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Delayed Crises and Slow Recoveries

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Keywords: Credit extension, synchronization, externality, economic recoveries
JEL Classification: G01, G20, E2, E44

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"When the music stops, in terms of liquidity, things will be complicated. But as long as the music is playing, you've got to get up and dance. We're still dancing."

Charles Prince, the former chairman and CEO of Citigroup

"Those who leave the dance too early leave money on the table and initially look incompetent. However, a fully committed player like Mr. Prince finds that he can't escape when the building's on fire and *everyone wants out at the same time*...As we've seen, hardly anybody knows how to call that timing very well."

Quoted from Wagner and Rieves (2009)

1 Introduction

The 2008-2009 global financial crisis has sparked revived interest in Minsky's financial instability hypothesis. Nowadays it has become commonplace for the financial media and practitioners to refer to the start of a financial crisis as a Minsky moment. According to Minsky, the financial system is intrinsically fragile. In prosperous times, increased profits encourage the tendency toward excessive indebtedness, and sooner or later debt will accumulate to a level that borrowers can no longer pay it off with their incoming revenues and then a financial crisis follows naturally. There is now substantial supporting evidence of Minsky's hypothesis in academic research.[1] The general pattern revealed by the evidence is that credit booms presage financial crises and the magnitude of booms can predict the severity and length of recessions in the aftermath of the financial crises. However, some natural questions remain: why would agents, whether firms, households, or intermediaries, take on so much debt in the first place if they are aware of the inevitable large risk? Why does a credit boom often last a very long time before busting if the financial system is intrinsically fragile? For instance, the subprime mortgage boom, one of the major causes of the recent financial crisis, experienced a prolonged period of rapid acceleration from 2000 to 2006, surging from $100 billion to $ 600 billion in the process. The market then suddenly collapsed during the 2008-2009 financial crisis to less than $ 20 billion.

One possible explanation is that not all agents are aware of the ultimate crisis. Minsky himself attributes this unawareness to the important role of over-optimism, which leads both borrowers and lenders to neglect crash risk, resulting in a gradual build-up of indebtedness and a slow moving but accelerating financial crisis. Indeed, the recent academic literature has found evidence of over-optimism in bond markets (Greenwood and Hanson, 2013), equity markets (Baron and Xiong, 2017), and housing markets (Cheng, Raina, and Xiong, 2014). Over-optimism or speculative euphoria (in Minsky's own terminology) undoubtedly plays an important role in exacerbating excessive indebtedness prior to a crisis. However, it is unlikely the only cause of a crisis with the kind of magnitude observed. As a matter of fact, amid ferocious competition, the stake or gain to have accurate unbiased beliefs in the financial market is simply very high. Thus, how over-optimism alone can generate the apparently repeated credit expansions and crises in financial history (Reinhart and Rogoff, 2009,

[1] See, e.g., the recent papers of Schularick and Taylor (2012), Baron and Xiong (2017), Hu (2017), Krishnamurthy and Muir (2017), López-Salido, Stein, and Zakrajšek (2017), and Li and Krishnamurthy (2021).

2011) remains an open question.[2]

On the other hand, the widespread outrage toward the Wall Street after the recent crisis manifests the common sentiment of the public that the financial institutions were well aware the excessive credit expansion would eventually turn into a crisis, yet they ignored the warning signs, failed to control evolving risks within the financial system, and sunk the entire economy together with them when the day of reckoning came. For instance, the Financial Crisis Inquiry Commission wrote in its report that "In the decade preceding the collapse, there were many signs that lending practices had spun out of control, that too many homeowners were taking on mortgages and debt they could ill afford, and that risks to the financial system were growing unchecked."

Building on the seminal work of Abreu and Brunnermeier (2002, 2003) on asynchronous awareness, this paper models credit expansions and financial crises as a Keynes' game of musical chairs. Credit expansion persists as individual players have incentive to exit right before the crash, neither too early nor too late, though all the players know that for sure some of the players will find themselves caught by the crisis in the end. Unlike the endowment economy of Abreu and Brunnermeier (2002, 2003), our model economy features real production, which provides a novel macroeconomic perspective on the dynamic interaction between credit expansions, crises, and recoveries.

One key insight of our paper is that individual banks rationally choose to delay responding to bad news about the economy by continuing credit extension, but the delay in timing generates negative externality across banks. Specifically, banks are sequentially aware of the start of the deterioration in the fundamentals of the economy. The status of the economy depends in part on the fundamentals and in part on the collective actions of banks. Given a fundamental value, if a sufficient number of banks pull out of the lending market, the crisis will occur. Due to asynchronous awareness, an individual bank is uncertain about when the crisis will come after its own awareness. In equilibrium, each individual bank optimally chooses to continue to extend credit for a while rather than cease lending immediately after learning the bad news. In particular, the delay in response for individual banks is an over-delay, which is socially inefficient. The inefficiency arises because negative externality exists across banks in their timing. Namely, when individual banks choose to extend credit for a longer time, the crisis is delayed for longer and, as a result, more banks get caught by the crisis in equilibrium, which depresses the fire-sale liquidation value for every caught bank and exacerbates capital misallocation, forming a negative externality.

We first present a baseline model — an investment game in a production economy. In our model economy, there are two types of business à la Minsky's narratives: speculative business and traditional business. The speculative business has higher profitability but is more fragile by nature. All banks initially operate in the speculative business sector. The status of the speculative sector depends on its fundamentals and the number of banks operating in it. The fundamentals are good enough ini-

[2] The recent paper of Li and Krishnamurthy (2021) finds the evidence that both the frictional intermediation mechanism and fluctuations in beliefs are needed to match the crisis cycle patterns.

tially but deteriorate gradually after a negative shock. Banks learn of the negative shock sequentially, and decide when to exit the speculative sector. For a given fundamental value, if more than a critical proportion of banks have exited, the speculative sector will collapse — the crisis occurs. Once the crisis occurs, the speculative sector will no longer generate profits. Moreover, banks exiting before the crisis (survival banks) can retrieve their full loan principal and thus are able to reallocate and reinvest in the traditional sector for sure, while banks exiting later and thus getting caught by the crisis (failure banks) can only recover an endogenous fire-sale liquidation value for their loans. Failure banks thus can reinvest in the traditional sector only with a probability, being an increasing function of the liquidation value. The equilibrium of the baseline model is characterized by the optimal waiting strategy for a bank — the length of delay between the awareness of the shock and the action of exiting. We study two equilibria: the social planner's second-best constrained equilibrium and the decentralized competitive equilibrium.

For the second-best constrained equilibrium, it is supposed that the social planner cannot observe the time of the shock either but can coordinate all banks to choose the same waiting length. The social planner faces the following tradeoff. On the one hand, an increase in the waiting time delays the crisis, which means all banks can receive the higher profit flow from the speculative sector (than that from the traditional sector) for a longer time. On the other hand, if banks stay longer in the speculative sector, the fundamentals will deteriorate more when the crisis strikes and thus more banks will get caught by the crisis, lowering the liquidation value for every caught bank. With the above tradeoff, the social planner has a unique optimal delay. For the decentralized competitive equilibrium, an individual bank faces a similar tradeoff. However, unlike the social planner who recognizes that the timing strategy of banks endogenously impacts the crisis arrival time and thus the fire-sale price, individual banks take the fire-sale price or the loan recovery value as given. The tradeoff also gives a unique optimal waiting time for individual banks. In comparing the two equilibria, we show that individual banks exit too late in the decentralized equilibrium relative to the second-best optimum. The fact that individual banks do not internalize the impact of their timing strategy on the fire-sale price results in their over-waiting in exiting.

We then study a full model with both entry and exit, the aim of which is to characterize the cycle of credit booms and crises. Specifically, we model the cycle of how banks start from the traditional business, enter the speculative business, and then exit it and re-enter the traditional business. Initially, all banks are operating in the traditional sector. The speculative sector begins to generate a high profit flow after a positive shock hits to its fundamentals. The information of the shock arrives to banks sequentially (like in the baseline model). All banks know that the good economic fundamentals of the speculative sector can last only for a certain period. Therefore, banks need to decide when best to enter and when best to exit the speculative sector. The equilibrium of the full model is characterized by the optimal length of time a bank stays in the speculative sector. As in the baseline model, there exists a unique equilibrium for both the second-best and the decentralized case. In particular, we show that individual banks stay in the speculative sector too long in the decentralized competitive equilibrium compared with the second-best optimum.

Finally, we extend our model to a macroeconomic growth setting with both entry and exit. The macroeconomic model explicitly examines the capital accumulation and consumption decision. The model highlights the dynamics. When the fundamentals of the speculative sector are initially deteriorating, the credit boom continues but the growth of the aggregate economy slows down. After a sufficient number of banks pull out, the speculative sector starts to decline and the growth of the aggregate economy slows down even further. Once it commences, the contraction of the speculative sector occurs at an increasing speed. The economy heads toward the financial crisis at an accelerated pace after a long period of slowly deteriorating fundamentals.[3] The rich dynamics of booms, slowdowns, crashes, and recoveries of our macroeconomic model demonstrates that asynchronous awareness could potentially be a powerful transmission and propagation mechanism in the standard DSGE models, which are well known for lacking a strong internal propagation mechanism in the absence of other frictions. Similar to the baseline model, we show that the credit boom lasts longer and the crisis is more delayed in the decentralized competitive equilibrium than in the second best. Once the crisis strikes, however, there is a bigger drop in capital and aggregate output than in the second best. It then takes a longer time for the capital to accumulate and for the aggregate output to recover to the pre-crisis level. This opens room for policy interventions.

We analyze two policy measures that can potentially mitigate or eliminate the inefficiency of the decentralized equilibrium. One is the tax policy and the other is the credit policy. For the tax policy, government can levy a tax on the asset fire-sale liquidation values of failure banks and distribute the tax revenue to all banks as a lump-sum subsidy. We show that there exists a unique tax rate that implements the second-best optimum. For the credit policy, failure banks can borrow from the government for restructuring or refinancing, but the government charges an interest rate that is higher than its funding cost. The extra interest rate charged forms the government's profit, which will be distributed to all banks as a lump-sum subsidy. We show that there exists a unique interest rate that implements the second-best optimum. The intuition for the two polices is similar. Both involve a penalty on failure banks, which makes individual banks have incentives to reduce the chance of being caught by the crisis by choosing to stay shorter in the speculative sector.

Related literature. Closely related to our paper is the work of Abreu and Brunnermeier (2002, 2003), who show that asynchronous awareness results in not only asynchronous responses to the shock but also a delay in the responses. In a general-equilibrium economy with production, our paper further shows that such a delay in the responses can be an over-delay, which is inefficient from the perspective of the social planner. In other words, the work of Abreu and Brunnermeier (2002,

[3]In the recent crisis, the subprime mortgage defaults had started to increase since the first quarter of 2007 when the S&P/Case-Shiller house price index recorded the first year-on-year decline since 1991. Yet, a full bloom crisis did not strike until the second half of 2008.

2003) explains why a bubble (boom) can persist, while our paper explains why an *inefficient* boom can persist. More importantly, building on their work, we apply the timing game to a business cycle model, which offers insights on why an inefficient credit boom can persist, causing an over-delayed crisis with the consequence of resource misallocations and slow economic recoveries. On the methodology front, our paper is the first to embed the microeconomic friction à la Abreu and Brunnermeier (2002, 2003) in a standard macroeconomic model. With capital accumulation and risk-aversion preference, modeling the timing game becomes much harder. Yet we are able to make the model highly tractable and extendable. Since asynchronous awareness is common in many important economic events such as bubbles, currency attacks, and bank runs, the macroeconomic model developed in this paper provides a first step to study the macroeconomic impact of asynchronous awareness in these events.

The macroeconomic model in our paper is a continuous-time model (see, e.g., Brunnermeier and Sannikov, 2014; Liu, Mian and Sufi, 2020; Bigio and Sannikov, 2021). A few papers in the literature follow the approach of Abreu and Brunnermeier (2003) and study asynchronous awareness in different directions and contexts. Doblas-Madrid (2012) aims to endogenize the asset prices in Abreu and Brunnermeier (2003). In his model, all agents are rational and prices reflect supply and demand at all times. He and Manela (2016) examine information acquisition in rumor-based bank runs, by adding uncertainty about the capacity of the bubble, decoupling the spreading rate from the length of the awareness window, and allowing agents to acquire additional noisy information upon awareness. Unlike these contributions where the model economy is still an endowment (exchange) economy, ours is a general-equilibrium production economy with the emphasis of the welfare implications, i.e., the decentralized equilibrium versus the constrained equilibrium under asynchronous awareness.

Our paper belongs to a broader literature that studies the causes of financial crises. As discussed earlier, some behavioral theories suggest that credit booms might lead to recessions or financial crises (see, e.g., Minsky, 1977, 1986; Kindleberger, 1978, and the recent contributions of Bordalo, Gennaioli, and Shleifer, 2018 and Bordalo et al., 2020). Two branches of rational expectations models are related to our paper. One branch is the theory based on financial frictions pioneered by Bernanke and Gertler (1989) and Kiyotaki and Moore (1997), which emphasizes the amplification and propagation mechanisms of an exogenous fundamental shock. The other branch of theory emphasizes that macroeconomic fluctuations and crises in particular can be self-fulfilling even in the absence of fundamental shocks to preferences, technologies, or economic endowments (e.g., Cass and Shell, 1983; Cooper and John, 1988; Benhabib and Farmer, 1994, Cole and Kehoe, 2000; Gertler and Kiyotaki, 2015; Martin and Ventura, 2012; Miao and Wang, 2018; and Benhabib, Liu, and Wang, 2019). The coordination game of timing in our paper is closely related to the particular literature on self-fulfilling beliefs in coordination games under frictions of imperfect information or imperfect communication; a leading example of this particular literature is models of sentiment-driven fluctuations (e.g., Angeletos and La'O, 2013; Benhabib, Wang, and Wen, 2015; Benhabib, Liu, and Wang, 2016). The financial crisis in our model is driven by both fundamentals and coordination and our model features a unique equilibrium. In other words, the financial crisis is triggered by a bad shock to fundamentals but amplified by the coordination problem. In particular, the coordination problem in our model concerns timing, which causes a more delayed and therefore more severe crisis. The insight of amplification through timing has not been shown in the extant literature.

