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Keywords: Parameter uncertainty, Bayesian learning, systematic consumption risk, investment-based asset pricing, SMM
JEL Classification: E2, E3, G12

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1 Introduction

The consumption-based asset pricing paradigm states that risk premia arise from the co-movement between consumption growth and returns. Despite its intuitive appeal, many early empirical tests did not find support for this prediction.[1] Inspired by a neoclassical investment model with a consumption-based pricing kernel, we propose a novel cash flow beta measured between consumption growth and productivity growth. We find strong support for a consumption-based pricing kernel because our cash flow betas predict investment rates, valuations ratios, and risk premia.

In our neoclassical investment model, firms' productivity is stochastic and correlated with consumption, rendering firms exposed to macroeconomic risk. We make two key assumptions about the parameter controlling the exposure of firms' productivity to consumption shocks. First, this parameter is constant over time but unknown. In response, agents learn about it through Bayesian updating. Second, firms in the same industry share the same exposure parameter. Because of the identical risk exposure among industry constituents, it is optimal for individual firms to incorporate peers' productivity as a signal into their learning about their own risk exposure. Using the collective observations of peers, firms learn from a rich source of cross-sectional information compared to learning from their own history only.[2]

Our model also features a consumption-based pricing kernel. As a result, the Bayesian beliefs about the productivity exposure parameter affect firm decisions and characteristics. Intuitively, an increase in risk exposure beliefs upon the arrival of new information means that the perceived covariance between productivity and consumption has increased. Due to the consumption-based pricing kernel, this elevated covariance translates into higher risk premia, even though the true risk exposure is constant. Higher discount rates also depress the value of new investment projects, thereby lowering investment rates and valuation ratios.

We test these predictions in the data using panel regressions and find strong support for them. We find that capital investment rates and valuation ratios, measured by Tobin's Q,

[1] Recent papers, which find support for the C-CAPM, include Lettau and Ludvigson (2001) and Hansen, Heaton, and Li (2008).

[2] While we are not the first to measure cash flow betas, see e.g. Bansal, Dittmar, and Lundblad (2005) and Da (2009), our cash flow betas are based on Bayesian learning using the cross section of productivity growth of industry constituents.

respond strongly negatively to the posterior mean risk exposure at the 1% significance level, controlling for other known cross-sectional determinants. These links are also economically significant. A one standard deviation increase in the mean risk exposure leads to, on average, a 4.2% decrease in investment and a 10.2% decrease in Tobin's Q.

In our model, these synchronous reactions are driven by endogenous shifts in discount rates that learning about risk exposure parameter induces. To test this conjecture, we employ both the implied cost of capital from accounting information[3] and realized stock returns as a proxy for discount rates. We find that both discount rate measures relate positively to the posterior mean risk exposure with statistical significance at the 1% level. Economically, a one standard deviation rise in the mean risk exposure is accompanied by, on average, a 0.76% increase in the annualized implied cost of capital and a 2.10% increase in the realized return.

A key identification assumption of the Bayesian learning is that firms learn from both their own history and from industry peers. In the data, we identify each firm's peers using the SIC, NAICS, and Hoberg and Phillips (2016) text-based industry classification systems. To illustrate the importance of industry peers, we consider three alternative forms of learning. First, firms learn from their own history only; second, they learn from their own history plus randomly assigned peer firms; and third, they learn from industry peer observations only, ignoring their own history. It turns out that the first two approaches, which do not include peer observations, result in insignificant links between firm variables and risk exposure beliefs. In contrast, beliefs in the last approach strongly predict firm variables, similar to our baseline results, highlighting information spillovers across peer firms. In sum, all three experiments support the assumption that industry peers share the same risk exposure parameter.

Our neoclassical investment model features a decreasing returns to scale production, convex adjustment costs, and Bayesian learning from peers about the risk exposure parameter. To quantitatively evaluate the model performance, we estimate with simulated method of moments the depreciation rate, capital share of production, adjustment cost parameter, idiosyncratic volatility, productivity exposure to consumption risk parameter and price of risk in the pricing kernel. We identify the six parameters based on eight moments. In the estimation, we include the mean and variance of the investment rate, equity returns, Tobin's Q,

[3]We measure the implied cost of capital following the approach of Hou, van Dijk, and Zhang (2012).

and the posterior mean of risk exposure.

Overall, the model matches the moments well. Both in the model and data, investment rates average around 27% annually with a standard deviation of 43%. Stock returns are also volatile at 60% in the model and data. The model can also match firm-level average excess returns of 11% with a price of risk of 1.65. The model also replicates fairly well average Q but it fails to match the volatility of Q. This shortcoming is shared by many neoclassical investment models, e.g., Nikolov and Whited (2014). The estimated risk exposure parameter is 7.4 even though the average posterior mean of risk exposure is only 5.6.

Even though we did not target any regression coefficients in the estimation, the model can quantitatively generate the negative response of investment rates and Tobin's Q to risk exposure beliefs. The model also produces a positive relationship between risk exposure beliefs and the cost of capital, which resembles the patterns in the data. In addition, we confirm that our model reproduces spillover effects. As in the data, beliefs about risk exposure driven only by industry peer observations negatively predicts both investment and the valuation ratio and, simultaneously, positively the cost of capital. These findings manifest the model's capability, that peer observations shape the beliefs about a firm's risk profile.

In a robustness test, we confirm that our main findings hold even when the true exposure to systematic risk is stochastic. Suppose that the true exposure changes over time, contrary to our baseline assumption. In this case, the learning-based estimate – derived from the constant-risk assumption – might misleadingly capture variations in true risk characteristics. To address this concern, we explicitly model the true risk exposure as an autoregressive process and estimate the beliefs distribution through the Kalman filter. In this setting, we focus on ambiguity about the unconditional mean of systematic risk, the true value of which is constant by nature. Similarly to the baseline findings, our empirical analysis establishes that the belief distribution for this particular parameter predicts firm observables. In essence, learning about the long-run mean of risk exposure continues to shape decisions in this alternative setting.

The remainder of this paper is organized as follows. In Section 2, we describe the dynamics of aggregate consumption and firm level productivity and derive the dynamics for the Bayesian learning. In Section 3, we present the empirical evidence linking beliefs about firms' risk exposure, investment, and valuation. These empirical results are rationalized by a neoclassical investment model with cross-sectional learning in Section 4. In Section 5, we show that our empirical results are robust to a setting with time variation in the true risk exposure.

Literature Review

Our paper builds on the literature, which studies parameter learning and its implications for asset valuations. Pastor and Veronesi (2009) provide a comprehensive review of learning models in finance. Jovanovic and Nyarko (1994), David (1997), Weitzman (2007), Collin-Dufresne, Johannes, and Lochstoer (2016), Johannes, Lochstoer, and Mou (2016) show that learning about parameters governing the economy generate regularities in asset returns and business cycles, which otherwise seem puzzling. Other papers study learning about an unknown aggregate state[4] or endogenous information acquisition such as Veldkamp (2006). Focusing on the aggregate level implications, however, the above studies have not examined how learning affects the cross section of corporate valuations and investments, which is the goal of this paper.

Our paper is also closely related to prior work, which highlights uncertainty about a firm-level parameter, including dividend growth (Veronesi (2000)), mean productivity (Pastor and Veronesi (2003); Alti (2003)), return-to-scale in the production function (Johnson (2007)), and mean cash flow (Andrei, Mann, and Moyen (2019)). Distinctively, we focus on ambiguity about exposure to macroeconomic risk, similar to the idea of Ai, Croce, Diercks, and Li (2018) and Li, Tsou, and Xu (2020). We complement these recent studies by elaborating upon the learning mechanism. In our model, agents learn from the history of realized productivity instead of noisy independent signals as in the prior studies. Furthermore, we propose learning from peer observations, which results in unique information spillovers.

The idea of learning from peers is similar to the study by Foucault and Fresard (2014), who document firm investment responding to peers' Tobin's Q. Our evidence further supports peer-inspired learning, linking it to a more comprehensive range of firm observables. In a different context, Boguth and Kuehn (2013) and Jurado, Ludvigson, and Ng (2015) underscore the importance of using a cross section of signals, which share a common truth. We extend the idea to the context of firm investment and show that cross-sectional information is crucial for

[4] These include Veldkamp (2005), Lettau, Ludvigson, and Wachter (2007), Ai (2010) and Croce, Lettau, and Ludvigson (2014).

optimal corporate decisions.

This study is also related to the literature on consumption-based asset pricing, including works by Bansal, Dittmar, and Lundblad (2005), Da (2009), and Boguth and Kuehn (2013). These studies reveal that the cross-sectional dispersion in expected returns is driven by the comovement between consumption growth and securities' cash flows. Distinct from these prior studies, we consider Bayesian learning about risk exposure from the comovement between productivity and consumption growth and expand the implications of consumption risk to the time series dimension.

More broadly, our work is also related to dynamic investment models. which examine the implications of firms' optimal decisions for asset returns. Prior studies, including Berk, Green, and Naik (1999), Gomes, Kogan, and Zhang (2003), Carlson, Fisher, and Giammarino (2004), Zhang (2005), and Kuehn and Schmid (2014), show that observed patterns in stock and bond returns arise as a result of corporate investment policy. We complement the literature by establishing new regularities about investment and returns caused by parameter learning.

Finally, our structural estimation is along the lines of Nikolov and Whited (2014) and Hennessy and Whited (2007), who apply the simulated method of moments to dynamic models. While the prior studies focus mainly on the impact of financing frictions on firm investment, we consider optimal firm policy under parameter uncertainty.

2 Bayesian Learning about Systematic Risk

In this section, we describe the dynamics of aggregate consumption and firm level productivity. Firms are exposed to aggregate consumption risk via their productivity process. Importantly, the exposure to consumption risk is unknown and has to be learned over time. Firms observe their own productivity growth and aggregate consumption growth. Moreover, firms observe productivity growth of their peers in the same industry as signals. This set of signals is informative because firms in the same industry share the same exposure to consumption risk. Based on this setup, we derive the dynamics for the Bayesian beliefs about the risk exposure parameter.

2.1 Dynamics

Aggregate consumption growth $g_{c,t+1}$ is normally distributed with mean μ and volatility σ_c and given by

$$g_{c,t+1} = \mu + \sigma_c \eta_{t+1}, \tag{1}$$

where η_{t+1} is an i.i.d. standard normal innovation. As in Kuehn and Schmid (2014), firm i productivity is stochastic and correlated with consumption, rendering firms exposed to macroeconomic risk. Specifically, firm-level productivity growth $g_{i,t+1}$ is a mixture of an idiosyncratic and aggregate shock and given by

$$g_{i,t+1} = \mu + b\sigma_c \eta_{t+1} + \sigma \varepsilon_{i,t+1}, \tag{2}$$

where $\varepsilon_{i,t+1}$ is a firm-specific i.i.d. standard normal innovation, σ quantifies the magnitude of idiosyncratic risk, and b controls the exposure of productivity to aggregate risk.

The risk exposure parameter b is our main focus. Intuitively, an increase in b amplifies the covariance between productivity and consumption and results in productivity displaying more systematic risk. In a consumption-based asset pricing framework, this elevated covariance translates into higher risk premia, thereby depressing optimal investment. We explore these theoretical channels in detail in Section 4.

We make two key assumptions about the risk exposure parameter. First, the parameter b is constant over time but unknown. In response, decision makers learn about it through Bayesian updating. Second, firms in the same industry share the same exposure parameter. These industry peers become heterogeneous ex post due to idiosyncratic productivity shocks, but they have a common characteristic.

A limitation of our specification is that the risk exposure does not change over time. This implies that in the long run agents can perfectly learn this parameter. In Section 5, we confirm that our main findings hold even when the true exposure to systematic risk is dynamic. We also provide empirical evidence, which justify the assumption that industry peers share the same risk exposure parameter, in Section 3.6 through three alternative forms of learning.

2.2 Learning

In this section, we derive the Bayesian beliefs about the risk exposure parameter b. Agents are equipped with prior beliefs about the parameter b, which are normally distributed with mean $m_{b,0}$ and standard deviation $\sigma_{b,0}$. Thereafter, they receive new information – the realized productivity of every industry constituent and consumption growth. Because of the identical risk exposure among industry constituents, peers' productivity should be informative about each other's risk exposure. Recognizing this, agents refer to the peers' collective observations in updating their parameter beliefs. Still, learning about b is nontrivial because productivity is subject to unobservable idiosyncratic shocks, which agents cannot distinguish from the systematic component.

To formulate the learning process, we let g_t denote the $(n_t \times 1)$ vector of productivity growth for n_t constituents of a specific industry at time t.[5] Conditional on the observations of productivity and consumption, beliefs about the parameter are revised according to Bayes' law. This learning mechanism induces a recursive structure of the posterior distribution

$$\text{Prob}(b|g_1, .., g_t, g_{c,1}, g_{c,t}) \propto \text{Prob}(g_t|b, g_{c,t}) \times \text{Prob}(b|g_1, .., g_{t-1}, g_{c,1}, g_{c,t-1})$$

where we use the fact that g_t depends only on current consumption growth and risk exposure. Since we assumed a Gaussian prior, the posterior distribution remains normal with mean $m_{b,t}$ and standard deviation $\sigma_{b,t}$. As we show in Appendix B, the mean and standard deviation follow a recursive structure given by

$$m_{b,t} = (1 - \kappa_t \sigma_{b,t}^2) m_{b,t-1} + \kappa_t \sigma_{b,t}^2 \hat{b}_t \tag{3}$$

$$\frac{1}{\sigma_{b,t}^2} = \frac{1}{\sigma_{b,t-1}^2} + \kappa_t, \tag{4}$$

where $\kappa_t = n_t \eta_t^2 \sigma_c^2 / \sigma^2 \geq 0$ is the scaled consumption shock and $\hat{b}_t = \text{Cov}(g_{i,t}, g_{c,t})/\text{Var}(g_{c,t})$ is the sample estimate based on time-t observations only.

In belief updates, the posterior mean $m_{b,t}$ is a weighted average of the prior mean $m_{b,t-1}$ and the sample estimate $\hat{b}_t$, with the weights being determined by parameter uncertainty $\sigma_{b,t}^2$ and the informativeness of the data as measured by κ_t. The revision is more sensitive to new observations when agents are more uncertain about the parameter, that is, when $\sigma_{b,t}$ is high,

[5] To save on notation, we do not include an index for industries.

and also when the new data has higher informativeness κ_t. The informativeness improves when there are more firms to learn from and when the ratio of consumption to productivity volatility is larger. Lastly, the precision of beliefs, $1/\sigma_{b,t}$, increases monotonically over time.

In our formulation, both each firm's own productivity and peers' observations constitute the source of learning by determining the sample estimate $\hat{b}_t$. Precisely, the updating puts equal weights between the firm's own observation and each of its peers. In a more generalized setting where accounting noise interferes, and the observations with respect to peers are less precise than firms' own, the learning mechanism would imply different weights. In this case, the Bayesian update imposes more weight on the accurate signal (own productivity) than it does on the inaccurate signal (peer's productivity).

