in \Gamma$ and individual i let $(s^1, \ldots, s^K)[\gamma](i)$ denote a sequence of institutions proposing to i during the sequential cut-offs mechanism. That is, s^1 proposes first and so on and so forth until s^K, which proposes last.

If the sequence $K = 2$, then there is no institution willing to block with i. Otherwise by Lemma 5, individual i holds a C-maximal institution at every step of the sequential cut-offs algorithm. Therefore, the outcome $\psi^{seq}[\gamma](i) \in \{s^1, \ldots, s^K\}$ is such that $C_i(\{\psi^{seq}[\gamma](i), s'\}) = \{\psi^{seq}[\gamma](i)\}$ for all $s' \in \{s^1, \ldots, s^K\} \setminus \{\psi^{seq}[\gamma](i)\}$.

The "only if" part: The sequential cut-offs mechanism ψ^{seq} is stable for every admissions problem only if the choice functions are weakly acylic and acceptable-consistent. The contrapositive is, if some choice functions are not weakly acylic and acceptable-consistent then the sequential cut-offs mechanism ψ^{seq} is not stable, that is, there exists at least one admission problem $\gamma \in \Gamma$ for which the outcome $\psi^{seq}[\gamma] \in \mathcal{M}$ is not stable.

Part 1. Acylicity is necessary.

Case 1: Suppose that for some C_i we have a cycle for $t \geq 3$ and distinct alternatives $s^1, s^2, \ldots, s^t$ in S such that $C_i(\{s^1, s^2\}) = \{s^2\}, \ldots C_i(\{s^{t-1}, s^t\}) = \{s^t\}$, and $C_i(\{s^t, s^1\}) = \{s^1\}$.

Consider an admission problem $\tilde{\gamma} \in \Gamma$ such that $i \ \tilde{\pi}_s \ i'$ for all $i' \in I \setminus \{i\}$ and $s \in \{s^1, s^2, \ldots, s^t\}$ and $\varnothing \ \tilde{\pi}_s \ i$ for all $s \in S \setminus \{s^1, s^2, \ldots, s^t\}$.

Given the definition of the sequential cut-offs mechanism, we have that $\psi^{seq}[\tilde{\gamma}](i) \in \{s^1, s^2, \ldots, s^t\}$, regardless of outcome we have a blocking pair proving the claim. If $\psi^{seq}[\tilde{\gamma}](i) = \varnothing$ the same holds, as all institutions in $\{s^1, s^2, \ldots, s^t\}$ are acceptable, leading to a blocking pair.

Case 2: Suppose that for some C_i we have a cycle for $t \geq 3$ and distinct alternatives $s^1, s^2, \ldots, s^t$ in S such that $C_i(\{s^1, s^2\}) = \{s^2\}, \ldots C_i(\{s^{t-1}, s^t\}) = \{s^t\}$, and $C_i(\{s^t, s^1\}) = \varnothing$.

Consider an admission problem $\tilde{\gamma} \in \Gamma$ such that $i \ \tilde{\pi}_s \ i'$ for all $i' \in I \setminus \{i\}$ and $s \in \{s^1, s^2, \ldots, s^t\}$ and $\varnothing \ \tilde{\pi}_s \ i$ for all $s \in S \setminus \{s^1, s^2, \ldots, s^t\}$.

In the sequential cut-offs mechanism, $s^1, s^2, \ldots, s^t$ will propose to i and no other institution will propose to i. Suppose s^1 proposes before s^t and s^t proposes before s^2 and s^2 proposes before s^3 and so on until s^{t-1}. We get $\psi^{seq}[\tilde{\gamma}](i) = \{s^{t-1}\}$. But in this case there is a blocking pair as $C_i(\{s^{t-1}, s^t\}) = \{s^t\}$.

Part 2. Acceptable-consistency is necessary.

Suppose that for some C_i we have $C_i(\{s\}) = \{s\}$ and $C_i(\{s'\}) = \varnothing$ but $C_i(\{s, s'\}) = \{s'\}$.

Consider an admission problem $\tilde{\gamma} \in \Gamma$ such that $i \ \tilde{\pi}_s \ i'$ for all $i' \in I \setminus \{i\}$ and $s \in \{s^1, s^2\}$ and $\varnothing \ \tilde{\pi}_s \ i$ for all $s \in S \setminus \{s^1, s^2\}$.