A branch of literature characterizes slow recoveries after crises. Veldkamp (2005) studies a model where information flow is endogenous and varies with the level of economic activity. When times are bad, a low level of economic activity implies scarce information, which slows agents' reactions and leads to a slow recovery (or boom). Van Nieuwerburgh and Veldkamp (2006) characterize slow recoveries in a real business cycle model where information flow about aggregate productivity is endogenously generated by production actions. Low production after a crisis yields noisy estimates of recovery, which impedes learning and slows recovery. Fajgelbaum, Schaal and Taschereau-Dumouchel (2017) show uncertainty traps: high uncertainty deters investment, and agents learn from the actions of others. As a result, a temporary shock can generate a long-lasing recession. In Kozlowski, Veldkamp and Venkateswaran (2020), the true distribution of shocks is unknown and agents estimate it with real-time data. Thus extreme events generate persistent changes in agents' beliefs and macro outcomes. The mechanism of slow recovery in our paper is different. When the crisis strikes, capital stock and aggregate output experience a big drop. It takes a long time for the capital to accumulate and for the aggregate output to recover to the pre-crisis level.

Our paper is also related to a growing literature that highlights the role of pecuniary externality in generating excessive financial fragility (see, e.g., Geanakoplos and Polemarchakis, 1986; Caballero and Krishnamurthy, 2003; Lorenzoni, 2008, Farhi, Golosov, and Tsyvinski, 2009; Jeanne and Korinek, 2010; Stein, 2012; He and Kondor, 2016). Dávila and Korinek (2018) build a general model to characterize pecuniary externalities in economies with financial frictions. The common insight from this literature is that individual borrowers over-borrow as they do not internalize the effect of their borrowing on the fire-sale price of assets during financial distress (see, e.g., Lorenzoni, 2008). The externality in our model concerns lenders' *timing choice* of credit extension and operates through affecting the number of borrowers that are under fire sales at the crisis time ("extensive margin"); in contrast, the externality in the extant literature concerns borrowers' *level choice* of leverage and operates through affecting the quantity of asset (debt) under fire sales per borrower at the crisis time ("intensive margin"). The different channels of operation may imply different policy interventions. More importantly, the timing dimension of externality in our paper has various forces with different signs. One key contribution of our paper is to analytically decompose the externality and characterize the sign and magnitude of each force, connected to the work of Dávila and Korinek (2018).

Our paper embeds a dynamic coordination game in a macroeconomic model. In this respect, our paper is connected to some of the papers in the microeconomic literature on dynamic coordination games. Frankel and Pauzner (2000) and He and Xiong (2012) study a dynamic game by adding a friction as in Calvo (1983). These papers assume that agents must make asynchronous choices, a Calvo-like friction. The friction in our paper is instead asynchronous awareness like in Abreu and Brunnermeier (2002, 2003); asynchronous choice is merely a byproduct of asynchronous awareness. Some other papers such as Angeletos, Hellwig, and Pavan (2007) and Liu (2019) study a repeated version of static global games pioneered by Carlsson and Van Damme (1993) and Morris and Shin (1998). The information structure of asynchronous awareness in our paper is different from that of dispersed noisy signals on a fundamental value in those papers. Our model framework has elements of both coordination and competition unlike in global games (see also Abreu and Brunnermeier, 2003). On the applied side, the aforementioned models do not allow capital accumulation and hence are not designed to explain the booms, crashes, and slow recoveries as our model is.

The paper is organized as follows. In Section 2, we present the baseline model. In Section 3, we study a full model with both entry and exit. In Section 4, we extend the model to a macroeconomic growth framework. In Section 5, we analyze policy implications. Section 6 concludes.

2 The baseline model

Our model economy consists of two types of business (sectors) in which banks can operate: speculative business and traditional business. The former, corresponding to "speculative investment" in Minsky's narratives, includes business such as subprime mortgage lending, CDS trading, and other shadow banking activities, while the latter can be normal commercial lending. We will model the cycle of how banks start from the traditional business, enter the speculative business, and then exit it and re-enter the traditional business. For clarity of exposition, in this section, we focus on the second half of the cycle — banks' decisions of exiting the speculative business and (re)entering the traditional business. In the next section, we will study the first half of the cycle.

Figure 1 illustrates the framework of the baseline model, where the notations will be explained in due course.

2.1 Setting

Time is continuous starting from $t = 0$. The risk-free net interest rate is r. There is a continuum of banks with unit mass. These banks are currently investing in the speculative business sector. There is a continuum of firms with unit mass in the speculative business sector. Without loss of generality, for our macroeconomic setting, we assume that one firm is matched with one bank (so we can call a pair as a "firm-bank").[4]

Following the static setting in Cooper and John (1988), Morris and Shin (2004), and Bebchuk and Goldstein (2011), the loan performance of a bank depends on the macroeconomic state as well as the number of other banks extending loans. In our dynamic context, the payoff of a bank in the speculative sector at time t depends on the fundamentals of the sector, $\theta(t)$, and the number of active

[4]For simplicity of our macroeconomic model, we can assume that one firm has one main creditor/investor — its bank. A firm has to close its business and liquidate assets to repay loans if its bank decides not to roll over.

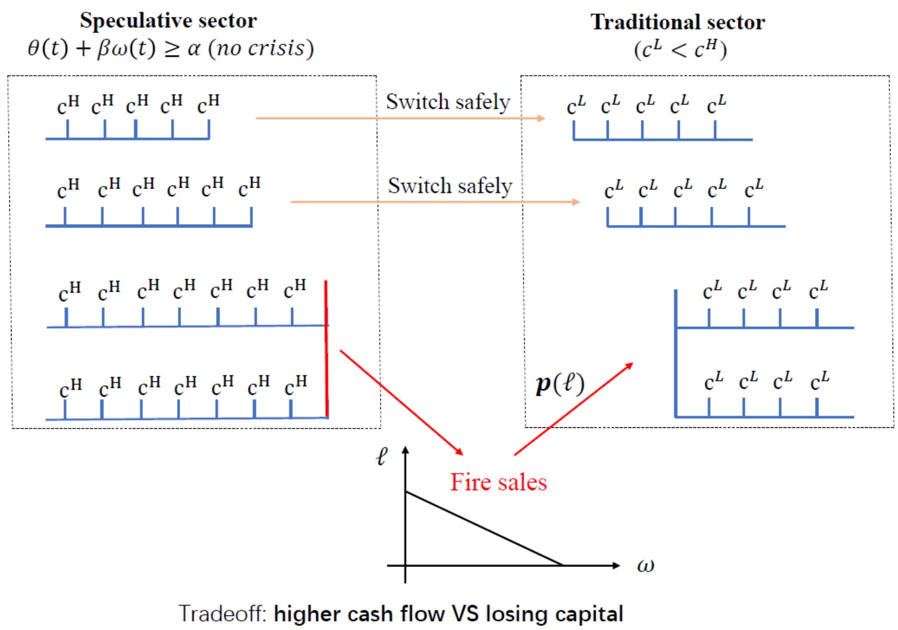

Figure 1: The framework of the baseline model

banks in the sector. Specifically, a bank's payoff (i.e., loan interests) is a continuous cash flow process, and at time t the flow is given by

$$c(t) = \begin{cases} c^H & \text{if } \theta(t) + \beta \cdot \omega(t) \geq \alpha \text{ (no crisis)} \\ 0 & \text{otherwise} \end{cases}, \tag{1}$$

where $c^H > r$, and $\omega(t)$ is the total measure of active banks at time t in the sector, $\beta > 0$ is a parameter representing the degree of (production) complementarity among firms in the sector, and α is a constant satisfying $\alpha > \beta$.[5] As we will show, the condition $\theta(t) + \beta \cdot \omega(t) \geq \alpha$ is not binding at the beginning but becomes binding after a certain time at which point we say a "crisis" occurs.

The fundamentals (the macroeconomic state) of the sector, $\theta(t)$, follow an exogenous process. $\theta(t)$ is initially high enough, but at $t = t_0 > 0$ a shock hits the sector. After the shock, $\theta(t)$ gradually declines, that is, $\theta'(\cdot) < 0$ for $t \geq t_0$. The arrival of the shock, the time t_0, follows an exponential distribution, with probability density function (pdf) $\phi(t_0) = \lambda e^{-\lambda(t_0 - m)}$ in the support $t_0 \in [m, +\infty)$, where the parameter $m \geq 0$ will be explained in detail in Section 3; in the baseline model we can simply set $m = 0$. Crucially, like in Abreu and Brunnermeier (2002, 2003), t_0 is not observable by banks. Nevertheless, after t_0, banks are sequentially informed (aware) of the event that the sector has been hit by the shock. The information spreads among banks over $[t_0, t_0 + \eta]$, following a uniform distribution. Ex ante, any bank is equally likely to become aware at any $t \in [t_0, t_0 + \eta]$. Since t_0 is random, an individual bank does not know its position in the queue (i.e., how many other banks are informed before or after it is informed). Sequential awareness can also be interpreted as a dispersion of beliefs or opinions as in Abreu and Brunnermeier (2002, 2003).

[5] Product market complementarity and search friction may generate this kind of payoff function (see, e.g., Hu and Varas, 2019).

All banks initially invest in the speculative business sector. After receiving private information about the shock time t_0, banks may decide to exit. Concretely, if a bank decides to exit (i.e., not to roll over its loan any longer) at time t, its firm liquidates assets and repays the principal of the bank loan. As long as crisis has not yet struck at time t, the assets can be sold *orderly* to investors in a related sector possessing a constant-returns-to-scale technology with productivity as 1.[6] In other words, if a bank exits before the crisis hits, it is able to get its full loan principal, 1, paid back. However, once the crisis hits, banks that are still active in the sector are "caught" by the crisis and their firms have to liquidate assets under fire sales. The fire-sale price function is a downward-sloping curve. Specifically, the liquidation value for a firm is given by

$$L = \begin{cases} 1 & \text{if no crisis} \\ \ell = g\left(\omega^C\right) & \text{if in the crisis} \end{cases},$$

where $g(\cdot)$ is the fire-sale price function with $g' \leq 0$, and ω^C, an endogenous variable, denotes the total measure of firms under fire sales at the crisis time.[7] The micro-foundation of $g(\cdot)$ is the following. When the crisis occurs, the assets have to be sold to outside investors in a less-related sector, which has a less efficient technology to use the assets. The technology is with (weakly) decreasing returns to scale. Specifically, the production function of the outside investor sector is $G(\omega)$ with the marginal productivity being $G'(\omega) = g(\omega)$.

After exiting and obtaining their liquidation value, those banks that successfully exit before the crisis (called "survival banks") as well as those banks that are caught by the crisis (called "failure banks") can enter the traditional business sector by reinvesting their liquidation value in an investment opportunity — a project of a fixed size. The payoff of the project is a continuous constant cash flow process c^L in perpetuity, where $c^L \in \left(r, c^H\right)$; that is, the project's present value (PV) is $\frac{c^L}{r} > 1$. The cost of this investment is 1.[8] A survival bank can certainly afford the investment cost and will thus reinvest. For a failure bank, if it is short of capital to finance the investment cost 1 (i.e., if $L = \ell < 1$), it can try to refinance the deficit. The probability of successful refinancing (to reach the required capital 1), which is independent across banks, is $p(L) \in [0, 1)$ for $L < 1$, where $p'(\cdot) > 0$. The probability $p(L)$ can also be interpreted as the success probability of restructuring a failure bank. In short, the expected payoff (PV) for a bank after obtaining its liquidation value with taking into account the reinvestment opportunity is summarized as

$$\Pi = \begin{cases} \frac{c^L}{r} & \text{if } L = 1 \\ \left[\frac{c^L}{r} - (1 - L)\right] \cdot p(L) + L\left(1 - p(L)\right) = L + \left(\frac{c^L}{r} - 1\right) \cdot p(L) & \text{if } L < 1 \end{cases}, \tag{2}$$

[6] As will become clear, at any time point t before the crisis, the measure of assets under sales is zero.

[7] In reality, banks themselves face the risk of being run by their short-term debtholders if their asset performance is in question. If such runs occur or are expected to occur, banks would be forced to fire sell (Liu, 2016, 2019). The fragile capital structure of banks plays a disciplining role (Calomiris and Kahn, 1991; Diamond and Rajan, 2001).

[8] Without changing the model results qualitatively, we assume that the interest incomes received by banks have been consumed or distributed to bank investors. Alternatively, we can assume that the interest incomes are non-storable. The consumption decision is endogenous in the macroeconomic growth model in Section 4, where the assumption of a fixed-size project is also not needed.

where the second line is the payoff for a failure bank when its L is lower than 1, i.e., it will not be able to reinvest with probability $1-p(L)$, in which case its payoff is the liquidation value L, and it will be able to reinvest with probability $p(L)$, in which case its payoff is $\frac{c^L}{r}$ net of the refinancing cost $1-L$.[9] The second line is intuitively rewritten in terms of the new investment's net present value (NPV), $\frac{c^L}{r}-1$.

To have closed-form solutions of the model, we use linear specifications throughout the paper. 1) The fundamental process is $\theta(t;t_0) = \begin{cases} \alpha_0 & \text{for } t \le t_0 \\ \alpha_0 - \kappa(t-t_0) & \text{for } t > t_0 \end{cases}$, where $\kappa > 0$ and $\alpha_0 \le \alpha$; without loss of generality we set $\alpha_0 = \alpha$. This means that the coordination problem of the macroeconomy arises only after $t = t_0$ (i.e., the period in which the fundamental θ is sufficiently weak in the spirit of Morris and Shin (1998) on currency attacks). 2) The fire-sale price function is $g(\omega) = \begin{cases} 1 & \text{when } \omega \le \omega_0 \\ 1-\gamma\cdot(\omega-\omega_0) & \text{when } \omega > \omega_0 \end{cases}$, where $\gamma > 0$ and $0 < \omega_0 < 1 - \frac{\underline{\zeta}}{\eta}$ with parameter $\underline{\zeta}$ to be given later in Lemma 1.[10,11] 3) $p(L) = \sigma L$, where $0 \le \sigma \le 1$; without loss of generality we set $\sigma = 1$.

We make two parameter assumptions.

Assumption 1 *Payoff (cash flow) parameters satisfy* $r < c^L < c^H$.

Assumption 2 *Assume that* $v\left(\frac{c^L}{r}-1\right) > c^H$, *where* $v \equiv \gamma\frac{\kappa}{\beta}$.