2.3 Measurement

To measure productivity growth in the data, we assume that firms employ a decreasing returns to scale production technology using capital as input. The output $Y_{i,t}$ of firm i at time t is given by

$$Y_{i,t} = X_{i,t}^{1-\alpha} K_{i,t}^{\alpha}, \tag{5}$$

where $X_{i,t}$ denotes the level of productivity, $K_{i,t}$ the capital stock, and $0 < \alpha < 1$ is the capital share of the production. Productivity follows a random walk $X_{i,t} = X_{i,t-1} e^{g_{i,t}}$, where its growth $g_{i,t+1}$ is given in equation (2). Given data on value-added and capital, the neoclassical production function implies that log productivity growth can be computed as

$$g_{i,t} = \frac{g_{y,i,t} - \alpha g_{k,i,t}}{1-\alpha}, \tag{6}$$

where $g_{y,i,t}$ denotes the log growth rate of value-added and $g_{k,i,t}$ the log growth rate of capital of firm i. We use the same functional form for output in the empirical exercise of Section 3 and model of Section 4.

3 Empirical Evidence

In this section, we present the empirical evidence linking beliefs about firms' risk exposure, investment, and valuation. These empirical results are rationalized by a neoclassical investment model with cross-sectional learning in Section 4.

3.1 Data

Our data consist of annual observations for non-financial and non-utility firms from the merged CRSP-Compustat dataset for the years 1964-2020. Among firm-year observations, we exclude data points with negative or missing values for sales, total assets, or the net value of property, plant, and equipment. Also, observations with missing stock prices or returns within a year are excluded. Finally, as in Belo, Lin, and Bazdresch (2014), we require firms to have December fiscal end to align the accounting, return, and consumption data. Our final dataset includes 79,421 firm-year observations.

In estimating firms' risk exposure, the key variables are firm-level productivity and consumption growth. To measure firm-level productivity growth as defined in equation (6), measures of value-added and capital are required. As in Imrohoroglu and Tuzel (2014) and Ai, Li, and Yang (2020), firms' value-added is measured as sales (SALE) minus the costs of goods sold (COGS) and capital is the net value of property, plant, and equipment (PPENT). We adjust the growth rates for value-added and capital for inflation using the GDP deflator. We also lag the growth rate of capital by one year to account for one-period time-to-build as in the neoclassical investment model. The capital share in production α is set to 0.65 as in Cooper and Ejarque (2003). For consumption, we use the real per capita growth rate of nondurable goods and services expenditures.

We identify each firm's peers using industry classifications, SIC or NAICS, as in the literature (e.g. Kahle and Walking (1996) and Krishnan and Press (2003)). We begin with four-digit classifications of each system. If a firm has too few peers at this granular level of industry, we relax the definition of peers to ensure a sufficient number of cross-sectional observations for learning. Specifically, for four-digit industries that have fewer than five constituents at any point in time, we expand the reference set to include firms in the same three-digit industries throughout the course of learning.[6] As an alternative definition of industries, we employ the text-based classification recently developed by Hoberg and Phillips (2010). This classification is based on product similarity among firms as measured through a text-based analysis of 10-K filings. This text-based network industry classification system

[6] In an alternative analysis, we use four-digit SIC and NAICS industries without relaxing the definition of peers. Our main empirical findings continue to hold with this alternative definition of peers.

(hereafter referred to as TNIC) is obtained from the Hoberg-Phillips Data Library.

The Bayesian learning about risk exposures, formulated in equations (3) and (4), requires a date-0 prior belief. We assume a diffuse prior by setting $m_{b,0}$ to 1 and $\sigma_{b,0}$ to 5. This large dispersion represents decision makers' ambiguity on risk exposure at the beginning of the learning process. In addition, we skip the first 5 years of beliefs so that date-0 prior's impact on the results is muted.[7] After initializing the belief distribution, we utilize the cross-section of firm-level productivity growth for each industry to update and estimate the belief distribution $m_{b,t}$ and $\sigma_{b,t}$.

We also consider an alternative estimation of risk exposures, which uses each firm's own history only. This estimation does not utilize observations of peer firms and therefore is more noisy than our benchmark estimation. The posterior mean obtained from this individual learning is denoted by $m^i_{b,t}$ and the posterior dispersion by $\sigma^i_{b,t}$.

The goal of the empirical exercises is to show that risk exposures correlate with investment rates and valuation ratios. We follow Clementi and Palazzo (2019) for the measurement of investment rates and Erickson, Jiang, and Whited (2014) for Tobin's Q. Appendix A describes the construction of all variables in detail. Table 1 reports summary statistics of the variables employed in our analysis.

3.2 Beliefs

Before studying the regression evidence, we present the estimated beliefs about risk exposure. In Figure 1, we plot the time series of the belief distribution for four industries selected for illustration: malt beverages (SIC 2082), distilled and blended liquors (SIC 2085), plastic foam products (SIC 3086), and laboratory analytical instruments (SIC 3826).

Based on this figure, we can make a couple of observations. First, each industry's estimated risk exposure changes substantially over time. For example, the mean risk exposure for the laboratory analytical instruments industry ranges from -9.24 to 9.04, with a standard deviation of 4.54. Furthermore, as the confidence intervals in the plots indicate, the precision of beliefs improves gradually due to an expanding dataset. Second, the historical paths of the parameter updates are strikingly different across industries. For instance, the risk

[7] As a robustness test, we alternatively choose different values ranging from 0.5 to 2 for $m_{b,0}$ and 3 to 10 for $\sigma_{b,0}$, and we find that the main empirical findings continue to hold.

estimates of the malt industry are negatively correlated with those of the distilled liquor industry with the coefficient of -0.53, even though these two industries belong to the same 2-digit SIC group. This distinction, however, is not surprising because belief updates are based on realized productivity, which is mainly idiosyncratic by nature. As a result, the learning process is remarkably unique to each industry.

The idiosyncratic nature of each industry's parameter learning contributes to the cross-sectional dispersion in the perceived risk exposures. Figure 2 illustrates this connection. Specifically, Panel A displays each year's distribution of mean risk exposure $m_{b,t}$ across industries. It appears that early estimates are particularly dispersed. As expected, these early beliefs are formed based on fewer observations and, therefore, more subject to measurement errors arising from idiosyncratic shocks.

In Panel B, we plot the distribution of the standard deviation of beliefs $\sigma_{b,t}$ across industries. As more observations become available, the belief precision for each industry improves over time. This improvement in precision also makes the cross-sectional distribution of the mean belief more concentrated, as can be seen in Panel A. Nonetheless, despite parameter learning over fifty years, risk estimates remain substantially distinct across industries. In the cross-section, the latest estimates of $m_{b,t}$ have a standard deviation is 5.30, comparable to the unconditional mean of 7.67.

In the following, we ignore the impact of beliefs' standard deviation on firm variables. Because the true risk exposure parameter is assumed to be constant, the beliefs' dispersion is a monotonically declining process. As a result, the dynamics do not capture interesting cross-sectional patterns.

3.3 Firm Investment

In this section, we want to test whether firms' investment policies respond to their estimated risk exposures. Intuitively, as the estimated risk exposure increases, firms are more exposed to aggregate productivity risk, which raises their discount rate and thus lowers the NPV of new investment projects. As a result, we expect a negative relationship between investment rates and risk exposures.

We test this hypothesis by running panel regressions of the form

$$\frac{I_{i,t}}{K_{i,t}} = \alpha_i + \beta \times m_{b,t-1} + \gamma \times \text{Controls}_{i,t-1} + \epsilon_{i,t}, \tag{7}$$

where $I_{i,t}/K_{i,t}$ denotes firm i's investment rate and α_i the firm fixed effect. Controls include variables, which have been found to affect investment, namely, firms' size, age, Tobin's Q, cash flow, leverage ratio, the indicator of financial constraints, and industries' Herfindahl-Hirschman index.[8] Considering the nature of learning, where parameter beliefs are updated over time, we expect firms' investment response to their risk exposure to be particularly pronounced in the time-series dimension. According to Pastor, Stambaugh, and Taylor (2017), panel regressions with firm fixed effects capture the time series response. Specifically, they show that in panel regressions with firm fixed effects, the OLS slope estimate is a weighted average across firms of the slope estimates from firm-by-firm time-series regressions.

We report the regression results in Table 2. In specification (1), firms' peers are identified by SIC codes. We find that the mean risk exposure $m_{b,t}$ forecasts investment with strong statistical significance, documented by a t-statistic of -3.60. This time-series association suggests that when beliefs about risk exposure are revised upward (downward), firms reduce (raise) investment.

This negative connection persists under alternative industry classifications. In specifications (2) and (3), we refer to NAICS or TNIC instead of SIC codes to identify industry peers. The resulting $m_{b,t}$ obtained from these alternative peers continues to be a significant and negative predictor of investment at the 1% level. Economically, a rise in the mean risk exposure by one standard deviation is accompanied by, on average, a 4.2% decrease in investment, i.e, the annual investment rate changes from 0.265 to 0.254.

It is worth noting that fluctuations in our risk estimate are only attributable to learning, while the true parameter is assumed to be constant. In contrast to this assumption, one might argue that the true risk exposure itself possibly changes over time. In such a case, our learning-based risk estimate might misleadingly capture variations in the true parameter. We address this concern in Section 5, where we extend the learning model to feature dynamic risk

[8] Gala, Gomes, and Liu (2020) find that firm size and cash flow contain information about the marginal value of capital beyond what average Q conveys. Also, investment is affected by agency conflicts associated with leverage, see Myers (1977), and financial constraints, see Whited and Wu (2006). Investment opportunities often change with firms' age as in Adelino, Ma, and Robinson (2017).

exposures. There, we show that beliefs about a constant parameter, the unconditional mean risk exposure, still predict empirical investment.

3.4 Firm Valuation

Considering that the risk exposure is a firm characteristic, we expect that learning about this parameter will also influence other firm variables beyond investment. Here, we consider firms' market valuation. If market participants engage in learning about risk exposure, the market value of firms in comparison to the book value is likely to respond to updates in the parameter beliefs. More specifically, a rising risk exposure raises discount rates and thus depresses valuations.

To examine this link empirically, we conduct a panel regression of the form

$$Q_{i,t} = \alpha_i + \beta \times m_{b,t} + \gamma \times \text{Controls}_{i,t} + \epsilon_{i,t}, \tag{8}$$

where Tobin's $Q_{i,t}$ represents the valuation ratio and α_i the firm fixed effect. Control variables are firm size, age, cash flow, leverage ratio, the indicator of financial constraints, and the industry Herfindahl-Hirschman index. This regression specification is similar to that of Pastor and Veronesi (2003), except that we additionally include the risk exposure beliefs and the competition index.[9] In this analysis, we test the contemporaneous link between the valuation and the parameter beliefs. Unlike investment, which needs to be measured from flow variables, firm values are measured at a snapshot of time, and thus parameter beliefs do not have to be lagged.

Table 3 reports the regression results. We find that Tobin's Q is strongly connected to the risk exposure beliefs. Across industry classifications, the mean risk exposure negatively predicts $Q_{i,t}$ at the 1% level. Furthermore, this association is economically significant. According to the coefficient estimates, a rise in the mean exposure by one standard deviation is associated with, on average, a 10.2% decline in the valuation ratio, i.e., Q changes from 2.787 to 2.502.

So far, we have reported that both investment and valuation respond strongly to the risk exposure beliefs. We conjecture that this synchronous reactions are driven by shifts in

[9] Pastor and Veronesi (2003) propose a model that predicts the market-to-book ratio of equity. In our model, firms' financing choices are not considered, so Tobin's Q represents the valuation ratio of total firm value.

firms' discount rate, which the parameter learning induces. The following sections tests this hypothesis.

3.5 Cost of Capital

In a consumption based asset pricing model, firms' cash flow exposure to consumption risk is a crucial determinant of their discount rates. If the risk exposure parameter is unknown, belief dynamics about this parameter will affect discount rates. In this section, we test this hypothesis by examining the direct link between risk exposure beliefs and discount rates. This analysis requires the measurement of discount rate, and we use both the implied cost of capital and realized stock returns as the proxy for discount rates.

We measure firms' implied cost of capital following Hou, van Dijk, and Zhang (2012). Their measure uses the information content implied by accounting data. They define the cost of capital as the discount rate, which makes the present value of expected future cash flows equal to the firms' market value. The forecast of future cash flows is obtained from a cross-sectional model, which relates firms' earnings to other accounting variables, such as total assets, dividends, and accruals.

We examine the contemporaneous relation between the implied cost of capital and the risk exposure beliefs through a panel regression

$$\mathrm{ICC}_{i,t} = \alpha_i + \beta \times m_{b,t} + \gamma \times \mathrm{Controls}_{i,t} + \epsilon_{i,t}, \tag{9}$$

where $\mathrm{ICC}_{i,t}$ denotes the implied cost of capital and α_i the firm fixed effect, as in Belo, Lin, and Bazdresch (2014). Control variables are the log market capitalization, log book-to-market equity ratio, investment rate, leverage ratio, growth rate in total assets, net operating assets, and accruals, following Hou, van Dijk, and Zhang (2012).

In Table 4, specifications (1) through (3) present the regression results of implied cost of capital on the mean risk exposure and other control variables. Importantly, the risk exposure belief is strongly positively related to the implied cost of capital across different industry classifications at the 1% significance level. Economically, a one standard deviation increase in the mean risk exposure leads to a rise in the implied cost of capital by 0.76% per year on average. Notice that this rise in the cost of capital is consistent with the response of investment and valuation that decrease with $m_{b,t}$.

Instead of using implied cost of capital, we next employ realized returns to proxy for discount rates. Considering a possible delay in the availability of accounting information, we examine the link between the beginning-of-year beliefs and the realized returns from July of the corresponding year until June next year. To this end, we conduct regressions similar to equation (9), replacing implied cost of capital by future realized returns.

Specifications (5) through (7) of Table 4 report the regression results. Consistent with the results for the implied cost of capital, realized future returns are also positively associated with risk exposure beliefs with statistical significance levels at the 1% or 5%. This statistical relationship is also economically significant. A one standard deviation increase in $m_{b,t}$ is accompanied by, on average, a 2.10% rise in realized annual returns.

In sum, we reveal that multiple proxies for the cost of capital are robustly connected to the mean risk exposure. This finding supports our prediction that the parameter beliefs influence firms' discount rates and, in particular, that a given firm's discount rate rises (falls) with risk exposure beliefs being revised upward (downward) over time.

3.6 Peer Learning

A key identification assumption of the Bayesian learning is that firms in the same industry share the same risk exposure parameter b. As a result, it is optimal for an individual firm to include the cross-section of productivity growth from its industry peers when it learns about its own risk exposure. In this section, we want to test whether this assumption holds in the data. We do so with three different approaches. First, we show that individual learning displays mixed results in explaining firm variables. Second, we test whether the industry classification systems are informative compared to a random industry assignment. Third, we measure the spillover effect from learning on firm variables when firms learn only from their peers ignoring their own signal. In sum, all three experiments support the assumption that industry peers share the same risk exposure parameter.

3.6.1 Learning from Firms' Individual History

Compared to the benchmark results, where firms learn from peers, we consider here an alternative form of Bayesian learning in which each firm uses only its own history of productivity growth as signal. This alternative form is worth considering because the classification systems, which we employ to identify industries, could be too noisy, see e.g. Bhojraj, Lee, and Oler (2003). Specifically, even firms in the same industry might have very different business profiles, so that peer observations might not accurately reflect a firm's own risk profile. If this is indeed the case, focusing instead on firm's individual history would result in more precise estimates. Considering this possibility, we test whether the parameter beliefs $m^i_{b,t}$ derived from individual learning predict firm variables.