In the sequential cut-offs mechanism, both s^1 and s^2 will propose to i and no other institution will propose to i. If s^1 proposes before s^2 we get $\psi^{seq}[\tilde{\gamma}](i) = \{s^2\}$ which violates individual rationality as $C_i(\{s^2\}) = \varnothing$. If s^2 proposes before s^1 we get $\psi^{seq}[\tilde{\gamma}](i) = \{s^1\}$. In this case there is a blocking pair as $C_i(\{s^1, s^2\}) = \{s^2\}$ and i has the highest priority at s^2. □

Proposition 9

Recall that, from an institutional viewpoint there are no complementarities between individuals, so the priority order π_s and capacity q_s of an institution s translate into a (partial order) preference over sets of individuals in a straightforward way. Formally, let $\succsim_s$ denote the preferences of institution s over 2^I, and $\succ_s$ denote strict preferences derived from it. We assume that college preferences are responsive. Formally, $\succsim_s$ is **responsive** (Roth (1985)) if,

(i) for any $I' \subset I$ with $|I'| < q_s$ and any $i \in I \setminus I'$,

$$(I' \cup \{i\}) \succ_s I' \iff i \,\pi_s\, \varnothing,$$

(ii) for any $I' \subset I$ with $|I'| < q_s$ and any $i, i' \in I \setminus I'$,

$$(I' \cup \{i\}) \succ_s (I' \cup \{i'\}) \iff i \,\pi_s\, i'.$$

We say matching μ is **blocked by a coalition** T of individuals and institutions, if there exists another matching ν and coalition T, such that for all $i \in T$ and $s \in T$,

(i) $\nu(i) \in T$,

(ii) $C_i(\{\nu(i), \mu(i)\}) = \nu(i)$ for all $i \in T$,

(iii) $\nu(s) \succ_s \mu(s)$ for all $s \in T$, and

(iv) if $j \in \nu(s)$, then $j \in T \cup \mu(s)$.

The first condition states that every individual in T who is matched by ν is matched to some institution in T. The second condition states that every individual in T chooses its assignment under ν over her assignment under μ. The third condition states that every institution in T strictly prefers its set of individuals under ν to that under μ. The last condition states that any new individual matched to an institution in the coalition must be a member of T.

A **group stable** matching is one that is not blocked by any coalition. Pairwise stability is equivalent to group stability in the standard setup of many-to-one matching markets (see Roth and Sotomayor (1990)). The following result shows that the same result holds in our setup.

Proposition 9. *A matching is group stable if and only if it is stable.*

Proof. Suppose μ is not stable due to an unacceptable individual (institution) assigned to an institution (individual), or a blocking pair. Then it is not group stable because it is blocked by the coalition consisting of the individual (institution), or the blocking pair respectively.

In the other direction, if μ is blocked by coalition T and matching ν. We show that we can always construct a blocking pair or a violation of individual rationality from T.

Suppose no institution is part of T, then there exists an individual $i \in T$ such that $\nu(i) = \varnothing$. By definition of the blocking coalition we have $C_i(\{\nu(i), \mu(i)\}) = \varnothing$ with $\mu(i) \in S$. Therefore, we have found a violation of individual rationality and thus pairwise stability.

Suppose no individual is part of T, then there exists an institution $s \in T$ such that $\nu(s) \subset \mu(s)$. By definition of the blocking coalition we have $\nu(s) \succ_s \mu(i)$ and by responsiveness some $i \in \mu(i) \setminus \nu(i)$ with $\varnothing \pi_s i$. Therefore, we have found a violation of individual rationality (on the institution side) and thus pairwise stability.

Finally, the coalition contains both individuals and institutions. Then pick institution $s \in T$ with $\nu(s) \succ_s \mu(s)$. Suppose there exists at least an individual $i \in \nu(s) \setminus \mu(s)$. Then we either have (i) $i \succ_s j$ for some $j \in \mu(s) \setminus \nu(s)$ or (ii) $i \succ_s \varnothing$ and $|\mu(s)|$. By the definition of a blocking coalition we have $C_i(\{s, \mu(i)\}) = \{s\}$, and therefore i and s form a blocking pair. Otherwise, if $\nu(s) \subset \mu(s)$ we again can construct a violation of individual rationality (on the institution side).

$\square$