Assumption 2 says that the full reinvestment value in the traditional business sector, $\frac{c^L}{r}$, is sufficiently high relative to the interest flow c^H in the speculative business sector. The role of this assumption will be clear in Proposition 1.

Before proceeding to solve the equilibrium, we consider a special case of Assumption 1.

Assumption 1' *Assume* $r < c^L < c^H$, *where* $r \to 0$, $c^L \to 0$, *and* $\frac{c^L}{r} \equiv \Sigma$ *is a constant.*

The only purpose of considering this special case of Assumption 1 is to make $r \to 0$ (so the discount factor term e^{-rt} in the analysis can be neglected) and at the same time to guarantee that $\frac{c^L}{r}$ is a finite number. This way, the presentation of the model equilibrium in the next subsection will become much clearer and cleaner. In the next subsection, we conduct the analysis under Assumption 1'. In Appendix B, we conduct the analysis under the general Assumption 1 and show all the results carry over. Moreover, in the full model of Section 3, we use Assumption 1 and again confirm that all results derived under Assumption 1' apply to the model under Assumption 1.

[9] A failure bank is short of $1-L$, which is refinanced by paying the continuous flow interest rate r.

[10] The corresponding production function is $G(\omega) = \begin{cases} \omega & \text{when } \omega \le \omega_0 \\ \omega - \frac{1}{2}\gamma\cdot(\omega-\omega_0)^2 & \text{when } \omega > \omega_0 \end{cases}$ for the outside investor sector, where ω_0 is the maximum absorbing capacity for the technology to maintain constant returns to scale.

[11] The only purpose of introducing the intercept ω_0 is to help characterize the condition for the non-corner solution of the competitive equilibrium (see Proposition 2 later).

2.2 Equilibrium

Suppose all banks use the same (symmetric) strategy of choosing a waiting period τ after being informed. That is, every bank decides to wait a time interval τ after being informed before existing. Denote the arrival time of the crisis by $t_0 + \zeta$.

We proceed to find ζ as a function of τ. Given τ, the measure of banks that have exited by time t is $\frac{t-(t_0+\tau)}{\eta}$ for $t \geq t_0 + \tau$. The measure of active banks, $\omega(t)$, is thus given by

$$\omega(t) = 1 - \frac{t - (t_0 + \tau)}{\eta} \quad \text{for } t \geq t_0 + \tau, \tag{3}$$

which is decreasing in t. Recall that the fundamental process is given by $\theta(t) = \alpha - \kappa(t - t_0)$ for $t > t_0$, which is also decreasing in t. The crisis condition $\theta(t) + \beta \cdot \omega(t) \geq \alpha$ is binding at $t = t_0 + \zeta$.[12] Substituting $\theta(t)$ and $\omega(t)$ in (3) into the crisis condition yields

$$\zeta = \frac{\tau + \eta}{1 + \frac{\kappa}{\beta}\eta}. \tag{4}$$

Lemma 1 follows.

Lemma 1. *The function $\zeta(\tau)$ is given by (4), which has the property that $\frac{d\zeta}{d\tau} = \frac{1}{1+\frac{\kappa}{\beta}\eta} \in (0,1)$. Moreover, τ has the domain $\tau \in [0, \bar{\zeta}]$ and ζ is bounded by $\zeta \in [\underline{\zeta}, \bar{\zeta}]$, where $\underline{\zeta} = \frac{\eta}{1+\frac{\kappa}{\beta}\eta}$ and $\bar{\zeta} = \frac{\beta}{\kappa}$.*

Next, we derive the liquidation value ℓ . When the crisis occurs, the total measure of firms under fire sales is given by

$$\omega^C = 1 - \frac{\zeta - \tau}{\eta}, \tag{5}$$

which is an increasing function of τ or ζ by (4). Hence, we obtain $\ell = g(\omega^C) = 1 - \gamma \cdot (\omega^C - \omega_0)$, which, by plugging (4) and (5), yields

$$\ell \equiv \ell(\zeta) = 1 - v \cdot (\zeta - \zeta_0), \tag{6}$$

where $v \equiv \gamma\frac{\kappa}{\beta}$ defined earlier in Assumption 2 and $\zeta_0 \equiv \omega_0\frac{\beta}{\kappa}$. Because we assume the parameter value ω_0 satisfying $\omega_0 < 1 - \frac{\zeta}{\eta}$, it follows that $\zeta_0 < \left(1 - \frac{\zeta}{\eta}\right)\frac{\beta}{\kappa} = \underline{\zeta}$ and hence $\ell(\zeta)$ is *strictly* less than 1 on the domain $\zeta \in [\underline{\zeta}, \bar{\zeta}]$. Lemma 2 follows.

Lemma 2. *The liquidation value or the loan recovery value for a bank caught by the crisis is given by (6), which has the properties that $\ell(\zeta) < 1$ and $\frac{d\ell}{d\zeta} = -v < 0$ on the domain $\zeta \in [\underline{\zeta}, \bar{\zeta}]$.*

The loan recovery value depends on how soon the crisis will occur. The sooner it occurs, the higher the loan recovery value. The parameter v measures the decline speed of the recovery value over time. Intuitively, if the crisis is delayed for longer, more banks will be caught by the crisis in equilibrium, in which case the fire-sale price for every caught bank becomes lower.

[12] Rigorously, the crisis occurs at $t = t_0 + \zeta^+$.

2.2.1 Benchmark: the social planner's second-best constrained problem

Suppose that the social planner cannot observe the shock time t_0 either. But the social planner can "coordinate" all individual banks to choose the same waiting length τ after being informed. Note that studying the social planner's choice is to have a benchmark, and we will study how to *indirectly* implement the social planner's τ in Section 5. Denote the arrival time of the crisis by $t = t_0 + \zeta$. The second-best constrained problem for the social planner is given by

$$\begin{aligned} \max_{\tau} \Psi(\tau, \zeta) \equiv & \int_{t_0}^{t_0+\zeta-\tau} \left[(t_i + \tau - t_0) c^H + \Sigma\right] \frac{1}{\eta} dt_i + \int_{t_0+\zeta-\tau}^{t_0+\eta} \left[\zeta c^H + \Pi(\ell)\right] \frac{1}{\eta} dt_i + \left(G\left(\omega^C\right) - \omega^C \ell\right) \\ \text{s.t. } \zeta = & \frac{\tau + \eta}{1 + \frac{\kappa}{\beta}\eta} \text{ given by (4)} \\ & \omega^C \equiv \omega(t = t_0 + \zeta) = 1 - \frac{\zeta - \tau}{\eta} \\ & \ell = \ell(\zeta) \text{ given by (6), and } \Pi(\ell) = \ell\Sigma. \end{aligned} \tag{7}$$

The social planner recognizes that τ endogenously impacts ζ, which is the first constraint. In the objective function, we count the payoffs starting from t_0 without loss of generality.[13] Banks fall into two categories: early banks receiving information at $t_i \in [t_0, t_0 + \zeta - \tau]$ and late banks receiving information at $t_i \in (t_0 + \zeta - \tau, t_0 + \eta]$. Early banks exit before the crisis and survive, while late banks are caught by the crisis and fail. The first term in the objective function is the payoff for the survival banks. A typical survival bank t_i gets the continuous payoff flow c^H in the period $[t_0, t_i + \tau]$ for time length $t_i + \tau - t_0$ until its exit time $t_i + \tau$, and gets the payoff Σ at its exit time by reinvesting its full liquidation value $L = 1$. The second term in the objective function is the payoff for the failure banks. A typical failure bank t_i gets the continuous payoff flow c^H in the period $[t_0, t_0 + \zeta]$ for time length ζ until the crisis arrival time point $t = t_0 + \zeta$, and gets the expected payoff $\Pi(\ell)$ at the crisis arrival time by reinvesting its partial liquidation value $L = \ell = \ell(\zeta) < 1$. The third term is the payoff for the outside investor sector. Figure 2 gives an illustration.

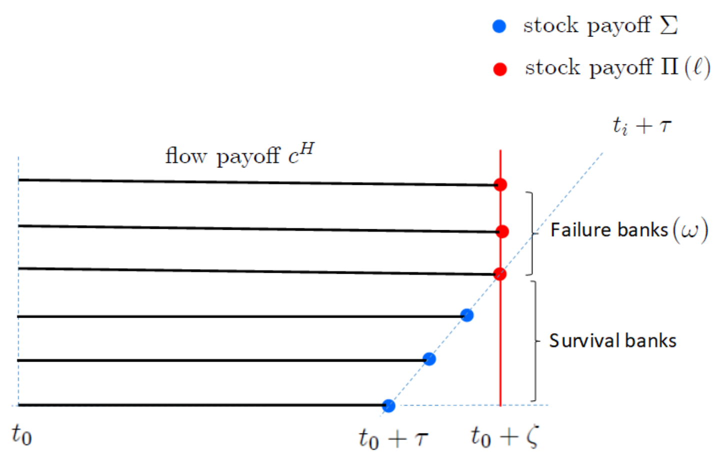

Figure 2: Illustration of the payoffs for banks

[13] If instead we count all the payoffs starting from time $t = 0$, the objective function of (7) is simply altered by adding a constant $c^H t_0$.

It is worth noting that because the social planner cannot observe t_0 either, the maximization problem can instead be written by adding the expectation operator over t_0. However, as we can see in the first line of (7) (define $s = t_i - t_0$ and replace t_i by s in the integral part), t_0 in the objective function can actually be cancelled out. So the expectation over t_0 is irrelevant.[14]

The first-order condition of Program (7) implies $F(\tau) = 0$, where

$$F(\tau) \equiv \frac{d\Psi(\tau, \zeta(\tau))}{d\tau} = (1-\omega) c^H + \left(c^H + \frac{d\Pi(\ell)}{d\ell}\frac{d\ell}{d\zeta}\right)\frac{d\zeta}{d\tau}\omega + (\Pi(\ell) - \Sigma)\frac{d\omega}{d\tau} + \left(-\frac{d\ell}{d\zeta}\right)\frac{d\zeta}{d\tau}\omega, \quad (8)$$

in which $\frac{d\zeta}{d\tau} = \frac{1}{1+\frac{\kappa}{\beta}\eta}$, $\frac{d\ell}{d\zeta} = -v$, $\frac{d\Pi(\ell)}{d\ell} = \Sigma$, and $\frac{d\omega}{d\tau} = \frac{d\left(1-\frac{\zeta-\tau}{\eta}\right)}{d\tau} = -\frac{1}{\eta}\left(\frac{d\zeta}{d\tau} - 1\right)$.

The first-order derivative (8) highlights the benefit-cost tradeoff for the social planner in choosing the optimal waiting length τ. An increase in τ has four effects on the payoffs in the objective function of (7). First, survival banks with $1-\omega$ mass obtain the interest flow c^H for a longer period because the crisis is delayed for longer, so the total incremental payoffs are given by the first term on the left-hand side (LHS) of (8). Second, failure banks with ω mass also obtain the interest flow c^H for a longer period; however, their expected reinvestment payoff $\Pi(\ell)$ is decreased due to a more delayed crisis. Third, a more delayed crisis results in some banks switching from survival banks to failure banks and each of such banks loses $\Sigma - \Pi(\ell)$, which is the third term. The fourth term represents the change in payoff for the outside investor sector. Note that combining the second term and the fourth term in (8) (i.e., the change in payoff for failure banks and for outside investors as a whole) yields $\left(c^H + \frac{d(\Pi(\ell)-\ell)}{d\ell}\frac{d\ell}{d\zeta}\right)\frac{d\zeta}{d\tau}\omega < 0$ by Assumption 2.

Proposition 1. *The social planner has a unique optimal τ, denoted by τ^{SB}, which lies in $\tau^{SB} \in [0, \bar{\zeta})$. Moreover, under the sufficient condition that $\frac{v(\Sigma-1)}{c^H} \geq \frac{\beta}{\kappa\eta} + 2$, it follows that $\tau^{SB} = 0$.*

The intuition for the sufficient condition to ensure $\tau^{SB} = 0$ in Proposition 1 is the following. One dollar of cash has different social values in the hand of failure banks (sellers) and in the hand of outside investors (buyers), because they have different marginal utilities/productivities, recalling that failure banks have valuable new investment opportunities while outside investors do not. (This is equivalent to buyers and sellers having different MRS, causing "distributive externality", in Dávila and Korinek (2018)). Hence, the wealth redistribution caused by the price change matters. The value (PV) from reinvesting the fire-sale price ℓ for failure banks is $\Pi(\ell) = \ell + \ell(\Sigma - 1)$, while the value for outside investors is ℓ itself. Hence, a decrease in price ℓ causes wealth redistribution, reducing the social surplus, and the effect can be captured by $\frac{d(\Pi(\ell)-\ell)}{d\ell} = \Sigma - 1$. A delayed crisis decreases the fire-sale price ℓ and thus reduces the surplus, captured by $\frac{d(\Pi(\ell)-\ell)}{d\zeta} = \frac{d\ell}{d\zeta}\frac{d(\Pi(\ell)-\ell)}{d\ell} = -v(\Sigma - 1)$, while the gain from a delayed crisis is captured by a function of cash flow c^H. Therefore, when $\frac{v(\Sigma-1)}{c^H}$ is high enough, the social planner would choose no delay, that is, $\tau^{SB} = 0$.

[14] If we instead count the payoffs starting from time $t = 0$ and consider the expectation over t_0, the objective function is simply altered to have an additional constant term regarding $c^H\mathbb{E}(t_0)$, so the optimum does not change.

First best Before closing this subsection, we discuss the first best, where the social planner can perfectly observe the shock time t_0 and coordinate all banks to choose the same waiting length τ. In this case, there is no asynchronous awareness and banks are essentially homogeneous. The crisis arrival time then is $t = t_0 + \zeta$ with $\zeta = \tau$; that is, all banks exit from the speculative sector simultaneously at time $t = t_0 + \zeta$, in contrast with the second best case where (survival) banks exit sequentially over $[t_0 + \tau, t_0 + \zeta)$. Denote by τ^{FB} the social planner's optimal τ in the first best, and by ζ^{FB} and ζ^{SB} the equilibrium ζ in the first best and in the second best, respectively. It is easy to show that if v is sufficiently high, ceteris paribus, the first best is such that banks stay in the speculative sector as long as possible and, at the same time, the liquidation price is ensured to be $\ell = 1$. So $t = t_0 + \tau^{FB}$ solves $\theta(t) + \beta \cdot \omega(t) = \alpha$, where $\theta(t) = \alpha - \kappa(t - t_0)$ and $\omega(t) = \omega_0$ (recalling that ω_0 is the maximum absorbing capacity of the outside investors to have no fire-sale discount).[15] This implies $\tau^{FB} = \zeta^{FB} = \omega_0 \frac{\beta}{\kappa} < \zeta^{SB}$ by noting $\zeta_0 \equiv \omega_0 \frac{\beta}{\kappa} < \underline{\zeta}$ and $\zeta^{SB} \in [\underline{\zeta}, \bar{\zeta}]$.