In Tables 2, 3, and 4, specifications (4) and (8) report regression results from individual learning. It turns out that the beliefs from individual learning display mixed performance in the prediction. The alternative beliefs strongly predict the valuation ratio, but they are insignificantly connected to firm investment and the implied cost of capital. This fluid performance contrasts with our baseline findings that beliefs based on peer learning are a robust predictor of the same variables.

Why do firms show only a weak response to the risk exposure estimates based on their own history? The main difference between the two forms of learning is the number of observations involved. The benchmark Bayesian beliefs include peer observations as an information source. It thus offers decision makers much richer data to learn from than does individual learning.

The primary source of information is realized productivity growth, which contains substantial noise. In the structural estimation of the model in Section 4, the volatility of idiosyncratic productivity shocks (noise) is approximately 30 times as large as the volatility of the systematic component (signal). Given this remarkably low signal-to-noise ratio, reliable identification of the risk exposure parameter will require a fairly large number of observations. In this regard, individual learning lacks sufficient observations and thus leads to inaccurate risk estimates, which are incapable of explaining some firm characteristics.

3.6.2 Counterfactual Industry Assignments

In the previous analysis, we showed that peer learning dominates individual learning in explaining firm variables. Next we examine the extent to which these industry classification systems help to identify informative signals in the cross section with respect to risk exposure. If the risk exposure is similar across firms irrespective of industry, the histories of any group of firms would be informative about this parameter. In this case, industry codes would not identify informative signals in the cross section.

Motivated by this conjecture, we compare the predictive power of risk exposure beliefs for firm variables based on learning from the actual systems of industry classification (SIC, NAICS, and TNIC) with random industry groupings of firms. The test is designed as follows. First, we create 385 hypothetical industries to match the total number of industries in the SIC system. Second, each firm is randomly assigned into an industry with five constituents, i.e., the average number of firms for SIC industries. Once assigned, each firm's industry assignment is fixed over time. Firms learn from past observations of their counterfactual peers and their own history to update beliefs about their risk exposure.

Based on the counterfactual peers, we conduct the regressions in equations (7) and (8) to see whether firms' investment and valuations respond to their risk exposure beliefs. We repeat this experiment 1,000 times and present the histograms of t-statistics for the slope coefficient on $m_{b,t}$ in Figure 3.

If the industry classifications are irrelevant, the risk exposure beliefs estimated from these counterfactual peers would strongly predict firm variables, as does our baseline estimate. Yet, we find that the predictive power derived from random peers is noticeably lower than that of the baseline estimates. In the investment regression, in which the risk exposure beliefs should predict negatively investment, the t-statistics of our baseline estimates – derived from SIC, NAICS, or TNIC – are lower than 996 out of 1,000 counterfactual estimates (larger in absolute magnitude).

The significance of the actual industry classifications is even more pronounced in predicting Tobin's Q. The estimates from every actual classification outperform all counterfactual estimates. In contrast, the parameter beliefs formed from the hypothetical peers lead to only an insignificant relation between the risk exposure and the valuation ratio; the median t-statistic of the counterfactual estimates is 0.42, contradicting the model prediction.

3.6.3 Spillover in Firm Variables

The cross-sectional learning that we propose induces an interdependence among firms: each firm's decisions are affected by its peers' observations. To highlight this nature, we examine whether firm variables exhibit spillover effects. The basic idea is to consider an alternative estimation of risk exposure beliefs based only on peers' observations without reference to each firm's own history. Specifically, for firm i, $m_{b,t}^{-i}$ denotes the mean risk exposure measured by its peers observations only. We expect that this alternative beliefs will predict firm i's decisions if agents do refer to industry peers in the parameter learning. To test this hypothesis, we conduct regressions as in equations (7), (8), and (9), replacing $m_{b,t}$ with $m_{b,t}^{-i}$.

Table 5 presents the regression results. We observe the spillover effect in all of the firm variables. In specifications (1) through (6), $m_{b,t}^{-i}$ is a significant negative predictor of investment and Tobin's Q. These results mean that when peers' productivity shocks imply a greater risk exposure, firms in the same industry see their market values decline and reduce their investment. Consistently, the cost of capital is also strongly connected to their peers-inspired beliefs. In specifications (7) through (9), the implied cost of capital is positively related with $m_{b,t}^{-i}$. All of these findings support our premise that peers' observations are used to form parameter beliefs.

Meanwhile, compared to our baseline estimation, which utilizes firms' own observations as well as peers', firm responses to the peers-inspired beliefs are relatively weaker, as indicated by regression coefficients that are slightly lower in absolute magnitude. This difference implies that firms' own observations constitute an important source of learning. However, firms' individual history alone does not suffice in this learning context as documented in Section 3.6.1, and its information is optimally utilized only in conjunction with peers' history.

4 Model

The goal of this section is to provide an economic model that explains the empirical link between firms' beliefs about their consumption risk exposure and their investment decisions. To this end, we solve a neoclassical investment model, where firms learn about their productivity exposure to aggregate risk over time. Firms observe their own productivity growth and aggregate consumption growth as signals. Moreover, firms observe productivity growth of their peers in the same industry. This set of signals is informative because firms in the same industry share the same exposure to consumption risk.

4.1 Stochastic Discount Factor

For tractability, we do not model a full general equilibrium model, instead, we specify an exogenous pricing kernel as in Berk, Green, and Naik (1999). While the majority of the production-based asset pricing literature specifies the pricing kernel as a function of aggregate productivity shocks, we model it as function of aggregate consumption shocks, similar to Kuehn and Schmid (2014). As such, this paper tries to bridge the gap between the production-based and consumption-based asset pricing literature.

Specifically, we assume that the log stochastic discount factor is given by

$$\log M_{t+1} = -r_f - \gamma\eta_{t+1} - 0.5\gamma^2, \tag{10}$$

where r_f denotes the risk-free rate, γ the price of consumption risk, and η_{t+1} is the aggregate shock to consumption growth as in equation (1). For simplicity, this pricing kernel does not feature time-variation in its conditional moments, such as the risk-free rate or price of risk. A similar specification can be found in Carlson, Fisher, and Giammarino (2004) and Hackbarth and Johnson (2015).

This pricing kernel could be motivated with Epstein-Zin preferences, when consumption growth follows an i.i.d. normal process. While risk preferences also affect the risk-free rate under power utility, Epstein-Zin preferences allow for separate risk and time preferences. As such, this specification is consistent with an Epstein-Zin agent, who has a very high elasticity of intertemporal substitution, rendering the risk-free rate being close to constant.

4.2 Firms' Problem

As in Section 2, firms generate output $Y_{i,t}$ according to a decreasing returns to scale production technology using capital as input, as specified in equation (5). Firm-level productivity growth $g_{i,t+1}$ is a mixture of an idiosyncratic and aggregate shock and given by equation (2). Firms do not know their productivity exposure to consumption risk b and learn about this parameter from the cross-section of productivity growth of firms within their industry. Since the Bayesian posterior is normal distributed, the mean and variance of beliefs follow a recursive structure, specified in equations (3) and (4), respectively.

The capital stock $K_{i,t}$ accumulates according to

$$K_{i,t+1} = (1-\delta)K_{i,t} + I_{i,t}, \tag{11}$$

where $I_{i,t}$ denotes investment and δ the depreciation rate. As is common in the literature, we assume that firms face convex adjustment costs, given by $\phi/2\,(I_{i,t}/K_{i,t})^2\,K_{i,t}$, where ϕ denotes the adjustment cost parameter.

Firms' net payouts $D_{i,t}$ equal output net of investment and adjustment costs and are given by

$$D_{i,t} = X_{i,t}^{1-\alpha}K_{i,t}^{\alpha} - I_{i,t} - \frac{\phi}{2}\left(\frac{I_{i,t}}{K_{i,t}}\right)^2 K_{i,t}. \tag{12}$$

Firms are all equity financed and chooses investment to maximize their market value given by

$$V(S_{i,t}) = \max_{I_{i,t}}\left\{D_{i,t} + \tilde{\mathbb{E}}_t\big[M_{t+1}V(S_{i,t+1})\big]\right\}, \tag{13}$$

where $S_{i,t} = (X_{i,t}, K_{i,t}, m_{b,t}, \sigma_{b,t})$ is the vector of state variables. The tilde above the expectation operator means that the integration over future shocks is done under the subjective beliefs of the agent. In addition to productivity and capital, the Bayesian learning about the risk exposure parameter implies that the dynamics for the mean and variance of the posterior distribution are state variables, whose dynamics are given by equations (3) and (4), respectively.

A few distinctive feature of this firms' problem are noteworthy. First, the optimal investment policy is a function of the agent's beliefs, i.e., $I_{i,t} = I(S_{i,t})$, which implies that revisions in beliefs affect real outcomes. Second, firm i's investment policy and valuation ratios are influenced by its peers' histories of productivity shocks via the updating of beliefs. Both of these model features are supported by the data.

Given firm value, we can compute returns $R_{i,t+1} = V_{i,t+1}/(V_{i,t} - D_{i,t})$ and excess returns $r_{i,t+1} = R_{i,t+1} - r_f$. We solve the firm's problem numerically and obtain the firm's investment rate $I_{i,t}/K_{i,t}$, Tobin's Q ratio $Q_{i,t} = (V_{i,t} - D_{i,t})/K_{i,t}$, and expected returns $\tilde{\mathbb{E}}_t[R_{i,t+1}]$ as a function of the state variables. Details about the numerical solution are provided in Appendix C.

4.3 Analytical Solutions

Before solving the firm's problem numerically, we consider a simplified model version, where the risk exposure parameter is known and capital adjustment is frictionless. This simplification lets us obtain analytic expressions for the investment policy of firms, market values and expected stock returns. As a result, we can analytically show how these firm variables depend on the risk exposure.

The following lemma presents the optimal investment policy and the valuation ratio.

Lemma 1 *When the risk exposure parameter is known and capital adjustment is frictionless, the optimal capital policy solving the value function* (13) *is given by*

$$K_{i,t+1} = \tau^* X_{i,t} \qquad \tau^* = \lambda e^{0.5[(1-\alpha)b\sigma_c - \gamma]^2/(1-\alpha)} \tag{14}$$

and average Q is

$$Q = \frac{\kappa_1}{1 - \kappa_2 e^{0.5(b\sigma_c - \gamma)^2}} + 1 \tag{15}$$

where λ, κ_1, and κ_2 are all positive constants.

The proof of the lemma is provided in Appendix D. Without frictions in capital adjustment, firms choose a constant capital-productivity ratio τ^*. Due to the i.i.d. stochastic discount factor and constant risk exposure, the discount rate is static, rendering the average Q constant. Importantly, $\partial\tau^*/\partial b < 0$ for $b < \gamma/\left[(1-\alpha)\sigma_c\right]$, and $\partial Q/\partial b < 0$ for $b < \gamma/\sigma_c$. Given that realistic values of b are well below these upper bounds, the theoretical model generates a negative influence of b on both optimal capital (therefore investment) and the valuation ratio.[10]

The firms' expected return is presented in the next lemma.

Lemma 2 *When the risk exposure parameter is known and capital adjustment is frictionless, the expected gross return is a constant given by*

$$\mathbb{E}\left[R_{i,t}\right] = \frac{1 - \kappa_2 e^{0.5(b\sigma_c - \gamma)^2}}{\kappa_1 + 1 - \kappa_2 e^{0.5(b\sigma_c - \gamma)^2}} \left[\kappa_3 e^{r_f + (1-\alpha)b\sigma_c\gamma} + (1-\delta) + \frac{\kappa_1 e^{\mu + 0.5b\sigma_c^2 + 0.5\sigma^2}}{1 - \kappa_2 e^{0.5(b\sigma_c - \gamma)^2}}\right]. \tag{16}$$

In Appendix D, we prove that $\partial\mathbb{E}\left[R_{i,t}\right]/\partial b$ is positive under specific conditions on b, which likely hold in reality. Thus, this analytic model explicitly states a positive link between b and the expected return.

[10] According to our SMM estimation, $\gamma/\sigma_c \approx 76$. Considering $0 < \alpha < 1$, $\gamma/\left[\sigma_c(1-\alpha)\right]$ is even greater.

Derived from the simplified setup, these predictions are qualitatively consistent with our empirical results. In the following, we will show that the same results hold quantitatively also in the full model, which features capital adjustment costs and learning.

4.4 Estimation

To quantitatively evaluate the model performance, we calibrate some parameters, which can be easily measured in the data, and structurally estimate the remaining parameters, for which the existing literature provides only weak priors. In particular, we calibrate the consumption process and risk-free rate to the data in Beeler and Campbell (2012). They report that consumption growth has an average growth rate μ of 1.93% and a volatility σ_c of 2.16% over the long sample of 1930-2008. The average risk-free rate r_f is 0.56%. These parameters are relevant for the pricing kernel.

The remaining model parameters are estimated with the simulated method of moments (SMM). Given the predefined parameters, we estimate the depreciation rate δ, capital share of production α, adjustment cost parameter ϕ, idiosyncratic volatility σ, productivity exposure to consumption risk b on the firm side and price of risk γ in the pricing kernel. For a given parameter vector $\theta = (\delta, \alpha, \phi, \sigma, b, \gamma)$, we solve the model numerically at an annual frequency. We simulate 1,000 economies each with 385 industries and 5 peer firms per industry for 57 years with 5 years of burn-in, as in the data. We set the initial prior beliefs to have a mean $m_{b,0}$ of one and dispersion $\sigma_{b,0}$ of 5, as in the empirical exercise.

Based on the simulated data, we calculate the model moments. The SMM objective function J equals a weighted distance metric between moments from actual data g^D and model moments $g^M(\theta)$, $J(\theta) = [g^D - g^M(\theta)]^\top W[g^D - g^M(\theta)]$. The efficient weighting matrix is the inverse of the sample covariance matrix of the moments, which we estimate using influence functions clustered at the firm-level as in Hennessy and Whited (2007). Since covariances are estimated with considerable noise, we use only its diagonal elements when computing the weighting matrix, similar to Schroth, Suarez, and Taylor (2014). The parameter estimate $\hat{\theta}$ is found by searching globally over the parameter space, which we implement via a particle swarm algorithm.

We identify the six parameters $(\delta, \alpha, \phi, \sigma, b, \gamma)$ based on eight moments. In the estimation, we include the mean and variance of the investment rate $I_{i,t}/K_{i,t}$, equity excess returns $r_{i,t}$, Tobin's $Q_{i,t}$, and the posterior mean belief $m_{b,t}$. The variances are demeaned at the firm level to remove persistent differences across firms, which the model cannot account for. While each parameter affects multiple moments, it is useful to discuss the main sources of identification. Table 6 reports the sensitivity of moment i with respect to parameter j, $\frac{dg_i^M}{d\theta_j}\frac{\theta_j}{g_i^M}$, evaluated at the point estimates from Table 7.

The depreciation rate is identified by the average investment rate because firms have to investment more when capital depreciates at a faster rate. The capital share of production is related to the mean and variance of returns and Tobin's Q. As the capital share rises, output growth becomes less volatile, see equation (5). This effect reduces the volatility of dividend growth. As a result, returns are less volatile and expected returns are lower.

A rise in the capital share also lower the growth rate of productivity, which is $(1-\alpha)\mu$. Lower discount rates lead to an increase in the valuation ratio Q, yet lower growth rates cause the valuation ratio to drop. Here the second effect dominates. The capital adjustment costs are pinned down by the variance of the investment rate. As capital adjustments become more costly, it is optimal for firms to make small investments more frequently.