2.2.2 The decentralized competitive equilibrium

Banks receive information regarding time t_0 in an asynchronous manner.[16] Suppose every bank uses the strategy of waiting for time length τ^* after receiving its information, and the crisis occurs at $t = t_0 + \zeta$. The decentralized equilibrium is characterized by the pair (τ^*, ζ). We find the equilibrium in three steps.

First, given τ^*, find ζ. Lemma 1 gives the result; that is,

$$\zeta = \frac{\tau^* + \eta}{1 + \frac{\kappa}{\beta}\eta}. \tag{9}$$

Second, given ζ, find the optimal strategy τ_i^* for an individual bank t_i. Because bank t_i can infer that t_0 must occur within $t_0 \in [t_i - \eta, t_i]$, the posterior pdf of t_0 conditional on the information $t_0 \in [t_i - \eta, t_i]$ from the perspective of bank t_i is given by

$$\phi(t_0|t_i) = \frac{\phi(t_0)\frac{1}{\eta}}{\int_{t_i-\eta}^{t_i} \phi(s)\frac{1}{\eta} ds} = \frac{\lambda e^{\lambda(t_i - t_0)}}{e^{\lambda\eta} - 1}, \tag{10}$$

by recalling that the prior pdf of t_0 is $\phi(t_0) = \lambda e^{-\lambda(t_0 - m)}$. Similarly, the conditional cumulative distribution function (cdf) of t_0 from the perspective of bank t_i is given by

$$\Phi(t_0|t_i) = \int_{t_i-\eta}^{t_0} \phi(s|t_i)\, ds = \frac{e^{\lambda\eta} - e^{\lambda(t_i - t_0)}}{e^{\lambda\eta} - 1}. \tag{11}$$

The individual bank t_i's optimization problem is given by

$$\tau_i^* = \arg\max_{\tau_i} \left\{ \begin{array}{l} \underbrace{\Pr\left(t_0 + \zeta \in (t_i + \tau_i, t_i + \zeta]\right)}_{\text{probability of survival}} \underbrace{\left(\tau_i c^H + \Sigma\right)}_{\text{payoff in the case of survival}} \\ + \int_{x=0}^{x=\tau_i} \underbrace{f\left(t_0 + \zeta = t_i + x\right)}_{\text{density of failure}} \underbrace{\left(x c^H + \Pi(\ell)\right)}_{\text{payoff in the case of failure}} dx \end{array} \right\}, \tag{12}$$

[15]In the first best, a proportion ω_0 of banks exit at $t_0 + \tau^{FB}$ while other banks exit an instant before $t_0 + \tau^{FB}$.

[16]As in Abreu and Brunnermeier (2002, 2003) and He and Manela (2016), in the main text we only consider the case where $t_0 \geq \eta + m$. We show robustness for the case where $t_0 < \eta + m$ in the proof of Proposition 2.

where $\ell = \ell(\zeta)$ given in (6) and an individual bank takes ℓ as given, and $\Pi(\ell) = \ell\Sigma$ by (2). The terms f and Pr in (12) are given by $f(t_0 + \zeta = t_i + x) = \phi(t_0 = t_i + x - \zeta | t_i) = \frac{\lambda e^{\lambda(\zeta - x)}}{e^{\lambda\eta} - 1}$ and $\Pr(t_0 + \zeta \in (t_i + \tau_i, t_i + \zeta]) = \Phi(t_0 = t_i | t_i) - \Phi(t_0 = t_i + \tau_i - \zeta | t_i) = \frac{e^{\lambda(\zeta - \tau_i)} - 1}{e^{\lambda\eta} - 1}$, by considering that the conditional pdf $\phi(t_0 | t_i)$ and cdf $\Phi(t_0 | t_i)$ are given in (10) and (11).

We explain (12). The individual bank which receives information at t_i knows that the crisis will occur at the earliest at $t = t_i^+$ and at the latest at $t = t_i + \zeta$ and hence that the crisis arrival time must fall into the interval $t_0 + \zeta \in (t_i, t_i + \zeta]$.[17] Thus, when the individual bank chooses its exiting time as $t_i + \tau_i$, it knows that there are two possibilities: $t_0 + \zeta \in (t_i, t_i + \tau_i] \cup (t_i + \tau_i, t_i + \zeta]$. For the possibility $t_0 + \zeta \in (t_i + \tau_i, t_i + \zeta]$, the crisis arrival time $t_0 + \zeta$ is after its exiting time $t_i + \tau_i$, in which case the bank survives and its payoff (starting from t_i) is given by the first term on the right-hand side (RHS) of (12). For the possibility $t_0 + \zeta \in (t_i, t_i + \tau_i]$, the crisis arrival time $t_0 + \zeta$ is before its exiting time $t_i + \tau_i$, in which case the bank fails at the crisis arrival time $t_i + x$, where $x \in (0, \tau_i]$. For each x, we can find the probability density for $t_0 + \zeta = t_i + x$ and the corresponding expected payoff, which gives the second term on the RHS of (12).

The first-order condition with respect to τ_i for (12) implies $\hat{F}(\tau_i = \tau_i^*; \zeta) = 0$, where

$$\hat{F}(\tau_i; \zeta) \equiv c^H \cdot \Pr(t_0 + \zeta \in (t_i + \tau_i, t_i + \zeta]) - (\Sigma - \Pi(\ell)) \cdot f(t_0 + \zeta = t_i + \tau_i), \tag{13}$$

which highlights the benefit-cost tradeoff for an individual bank: the first term corresponds to the expected benefit of delay and the second term corresponds to the expected cost. Hence, the first-order condition $\hat{F}(\tau_i = \tau_i^*, \zeta) = 0$ implies

$$\frac{f(t_0 + \zeta = t_i + \tau_i^*)}{\Pr(t_0 + \zeta \in (t_i + \tau_i^*, t_i + \zeta])} = \frac{c^H}{\Sigma - \Pi(\ell)}, \tag{14}$$

that is,

$$\frac{\lambda}{1 - \exp[-\lambda(\zeta - \tau_i^*)]} = \frac{c^H}{\Sigma - \Pi(\ell)}. \tag{15}$$

The intuition behind (15) can be explained as in Abreu and Brunnermeier (2002). That is, the conditional density that the crisis occurs at the next instant is given by the hazard rate $h(t_0 + \zeta = t_i + \tau_i | t_i, \tau_i) = \frac{\lambda}{1 - \exp[-\lambda(\zeta - \tau_i)]}$. Hence, τ_i^* is given by the indifference condition

$$\underbrace{\Sigma}_{\text{exit at } t = t_i + \tau_i^*} = \underbrace{(1 - \Delta h)\left(\Delta c^H + \Sigma\right) + (\Delta h)\,\Pi(\ell)}_{\text{postpone for an instant } \Delta},$$

which implies (15) under $\Delta \to 0$. We can see that the tradeoff is between *flow* payoff $\left(c^H - c^L\right) \cdot dt$ (where $c^L \to 0$) and *stock* payoff $(1 - \ell)\Sigma$.

It is interesting to note that the first-order derivative (13) for an individual bank and its counterpart, the first-order derivative (8) for the social planner, look similar but are different. Indeed, the

[17] In the proof of Proposition 2 in the appendix, we will distinguish the two cases of $\zeta < \eta$ and $\zeta \geq \eta$, which nevertheless give the same first-order condition.

two first-order derivatives are linked economically. When we consider the limiting case $\lambda \to 0$, we can formally characterize their relation. Lemma 3 follows.

Lemma 3. *When $\lambda \to 0$ (and $r \to 0$), it follows that $\hat{F}\left(\tau_i = \tau, \zeta\right) = \frac{\partial \Psi(\tau,\zeta)}{\partial \tau}$; that is, an individual bank's first-order derivative coincides with the social planner's if she took ζ as given.*

In fact, when $\lambda \to 0$ (meaning that t_0 has an improper uniform prior distribution), the posterior probability and density become $\Pr\left(t_0 + \zeta \in \left(t_i + \tau, t_i + \zeta\right]\right) \to \frac{\zeta - \tau}{\eta} = 1 - \omega$ and $f\left(t_0 + \zeta = t_i + \tau\right) \to \frac{1}{\eta}$. That is, under the limit $\lambda \to 0$, even with Bayesian updating, posteriorly an individual bank t_i perceives that it can be located at any point in the queue $[t_0, t_0 + \eta]$ with *equal probability*. This coincides with the *equal weight* that the social planner assigns to individual banks when calculating the aggregate payoff for all banks together.

Third, by symmetric equilibrium, we have

$$\tau_i^* = \tau^*. \tag{16}$$

With the above three steps, the competitive equilibrium is given by the system of equations (9), (15) and (16). Combining the three equations, we have the following equation:

$$\Gamma\left(\tau\right) \equiv h\left(\tau, \zeta\right) - \frac{c^H}{\left[1 - \ell\left(\zeta\right)\right]\Sigma}, \tag{17}$$

where $h\left(\tau, \zeta\right) = \frac{\lambda}{1 - \exp\left[-\lambda\left(\zeta - \tau\right)\right]}$ and $\zeta = \zeta\left(\tau\right)$ given by (4). The competitive equilibrium solves equation $\Gamma\left(\tau^*\right) = 0$ with taking into account the possible corner solution. Because $h\left(\tau, \zeta\left(\tau\right)\right)$ is increasing in τ and $\frac{c^H}{\left[1-\ell(\zeta)\right]\Sigma}$ is decreasing in τ, $\Gamma\left(\tau\right)$ is increasing in τ. Moreover, when $\tau = \bar{\zeta}$, it follows that $h\left(\tau, \zeta\right) = \infty$ and thus $\Gamma\left(\tau = \bar{\zeta}\right) > 0$. Therefore, if $\Gamma\left(\tau = 0\right) < 0$, there is a unique equilibrium with non-corner solution $\tau^* > 0$; otherwise, there is a unique equilibrium with corner solution $\tau^* = 0$.

Proposition 2. *The decentralized competitive equilibrium, characterized by the pair (τ^*, ζ), is given by (9), (15) and (16). There exists a unique equilibrium. Moreover, if parameter ζ_0 is close to $\underline{\zeta}$ enough such that $\Gamma\left(\tau = 0\right) < 0$, the unique equilibrium satisfies $\tau^* > 0$ (non-corner solution).*

Proposition 2 shows banks' delay (i.e., $\tau^* > 0$) in response to information in an investment game. The delay in response is due to asynchronous awareness which introduces an individual bank's uncertainty about when the crisis will come after its own awareness. As long as the cost in the case of being caught by the crisis is not particularly high (i.e., the condition for the non-corner solution), an individual bank has incentives to wait. We can immediately verify that $\tau^* = 0$ is not an equilibrium when $\zeta_0 \to \underline{\zeta}$.[18] Intuitively, if other banks set $\tau^* = 0$, then $\Sigma - \Pi\left(\ell\right) \to 0$ under $\zeta_0 \to \underline{\zeta}$; that is, there is no cost of waiting for a particular individual bank t_i and thus it has incentives to wait (i.e., $\tau_i^* > 0$). That is, $\tau^* = 0$ is not incentive compatible for the decentralized competitive equilibrium.

[18]To guarantee the non-corner solution $\tau^* > 0$, the paper of Abreu and Brunnermeier (2002) also implicitly assumes a parameter restriction, in a similar spirit to ours. Concretely, it requires $\left.\frac{\lambda}{1-\exp\left[-\lambda\left(\varphi(\tau^*) - \tau^*\right)\right]}\right|_{\tau^*=0} \leq \frac{c}{\beta}$ in their paper (Figure 2 and Proposition 1 in their paper).

2.2.3 Comparison of the second best and the competitive equilibrium

We have the following result.

Proposition 3. *Under the sufficient condition that $\frac{v(\Sigma-1)}{c^H}$ is high enough, it follows that $\tau^{SB} \leq \tau^*$ with strict inequality holding whenever $\tau^* > 0$ (non-corner solution).*[19] *That is, the banks exit too late compared with the second-best optimum.*

The key intuition behind the result of the over-waiting for banks in the competitive equilibrium is externality. To better understand the mechanism behind Proposition 3, we compare the benefit-cost tradeoff in the two equilibria, given by the two first-order conditions of (8) and (13). We reiterate them here:

$$F(\tau) = c^H \underbrace{(1-\omega)}_{\text{survival probability}} - (\Sigma - \Pi(\ell)) \underbrace{\frac{d\omega}{d\tau}}_{\text{failure density}} + \underbrace{\left(c^H + \frac{d(\Pi(\ell)-\ell)}{d\ell}\frac{d\ell}{d\zeta}\right)\frac{d\zeta}{d\tau}\omega}_{\text{for failure banks and outsiders as a whole } (-)}$$

$$\hat{F}(\tau_i, \zeta) = c^H \underbrace{\Pr}_{\text{survival probability}} - (\Sigma - \Pi(\ell)) \underbrace{\frac{\partial(1-\Pr)}{\partial \tau_i}}_{\text{failure density}},$$

where $\frac{d\omega}{d\tau} = -\frac{1}{\eta}\left(\frac{d\zeta}{d\tau} - 1\right)$ given in (8) and $\Pr = \frac{e^{\lambda(\zeta-\tau_i)}-1}{e^{\lambda\eta}-1}$ given in (13). When the first-order condition for the social planner is evaluated at the competitive equilibrium solution pair $(\tau^*, \zeta(\tau^*))$, it follows that $\hat{F}(\tau_i^* = \tau^*, \zeta(\tau^*)) = 0$ and

$$F(\tau^*) = F(\tau^*) - \hat{F}(\tau_i^* = \tau^*, \zeta(\tau^*)) = \left\{\left[c^H + \underbrace{(-v)(\Sigma-1)}_{\text{not internalized } (-)}\right]\frac{d\zeta}{d\tau}\omega + \varkappa\right\}\Bigg|_{(\tau,\zeta)=(\tau^*,\zeta(\tau^*))}, \quad (18)$$

where $\varkappa$ is a residual term independent of v, provided in the proof in the appendix. In particular, individual banks do not internalize the "price impact" of a more delayed crisis, i.e., the term $(-v)(\Sigma-1)$ in (18), by noting $\frac{d\ell}{d\zeta}\frac{d(\Pi(\ell)-\ell)}{d\ell} = (-v)(\Sigma-1)$. Overall, from (18), under the sufficient condition that $\frac{v(\Sigma-1)}{c^H}$ is high enough, $F(\tau^*; \zeta(\tau^*)) < 0$. That is, the equilibrium solution pair $(\tau^*; \zeta(\tau^*))$ is not the optimal choice for the social planner, who can achieve a higher value of the second-best objective function by choosing a lower τ, which implies that $\tau^{SB} < \tau^*$ whenever $\tau^* > 0$.