Idiosyncratic risk positively impacts the variance of investment rates, returns, Tobin's Q, and average Bayesian beliefs. The risk exposure parameter is related to average returns and beliefs. As firms are more exposed to aggregate consumption risk, risk premia rise. As the true risk exposure parameter increases, the average Bayesian belief does so too. The price of risk is identified by average returns because this parameter increases the curvature of the pricing kernel.

4.5 Estimation Results

Table 7 summarizes the point estimates and moment conditions. Overall, the model matches the moments well. Both in the model and data, investment rates average around 27% annually with a standard deviation of 43%. Stock returns are also volatile at 60% in the model and data. The model can match these moments with a large depreciation rate of 40%, idiosyncratic volatility of 74%, and adjustment cost parameter of 0.22. The magnitude of the adjustment costs parameter implies that firms spend around 2.5% of their output on capital adjustments.

The model can also match firm-level average excess returns of 11% with a price of risk of 1.65. The model also replicates fairly well average Q of 2.8 with a capital share of 0.59. But the model fails to match the volatility of Q. This shortcoming is shared by many neoclassical investment models, e.g., Nikolov and Whited (2014). In the data, time variation of valuation ratios such as Tobin's Q is driven by time variation in discount rates. Learning about fundamentals generates time variation in discount rates, even though the true parameter is constant. In a model, where firms do not share the same risk exposure within their industry, so that firms cannot learn from their peers, learning can generate a realistic volatility of valuation ratios. Yet learning from peers mitigates this channel because these additional signals are very informative.

The estimated risk exposure parameter is 7.4 even though average risk exposure is only 5.6. The reason for this wedge is that we assume a date-0 prior of 1 in the model and data. As a comparison, Bansal and Yaron (2004) assume a volatility scaling parameter of 4.5 for aggregate dividends. Even though the economic fit of the model is very good, statistically the model is rejected with a J-value of 311.

Figure 4 depicts the investment rate I_t/K_t, Tobin's Q and expected returns as a function of the mean risk exposure $m_{b,t}$, evaluated at the average capital stock. Expected returns increases in the average risk exposure because a large fraction of the volatility of productivity arises from systematic risk. As discount rates rise in average risk exposure, they depress investment and the Q valuation ratio. These relationships are consistent with the theoretical predictions from the simplified model in Section 4.3.

4.6 Model Implications

We now explore the model implications on how risk exposure beliefs affect firm decisions. Using the same 1,000 simulated economies as for the estimation, we replicate regressions (7), (8), and (9) on simulated data. In the model, we measure size as the logarithm of the capital stock, cash flow as dividends divided by capital, market equity (ME) as the logarithm of firm value, book-to-market equity ratio (BM) as the reciprocal of Tobin's Q, and asset growth as the growth rate of the capital stock.

Table 8 reports the cross simulation averages of regression coefficients. Note that we did not target these regression coefficients in the estimation, so they should be interpreted as out-of-sample model evidence. Overall, the model generates responses in firm variables to risk exposure beliefs, which are similar to the empirical patterns. The model implied link between investment and $m_{b,t}$ is negative. The slope coefficient of $m_{b,t}$ is -0.010, implying that a one standard deviation increase in $m_{b,t}$ leads to a 10.9% decrease in the investment rate. The magnitude is comparable to the empirical response of 4.2%.

The model's ability to generate this negative association depends particularly on the capital share α in the production function. If the production technology were constant returns to scale, i.e., $Y_{i,t} = X_{i,t}K_{i,t}$, then average and marginal Q would align and marginal Q would be a sufficient statistic for investment (Hayashi (1982)). Therefore, $m_{b,t}$, which affects marginal Q, would not have additional predictive power beyond average Q in model regressions. In contrast, our structural estimation finds α to be distinctly lower than one, allowing the model to replicate the negative impact of risk exposure beliefs on investment.

Furthermore, the model produces a positive relationship between risk exposure beliefs and the cost of capital, which resembles the regularity in the data. In the simulated model, both the expected excess returns and future realized excess returns increase with the posterior mean risk exposure. Quantitatively, a one standard deviation increase in $m_{b,t}$ raises the expected return (realized return) by 2.33% (3.76%) per year. These model responses are reasonably close to the empirical reaction. For the same change in $m_{b,t}$, the ICC (realized return) moves upward by 0.76% (2.10%) on average in the data.

In addition, we confirm that our model reproduces spillover effects. Table 9 presents the model implied responses to peer observation driven beliefs about risk exposures. As in the data, $m_{b,t}^{-i}$ negatively predicts both investment and the valuation ratio and, simultaneously, positively the cost of capital. These findings manifest the model's capability, that peer observations shape the beliefs about a firm's risk profile. Compared to Table 8, where both firms' own and peers' observations are used in learning, firms respond relatively weakly to the beliefs influenced by peers only. This comparison also aligns with the empirical evidence.

5 Time Variation in Risk Exposure

One of the stylized assumptions in our model is that the true exposure of productivity to consumption shocks is constant over time. Changes in risk exposure beliefs are not driven by shifts in true risk profile but induced by the continuous updating of beliefs from growing observations. Meanwhile, what if the true exposure itself fluctuates over time, contrary to our assumption? If so, our learning-based risk estimate might misleadingly capture variation in the true exposure. Consequently, our empirical evidence might not be interpretable as evidence of learning. To address this concern, we extend our model to incorporate possible shifts of true risk exposure and examine the learning impact in this context.

Specifically, we now assume that the true risk exposure follows a first-order autoregressive process

$$b_{t+1} = \varphi b_t + (1-\varphi)\bar{b} + \sigma_b \xi_{t+1}, \tag{17}$$

where ξ_{t+1} is a standard normal innovation, which is independent of η_{t+1} and $\varepsilon_{i,t+1}$. This dynamic exposure b_{t+1} enters the law of motion of productivity in equation (2), replacing the constant exposure b. Similarly to the baseline model, here we consider an agent who observes neither $\bar{b}$ nor b_t and instead infers them from realized productivity.

Appendix E describes the updating of beliefs about risk exposure in this setting. Briefly speaking, agents revise their beliefs about the risk exposure vector $[b_t, \bar{b}]^T$ over time through the Kalman filter. To conduct this filtering, we estimate the parameters in equation (17) using the expectation-maximization algorithm. Based on the parameter estimates, we obtain the posterior means of $\bar{b}$ and b_t conditional on all observations available at time t and denote them by $m^{\mathrm{KF}}_{\bar{b},t}$ and $m^{\mathrm{KF}}_{b_t,t}$. Next, we test whether the firm variables respond to these beliefs. In particular, we focus on their responses to the beliefs about the unconditional mean of risk exposure $m^{\mathrm{KF}}_{\bar{b},t}$ because the true value of $\bar{b}$ is constant, so that any change in the estimate of this parameter is entirely driven by learning.

We report the regression results in Table 10. Interestingly, the results here echo our baseline findings. The posterior mean $m^{\mathrm{KF}}_{\bar{b},t}$ negatively predicts investment and Tobin's Q and positively the cost of capital. Most of these associations are highly significant at the 5% level. By contrast, firms respond distinctively weakly to the conditional risk exposure. In specifications (2), (4), and (6), where we conduct a "horse race" between $m^{\text{KF}}_{\bar{b},t}$ and $m^{\text{KF}}_{b_t,t}$, beliefs about the conditional risk exposure $m^{\text{KF}}_{b_t,t}$ is only insignificantly linked to all firm variables. These findings show that corporate decisions and market valuations are particularly sensitive to learning about the constant parameter that determines firms' long-run risk characteristics.

In sum, we conclude that the evolution of risk exposure beliefs is an important consideration in practice, irrespective of whether the true risk exposure is static or dynamic.

6 Conclusion

In the consumption-based asset pricing paradigm, firms' exposure to consumption risk is a crucial characteristic and thus should impact firm decisions and valuations. Despite its importance, estimations of the risk exposure parameter have been elusive. Firm observables, which are potentially informative about this parameter, are often primarily driven by idiosyncratic innovations, thus hampering the identification of the systematic component of cash flows. In recognition of this challenge, prior studies, which have attempted to measure the consumption cash flow betas, have relied on specific assumptions to deal with the noisy information – employing portfolio-level analysis (Bansal, Dittmar, and Lundblad (2005)) or using very long firm-level data including future earnings (Da (2009)). As a solution, we propose a neoclassical investment model, where agents gradually learn about the parameter through Bayesian updating. In particular, parameter beliefs are updated from firms' and industry peers' comovement between their productivity and consumption growth.

We empirically establish that this parameter learning shapes firms' real decisions and market valuations. As the Bayesian mean risk exposure is continuously revised over time, discount rates respond positively, while the investment rate and Tobin's Q negatively. We find that a key source of learning in this context is cross-sectional information from peers. Alternative beliefs about risk exposure, which ignore peer observations, do not predict firm variables. In further support to these empirical findings, our structurally estimated model reproduces the quantitative links between risk exposure beliefs and firm variables. All these findings suggest that consumption risk is an important consideration in practice and the evolution of risk exposure beliefs is priced in financial markets.

Appendix

A Data

In this section, we describe in detail the construction of all variables. Following Clementi and Palazzo (2019), we measure investment rates as the growth rate of the book value of property, plant and equipment (PPENT) plus capital depreciation

$$\frac{\text{PPENT}_{i,t} - \text{PPENT}_{i,t-1}}{\text{PPENT}_{i,t-1}} + \delta,$$

where δ is the average depreciation rate of a particular industry to which firm i belongs. Each industry's depreciation rate is estimated from data provided by Bureau of Economic Analysis (BEA). The BEA provides data on the net stock and depreciation of private fixed asset for 63 industries, each of which matches NAICS industries. For each industry, we compute the average depreciation rate for the years 1964-2020.

Firm size is defined as the logarithm of total assets (AT). Book debt is the sum of debt in current liabilities (DLC) and long-term debt (DLTT). The market value of total assets is book debt plus market equity, which is the product of common shares outstanding (CSHO) and the stock price (PRCC). The leverage ratio is book debt divided by the market value of total assets.

We measure cash flow as the sum of income before extraordinary items (IB) and depreciation and amortization (DP) divided by lagged total assets. We measure Tobin's Q as book debt plus market equity minus current assets (ACT) divided by total assets minus current assets.

For the book-to-market ratio of equity and firm age, we follow Pastor and Veronesi (2003). The book-to-market equity ratio is book equity dividend by market equity. Book equity is shareholders' equity (SEQ) plus deferred taxes and investment tax credit from balance sheets (TXDITC) minus the book value of the preferred stock (PSTKRV). Firm age is minus the reciprocal of one plus the number of years since the firm's stock price first appeared in CRSP.

For net operating assets and accruals, we follow Hirshleifer, Hou, Heoh, and Zhang (2004). Net operating assets are the difference between operating assets and operating liabilities, divided by lagged total assets. Net operating assets are total assets minus cash and short-term investment (CHE). Operating liabilities are total assets minus book debt minus minority interest (MIB) minus the book value of preferred stock redemption (PSTKRV) minus common equity (CEQ). Accruals are the change in non-cash current assets minus the change in current liabilities excluding the change in short-term debt and the change in taxes payable minus depreciation, scaled by lagged total assets.

We measure firms' financial constraints as in Whited and Wu (2006), which is a composite index of firms' cash flow, dividends, leverage ratio, total assets, and sales growth.

For each industry, we calculate the Herfindahl-Hirschman index (HHI) to measure the level of competition among industry constituents. As in Giroud and Mueller (2011), we calculate the market share of each constituent using sales (SALE) and the sum of the squared shares is the HHI.

B Derivation of the Bayesian Posterior

To simplify formulation, we define the demeaned growth in consumption and firm i's productivity

$$\begin{aligned} \bar{g}_{c,t} &= g_{c,t} - \mu = \sigma_c \eta_t \\ \bar{g}_{i,t} &= g_{i,t} - \mu = b\sigma_c \eta_t + \sigma \epsilon_{i,t}. \end{aligned}$$

Consider an industry with n constituting firms. Let $\bar{g}_t$ denote the vector consisting of the constituents' growth, $\bar{g}_t = [\bar{g}_{1,t} \quad \bar{g}_{2,t} \quad \cdots \quad \bar{g}_{n,t}]$. According to Bayes' law, the probability of b conditional on all observations until time t is

$$\begin{aligned} \text{Prob}\left(b|\bar{g}_1, \cdots, \bar{g}_t, \bar{g}_{c,1}, \cdots, \bar{g}_{n,t}\right) &\propto \text{Prob}\left(b, \bar{g}_t, \bar{g}_{c,t}|\bar{g}_1, \cdots, \bar{g}_{t-1}, \bar{g}_{c,1}, \cdots, \bar{g}_{n,t-1}\right) \\ &\propto \text{Prob}\left(\bar{g}_t|b, \bar{g}_{c,t}\right) \times \text{Prob}\left(b|\bar{g}_1, \cdots, \bar{g}_{t-1}, \bar{g}_{c,1}, \cdots, \bar{g}_{n,t-1}\right) \end{aligned}$$

where we use the fact that $\bar{g}_{c,t}$ is independent of past shocks and $\bar{g}_t$ is conditionally independent of past observations given b and $\bar{g}_{c,t}$.

Suppose that beliefs about b based on observations until time $t-1$ are normally distributed with mean $m_{b,t-1}$ and standard deviation $\sigma_{b,t-1}$. With new observations in time t, the Bayesian posterior becomes

$$\begin{aligned}
& \text{Prob}\left(b|\bar{g}_1, \bar{g}_2, \cdots, \bar{g}_t, \bar{g}_{c,1}, \bar{g}_{c,2}, \cdots, \bar{g}_{n,t}\right) \\
\propto\ & \prod_{i=1}^{n} \exp\left(-\frac{(\bar{g}_{i,t} - b\bar{g}_{c,t})^2}{2\sigma^2}\right) \times \exp\left(-\frac{(b - m_{b,t-1})^2}{2\sigma_{b,t-1}^2}\right) \\
=\ & \exp\left(-\frac{\sum_{i=1}^{n}(\bar{g}_{i,t} - b\bar{g}_{c,t})^2}{2\sigma^2}\right) \times \exp\left(-\frac{(b - m_{b,t-1})^2}{2\sigma_{b,t-1}^2}\right) \\
=\ & \exp\left(-\frac{\sum_{i=1}^{n}(\bar{g}_{i,t} - b\bar{g}_{c,t})^2}{2\sigma^2} - \frac{(b - m_{b,t-1})^2}{2\sigma_{b,t-1}^2}\right) \\
=\ & \exp\left(-\frac{\sigma_{b,t-1}^2 \sum_{i=1}^{n}(\bar{g}_{i,t} - b\bar{g}_{c,t})^2 + \sigma^2(b - m_{b,t-1})^2}{2\sigma_{b,t-1}^2\sigma^2}\right) \\
=\ & \exp\left(-\frac{\sigma_{b,t-1}^2 \sum_{i=1}^{n}(b^2\bar{g}_{c,t}^2 - 2b\bar{g}_{i,t}\bar{g}_{c,t} + (\bar{g}_{i,t})^2) + \sigma^2\left(b^2 - 2bm_{b,t-1} + m_{b,t-1}^2\right)}{2\sigma_{b,t-1}^2\sigma^2}\right) \\
=\ & \exp\left(-\frac{(\sigma_{b,t-1}^2 \sum_{i=1}^{n}\bar{g}_{c,t}^2 + \sigma^2)b^2 - 2(\sigma_{b,t-1}^2 \sum_{i=1}^{n}\bar{g}_{i,t}\bar{g}_{c,t} + \sigma^2 m_{b,t-1})b}{2\sigma_{b,t-1}^2\sigma^2}\right) \\
& \times \exp\left(-\frac{\sigma_{b,t-1}^2 \sum_{i=1}^{n}(\bar{g}_{i,t})^2 + \sigma^2 m_{b,t-1}^2}{2\sigma_{b,t-1}^2\sigma^2}\right) \\
\propto\ & \exp\left(-\frac{b^2 - 2\frac{\sigma_{b,t-1}^2 \sum_{i=1}^{n}\bar{g}_{i,t}\bar{g}_{c,t} + \sigma^2 m_{b,t-1}}{\sigma_{b,t-1}^2 \sum_{i=1}^{n}\bar{g}_{c,t}^2 + \sigma^2}b}{2\frac{\sigma_{b,t-1}^2\sigma^2}{\sigma_{b,t-1}^2 \sum_{i=1}^{n}\bar{g}_{c,t}^2 + \sigma^2}}\right).
\end{aligned}$$