To provide clearer economic insight, we consider the limiting case of $\lambda \to 0$. Lemma 4 follows.

Lemma 4. *Under the limiting case of $\lambda \to 0$ (and $r \to 0$), it follows that $\frac{d\Psi(\tau,\zeta(\tau))}{d\tau} = \frac{\partial\Psi(\tau,\zeta)}{\partial\tau} + \frac{d\zeta}{d\tau}\frac{\partial\Psi(\tau,\zeta)}{\partial\zeta}$ implies*

$$F(\tau) - \hat{F}(\tau_i = \tau, \zeta(\tau)) = \underbrace{\frac{d\zeta}{d\tau}\frac{\partial\Psi(\tau,\zeta)}{\partial\zeta}}_{\textit{not internalized}} \quad (19)$$

[19] Clearly, a sufficient condition for the strict inequality $\tau^{SB} < \tau^*$ is the condition in Proposition 1 to guarantee $\tau^{SB} = 0$ jointly with the condition in Proposition 2 to guarantee $\tau^* > 0$.

at any pair $(\tau, \zeta(\tau))$, where $\frac{d\zeta}{d\tau} = \frac{1}{1+\frac{\kappa}{\beta}\eta}$ and

$$\frac{\partial \Psi(\tau,\zeta)}{\partial \zeta} = \underbrace{(\Sigma - \Pi(\ell))\frac{1}{\eta}}_{\text{part 1 externality } (+)} + \underbrace{c^H \omega}_{\text{part 2 } (+)} + \underbrace{(\Sigma - 1)\,\omega\,(-v)}_{\text{part 3 } (-)}. \tag{20}$$

The magnitude of the part-1 externality, called "quantity effect", is determined by the product of two variables:

(Q1) The incremental value for a bank switching to survival from failure, $\Sigma - \Pi(\ell)$,

(Q2) The sensitivity of the mass of survival banks to a change in the crisis time, $\frac{\partial(1-\omega)}{\partial \zeta} = \frac{1}{\eta}$;

The magnitude of the part-3 externality, called "price effect", is determined by the product of three variables:

(P1) The difference in marginal utility/productivity between failure banks and outside investors, $\frac{d(\Pi(\ell)-\ell)}{d\ell} = \Sigma - 1$,

(P2) The aggregate quantity of fire sales (the mass of failure banks), ω,

(P3) The sensitivity of the fire-sale price to a change in the crisis time, $\frac{d\ell}{d\zeta} = -v$.

The expression (19) analytically characterizes the wedge between the two first-order conditions and shows that the wedge maps one-to-one to the externality. Recall that $\Psi(\tau,\zeta)$ is the social planner's objective function. From Lemma 4, we can see that while the social planner considers both the direct and the indirect effects of increasing τ (i.e., $F(\tau) = \frac{d\Psi(\tau,\zeta(\tau))}{d\tau}$), individual banks only consider the direct effect (i.e., $\hat{F}(\tau_i = \tau, \zeta) = \frac{\partial \Psi(\tau,\zeta)}{\partial \tau}$; see also Lemma 3). The externality is fully captured by the indirect effect, namely the term $\frac{d\zeta}{d\tau}\frac{\partial \Psi(\tau,\zeta)}{\partial \zeta}$.

Equation (20) further decomposes the externality into three parts. When individual banks increases τ^*, the crisis is delayed for longer (i.e., ζ is increased). An increase in ζ has three effects (externality) on other banks which keep τ^*. First, a more delayed crisis causes some among the other banks, which would otherwise fail, to be able to successfully escape from being caught by the crisis and each of such banks gains $\Sigma - \Pi(\ell)$ (part 1), by noting $\frac{\partial(1-\omega)}{\partial \zeta} = \frac{1}{\eta}$ and $1-\omega$ is the mass of survival banks. Second, a more delayed crisis also causes those eventually failed banks among the other banks, with a total mass ω, to obtain the higher interest flow c^H for a longer period (part 2). Third, a more delayed crisis results in a lower liquidation price $\ell(\zeta)$ for those eventually failed banks and thereby a lower social surplus $\Pi(\ell) - \ell$ (part 3), by noting $\frac{d\ell}{d\zeta}\frac{d(\Pi(\ell)-\ell)}{d\ell} = (-v)(\Sigma - 1)$. Under a sufficient condition that $\frac{v(\Sigma-1)}{c^H}$ is high enough, the negative externality outweighs the positive externality (so the net externality is negative), which is the root cause of the result $\tau^{SB} < \tau^*$.

It is worth noting that when some banks (say, group A) decide to increase their τ for some reason, their action will make the other banks (group B) survive more likely, but the *total* number of banks caught by the crisis in the system (including those among group B *and* those among group A) increases, decreasing the fire-sale price.

Our paper studies the timing dimension of externality, which is different from prior researches. Our decomposition of externality can be connected to the decomposition in Dávila and Korinek

(2018). The part 3 of externality in our model corresponds to "distributive externality" in their paper (see also Caballero and Krishnamurthy, 2003; Lorenzoni, 2008; He and Kondor, 2016). The three variables that determine the magnitude are also similar. The "collateral externality" in their decomposition is the effect of the price-change triggered tightness of collateral constraints and consequent asset reallocation between buyers and sellers with different valuations/productivities on assets. The part 1 of externality in our model resembles the "collateral externality" in their paper. But the tightness of the constraint in our model is not triggered by the change of the asset price. More importantly, the sign of "collateral externality" is often negative, i.e., individual agents engage in overborrowing and overinvestment. In our model, the sign of the part 1 of externality is positive, i.e., some individual banks' longer delay benefits other banks by making other banks survive more likely.

Two remarks regarding the externality in our model are in order. First, our paper emphasizes the *possibility* of the outcome of negative net externality in the timing strategy, not the *necessity*. Indeed, our model shows that there exist both positive and negative forces of externality in the timing strategy, and the net effect can be either positive or negative. However, received wisdom and direct intuition would suggest that the externality should only be positive as implied by the classic static coordination game such as Morris and Shin (2004), Cooper and John (1988), and Angeletos and Pavan (2007), where banks benefit each other by extending credit. Our paper identifies a new source of externality in the timing strategy that is negative and showing that the net externality in the timing strategy can be negative. Second, the externality we identify in the dynamic context depends on the *interaction* of two ingredients of the model characterized by two parameters $\beta > 0$ and $\gamma > 0$. In fact, based on Lemma 4, the negative externality in the timing strategy is due to $\frac{d\ell}{d\zeta} = -v$ where $v \equiv \gamma\frac{\kappa}{\beta}$, while we can decompose $\frac{d\ell}{d\zeta} = \frac{d\omega^C}{d\zeta}\frac{d\ell}{d\omega^C}$ (recalling that ω^C denotes the mass of banks caught by the crisis) with $\frac{d\omega^C}{d\zeta} = \frac{\kappa}{\beta}$ and $\frac{d\ell}{d\omega^C} = -\gamma$. Hence, the production complementarity parameter $\beta > 0$ guarantees that, after the fundamental deterioration starts, a more delayed crisis leads to more banks being caught by the crisis (i.e., $\frac{d\omega^C}{d\zeta} > 0$), while the downward-slope parameter $\gamma > 0$, which is emphasized by the existing literature, guarantees that more banks being caught leads to a lower fire-sale price for every caught bank (i.e., $\frac{d\ell}{d\omega^C} < 0$).

Corollary 1 follows directly from Proposition 3.

Corollary 1. *The measure of banks that reallocate resources to invest in the traditional business sector is given by* $\frac{\zeta-\tau}{\eta} + \left(1 - \frac{\zeta-\tau}{\eta}\right) p(\ell)$*, which is lower in the decentralized competitive equilibrium than in the second-best optimum.*

In fact, a more delayed crisis results in fewer survival banks and a lower liquidation value for failure banks, and both forces contribute to the decrease in the number of banks in the end investing in the traditional business sector.

Proposition 3 and Corollary 1 show that in the general-equilibrium production economy, individual banks' delay in response to information in the decentralized competitive equilibrium (given in Proposition 2) is an *over-delay* from the social welfare point of view. The welfare consequence of the over-delay is resource misallocation in the sense that the resource in the speculative business sector cannot be timely and thus efficiently reallocated to capture the investment opportunities in the traditional business sector.

Discussions of the assumptions 1) We assume that a bank cannot observe other banks' exit or entry decisions, so the information regarding t_0 is not revealed. In fact, some earlier works in the literature address the issue why the information regarding t_0 in the model framework of Abreu and Brunnermeier (2003) is not revealed through some devices. For example, by introducing multidimensional uncertainty into the model of Abreu and Brunnermeier (2003), Doblas-Madrid (2012) shows that the economy can have a sufficient amount of noise which makes it difficult for agents to infer information from endogenous asset prices. Along this line, a more complicated model could explicitly formalize that banks' exit or entry decisions involve many different motives, so agents face multidimensional uncertainty and it is difficult for them to infer the information regarding t_0 through banks' exit or entry actions. 2) As long as the fire-sale price at the crisis time has a discount (i.e., $\ell < 1$, so there is a gap between Σ and $\Pi(\ell)$) plus $c^H > c^L$, there is a tradeoff in timing for individual banks. 3) When $\beta > 0$ plus $\gamma > 0$, the timing dimension of externality arises.

A numerical exercise We provide a simple numerical exercise to compare the second best and the decentralized equilibrium. The numerical exercise is based on the baseline model under the general Assumption 1, rather than under Assumption 1′ (see the details in Appendix B). Whenever possible, we choose parameter values according to the standard literature on quantitative research. As it is hard to find the standard literature to refer to for some of other parameters, we will try to provide reasonable values for them. Table 1 summarizes the parameter values.

Parameter	Description	Value
r	Risk-free interest rate	0.02
c^L	Cash flow of the traditional sector	0.07
c^H	Cash flow of the speculative sector	0.4
$\alpha \equiv \alpha_0$	The economic fundamentals prior to the negative shock	2
β	The parameter of (production) complementarity among banks	1
κ	The decline rate of economic fundamentals after the negative shock	0.4
λ	The parameter of the prior distribution of t_0	0.01
η	Banks' sequential awareness window $[t_0, t_0+\eta]$	3
γ	The downward slope of the fire-sale price function	2.5
ω_0	The intercept of the fire-sale price function	0.54
$\zeta_0 \equiv \omega_0 \frac{\beta}{\kappa}$	In equilibrium no fire sale discount if the crisis occurs before $t_0+\zeta_0$	1.35
$v \equiv \gamma \frac{\kappa}{\beta}$	The decline rate of the equilibrium fire-sale price over time after $t_0+\zeta$	1

Table 1: List of parameter values for calibration

Given these parameter values, for the second-best equilibrium, the social planner chooses $\tau^{SB} = 0$, under which $\zeta = 1.36$ the equilibrium fire-sale price is $\ell = 0.99$. For the decentralized equilibrium, individual banks choose $\tau^* = 0.23$ (namely, delaying for 0.23 years), under which $\zeta = 1.47$ the equilibrium fire-sale price is $\ell = 0.88$. The social welfare function $\Psi(\tau, \zeta(\tau))$ evaluated at $\tau^{SB} = 0$ is $\Psi(\tau, \zeta(\tau))|_{\tau=0} = 3.83$, while the social welfare function evaluated at $\tau^* = 0.23$ is $\Psi(\tau, \zeta(\tau))|_{\tau=0.23}$ $= 3.73$. That is, the welfare loses by 2.61% for the decentralized equilibrium relative to the second best efficiency. In the literature quantifying the effect of financial frictions, the welfare cost due to the externalities (for the decentralized equilibrium allocation relative to the constrained efficient allocation) ranges from 0.05% to 0.135% in Bianchi and Mendoza (2010) and Bianchi (2011) and is 0.15% in Boissay, Collard, and Smets (2016).

3 The full model with both entry and exit decisions

In this section, we extend the baseline model by allowing banks to enter the speculative business sector from the traditional business sector. The purpose of studying the entry decisions is to shed light on the build-up of credit booms.

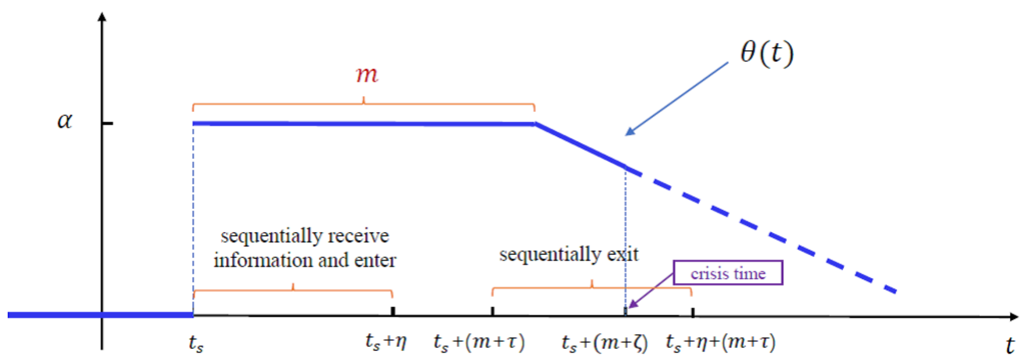

Figure 3: The setup of the full model

3.1 Setting

We add some minimum elements to the baseline model. There is a continuum of banks with unit mass. These banks are currently investing in the traditional business sector. But all banks know that there is an alternative investment opportunity in the speculative business sector and that the fundamentals of the alternative investment opportunity follow the process:

$$\theta(t) = \begin{cases} \alpha & \text{for } t \in [t_s, t_0 \equiv t_s + m] \\ \alpha - \kappa(t - t_0) & \text{for } t > t_0 \end{cases}, \tag{21}$$

that is, the alternative investment opportunity starts to have the good fundamentals, α, from $t = t_s$, and maintains the good fundamentals for time length $m > 0$, and then declines after $t = t_0$ as in the baseline model. Banks know the evolution of the process, but do not know when it starts. More specifically, the arrival of t_s is not observable by banks, and follows a prior exponential distribution with pdf $\phi(t_s) = \lambda e^{-\lambda t_s}$ in the support $t_s \in [0, +\infty)$. (This implies that t_0 has pdf $\phi(t_0) = \lambda e^{-\lambda(t_0 - m)}$ in the support $t_0 \in [m, +\infty)$, in line with the baseline model.) After t_s, banks sequentially become informed of the arrival of t_s, and the information spreads among banks over $[t_s, t_s + \eta]$, following a uniform distribution. We assume that $\eta < m$. Figure 3 gives an illustration.

As in the baseline model, a bank's payoff at time t by investing in the speculative sector is still given by (1) while the return on the investment in the traditional business sector is c^L; payoff parameters now satisfy the general Assumption 1 instead of Assumption 1′. The fire-sale price function is still $g(\omega) = \begin{cases} 1 & \text{when } \omega \leq \omega_0 \\ 1 - \gamma \cdot (\omega - \omega_0) & \text{when } \omega > \omega_0 \end{cases}$, where $\gamma > 0$ and parameter ω_0 is a small positive number. Other setups are the same as those in the baseline model.

3.2 Equilibrium

Banks make the entry and exit decisions, that is, they decide when to enter the speculative sector and when to exit. We focus on the equilibrium in which banks immediately enter the speculative sector upon receiving their private information. Given the entry strategy, we find the exit strategy of banks. Specifically, a bank sets the strategy of staying in the speculative sector for time length $m + \tau$ after its entry at $t = t_i$,[20] where τ is to be solved.

As in the baseline model, we study the social planner's second-best constrained problem and the decentralized competitive equilibrium. To save the space, the details are relegated to the appendix. Proposition 4 summarizes the results.

Proposition 4. *1) Under certain conditions (given in the proof), banks immediately enter the speculative sector upon receiving their private information about the arrival of t_s; 2) There exists a unique optimum τ^{SB} for the social planner; 3) There exists a unique τ^* for individual banks in the decentralized competitive equilibrium; 4) Under the sufficient condition that $v\left(\frac{c^L}{r} - 1\right)/c^H$ is high enough, where $v \equiv \gamma\frac{\kappa}{\beta}$, it follows that $\tau^{SB} < \tau^*$. That is, individual banks stay in the speculative sector too long in the decentralized equilibrium in comparison with the second-best optimum.*

The full model is highly similar to the baseline model mathematically, but the full model gives the process of the rise and fall of the speculative sector. Banks gradually enter the speculative sector, forming the rise. Individual banks stay in the speculative sector until they expect that the speculative sector will collapse shortly. Individual banks gradually exit the speculative sector until the bust with a crash at the crisis time. Some banks are caught by the crisis. The boom period before the crash lasts longer and the crisis comes later for the decentralized equilibrium than for the second best, and the magnitude of the crash is also larger (see also Figure 4 later).

4 The macroeconomic model

We now consider a standard macroeconomic growth model with both entry and exit. This model explicitly shows the capital accumulation and consumption process and thus demonstrates the business cycle dynamics under the microeconomic friction à la Abreu and Brunnermeier (2002, 2003).

[20]The reason we characterize the strategy by waiting length $m + \tau$ is to make the presentation of the equilibrium symmetric to that of the baseline model. Concretely, given the strategy of waiting length $m + \tau$, the first bank exits from the speculative sector at time $t = t_s + (m + \tau) = t_0 + \tau$, which is exactly the same as in the baseline model.

4.1 Setting

We use a simple textbook macroeconomic growth setting, by embedding two modified elements of the baseline model. First, the production that generates a constant dividend process (c^H or c^L) in the baseline model is modified as an A-K technology (see, e.g., Jones, 1995). Second, when a bank is caught by a crisis, it loses a proportion of its capital in fire sales. The tradeoff in the baseline model — a higher cash flow vs. losing capital — becomes a higher growth rate vs. losing capital in the macroeconomic model.

One investor is matched with one bank, and one bank is matched with one firm (so we can call a team as a "investor-bank-firm"). This simplified setup is to capture the following realism: if investors withdraw funding from banks, banks suffer a creditor run and must liquidate their loan portfolios, which would in turn affect or interrupt the business operation of firms on the real side of the economy. Each agent ("investor-bank-firm") j has the following utility function

$$\int_0^\infty e^{-\rho t} \log C_t^j dt. \tag{22}$$

The production takes the form of an A-K technology. If bank j operates in the traditional business sector, its production technology (production function) is

$$y_t^j = ZK_t^j, \tag{23}$$

where Z represents productivity. If it operates in the speculative business sector, its production technology is

$$y_t^j = \begin{cases} AK_t^j & \text{if } \theta(t) + \beta \cdot \omega(t) \geq \alpha \text{ (no crisis)} \\ 0 \cdot K_t^j & \text{otherwise} \end{cases}, \tag{24}$$

where $A > Z$, and $\omega(t)$ is the total measure of active banks at time t in the speculative business sector. The setup (24) is a modified version of (1) in the baseline model. The fundamentals of the speculative business sector, $\theta(t)$, follow the process given in (21). The setup of the information structure is the same as in the model in Section 3.

A bank faces the budget constraint

$$C_t^j + I_t^j = ZK_t^j$$

if it operates in the traditional business sector and

$$C_t^j + I_t^j = AK_t^j$$

if it operates in the speculative business sector, where I_t^j is the investment (saving). Crucially, if a bank is caught by the crisis, its capital is under fire sales, in which case a proportion of its capital is lost; that is,

$$\begin{cases} K_t \to K_t \cdot 1 & \text{if not caught by crisis} \\ K_t \to K_t \cdot \ell \leq K_t & \text{if caught} \end{cases}, \tag{25}$$

where ℓ is the recovery rate in fire sales (equivalently, $1-\ell$ can be interpreted as the fire-sale discount rate). Formally, if a bank is not caught by the crisis, its capital evolves according to

$$dK_t^j = -\delta K_t^j dt + I_t^j dt,$$

where δ is the depreciation rate; if a bank is caught by the crisis at time t, its capital evolves according to

$$dK_t^j = -\delta K_t^j dt + I_t^j dt - (1-\ell) K_t^j.$$

By changing the "simple discount" to the "compound discount", we redefine the downward-sloping fire-sale price function:

$$\ell = \tilde{g}(\omega) = \begin{cases} 1 & \text{when } \omega \le \omega_0 \\ \exp\left[-\gamma \cdot (\omega - \omega_0)\right] & \text{when } \omega > \omega_0 \end{cases}, \tag{26}$$

where ω is the measure of banks under fire sales. The fire-sale price function (26) can be equivalently written as

$$\ell = \tilde{g}(q;t) = \begin{cases} 1 & \text{when } \frac{q}{\bar{K}(t)} \le \omega_0 \\ \exp\left[-\gamma \cdot \left(\frac{q}{\bar{K}(t)} - \omega_0\right)\right] & \text{when } \frac{q}{\bar{K}(t)} > \omega_0 \end{cases},$$

where $q = \omega \bar{K}(t)$ is the aggregate quantity of capital under fire sales and $\bar{K}(t)$ is the average quantity of capital per bank under fire sales.[21]

The outside investors also have utility function (22) and production function (23). The capital conversion technology of the outside investors is the following. When the outside investors buy q units of capital from the banking sector, the outside investors can convert them to $G(q)$ units of new capital, where $\ell = G'(q) > 0$ and $G''(q) < 0$. The outside investors pay the banking sector $q\ell$ and retain $G(q) - q\ell$ as profit. We assume that the outside investors are less efficient in using capital in the spirit of Kiyotaki and Moore (1997), and only $\mu(G(q) - q\ell)$ units of capital are finally put into production, where $\mu < 1$. In addition, we assume the outside investors have initial endowment $W_0 > 0$ in terms of capital at $t = 0$.

4.2 Equilibrium

As in the model in Section 3, banks make the entry and exit decisions. Again, we focus on the equilibrium in which banks immediately enter the speculative sector upon receiving their private information. Given the entry strategy, we find the exit strategy of banks. Specifically, a bank sets the strategy of staying in the speculative sector for time length $m + \tau$ after its entry at $t = t_i$, where τ is to be solved. Denote the arrival time of the crisis by $t = t_0 + \zeta$.

Based on the modified fire-sale price function (26), we have Lemma 5.

[21] The corresponding production function is $G(q) = \begin{cases} q & \text{when } \frac{q}{\bar{K}(t)} \le \omega_0 \\ \bar{K}(t)\omega_0 + \frac{\bar{K}(t)}{\gamma}\left[1 - \left(\exp\left[-\gamma \cdot \left(\frac{q}{\bar{K}(t)} - \omega_0\right)\right]\right)\right] & \text{when } \frac{q}{\bar{K}(t)} > \omega_0 \end{cases}$ for the outside investor sector.

Lemma 5. *The liquidation value or the loan recovery value for a bank caught by the crisis in the macroeconomic model is given by*

$$\ell \equiv \ell(\zeta) = \begin{cases} 1 & \text{when } \zeta \leq \zeta_0 \\ \exp[-v(\zeta - \zeta_0)] & \text{when } \zeta > \zeta_0 \end{cases}, \tag{27}$$

where $v \equiv \gamma \frac{\kappa}{\beta}$ *and* $\zeta_0 \equiv \omega_0 \frac{\beta}{\kappa}$*, which has the property that* $\frac{d\ell}{d\zeta} = -\ell v < 0$ *for* $\zeta > \zeta_0$*.*

The utility maximization of (22) subject to the budget constraint gives the decision rule:

$$C_t^j = \rho K_t^j.$$

Hence, if a bank operates in the speculative business sector and is not caught by the crisis, its capital evolves according to

$$dK_t^j = -\delta K_t^j dt + (A - \rho) K_t^j dt,$$

which implies

$$k_{t+x} = k_t + ax \quad \text{for any } x > 0, \tag{28}$$

where we define $k_t \equiv \log K_t$ and $a \equiv A - \rho - \delta > 0$. The value function for such a bank is then given by

$$V(k_t) = \int_0^\infty e^{-\rho x} (\log \rho + k_{t+x})\, dx = \frac{\log \rho + k_t}{\rho} + \frac{a}{\rho^2}.$$

Similarly, we can find the value function for a bank that stays in the speculative sector in the period $[t, t+x]$ and then *safely* moves to the traditional sector from $t + x$, that is,

$$V(k_t) = \frac{\log \rho + k_t}{\rho} + \frac{a}{\rho^2} - e^{-\rho x} \frac{a - z}{\rho^2},$$

where we define $z \equiv Z - \rho - \delta > 0$. Finally, the value function for a bank that stays in the speculative business sector in the period $[t, t+x]$ and is caught by the crisis at time $t + x$ and then moves to the traditional business sector by using its recovery capital from fire sales is given by

$$V(k_t) = \frac{\log \rho + k_t}{\rho} + \frac{a}{\rho^2} - e^{-\rho x} \frac{a - z}{\rho^2} + e^{-\rho x} \frac{\log \ell}{\rho}.$$

4.2.1 The social planner's second-best constrained problem

Suppose the social planner can coordinate all banks to choose the same length of stay, $m + \tau$, in the speculative sector conditional on banks' entry. Given the exit strategy, banks are individually rational in the entry decision ex ante. Conditional on entry, the social planner chooses the optimum

τ^{SB} as follows:

$$\tau^{SB} = \arg\max_{\tau} \left\{ \begin{array}{l} \frac{\log\rho + k(t_s)}{\rho} + \frac{z}{\rho^2} + \int_{t_s}^{t_s+\zeta-\tau} \left[e^{-\rho(t_i - t_s)} \frac{a-z}{\rho^2} \left(1 - e^{-\rho(m+\tau)}\right) \right] \frac{1}{\eta} dt_i \\ + \int_{t_s+\zeta-\tau}^{t_s+\eta} \left[e^{-\rho(t_i - t_s)} \frac{a-z}{\rho^2} \left(1 - e^{-\rho[(m+\zeta)-(t_i-t_s)]}\right) + e^{-\rho(m+\zeta)} \frac{\log\ell}{\rho} \right] \frac{1}{\eta} dt_i \\ + \left[\frac{\log\rho + (\log W_0 + t_s z)}{\rho} + z\frac{1}{\rho^2} + e^{-\rho(m+\zeta)} \frac{\log\left[1 + \frac{\mu(G(q)-q\ell)}{\exp[\log W_0 + (t_0+\zeta)z]}\right]}{\rho} \right] \end{array} \right\}$$

$$\begin{aligned} &\text{s.t. } \zeta = \frac{\tau+\eta}{1+\frac{\kappa}{\beta}\eta} \text{ given by (4)} \\ &\quad \omega^C \equiv \omega(t = t_0 + \zeta) = 1 - \frac{\zeta - \tau}{\eta} \text{ and } q = \omega^C \bar{K}(t) \\ &\quad \ell = \ell(\zeta) \text{ given by (27).} \end{aligned} \tag{29}$$

In the objective function, all the utility terms are discounted back to time t_s without loss of generality. On top of the growth rate z in the traditional sector, banks obtain the additional growth rate $a - z$ if they operate in the speculative sector. A typical early bank receiving information at $t_i \in [t_s, t_s + \zeta - \tau]$ operates in the speculative sector during the period $[t_i, t_i + m + \tau]$ for time length $m + \tau$. A typical late bank receiving information at $t_i \in (t_s + \zeta - \tau, t_s + \eta]$ operates in the speculative sector during the period $[t_i, t_0 + \zeta = t_s + m + \zeta]$ for time length $(m + \zeta) - (t_i - t_s)$; in particular, it is caught by the crisis at time at $t = t_s + m + \zeta$. For the outside investors, they have saved up $\exp[\log W_0 + (t_0 + \zeta) z]$ units of capital at $t = t_0 + \zeta$, so the profit gained from banks' fire sales increases that capital by a proportion of $\frac{\mu(G(q)-q\ell)}{\exp[\log W_0 + (t_0+\zeta)z]}$.