In the exponential function, the denominator is

$$\frac{\sigma_{b,t-1}^2\sigma^2}{\sigma_{b,t-1}^2 \sum_{i=1}^{n}\bar{g}_{c,t}^2 + \sigma^2} = \frac{1}{\sum_{i=1}^{n}\bar{g}_{c,t}^2/\sigma^2 + 1/\sigma_{b,t-1}^2} = \frac{1}{\underbrace{n\sigma_c^2\eta_t^2/\sigma^2}_{\equiv\kappa_t} + 1/\sigma_{b,t-1}^2} \equiv \frac{1}{1/\sigma_{b,t}^2} = \sigma_{b,t}^2.$$

The second term in the numerator is

$$\begin{aligned}
\frac{\sigma_{b,t-1}^2 \sum_{i=1}^n \bar{g}_{i,t}\bar{g}_{c,t} + \sigma^2 m_{b,t-1}}{\sigma_{b,t-1}^2 \sum_{i=1}^n \bar{g}_{c,t}^2 + \sigma^2} &= \frac{\sigma_{b,t-1}^2 \sum_{i=1}^n \bar{g}_{i,t}\bar{g}_{c,t} + \sigma^2 m_{b,t-1}}{\sigma_{b,t-1}^2 \sum_{i=1}^n \bar{g}_{c,t}^2 + \sigma^2} \times \frac{1/(\sigma^2\sigma_{b,t-1}^2)}{1/(\sigma^2\sigma_{b,t-1}^2)} \\
&= \frac{\sum_{i=1}^n \bar{g}_{i,t}\bar{g}_{c,t}/\sigma^2 + m_{b,t-1}/\sigma_{b,t-1}^2}{\sum_{i=1}^n \bar{g}_{c,t}^2/\sigma^2 + 1/\sigma_{b,t-1}^2} \\
&= \frac{(\sum_{i=1}^n \bar{g}_{c,t}^2)^{-1}\sum_{i=1}^n \bar{g}_{i,t}\bar{g}_{c,t} \times \sum_{i=1}^n \bar{g}_{c,t}^2/\sigma^2 + m_{b,t-1}/\sigma_{b,t-1}^2}{\sum_{i=1}^n \bar{g}_{c,t}^2/\sigma^2 + 1/\sigma_{b,t-1}^2} \\
&= \frac{\overbrace{(\sum_{i=1}^n \bar{g}_{c,t}^2)^{-1}\sum_{i=1}^n \bar{g}_{i,t}\bar{g}_{c,t}}^{\equiv \widehat{b}_t} \times \kappa_t + m_{b,t-1}/\sigma_{b,t-1}^2}{\kappa_t + 1/\sigma_{b,t-1}^2} \\
&= \sigma_{b,t}^2\left(\kappa_t \widehat{b}_t + (1/\sigma_{b,t}^2 - \kappa_t) m_{b,t-1}\right) \\
&= \kappa_t\sigma_{b,t}^2 \widehat{b}_t + \left(1 - \kappa_t\sigma_{b,t}^2\right) m_{b,t-1} \\
&\equiv m_{b,t}.
\end{aligned}$$

Using $m_{b,t}$ and $\sigma_{b,t}$ defined above, we can express the Bayesian posterior

$$\text{Prob}\left(b|\bar{g}_1, \bar{g}_2, \cdots, \bar{g}_t, \bar{g}_{c,1}, \bar{g}_{c,2}, \cdots, \bar{g}_{n,t}\right) \propto \exp\left(-\frac{(b - m_{b,t})^2}{2\sigma_{b,t}^2}\right).$$

C Numerical Solution

To solve the firms' problem (13) numerically, we first state the value function in terms of stationary variables. To this end, we define

$$k_{i,t} = \frac{K_{i,t}}{X_{i,t}}, \qquad \tau_{i,t} = \frac{K_{i,t+1}}{X_{i,t}}, \qquad i_{i,t} = \frac{I_{i,t}}{X_{i,t}}.$$

Then the stationary version of the law of motion for capital becomes $\tau_{i,t} = (1-\delta)k_{i,t} + i_{i,t}$ and the stationary value function is given by

$$v(s_{i,t}) = \max_{\tau_{i,t}} \left\{ k_{i,t}^\alpha - i_{i,t} - \frac{\phi}{2}\left(\frac{i_{i,t}}{k_{i,t}}\right)^2 k_{i,t} + \tilde{\mathbb{E}}_t\left[M_{t+1}e^{g_{i,t+1}}v(s_{i,t+1})\right]\right\},$$

where $s_{i,t} = (k_{i,t}, m_{b,t}, \sigma_{b,t})$ is the vector of state variables for the stationary problem. Since the agent does not know the true b, she forms Bayesian beliefs over b denoted by $b_{t+1|t} \sim N(m_{b,t}, p_{b,t})$. Consequently, the perceived productivity process follows

$$\tilde{g}_{i,t+1} = \mu + b_{t+1|t}\sigma_c\eta_{t+1} + \sigma\varepsilon_{i,t+1}.$$

We solve this firm problem using the value function iteration, where we discretize the grid for capital with 100 points, average Bayesian belief with 15 points and dispersion with 5 points. To compute the continuation value of the value function, we use Gauss-Hermite quadrature with 5 points to integrate over a 4-dimensional space: the idiosyncratic $\varepsilon_{i,t+1}$ and aggregate shock η_{t+1}, the subjective distribution over risk exposure $b_{t+1|t}$, and the shock distribution of peer firms. In particular, the mean belief can be written as

$$m_{b,t} = \sigma_{b,t}^2 \left(\frac{m_{b,t-1}}{\sigma_{b,t-1}^2} + \frac{g_{c,t} \left[bg_{c,t} + \sigma\varepsilon_{i,t} + (n-1)bg_{c,t} + \sigma \sum_{j \neq i} \varepsilon_{j,t} \right]}{\sigma^2} \right),$$

where n is the number of constituents in the industry. This result implies that firm i's updating of $m_{b,t}$ depends on its peers' idiosyncratic shocks $\varepsilon_{j,t}$. However, only the sum of its peers' idiosyncratic shocks matters. This sum is normally distributed $\sum_{j \neq i} \varepsilon_{j,t} \sim N\left(0, \sqrt{n-1}\right)$, because peers' idiosyncratic shocks are independent of each other and follow a standard normal distribution.

D Analytic Solutions

When the risk exposure parameter is known and capital adjustment is frictionless, the firms' problem simplifies to

$$v(k_{i,t}) = \max_{\tau_{i,t}} \left\{ k_{i,t}^{\alpha} - i_{i,t} + \mathbb{E}_t \left[M_{t+1} e^{g_{i,t+1}} v(\tau_{i,t} e^{-g_{i,t+1}}) \right] \right\}.$$

The Envelope condition is

$$v'(k_{i,t}) = \alpha k_{i,t}^{\alpha-1} + (1-\delta).$$

Integrating $v(k_{i,t})$ with respect to $k_{i,t}$, we solve for the firm value:

$$v(k_{i,t}) = \int v'(k_{i,t}) dk_{i,t} = k_{i,t}^{\alpha} + (1-\delta)k_{i,t} + c,$$

where c is a constant to be determined.

The first order condition is

$$\begin{aligned} 0 &= -1 + \mathbb{E}_t \left[M_{t+1} e^{g_{i,t+1}} v' \left(\tau_t e^{-g_{i,t+1}} \right) e^{g_{i,t+1}} \right] \\ &= -1 + \mathbb{E}_t \left[M_{t+1} \alpha \left(e^{g_{i,t+1}} \right)^{\alpha-1} \right] \tau_{i,t}^{\alpha-1} + (1-\delta) e^{-r_f}. \end{aligned}$$

Thus, the optimal capital $\tau^*_{i,t}$ for the next period satisfies

$$
\begin{aligned}
(\tau^*_{i,t})^{1-\alpha} &= \frac{\mathbb{E}_t\left[M_{t+1}\alpha e^{g_{i,t+1}(1-\alpha)}\right]}{1-(1-\delta)e^{-r_f}} \\
&= \frac{\alpha\mathbb{E}_t\left[e^{-r_f-\gamma\eta_{t+1}-0.5\gamma^2}e^{(1-\alpha)\mu+(1-\alpha)b\sigma_c\eta_{t+1}+(1-\alpha)\sigma\epsilon_{i,t+1}}\right]}{1-(1-\delta)e^{-r_f}} \\
&= \frac{\alpha}{1-(1-\delta)e^{-r_f}}e^{-r_f-0.5\gamma^2+(1-\alpha)\mu+0.5(1-\alpha)^2\sigma^2+0.5[(1-\alpha)b\sigma_c-\gamma]^2}.
\end{aligned}
$$

As a result, $\tau^*_{i,t}$ is a constant given by

$$
\begin{aligned}
\tau^* &= \left[\frac{\alpha}{1-(1-\delta)e^{-r_f}}\right]^{1/(1-\alpha)} e^{-r_f/(1-\alpha)-0.5\gamma^2/(1-\alpha)+\mu+0.5(1-\alpha)\sigma^2+0.5[(1-\alpha)b\sigma_c-\gamma]^2/(1-\alpha)} \\
&= \underbrace{\left[\frac{\alpha}{1-(1-\delta)e^{-r_f}}\right]^{1/(1-\alpha)} e^{-r_f/(1-\alpha)-0.5\gamma^2/(1-\alpha)+\mu+0.5(1-\alpha)\sigma^2}}_{\equiv\lambda} e^{0.5[(1-\alpha)b\sigma_c-\gamma]^2/(1-\alpha)}.
\end{aligned}
$$

Note that $\lambda > 0$ is positive for $0 < \alpha < 1$ and $0 < \delta < 1$. To see the dependence of τ^* on b, let us differentiate τ^* with respect to b:

$$
\frac{\partial\tau^*}{\partial b} = \lambda e^{0.5[(1-\alpha)b\sigma_c-\gamma]^2/(1-\alpha)}\left[(1-\alpha)b\sigma_c-\gamma\right].
$$

This derivative is negative if $b < \gamma/\left[(1-\alpha)\sigma_c\right]$.

Next, we solve for the unknown constant c in the firm value. Plugging the functional form of $v(\tau^* e^{-g_{i,t+1}})$ into firm problem, we obtain

$$
\begin{aligned}
v(k_t) &= k_t^\alpha + (1-\delta)k_t - \tau^* + \mathbb{E}_t\left[M_{t+1}e^{g_{i,t+1}}\left((\tau^*)^\alpha e^{-\alpha g_{i,t+1}} + (1-\delta)\tau^* e^{-g_{i,t+1}} + c\right)\right] \\
&= k_t^\alpha + (1-\delta)k_t - \tau^* + \mathbb{E}_t\left[M_{t+1}e^{(1-\alpha)g_{i,t+1}}\right](\tau^*)^\alpha + e^{-r_f}(1-\delta)\tau^* + c\mathbb{E}_t\left[M_{t+1}e^{g_{i,t+1}}\right] \\
&= k_t^\alpha + (1-\delta)k_t - \tau^* + \frac{1-(1-\delta)e^{-r_f}}{\alpha}(\tau^*)^{1-\alpha}(\tau^*)^\alpha + e^{-r_f}(1-\delta)\tau^* + c\mathbb{E}_t\left[M_{t+1}e^{g_{i,t+1}}\right].
\end{aligned}
$$

We can now pin down c:

$$
c = \frac{\overbrace{\left[-1+1/\alpha+e^{-r_f}(1-\delta)(1-1/\alpha)\right]}^{\equiv\kappa_1}\tau^*}{1-\mathbb{E}_t\left[M_{t+1}e^{g_{i,t+1}}\right]}.
$$

Tobin's Q is ex-dividend firm value divided by capital:

$$
\begin{aligned}
q_{i,t} &= \frac{v(k_{i,t}) - k_{i,t}^{\alpha} - (1-\delta)k_{i,t} + \tau^*}{\tau^*} \\
&= \frac{k_{i,t}^{\alpha} + (1-\delta)k_{i,t} + c - k_{i,t}^{\alpha} - (1-\delta)k_{i,t} + \tau^*}{\tau^*} \\
&= \frac{c+\tau^*}{\tau^*} \\
&= \frac{\kappa_1 \tau^*/\tau^*}{1-\mathbb{E}_t\left[M_{t+1}e^{g_{i,t+1}}\right]} + \frac{\tau^*}{\tau^*} \\
&= \frac{\kappa_1}{1-\underbrace{e^{-r_f-0.5\gamma^2+\mu+0.5\sigma^2}}_{\equiv \kappa_2} e^{0.5(b\sigma_c-\gamma)^2}} + 1,
\end{aligned}
$$

where the last line holds because $\mathbb{E}_t\left[M_{t+1}e^{g_{i,t+1}}\right] = e^{-r_f-0.5\gamma^2+\mu+0.5(b\sigma_c-\gamma)^2+0.5\sigma^2}$. The result implies that Tobin's Q is a constant, which we denote by q. Also, note that κ_1 and κ_2 are all positive for $0<\alpha<1$ and $0<\delta<1$. The derivative of q with respect to b is

$$
\frac{\partial q}{\partial b} = \frac{\kappa_1\kappa_2 e^{0.5(b\sigma_c-\gamma)^2}(b\sigma_c-\gamma)\sigma_c}{\left[1-\kappa_2 e^{0.5(b\sigma_c-\gamma)^2}\right]^2}.
$$

The derivative is negative if $b<\gamma/\sigma_c$. In addition, q is positive only if $e^{(b\sigma_c-\gamma)^2/2} < 1/\kappa_2$ or $e^{(b\sigma_c-\gamma)^2/2} > (1+\kappa_1)/\kappa_2$. Thus, in the following analysis we only consider the range of b values that satisfy this condition.