Proposition 5. *The second-best equilibrium of the macroeconomic model, characterized by the pair (τ^{SB}, ζ), is given by (4) and (29) and satisfies two entry conditions (given in the proof).*

1) When λ is small enough and m is high enough, banks immediately enter the speculative sector upon receiving their private information about the arrival of t_s.

2) Suppose $a - z < v$. Under the sufficient condition that ρ is small enough and μ is small enough, the social planner has a unique optimum τ^{SB} in choosing stay length $m + \tau$.

4.2.2 The decentralized competitive equilibrium

Conditional on entry, the decentralized equilibrium is characterized by the pair (τ^*, ζ). Given ζ, find the optimal strategy τ_i^* for an individual bank t_i. The individual bank t_i's optimization problem in choosing its length of stay, $m + \tau_i$, is given by

$$\tau_i^* = \arg\max_{\tau_i} \left\{ \begin{array}{l} \underbrace{\Pr\left(t_0 + \zeta \in (t_i + m + \tau_i, t_i + m + \zeta]\right)}_{\text{probability of survival}} \underbrace{U(t_i + m + \tau_i)}_{\text{value in the case of survival}} \\ + \int_{x=0}^{x=\tau_i} \underbrace{f\left(t_0 + \zeta = t_i + m + x\right)}_{\text{density of failure}} \underbrace{\hat{U}(t_i + m + x)}_{\text{value in the case of failure}} dx \end{array} \right\}, \tag{30}$$

where

$$\begin{aligned} U(t_i + m + \tau_i) &= \frac{\log\rho + k(t_i)}{\rho} + \frac{a}{\rho^2} - e^{-(m+\tau_i)\rho} \frac{a-z}{\rho^2} \\ \hat{U}(t_i + m + x) &= \frac{\log\rho + k(t_i)}{\rho} + \frac{a}{\rho^2} - e^{-(m+x)\rho} \frac{a-z}{\rho^2} + e^{-(m+x)\rho} \frac{\log\ell}{\rho}, \end{aligned}$$

with $\ell = \ell(\zeta)$ given in (27) and an individual bank taking ℓ as given.

The first-order condition of the optimization problem (30) implies

$$\frac{\lambda}{1 - \exp\left[-\lambda\left(\zeta - \tau_i^*\right)\right]} = \frac{a - z}{-\log \ell}. \tag{31}$$

The result of (31) is intuitive. A bank faces the following tradeoff in choosing its optimal τ_i. On the one hand, an increase in τ_i, meaning that the bank stays in the speculative sector longer, makes its capital grow at a higher rate (i.e., from z to a). In other words, if the bank experiences such an increase in capital growth rate over the period $[t, t + dt]$, the gain is $(a - z)\, dt$ in terms of $\log$-capital (i.e., $k_{t+dt} = k_t + (a - z)\, dt$ by recalling (28)). On the other hand, an increase in τ_i raises the chance to be caught by the crisis; if that happens, the loss is $\log \ell$ in terms of $\log$-capital (i.e., $k_{t+dt} = k_t + \log \ell$). In terms of the impact on the $\log$ utility given by the value function, the tradeoff is then between the effect $\frac{(a-z)dt}{\rho}$ and the effect $\frac{-\log \ell}{\rho}$. This exactly maps onto the tradeoff in (15) for the baseline model: flow payoff $\left(c^H - c^L\right) dt$ (where $c^L \to 0$) versus stock payoff $(1 - \ell)\,\Sigma$.

Proposition 6. *The decentralized competitive equilibrium of the macroeconomic model, characterized by the pair (τ^*, ζ), is given by (9), (31) and (16), and satisfies two entry conditions (given in the proof).*

1) When λ is small enough and m is high enough, banks immediately enter the speculative sector upon receiving their private information about the arrival of t_s.

2) There exists a unique equilibrium.

Proposition 7 compares the second best and the competitive equilibrium.

Proposition 7. *Under the sufficient condition that ρ is small enough, μ is small enough, and v is high enough, ceteris paribus, it follows that $\tau^{SB} < \tau^*$. That is, individual banks stay in the speculative sector too long in the decentralized equilibrium in comparison with the second-best optimum.*

4.3 Aggregate Output

In this subsection, we work out the aggregate output. For simplicity, we assume that $m - \eta$ is sufficiently high such that $t_s + m + \tau > t_s + \eta$ is true, which means that only after all banks have already entered the speculative sector do they start to gradually exit the speculative sector. Denote by $K_0 \equiv \exp k_0$ the capital each bank possesses at time $t = 0$ and by $Y(t)$ the aggregate output of the two sectors together at time t.

We divide time into five stages: $t \in [0, t_s)$, $[t_s, t_s + \eta)$, $[t_s + \eta, t_s + m + \tau = t_0 + \tau)$, $[t_0 + \tau, t_0 + \zeta)$, and $[t_0 + \zeta, +\infty)$. The expression of the aggregate $Y(t)$ in the five stages is provided in Appendix B. We also calculate the aggregate output of the speculative sector only, denoted by $Y_s(t)$, the expression of which is provided in Appendix B.

Figure 4 depicts the process of the aggregate output of $Y(t)$ and $Y_s(t)$ under the second-best equilibrium and the decentralized equilibrium. Note that the second-best equilibrium and the decentralized equilibrium have the same entry decision (i.e., banks immediately enter the speculative

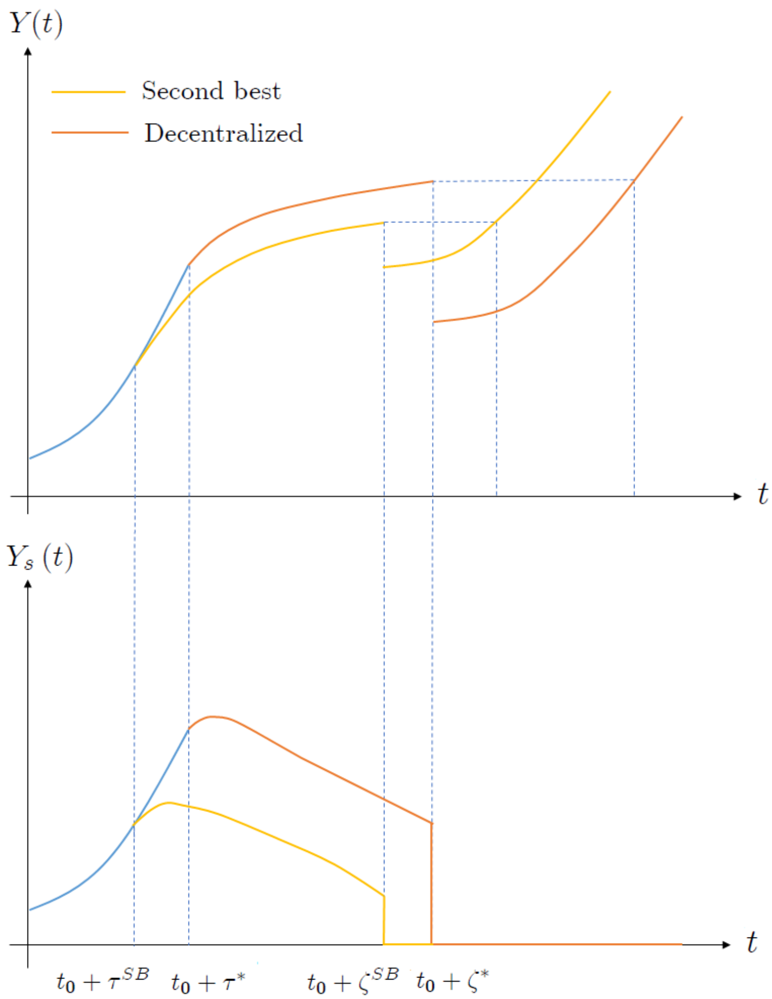

Figure 4: Aggregate output ($Y(t)$ for the two sectors together and $Y_s(t)$ for the speculative sector)

sector upon receiving their private information) and the two equilibria differ only in the exit decision (i.e., length of stay, $m+\tau$). Hence, the aggregate output, $Y(t)$ or $Y_s(t)$, coincides for the first three stages but diverges for the fourth and fifth stages under the two different equilibria. Figure 4 depicts only the process for the last three stages.[22]

From Figure 4, we can see that under the decentralized equilibrium (relative to the second best) the more delayed crisis causes a bigger drop in the aggregate capital and a bigger crash of the aggregate output $Y(t)$ when the crisis hits at $t=t_0+\zeta$. As a result, it takes longer for the aggregate output $Y(t)$ to recover to the pre-crisis level. In short, the decentralized equilibrium features a more delayed crisis and a slower recovery.

The aggregate output $Y_s(t)$ of the speculative sector exhibits a "∩" shape over the fourth stage before the crash. At the beginning of the fourth stage, because only a few banks have started to exit

[22]We assume that no precise and timely information, signal or index about the aggregate output is available, so the shock time t_s is not revealed. This is similar to Abreu and Brunnermeier (2002, 2003) where the asset price cannot reveal the information of the shock time.

the speculative sector and thus the growth in capital of staying banks in that sector can sufficiently offset the loss of capital from exiting banks, the aggregate output $Y_s(t)$ continues to increase for a while before declining.[23] In other words, in the fourth stage, $Y_s(t)$ continues the boom of the third stage but with a slowdown in the growth rate before experiencing a decline, and finally collapses.

5 Policy implications

In this section, we discuss policy measures that can potentially mitigate or eliminate the inefficiency of the decentralized equilibrium relative to the second best, that is, we find policies that might implement the second-best efficiency. To illustrate the idea, we use our baseline model to study policy implications.[24] We analyze two policy measures: tax policy and credit policy.

Tax policy Recall that in the baseline model, a failure bank's payoff is $\Pi(\ell) = \ell + \ell(\Sigma - 1) = \Sigma \cdot \ell$ while a survival bank's payoff is $\Sigma \cdot 1$. Hence, we can alternatively interpret the setup of the baseline model in the way that the investment project in the traditional sector is scalable when banks re-invest their asset liquidation value from the speculative sector. This alternative interpretation is also in line with the setup of our macroeconomic growth model.

We consider the following policy: the government imposes capital tax on failure banks while distributing the tax revenue among all banks as a lump-sum subsidy. Specifically, the government sets the tax rate χ on the asset liquidation value of failure banks, that is, a failure bank retains $(1-\chi)\ell$ amount of capital and the government collects $\chi\ell$ as tax. The total tax revenue for the government thus is $\chi\ell\omega$, where ω is the proportion of failure banks in the system. The government then distributes the tax revenue at time $t = t_0 + \zeta$ such that all banks receive a lump-sum capital subsidy, Λ. The tax policy is characterized by $\{\chi, \Lambda\}$, where Λ is a function of χ determined by the break-even condition of the government, that is, $\Lambda = \chi\ell\omega$. Under this policy, the decentralized equilibrium condition (15) is replaced by

$$\frac{\lambda}{1 - \exp\left[-\lambda\left(\zeta - \tau^*\right)\right]} = \frac{c^H}{\left[\Sigma\left(1+\Lambda\right)\right] - \left[\Sigma\left[\left(1-\chi\right)\ell + \Lambda\right]\right]} \tag{32}$$

or $\frac{\lambda}{1-\exp[-\lambda(\zeta-\tau^*)]} = \frac{c^H}{\Sigma - \Sigma\ell(1-\chi)}$, where $\Sigma\left[(1-\chi)\ell + \Lambda\right]$ is the payoff in the case of failure and $\Sigma(1+\Lambda)$ is the payoff in the case of survival, and an individual bank understands that there is a tax penalty in the case of failure and takes subsidy Λ as a given constant. Then, we can find a unique χ such that τ^* given by (32) together with (9) satisfies $\tau^* = \tau^{SB}$.

Moreover, under the policy, the second-best efficiency (i.e., the value of the objective function of (7) evaluated at $\tau = \tau^{SB}$) is implemented as long as τ^{SB} is implemented. Intuitively, as failure banks and survival banks have the same productivity in using capital, the capital transfer from the failure

[23]For the fourth stage of $Y_s(t)$, if the capital of staying banks does not grow fast enough to offset the loss of capital caused by some banks that exit, the aggregate output $Y_s(t)$ will start to decline from the very beginning at $t = t_0 + \tau$.

[24]The macroeconomic model involves the concave utility function (i.e., the log utility), which complicates the analysis of the policy implementing the second best in terms of utility. However, if we focus on implementing the second best in terms of aggregate output, our analysis in this section based on the baseline model setting applies to the macroeconomic model setting.

banks to the survival banks at $t = t_0 + \zeta$ under the tax policy will not affect the aggregate output. Corollary 2 follows.

Corollary 2. *There exists a unique tax rate χ that can implement the second best in the baseline model.*

The intuition behind the tax policy is easy to understand. Since failure banks must pay a tax (which essentially is a penalty) on their fire-sale value, then the cost of being caught by the crisis becomes higher under the tax policy. Consequently, individual banks will optimally choose to lower the chance of being caught by choosing to exit the speculative sector sooner, which implements second best optimum.

Credit policy In our baseline model, we assume that a failure bank which is short of capital for financing the investment cost 1 can refinance the deficit part through borrowing from external investors. Here we assume that a part or all of the external finance for a failure bank comes from the government (perhaps because the crisis is a systemic crisis, the private sector as a whole is short of funding). The government's funding is costly, e.g., the government has to forgo other projects (opportunity cost) or it may have to borrow from foreign governments. For simplicity and without loss of generality, we assume that the cost of the government's funding is the risk-free interest rate r. As $r \to 0$ under Assumption 1′, we can regard the government's funding cost as approaching zero. When the government lends to failure banks, it can charge a higher net interest ϱ, with $\varrho > 0$. The gain from the interest rate gap ϱ will form the government's interest income, which will be distributed to all banks in the system as a lump-sum subsidy.

For simplicity, we also assume that the government can provide the deficit $1 - \ell$ for sure as long as a failure bank is willing to pay the interest rate ϱ; that is, we set $p(\ell) = 1$ for simplicity. It is easy to show that the total amount of interest income for the government is $\varrho (1 - \ell) \omega$ at time $t = t_0 + \zeta$,[25] where ω is the proportion of failure banks in the system.