Next, let us determine the expected return. The expected gross return is

$$
\begin{aligned}
\mathbb{E}_t\left[R_{i,t}\right] &= \mathbb{E}_t\left[\frac{V(X_{i,t+1},K_{i,t+1})}{V(X_{i,t},K_{i,t})-D_{i,t}}\right] = \mathbb{E}_t\left[\frac{v(k_{i,t+1})X_{i,t+1}}{(v(k_{i,t})-d_{i,t})X_{i,t}}\right] = \mathbb{E}_t\left[\frac{v(k_{i,t+1})e^{g_{i,t+1}}}{v(k_{i,t})-d_{i,t}}\right] \\
&= \mathbb{E}_t\left[\frac{\left(\tau^* e^{-g_{i,t+1}}\right)^{\alpha} e^{g_{i,t+1}} + (1-\delta)\tau^* e^{-g_{i,t+1}} e^{g_{i,t+1}} + c e^{g_{i,t+1}}}{q\tau^*}\right] \\
&= \frac{\mathbb{E}_t\left[\left(\tau^*\right)^{\alpha-1} e^{(1-\alpha)g_{i,t+1}} + (1-\delta) + \frac{c}{\tau^*}e^{g_{i,t+1}}\right]}{q}.
\end{aligned}
$$

Let $N(b)$ denote the numerator in the expected return. We expand it as below:

$$
\begin{aligned}
N(b) =& \mathbb{E}_t\left[\left(\tau^*\right)^{\alpha-1} e^{(1-\alpha)g_{i,t+1}} + (1-\delta) + \frac{c}{\tau^*}e^{g_{i,t+1}}\right] \\
=& \left(\tau^*\right)^{\alpha-1}\mathbb{E}_t\left[e^{(1-\alpha)g_{i,t+1}}\right] + (1-\delta) + \frac{\kappa_1}{1-\mathbb{E}_t\left[M_{t+1}e^{g_{i,t+1}}\right]}\mathbb{E}_t\left[e^{g_{i,t+1}}\right] \\
=& \frac{1-(1-\delta)e^{-r_f}}{\alpha\mathbb{E}_t\left[M_{t+1}e^{(1-\alpha)g_{i,t+1}}\right]}\mathbb{E}_t\left[e^{(1-\alpha)g_{i,t+1}}\right] + (1-\delta) + \frac{\kappa_1}{1-\mathbb{E}_t\left[M_{t+1}e^{g_{i,t+1}}\right]}\mathbb{E}_t\left[e^{g_{i,t+1}}\right] \\
=& \underbrace{\frac{1-(1-\delta)e^{-r_f}}{\alpha}}_{\equiv\kappa_3} e^{r_f+(1-\alpha)b\sigma_c\gamma} + (1-\delta) + \frac{\kappa_1}{1-e^{-r_f-0.5\gamma^2+\mu+0.5\sigma^2+0.5(b\sigma_c-\gamma)^2}} e^{\mu+0.5b\sigma_c^2+0.5\sigma^2} \\
=& \kappa_3 e^{r_f+(1-\alpha)b\sigma_c\gamma} + (1-\delta) + \frac{\kappa_1 e^{\mu+0.5b\sigma_c^2+0.5\sigma^2}}{1-\kappa_2 e^{0.5(b\sigma_c-\gamma)^2}},
\end{aligned}
$$

where we use the fact

$$\frac{\mathbb{E}_t\left[e^{g_{i,t+1}}\right]}{\mathbb{E}_t\left[M_{t+1}e^{g_{i,t+1}}\right]} = \frac{e^{\mu+0.5b\sigma_c^2+0.5\sigma^2}}{e^{-r_f-0.5\gamma^2+\mu+0.5\sigma^2+0.5(b\sigma_c-\gamma)^2}} = e^{r_f+b\sigma_c\gamma}$$
$$\frac{\mathbb{E}_t\left[e^{(1-\alpha)g_{i,t+1}}\right]}{\mathbb{E}_t\left[M_{t+1}e^{(1-\alpha)g_{i,t+1}}\right]} = \frac{e^{(1-\alpha)\mu+0.5(1-\alpha)^2b\sigma_c^2+0.5(1-\alpha)^2\sigma^2}}{e^{-r_f-\gamma^2/2+(1-\alpha)\mu+0.5(1-\alpha)^2\sigma^2+0.5[(1-\alpha)b\sigma_c-\gamma]^2}} = e^{r_f+(1-\alpha)b\sigma_c\gamma}.$$

Combining the expressions for $N(b)$ and q, we obtain the expected return:

$$\begin{aligned}\mathbb{E}_t\left[R_{i,t+1}\right] &= \frac{N(b)}{q} \\ &= \frac{1-\kappa_2 e^{0.5(b\sigma_c-\gamma)^2}}{\kappa_1+1-\kappa_2 e^{0.5(b\sigma_c-\gamma)^2}}\left[\kappa_3 e^{r_f+(1-\alpha)b\sigma_c\gamma} + (1-\delta) + \frac{\kappa_1 e^{\mu+0.5b\sigma_c^2+0.5\sigma^2}}{1-\kappa_2 e^{0.5(b\sigma_c-\gamma)^2}}\right].\end{aligned}$$

The derivative of the expected return with respect to b is

$$\begin{aligned}&\frac{\partial \mathbb{E}_t\left[R_{i,t+1}\right]}{\partial b} \\ =&\frac{\sigma_c(b\sigma_c-\gamma)e^{(b\sigma_c-\gamma)^2/2}\kappa_2\left(1-\kappa_2 e^{(b\sigma_c-\gamma)^2/2}\right)\left(1-\delta+\kappa_3 e^{r_f+(1-\alpha)b\sigma_c\gamma}+\frac{\kappa_1 e^{\mu+b^2\sigma_c^2/2+\sigma^2/2}}{1-\kappa_2 e^{(b\sigma_c-\gamma)^2/2}}\right)}{\left(1+\kappa_1-\kappa_2 e^{(b\sigma_c-\gamma)^2/2}\right)^2} \\ &+\frac{\kappa_3 e^{r_f+(1-\alpha)b\sigma_c\gamma}(1-\alpha)\gamma\sigma_c + b\sigma_c^2\frac{\kappa_1 e^{\mu+b^2\sigma_c^2/2+\sigma^2/2}}{1-\kappa_2 e^{(b\sigma_c-\gamma)^2/2}}}{1+\kappa_1-\kappa_2 e^{(b\sigma_c-\gamma)^2/2}} \\ &+\frac{(1-\delta)(\gamma-b\sigma_c)+\kappa_3 e^{r_f+(1-\alpha)b\sigma_c\gamma}(-b\sigma_c^2+\alpha\gamma\sigma_c)+\frac{\kappa_1 e^{\mu+b^2\sigma_c^2/2+\sigma^2/2}}{1-\kappa_2 e^{(b\sigma_c-\gamma)^2/2}}\left(\sigma_c\gamma-2b\sigma_c^2\right)}{1+\kappa_1-\kappa_2 e^{(b\sigma_c-\gamma)^2/2}}\kappa_2 e^{(b\sigma_c-\gamma)^2/2} \\ =&\frac{\kappa_3 e^{r_f+(1-\alpha)b\sigma_c\gamma}(1-\alpha)\gamma\sigma_c + b\sigma_c^2\frac{\kappa_1 e^{\mu+b^2\sigma_c^2/2+\sigma^2/2}}{1-\kappa_2 e^{(b\sigma_c-\gamma)^2/2}}}{1+\kappa_1-\kappa_2 e^{(b\sigma_c-\gamma)^2/2}} \\ &+\frac{(1-\delta)(\gamma-b\sigma_c)+\kappa_3 e^{r_f+(1-\alpha)b\sigma_c\gamma}(-b\sigma_c^2+\alpha\gamma\sigma_c)+\frac{\kappa_1 e^{\mu+b^2\sigma_c^2/2+\sigma^2/2}}{1-\kappa_2 e^{(b\sigma_c-\gamma)^2/2}}\left(\sigma_c\gamma-2b\sigma_c^2\right)}{\left(1+\kappa_1-\kappa_2 e^{(b\sigma_c-\gamma)^2/2}\right)^2}\kappa_1\kappa_2 e^{(b\sigma_c-\gamma)^2/2} \\ &+\frac{e^{(b\sigma_c-\gamma)^2/2}\kappa_2\left(1-\kappa_2 e^{(b\sigma_c-\gamma)^2/2}\right)}{\left(1+\kappa_1-\kappa_2 e^{(b\sigma_c-\gamma)^2/2}\right)^2}\left[(1-\delta)(1-\sigma_c)(\gamma-b\sigma_c)-b\sigma_c^2\frac{\kappa_1 e^{\mu+b^2\sigma_c^2/2+\sigma^2/2}}{1-\kappa_2 e^{(b\sigma_c-\gamma)^2/2}}\right].\end{aligned}$$

When $e^{(b\sigma_c-\gamma)^2/2} < 1/\kappa_2$, it follows that $e^{(b\sigma_c-\gamma)^2/2} < (1+\kappa_1)/\kappa_2$ because $\kappa_1 > 0$. Then, the derivative of return is positive if b satisfies

$$b \leq \max\left(\frac{\alpha\gamma}{\sigma_c}, \frac{\gamma}{2\sigma_c}\right) \qquad \text{and} \qquad (1-\delta)(1-\sigma_c)\gamma \geq (1-\delta)(1-\sigma_c)\sigma_c b + b\sigma_c^2\frac{\kappa_1 e^{\mu+b^2\sigma_c^2/2+\sigma^2/2}}{1-\kappa_2 e^{(b\sigma_c-\gamma)^2/2}}.$$

E Learning about Dynamic Risk Exposure

In this section, we formulate the updating of beliefs about risk exposure when the true parameter is stochastic, following an autoregressive process as in equation (17). To facilitate the formulation, we express the law of motion of risk exposure using a state-space representation:

$$\underbrace{\begin{bmatrix} b_{t+1} \\ \bar{b} \end{bmatrix}}_{\equiv B_{t+1}} = \underbrace{\begin{bmatrix} \varphi & 1-\varphi \\ 0 & 1 \end{bmatrix}}_{\equiv \Phi} \underbrace{\begin{bmatrix} b_t \\ \bar{b} \end{bmatrix}}_{\equiv B_t} + \sigma_b \begin{bmatrix} \xi_{t+1} \\ 0 \end{bmatrix}. \tag{E.1}$$

Suppose that B_t conditional on all observations until time t is normally distributed with mean $m_{B,t}$ and covariance $\Sigma_{B,t}$. Our goal is to update beliefs about B_{t+1} using new observations in time $t+1$ along with the above transition equation.

From the new observation, we calculate the sample estimate $\widehat{b_{t+1}}$ defined as in Section 2. This estimate can be written as

$$\begin{aligned} \widehat{b_{t+1}} &= \frac{\sum_{i=1}^{n_{t+1}} (g_{c,t+1} - \mu)(g_{i,t+1} - \mu)}{\sum_{i=1}^{n_{t+1}} (g_{c,t+1} - \mu)^2} \\ &= \frac{\sum_{i=1}^{n_{t+1}} (g_{c,t+1} - \mu)\left(b_{t+1}(g_{c,t+1} - \mu) + \sigma\epsilon_{i,t+1}\right)}{\sum_{i=1}^{n_{t+1}} (g_{c,t+1} - \mu)^2} \\ &= b_{t+1} + \frac{\sigma_c \eta_{t+1} \sum_{i=1}^{n_{t+1}} \sigma\epsilon_{i,t+1}}{n_{t+1}\sigma_c^2 \eta_{t+1}^2}. \end{aligned}$$

Consequently, $\mathbb{E}_{t+1}\left[\widehat{b_{t+1}}\right] = b_{t+1}$, and $\mathrm{Var}_{t+1}\left(\widehat{b_{t+1}}\right) = \sigma^2/\left(n_{t+1}^2\sigma_c^2\eta_{t+1}^2\right)$. Based on these properties of $\widehat{b_{t+1}}$, we can obtain an observation equation

$$\widehat{b_{t+1}} = \underbrace{\begin{bmatrix} 1 & 0 \end{bmatrix}}_{\equiv N} \underbrace{\begin{bmatrix} b_{t+1} \\ \bar{b} \end{bmatrix}}_{=B_{t+1}} + \underbrace{\frac{\sigma}{n_{t+1}\sigma_c|\eta_{t+1}|}}_{\equiv s_{t+1}} \zeta_{t+1} \tag{E.2}$$

where ζ_{t+1} is a standard normal innovation that is independent of the innovation ξ_{t+1} in B_{t+1}.

We now have the state-space model consisting of the transition equation (E.1) and observation equation (E.2). Next, we apply the Kalman filter to derive the distribution of B_{t+1} conditional on all observations until time $t+1$. Here, the relevant statistics are as follows:

$$\begin{aligned} \mathrm{Cov}_t\left(B_{t+1}, \widehat{b_{t+1}}^T\right) &= \left(\Phi\Sigma_{B,t}\Phi^T + \begin{bmatrix} \sigma_b^2 & 0 \\ 0 & 0 \end{bmatrix}\right) N^T \equiv \Sigma_{B,t+1|t} N^T, \\ \mathrm{Var}_t\left(\widehat{b_{t+1}}\right) &= N\Sigma_{B,t+1|t}N^T + s_{t+1}^2. \end{aligned}$$

Finally, based on the property of a joint-normal distribution, B_{t+1} conditional on new observation $\widehat{b_{t+1}}$ in addition to the previous information is also normally distributed. Its mean and covariance are updated as follows:

$$m_{B,t+1} = \Phi m_{B,t} + \left[\text{Cov}_t\left(B_{t+1}, \widehat{b_{t+1}}^T\right)\right]\left[\text{Var}_t\left(\widehat{b_{t+1}}\right)\right]^{-1}\left(\widehat{b_{t+1}} - N\Phi m_{B,t}\right)$$

$$= \Phi m_{B,t} + \underbrace{\left[\Sigma_{B,t+1|t}N^T\right]\left[N\Sigma_{B,t+1|t}N^T + s_{t+1}^2\right]^{-1}}_{\equiv \mathbb{K}_{t+1}}\left(\widehat{b_{t+1}} - N\Phi m_{B,t}\right),$$

$$\Sigma_{B,t+1} = \left(I - \mathbb{K}_{t+1}N\right)\Sigma_{B,t+1|t}.$$

Of course, this filtering depends on the model parameters. We estimate the parameters $(\varphi, \sigma_b, \sigma)$ for each industry using the expectation-maximization algorithm. Plugging these parameter estimates into the updating equation, we obtain the posterior beliefs about the risk exposure vector B_t. The time-t beliefs are normally distributed with the following conditional mean and covariance:

$$m_{B,t}^{\text{KF}} \equiv \begin{bmatrix} m_{b_t,t}^{\text{KF}} \\ m_{\bar{b},t}^{\text{KF}} \end{bmatrix}, \quad \Sigma_{B,t}^{\text{KF}} \equiv \begin{bmatrix} \left(\sigma_{b_t,t}^{\text{KF}}\right)^2 & \rho\sigma_{b_t,t}^{\text{KF}}\sigma_{\bar{b},t}^{\text{KF}} \\ \rho\sigma_{b_t,t}^{\text{KF}}\sigma_{\bar{b},t}^{\text{KF}} & \left(\sigma_{\bar{b},t}^{\text{KF}}\right)^2 \end{bmatrix}.$$

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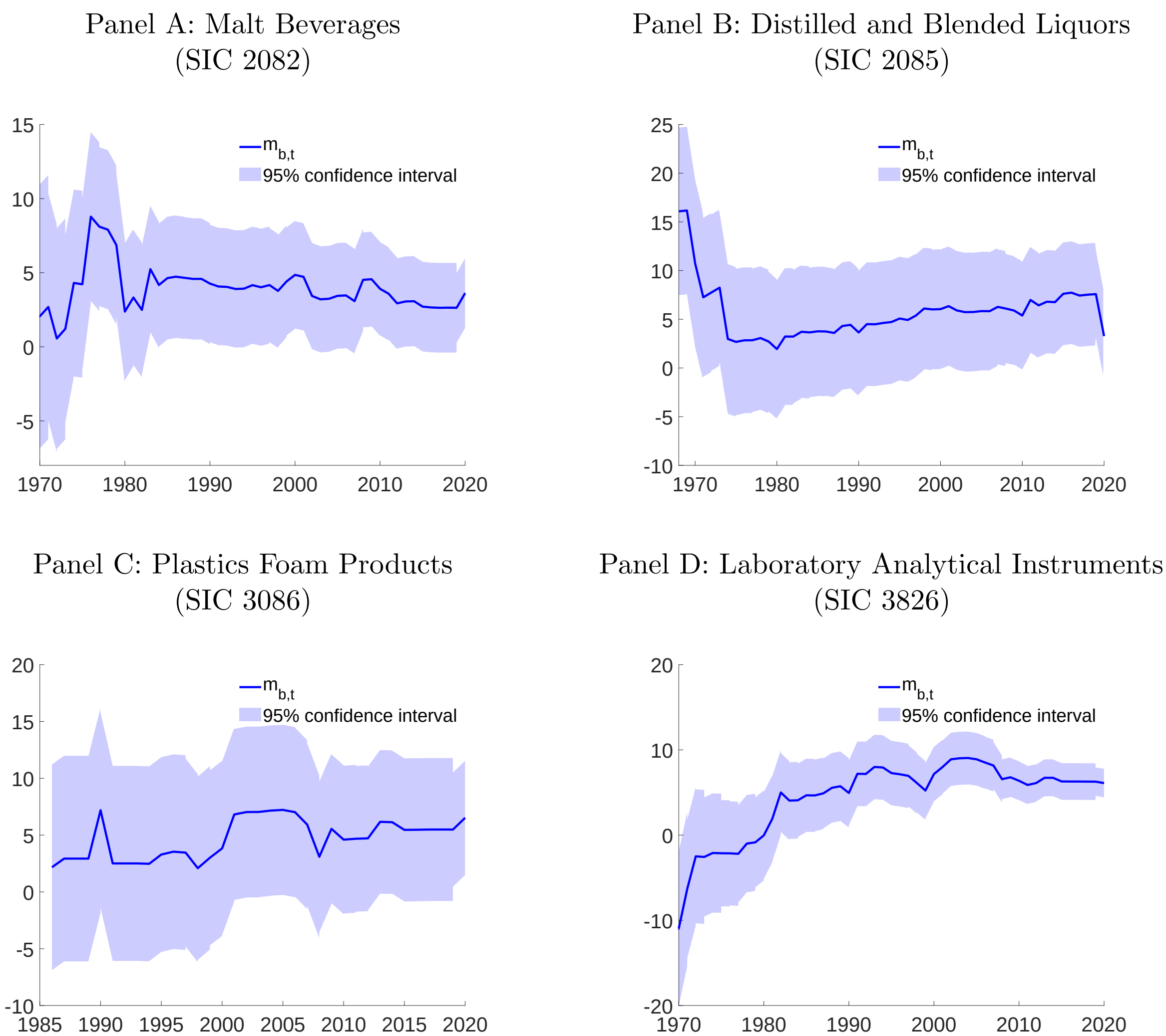
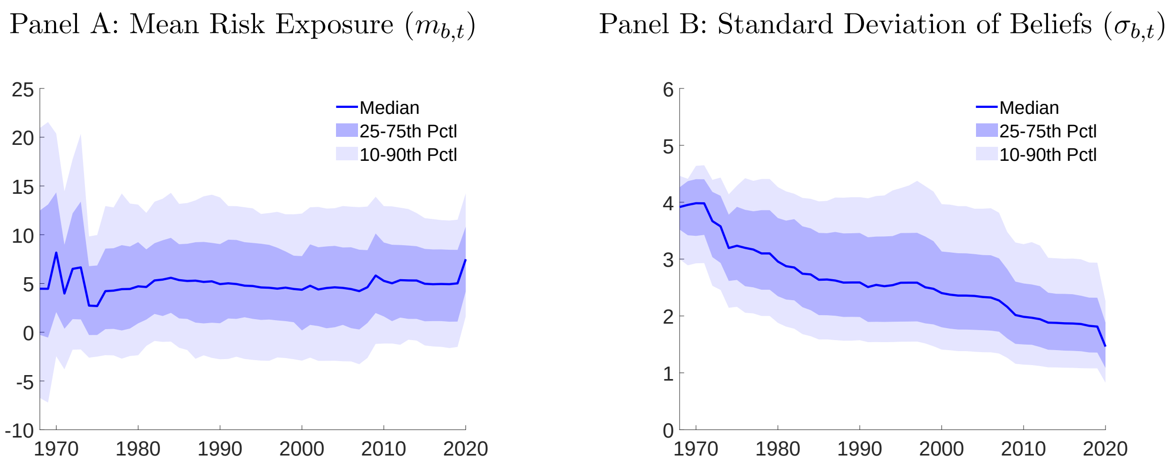
Figure 1: Examples of Risk Exposure Beliefs



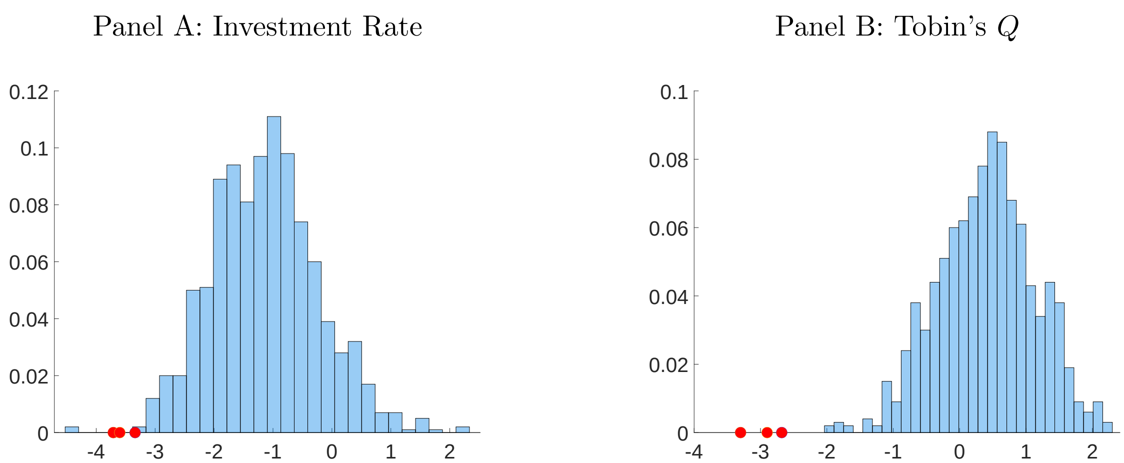
This figure presents the time series of the posterior beliefs about consumption risk exposure for selected industries: malt beverages (SIC 2082), distilled and blended liquors (SIC 2085), plastic foam products (SIC 3086), and laboratory analytical instruments (SIC 3826). The blue line represents the mean belief and the shaded area indicates the 95% confidence interval of the estimate, based on the standard deviation of the belief distribution.

Figure 2: Cross-Sectional Distribution of Risk Exposures



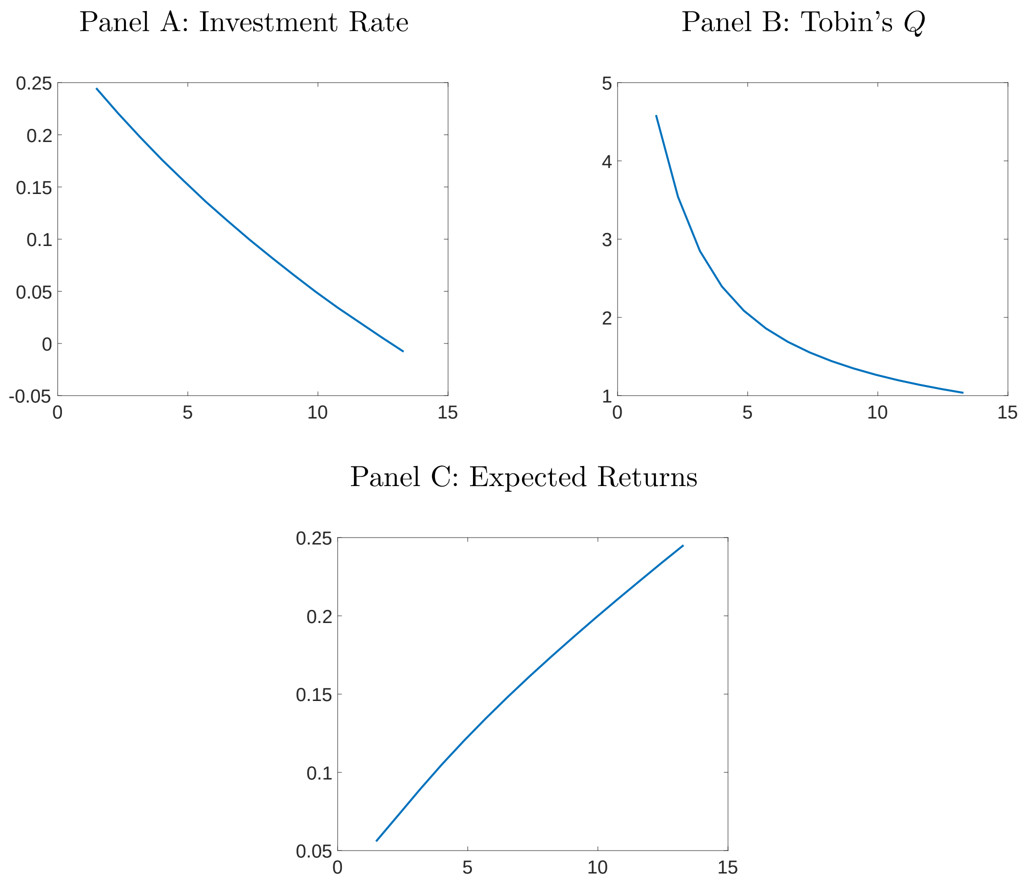
This figure depicts the annual cross-sectional distribution of mean risk exposure $m_{b,t}$ and the beliefs' standard deviation $\sigma_{b,t}$ across industries. The solid line is the median of the distribution for each year. The dark shaded area indicates the 25th and 75th percentiles and the light shaded area the 10th and 90th percentiles of the distribution.

Figure 3: Learning about Risk Exposures from Counterfactual Industry Peers



This figure plots the histograms of t-statistics of regression slope coefficients based on 1,000 counterfactual experiments. We regress investment rates in Panel A and Tobin's Q in Panel B on risk exposure beliefs and control variables as in regression models (7) and (8), respectively. Beliefs about each firm's risk exposure is updated from observations of counterfactual industry peers, which are randomly assigned. The dots on the x-axis indicate the t-statistics of our baseline results, which are based on actual SIC, NAICS, and TNIC industry classifications.

Figure 4: Risk Exposure Beliefs and Firm Variables



This figure depicts the investment rate I_t/K_t, Tobin's Q and expected excess returns as a function of the mean risk exposure $m_{b,t}$, evaluated at the average capital stock.

Table 1: Summary Statistics

This table presents the descriptive statistics of the annualized variables.

	Across-Panel Statistics					Within-Firm Statistics
Variable	Mean	Std.Dev.	25%	50%	75%	Std.Dev.
$m_{b,t}$ (SIC)	5.583	5.683	1.913	5.449	9.209	2.859
$m_{b,t}$ (NAICS)	5.602	5.339	2.739	5.246	8.649	2.840
$m_{b,t}$ (TNIC)	6.159	6.914	1.576	5.372	10.405	4.346
$m_{b,t}^{-i}$ (SIC)	5.915	5.660	2.940	5.738	9.178	3.004
$m_{b,t}^{-i}$ (NAICS)	6.193	4.643	3.518	6.043	8.772	2.763
$m_{b,t}^{-i}$ (TNIC)	6.433	7.880	1.589	5.521	10.699	5.105
$m_{b,t}^{i}$	3.676	8.464	-0.530	3.032	7.543	4.7078
$1/\sigma_{b,t}$ (SIC)	0.895	0.541	0.501	0.767	1.149	0.192
$1/\sigma_{b,t}$ (NAICS)	1.137	0.576	0.722	1.039	1.430	0.239
$1/\sigma_{b,t}$ (TNIC)	0.860	0.468	0.483	0.782	1.157	0.218
$1/\sigma_{b,t}^{-i}$ (SIC)	0.853	0.522	0.479	0.726	1.083	0.188
$1/\sigma_{b,t}^{-i}$ (NAICS)	1.089	0.554	0.692	0.986	1.360	0.240
$1/\sigma_{b,t}^{-i}$ (TNIC)	0.540	0.263	0.336	0.472	0.670	0.136
$1/\sigma_{b,t}^{i}$	0.249	0.033	0.219	0.246	0.271	0.020
Investment rate	0.265	0.477	0.066	0.165	0.316	0.428
Q	2.787	7.208	0.462	1.053	2.352	5.005
ICC	0.111	0.137	0.035	0.068	0.134	0.097
r	0.115	0.639	-0.245	0.024	0.321	0.601
Size	1.396	2.031	-0.063	1.321	2.815	0.625
Cashflow	0.069	0.155	0.038	0.090	0.141	0.100
Leverage	0.270	0.245	0.055	0.211	0.430	0.142
Age	-0.106	0.069	-0.143	-0.083	-0.050	0.052
WW index	-0.092	2.018	-0.243	-0.157	-0.057	1.220
HHI_t	0.354	0.243	0.165	0.295	0.468	0.108
Market capitalization	5.264	2.323	3.524	5.216	6.896	0.957
Book-to-market equity ratio	0.790	0.880	0.318	0.600	1.058	0.637
Asset growth	0.128	0.363	-0.023	0.065	0.181	0.326
Net operating assets	0.680	0.495	0.514	0.684	0.820	0.420
Accruals	-0.036	0.180	-0.082	-0.040	0.002	0.160

Table 2: Risk Exposure Beliefs and Capital Investment

This table presents panel regressions of investment rates on risk exposure beliefs and controls. In specifications (1) to (3), mean risk exposure beliefs $m_{b,t}$ are calculated from cross-sectional observations of industry constituents, which are identified based on SIC or NAICS codes or the text-based classification system TNIC. In specification (4), mean risk exposure beliefs $m^i_{b,t}$ are alternatively estimated based on individual learning. The controls are Tobin's Q, size, age, leverage, cash flow, the Whited-Wu (WW) index, and industries' Herfindahl-Hirschman index (HHI). Firm fixed effects are included and standard errors clustered by firms. t-statistics are presented in parentheses below the parameter estimates. *, **, and *** denote significance at the 10%, 5%, and 1% levels, respectively.

Specification Industry classification	(1) SIC	(2) NAICS	(3) TNIC	(4)
$m_{b,t-1}$	-0.0017***	-0.0020***	-0.0015***	
	(-3.59)	(-3.71)	(-3.08)	
$m^i_{b,t-1}$				0.00007
				(0.17)
Q	0.0092***	0.0092***	0.0084***	0.0092***
	(12.36)	(12.32)	(10.18)	(12.37)
Size	-0.0874***	-0.0861***	-0.0737***	-0.0877***
	(-20.37)	(-19.81)	(-15.55)	(-20.43)
Age	-0.0584	-0.0792*	-0.0490	-0.0570
	(-1.27)	(-1.65)	(-0.87)	(-1.23)
Cashflow	0.414***	0.410***	0.376***	0.416***
	(14.59)	(14.31)	(11.22)	(14.63)
Leverage	-0.422***	-0.425***	-0.395***	-0.421***
	(-27.28)	(-26.27)	(-22.24)	(-27.27)
WW index	-0.0013	-0.0012	0.00008	-0.0013
	(-0.87)	(-0.85)	(0.07)	(-0.87)
HHI	0.00510	0.00611	0.00784	0.00481
	(0.32)	(0.38)	(0.45)	(0.31)
N	69,462	65,102	46,380	69,462
adj. R^2	0.105	0.105	0.104	0.105

Table 3: Risk Exposure Beliefs and Tobin's Q

This table presents panel regressions of Tobin's Q on risk exposure beliefs and controls. In specifications (1) to (3), mean risk exposure beliefs $m_{b,t}$ are calculated from cross-sectional observations of industry constituents, which are identified based on SIC or NAICS codes or the text-based classification system TNIC. In specification (4), mean risk exposure beliefs $m^i_{b,t}$ are alternatively estimated based on individual learning. The controls are Tobin's Q, size, age, leverage, cash flow, the Whited-Wu (WW) index, and industries' Herfindahl-Hirschman index (HHI). Firm fixed effects are included and standard errors clustered by firms. t-statistics are presented in parentheses below the parameter estimates. *, **, and *** denote significance at the 10%, 5%, and 1% levels, respectively.

Specification Industry classification	(1) SIC	(2) NAICS	(3) TNIC	(4)
$m_{b,t}$	-0.0291***	-0.0342***	-0.0731***	
	(-3.00)	(-2.71)	(-3.45)	
$m^i_{b,t}$				-0.0637***
				(-3.68)
Size	-1.137***	-1.156***	-1.119***	-1.131***
	(-6.60)	(-6.55)	(-4.64)	(-6.59)
Age	-3.132***	-3.056**	-4.520**	-2.494**
	(-2.80)	(-2.59)	(-2.33)	(-2.32)
Cashflow	7.344***	7.373***	8.050***	7.273***
	(6.20)	(6.16)	(5.27)	(6.17)
Leverage	-4.148***	-4.337***	-3.957***	-4.153***
	(-10.74)	(-10.68)	(-7.93)	(-10.77)
WW index	-0.0057	-0.0052	-0.0061	-0.0060
	(-0.42)	(-0.38)	(-0.40)	(-0.45)
HHI	0.0810	0.0509	0.306	0.137
	(0.19)	(0.11)	(0.59)	(0.32)
N	75,293	70,141	48,706	75,293
adj. R^2	0.033	0.033	0.030	0.034

Table 4: Risk Exposure Beliefs and the Cost of Capital

This table presents panel regressions of the cost of capital on risk exposure beliefs and controls. The cost of capital is measured by either the implied cost of capital $\text{ICC}_{i,t}$ from accounting information or realized excess return $r_{i,t+1}$. $\text{ICC}_{i,t}$ and all regressors are measured at the beginning of each year and $r_{i,t+1}$ is the excess return from July of the corresponding year to June of the following year. In specifications (1) to (3) and (5) to (7), mean risk exposure beliefs $m_{b,t}$ are calculated from cross-sectional observations of industry constituents, which are identified based on SIC or NAICS codes or the text-based classification system TNIC. In specifications (4) and (8), mean risk exposure beliefs $m^i_{b,t}$ are alternatively estimated based on individual learning. The controls are the log market capitalization (ME), log book-to-market equity ratio (BM), investment rate, leverage ratio, growth rate in total assets, net operating assets (NOA), and accruals. All regressors are standardized. Firm fixed effects are included and standard errors are clustered by firms. t-statistics are presented in parentheses below the parameter estimates. *, **, and *** denote significance at the 10%, 5%, and 1% levels, respectively.

Dependent variable	$\text{ICC}_{i,t}$				$r_{i,t+1}$			
Specification	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Industry classification	SIC	NAICS	TNIC		SIC	NAICS	TNIC	
$m_{b,t}$	0.0088***	0.0103***	0.0037***		0.0202***	0.0241***	0.0186**	
	(6.19)	(9.18)	(4.59)		(3.74)	(4.86)	(2.66)	
$m^i_{b,t}$				0.0006				0.0035
				(0.98)				(0.78)
ME	0.0030*	0.0031*	0.0086***	0.0019	-0.2207***	-0.2224***	-0.2353***	-0.2204***
	(1.97)	(2.13)	(5.78)	(1.28)	(-21.97)	(-21.95)	(-18.17)	(-21.96)
BM	0.0418***	0.0421***	0.0361***	0.0413***	0.0666***	0.0625***	0.0562***	0.0660***
	(32.16)	(32.66)	(27.04)	(32.12)	(10.93)	(10.03)	(7.27)	(10.88)
I/K	0.0013**	0.0014**	0.0006	0.0012**	-0.0117**	-0.0119**	-0.0084	-0.0117**
	(2.70)	(2.80)	(1.15)	(2.43)	(-2.61)	(-2.64)	(-1.37)	(-2.61)
Leverage	0.0263***	0.0263***	0.0266***	0.0262***	0.0377***	0.0385***	0.0555***	0.0376***
	(17.66)	(17.78)	(16.09)	(17.61)	(6.16)	(6.12)	(6.76)	(6.14)
Asset growth	0.0041***	0.0043***	0.0047***	0.0036***	-0.0389***	-0.0365***	-0.0423***	-0.0392***
	(3.48)	(3.66)	(3.61)	(3.02)	(-8.48)	(-7.96)	(-6.96)	(-8.53)
NOA	-0.0163***	-0.0165***	-0.0165***	-0.0156***	-0.0013	-0.0059**	-0.0110***	-0.0012
	(-11.44)	(-11.60)	(-10.57)	(-10.95)	(-0.43)	(-2.49)	(-3.26)	(-0.42)
Accruals	0.0019***	0.0019***	0.0015**	0.0018***	-0.0040	-0.0151***	-0.0250***	-0.0039
	(3.03)	(3.04)	(2.24)	(2.97)	(-0.69)	(-2.73)	(-2.74)	(-0.69)
N	57,407	57,405	43,186	57,407	66,666	62,598	43,320	66,666
adj. R^2	0.454	0.455	0.414	0.453	0.071	0.073	0.083	0.071

Table 5: Spillovers in Firm Variables

This table presents panel regressions of firm variables on risk exposure beliefs. Across nine specifications, we regress investment rates $I_{i,t}/K_{i,t}$, Tobin's $Q_{i,t}$, or the implied cost of capital $\text{ICC}_{i,t}$ on the posterior mean belief $m_{b,t}^{-i}$, which is based on peer observations only, excluding each firm's own history, and controls. The industry peers are identified by SIC or NAICS codes or the text-based classification system TNIC. In specifications (1) to (3), the controls are firm age, Tobin's Q, size, leverage, cash flow, the Whited-Wu index, and the Herfindahl-Hirschman index. The same controls excluding Tobin's Q are used in specifications (4) to (6). In specifications (7) to (9), the controls are the log of market capitalization, the log of book-to-market equity ratio, investment rate, leverage ratio, growth rate in total assets, net operating assets, and accruals. Firm fixed effects are included and standard errors are clustered by firms. t-statistics are presented in parentheses below the parameter estimates. *, **, and *** denote significance at the 10%, 5%, and 1% levels, respectively.

Dependent variable	$I_{i,t}/K_{i,t}$			$Q_{i,t}$			$\text{ICC}_{i,t}$		
Specification	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
Industry classification	SIC	NAICS	TNIC	SIC	NAICS	TNIC	SIC	NAICS	TNIC
$m_{b,t}^{-i}$	-0.0012**	-0.0015***	-0.0012**	-0.0249**	-0.0302***	-0.0522***	0.0068**	0.0096***	0.0045***
	(-2.41)	(-2.60)	(-2.57)	(-2.43)	(-2.70)	(-2.87)	(5.44)	(9.03)	(5.93)
Controls	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
N	67,557	64,661	37,728	67,789	69,649	42,246	56,513	56,785	38,215
adj. R^2	0.105	0.105	0.110	0.046	0.045	0.030	0.455	0.456	0.419

Table 6: Sensitivity Matrix

This table shows the sensitivity of model-implied moments (in rows) with respect to model parameters (in columns). The sensitivity of moment i with respect to parameter j equals $\frac{dg_i^M}{d\theta_j}\frac{\theta_j}{g_i^M}$ and is evaluated at the vector of point estimates from Table 7.

		δ	α	ϕ	σ	b	γ
Investment rate	*mean*	1.66	-0.25	-0.16	-1.01	0.01	-0.01
	var.	0.53	-0.81	-0.50	1.45	0.07	-0.01
Stock return	*mean*	0.43	-1.16	-0.06	-0.79	0.90	0.98
	var.	0.76	-2.07	-0.09	2.41	-0.52	-0.49
Tobin's Q	*mean*	0.79	-1.80	-0.00	0.27	-0.84	-0.76
	var.	1.82	-4.32	0.06	3.59	-2.33	-2.11
Bayesian posterior mean	*mean*	0.00	0.00	0.00	-0.42	0.94	0.00
	var.	0.00	0.00	0.00	1.43	0.27	0.00

Table 7: SMM Estimation

This tables summarizes the SMM estimation of six model parameters: depreciation rate δ, capital share of production α, adjustment cost parameter ϕ, volatility of idiosyncratic productivity σ, exposure to consumption risk b, and the price of risk γ. The estimation targets the mean and variance of investment rates, Tobin's Q, stock returns, and the Bayesian posterior mean risk exposure. Standard errors are reported in parenthesis and based on the sample covariance matrix of the moments, which we estimate using influence functions clustered at the firm-level as in Hennessy and Whited (2007).

Panel A: Parameter Estimates

Parameter		Estimates
Depreciation rate	δ	0.3955
		(0.0044)
Capital share of production	α	0.5916
		(0.0070)
Adjustment cost parameter	ϕ	0.2201
		(0.0156)
Volatility of idiosyncratic productivity	σ	0.7374
		(0.0073)
Exposure to consumption risk	b	7.3838
		(0.1092)
Price of risk	γ	1.6515
		(0.0349)

Panel B: Moments

Moments		Data	Model
Investment rate	*mean*	0.2647	0.2652
	s.d.	0.4282	0.4262
Stock return	*mean*	0.1150	0.1138
	s.d.	0.6020	0.6060
Tobin's Q	*mean*	2.7870	2.9608
	s.d.	5.0049	2.6533
Bayesian posterior mean	*mean*	5.5833	5.5411
	s.d.	2.8595	2.9139

Table 8: Risk Exposure Beliefs and Firm Variables in the Model

This table presents panel regressions of firm variables on risk exposure beliefs on simulated data. We simulate 1,000 economies each with 385 industries and 5 peer firms per industry for 57 years with 5 years of burn-in, as in the data. The table reports cross simulation averages of regression coefficients and adjusted R^2. Across four specifications, we regress investment rates $I_{i,t}/K_{i,t}$, Tobin's $Q_{i,t}$, expected excess returns $\mathbb{E}_t[r_{i,t+1}]$, or realized excess returns $r_{i,t+1}$ on risk exposure beliefs $m_{b,t}$ and controls. In specification (1), the controls are Tobin's Q, size, and cash flow. The same controls excluding Tobin's Q are used in specification (2). In specifications (3) and (4), the controls are log market capitalization (ME), log book-to-market equity ratio (BM), investment rate, and growth rate in total assets.

Specification	(1)	(2)	(3)	(4)
Dependent variable	$I_{i,t}/K_{i,t}$	$Q_{i,t}$	$\mathbb{E}_t\left[r_{i,t+1}\right]$	$r_{i,t+1}$
$m_{b,t}$	-0.0100	-0.3755	0.0233	0.0376
Q	0.0600			
Size	0.1292	1.1440		
Cash flow	1.764	13.8550		
ME			-0.0412	-0.0685
BM			-0.0029	0.1271
I/K			0.0055	1.1191
Asset growth			-0.00003	-0.2464
adj. R^2	0.998	0.746	0.984	0.934

Table 9: Spillovers in Firm Variables in the Model

This table presents panel regressions of firm variables on risk exposure beliefs on simulated data. We simulate 1,000 economies each with 385 industries and 5 peer firms per industry for 57 years with 5 years of burn-in, as in the data. The table reports cross simulation averages of regression coefficients and adjusted R^2. Across four specifications, we regress investment rates $I_{i,t}/K_{i,t}$, Tobin's $Q_{i,t}$, expected excess returns $\mathbb{E}_t[r_{i,t+1}]$, or realized excess returns $r_{i,t+1}$ on the posterior mean belief $m_{b,t}^{-i}$, which is based on peer observations only, excluding each firm's own history, and controls. In specification (1), the controls are Tobin's Q, size, and cash flow. The same controls excluding Tobin's Q are used in specification (2). In specifications (3) and (4), the controls are log market capitalization (ME), log book-to-market equity ratio (BM), investment rate, and growth rate in total assets.

Specification	(1)	(2)	(3)	(4)
Dependent variable	$I_{i,t}/K_{i,t}$	$Q_{i,t}$	$\mathbb{E}_t\left[r_{i,t+1}\right]$	$r_{i,t+1}$
$m_{b,t}^{-i}$	-0.0082	-0.3411	0.0063	0.0016
Q	0.0617			
Size	0.1169	0.7981		
Cashflow	1.7008	12.4740		
ME			-0.0574	-0.1021
BM			0.0250	0.1850
I/K			0.0146	1.1371
Asset growth			-0.00007	-0.2464
adj. R^2	0.998	0.733	0.970	0.934

Table 10: Time Variation in Risk Exposure and Firm Variables

This table presents panel regressions of firm variables on risk exposure beliefs. Across six specifications, we regress investment rates $I_{i,t}/K_{i,t}$, Tobin's $Q_{i,t}$, or the implied cost of capital $\text{ICC}_{i,t}$ on beliefs about time-varying risk exposures and controls. The beliefs consist of the posterior mean about the unconditional risk exposure $\bar{b}$ denoted by $m^{\text{KF}}_{\bar{b},t}$ and the posterior mean about the conditional risk exposure b_t denoted by $m^{\text{KF}}_{b_t,t}$, which are estimated using the Kalman filter based on SIC industry peers. In specifications (1) and (2), controls are firm age, Tobin's Q, size, leverage, cash flow, the Whited-Wu index, and the Herfindahl-Hirschman index. The same controls excluding Tobin's Q are used in specifications (3) and (4). In specifications (5) and (6), controls are the log market capitalization, log book-to-market equity ratio, investment rate, leverage ratio, growth rate in total assets, net operating assets, and accruals. Firm fixed effects are included and standard errors are clustered by firms. t-statistics are presented in parentheses below the parameter estimates. *, **, and *** denote significance at the 10%, 5%, and 1% levels, respectively.

Dependent variable	$I_{i,t}/K_{i,t}$		$Q_{i,t}$		$\text{ICC}_{i,t}$	
Specification	(1)	(2)	(3)	(4)	(5)	(6)
$m^{\text{KF}}_{\bar{b},t}$	-0.0078**	-0.0118*	-0.2772***	-0.2449**	0.0051***	0.0069**
	(-2.59)	(-1.80)	(-3.65)	(-2.35)	(3.39)	(2.10)
$m^{\text{KF}}_{b_t,t}$		0.0038		-0.0307		-0.0019
		(0.69)		(-0.47)		(-0.63)
Controls	Yes	Yes	Yes	Yes	Yes	Yes
N	68,698	68,698	74,418	74,418	56,854	56,854
adj. R^2	0.108	0.108	0.035	0.035	0.453	0.453