Recalling (2), the payoff for a failure bank which borrows $1 - \ell$ at interest rate $1 + \varrho$ becomes

$$\Sigma - (1 - \ell)(1 + \varrho) = \ell + \underbrace{(\Sigma - 1)}_{\text{NPV of project}} - \underbrace{\varrho (1 - \ell)}_{\text{PV of extra interest paid}} .$$

Similar to the earlier analysis, the government will distribute monetary subsidy Λ to every bank at $t_0 + \zeta$. Under the credit policy, the decentralized equilibrium condition (15) is replaced by

$$\frac{\lambda}{1 - \exp[-\lambda(\zeta - \tau^*)]} = \frac{c^H}{(\Sigma + \Lambda) - [\Sigma - (1 - \ell)(1 + \varrho) + \Lambda]} \tag{33}$$

or $\frac{\lambda}{1 - \exp[-\lambda(\zeta - \tau^*)]} = \frac{c^H}{(1 - \ell)(1 + \varrho)}$, where $\Sigma - (1 - \ell)(1 + \varrho) + \Lambda$ is the payoff in the case of failure and $\Sigma + \Lambda$ is the payoff in the case of survival, and an individual bank understands that the refinancing is costly in the case of failure and takes subsidy Λ as a given constant. It is easy to show that there is a

[25] The interest rate ϱ is effectively a one-short interest rate (in terms of PV) at the borrowing time $t = t_0 + \zeta$. It is equivalent to the flow interest rate ϱr in terms of perpetuity on the time interval $[t_0 + \zeta, +\infty)$.

unique ϱ that implements $\tau^* = \tau^{SB}$. Also, as long as τ^{SB} is implemented, the second best efficiency is implemented. Corollary 3 follows.

Corollary 3. *There exists a unique interest rate ϱ that can implement the second best in the baseline model.*

The credit policy is essentially a tax policy. We can regard the extra interest paid, the term $\varrho(1-\ell)$ in the denominator on the RHS of (33), as an *income* tax on a failure bank, and the tax revenue is then distributed among all banks. The difference here is that the tax is imposed on the output of the investment (i.e., income tax), whereas the tax discussed earlier is imposed on the input of the investment (i.e., capital tax).

6 Conclusion

We present a rational-expectations model of credit-driven crises, providing a new perspective to explain why credit booms are often followed by a financial crisis. In particular, our model provides a novel macroeconomic perspective on the dynamic interaction between credit expansions, crises, and recoveries. Our model has both positive and normative implications. On the positive side, we show that asynchronous awareness in the macroeconomic environment, which naturally requires coordination, results in a delay in the responses of banks to their information, which in turn leads to a delayed financial crisis. On the normative side, we show that such a delay in the responses is an over-delay, which is socially inefficient. This is because of the existence of a negative externality: when individual banks choose to extend credit for a longer time, the crisis is delayed for longer and consequently more banks are caught by the crisis, which depresses the fire-sale liquidation values for all caught banks. At the macroeconomic level, fewer survival banks and lower liquidation values for failure banks both contribute to severer capital misallocations, so that it takes a longer time for the capital to accumulate and for the aggregate output to recover to the pre-crisis level. We analyze policy implications of the model.

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Appendix

A Proofs

Proof of Lemmas 1 and 2: The proof is straightforward based on the main text.

Proof of Propositions 1 to 3: We provide the proof for the general case under Assumption 1 in Appendix B.

Proof of Lemmas 3 and 4: We prove Lemma 3 and Lemma 4 together. Recalling Proposition 1, under $r \to 0$, the total derivative is $\digamma(\tau) \equiv \frac{d\Psi(\tau,\zeta(\tau))}{d\tau} = (1-\omega)c^H + (\Pi(\ell) - \Sigma)\frac{d\omega}{d\tau} + \left[c^H + (-v)(\Sigma - 1)\right]\frac{d\zeta}{d\tau}\omega$, where $\frac{d\zeta}{d\tau} = \frac{1}{1+\frac{\kappa}{\beta}\eta}$, $\omega = 1 - \frac{\zeta-\tau}{\eta}$, and $\frac{d\omega}{d\tau} = -\frac{1}{\eta}\left(\frac{d\zeta}{d\tau} - 1\right)$. Also, we can calculate that under $r \to 0$, the two partial derivatives are $\frac{\partial\Psi(\tau,\zeta)}{\partial\zeta} = (\Sigma - \Pi(\ell))\frac{1}{\eta} + \omega\left[c^H + (-v)(\Sigma-1)\right]$ and $\frac{\partial\Psi(\tau,\zeta)}{\partial\tau} = (1-\omega)c^H - \frac{1}{\eta}(\Sigma - \Pi(\ell))$. Clearly, $\digamma(\tau) \equiv \frac{d\Psi(\tau,\zeta(\tau))}{d\tau} = \frac{\partial\Psi(\tau,\zeta)}{\partial\tau} + \frac{d\zeta}{d\tau}\frac{\partial\Psi(\tau,\zeta)}{\partial\zeta}$.

Recalling (13), $\hat{\digamma}(\tau_i,\zeta) = \Pr(t_0 + \zeta \in (t_i + \tau_i, t_i + \zeta])\, c^H - f(t_0 + \zeta = t_i + \tau_i)(\Sigma - \Pi(\ell))$. Hence, $\lim_{\lambda\to 0}\hat{\digamma}(\tau_i = \tau, \zeta) = (1-\omega)c^H - \frac{1}{\eta}(\Sigma - \Pi(\ell))$, by considering $\lim_{\lambda\to 0}\Pr(t_0 + \zeta \in (t_i + \tau, t_i + \zeta]) = \lim_{\lambda\to 0}\frac{e^{\lambda(\zeta-\tau)}-1}{e^{\lambda\eta}-1} = \frac{\zeta-\tau}{\eta} = 1-\omega$ and $\lim_{\lambda\to 0} f(t_0 + \zeta = t_i + \tau) = \lim_{\lambda\to 0}\frac{\lambda e^{\lambda(\zeta-\tau)}}{e^{\lambda\eta}-1} = \frac{1}{\eta}$. Therefore, under $\lambda \to 0$, it follows that $\hat{\digamma}(\tau_i = \tau,\zeta) = \frac{\partial\Psi(\tau,\zeta)}{\partial\tau}$ and $\digamma(\tau) - \hat{\digamma}(\tau_i = \tau,\zeta) = \frac{d\zeta}{d\tau}\frac{\partial\Psi(\tau,\zeta)}{\partial\zeta}$.

Proof of Corollary 1: The measure of banks that reallocate resource to invest in the traditional business sector is given by $\frac{\zeta-\tau}{\eta}[1-p(\ell)] + p(\ell)$. The first-order derivative with respect to τ is

$$\frac{d\left\{\frac{\zeta-\tau}{\eta}[1-p(\ell)] + p(\ell)\right\}}{d\tau} = \frac{d\left(\frac{\zeta-\tau}{\eta}\right)}{d\tau}[1-p(\ell)] + \frac{dp(\ell(\zeta))}{d\tau}\left[1 - \left(\frac{\zeta-\tau}{\eta}\right)\right] < 0$$

because $\frac{d\left(\frac{\zeta-\tau}{\eta}\right)}{d\tau} < 0$, $\frac{dp(\ell(\zeta))}{d\tau} < 0$, and $\frac{\zeta-\tau}{\eta} < 1$. By Proposition 3, τ is higher in the competitive case than in the second best case and, therefore, fewer banks will grab the new investment opportunity in the competitive case.

Proof of Proposition 4: i) Given that all banks use the same (symmetric) strategy by choosing a length of stay $m + \tau$, find the arrival time of the crisis, denoted by $t_0 + \zeta$. Given the strategy, the first bank exits from the speculative sector at $t = t_s + (m + \tau) = t_0 + \tau$. Similar to the procedure to derive Lemma 1, we can find that ζ is given by (4), i.e., $\zeta = \frac{\tau+\eta}{1+\frac{\kappa}{\beta}\eta}$. Clearly, $\frac{d\zeta}{d\tau} = \frac{1}{1+\frac{\kappa}{\beta}\eta} \in (0,1)$. Because the crisis must occur after $t = t_0$, we have $\zeta \geq 0$, which implies $\tau \geq -\eta$. That is, we have the lower bound of τ. We revise Lemma 1 by expanding the support of τ with allowing τ to take a negative value. Hence, ζ is bounded by $\zeta \in \left[0, \bar{\zeta}\right]$, where $\zeta = 0$ at $\tau = -\eta$ and $\zeta = \bar{\zeta} = \frac{\beta}{\kappa}$ at $\tau = \bar{\zeta}$.

ii) The liquidation value is still given by (6) (see Lemma 2). That is, the liquidation value or the loan recovery value for a bank caught by the crisis is given by

$$\ell \equiv \ell(\zeta) = \begin{cases} 1 & \text{when } \zeta \leq \zeta_0 \\ 1 - v\cdot(\zeta - \zeta_0) & \text{when } \zeta > \zeta_0 \end{cases}, \tag{A.1}$$

where $v \equiv \gamma\frac{\kappa}{\beta}$ and $\zeta_0 \equiv \omega_0\frac{\beta}{\kappa}$, which has the property that $\frac{d\ell}{d\zeta} = -v < 0$ for $\zeta > \zeta_0$.

The liquidation value in (A.1) can be $\ell = 1$ (i.e., if $\zeta \leq \zeta_0$), while the liquidation value in Lemma 2 always satisfies $\ell < 1$. For future reference, we define τ_0 such that $\zeta(\tau = \tau_0) = \zeta_0$ based on (4), which gives $\tau_0 \equiv \left[1 + \frac{\kappa}{\beta}\eta\right]\zeta_0 - \eta > -\eta$. As ω_0 is a small positive constant, τ_0 is slightly above $-\eta$. The economic meaning of τ_0 is the threshold of τ above which the equilibrium fire-sale price $\ell < 1$ (i.e., having a discount).

iii) **The social planner's second-best constrained problem** Conditional on the entry strategy of individual banks, the social planner chooses the optimum τ^{SB} as follows:

$$\max_{\tau} \left\{ \begin{array}{l} \int_{t_s}^{t_s+\zeta-\tau} \left[\int_{t_s}^{t_i} e^{-r(s-t_s)} c^L ds + \int_{t_i}^{t_i+m+\tau} e^{-r(s-t_s)} c^H ds + e^{-r(t_i+m+\tau-t_s)} \cdot \frac{c^L}{r} \right] \frac{1}{\eta} dt_i \\ + \int_{t_s+\zeta-\tau}^{t_s+\eta} \left[\int_{t_s}^{t_i} e^{-r(s-t_s)} c^L ds + \int_{t_i}^{t_s+m+\zeta} e^{-r(s-t_s)} c^H ds + e^{-r(t_s+m+\zeta-t_s)} \cdot \Pi(\ell) \right] \frac{1}{\eta} dt_i \\ + e^{-r(m+\zeta)} \left(G\left(\omega^C\right) - \omega^C \ell \right) \end{array} \right\}$$

$$\text{s.t. } \zeta = \frac{\tau+\eta}{1+\frac{\kappa}{\beta}\eta} \text{ given by (4)}, \omega^C \equiv \omega\left(t = t_0+\zeta\right) = 1 - \frac{\zeta-\tau}{\eta}$$

$$\ell = \ell(\zeta) \text{ given by (A.1), and } \Pi(\ell) = \ell + \ell\left(\frac{c^L}{r} - 1\right). \tag{A.2}$$

In the objective function, all the payoffs are discounted back to time t_s without loss of generality. Banks fall into two categories: early banks receiving information at $t_i \in [t_s, t_s+\zeta-\tau]$ and late banks receiving information at $t_i \in (t_s+\zeta-\tau, t_s+\eta]$. The first term in the objective function is the payoff for the first category of banks which exit before the crisis. A typical bank t_i stays in the traditional sector in the period $[t_s, t_i]$ with cash flow c^L, and then enters and stays in the speculative sector in the period $[t_i, t_i+m+\tau]$ for time length $m+\tau$ with cash flow c^H, and finally exits safely at $t = t_i+m+\tau$ with the reinvestment value $\frac{c^L}{r}$. The second term is the payoff for the second category of banks which are caught by the crisis. A typical bank t_i stays in the traditional sector in the period $[t_s, t_i]$ with cash flow c^L, and then enters and stays in the speculative sector until the crisis arrival time $t = t_s+m+\zeta$ with cash flow c^H, and finally exits at the crisis arrival time with the expected reinvestment value $\Pi(\ell)$. The third term is the payoff for the asset buyers (outside investors).

The first-order condition of Program (A.2) implies

$$F(\tau) = \left\{ \begin{array}{r} \left(c^H - c^L\right) \frac{1}{\eta} \frac{e^{r(\zeta-\tau)}-1}{r} + \left[c^H - r\Pi(\ell) + \frac{d\Pi(\ell)}{d\ell}\frac{d\ell}{d\zeta}\right] \frac{d\zeta}{d\tau}\omega + \left(\Pi(\ell) - \frac{c^L}{r}\right)\frac{d\omega}{d\tau} \\ + \left[(-r)\frac{d\zeta}{d\tau}\left(G(\omega) - \omega\ell\right) - \frac{d\ell}{d\zeta}\frac{d\zeta}{d\tau}\omega\right] \end{array} \right\} = 0, \tag{A.3}$$

which is in the exact same form as the first-order condition (B.9).

We discuss two ranges of τ, namely, $\tau \in [-\eta, \tau_0)$ and $\tau \in \left[\tau_0, \bar{\zeta}\right]$. Over the first range $\tau \in [-\eta, \tau_0)$, we have $F(\tau) > 0$, by considering that in this case $\ell(\zeta) = 1$ and $\Pi(\ell) = \frac{c^L}{r}$. Intuitively, there is no cost by setting a higher τ in this first range for the social planner. So the optimum must be $\tau^{SB} \geq \tau_0$. Over the second range $\tau \in [-\eta, \tau_0)$, it follows that $\frac{d\ell}{d\zeta} = -v$, so the signs of all terms of $F(\tau)$ in (A.3) are the same as in (B.9). Consequently, the condition for $F'(\tau) < 0$ is the same as that in Proposition 8; namely, under a sufficient condition that r relative to $\frac{v\left(\frac{c^L}{r}-1\right)}{c^H}$ is small enough (e.g., $r < \frac{\kappa}{\beta}\left(2 - \frac{\frac{v}{c^H}+1}{\frac{v}{c^H}+\frac{v\left(\frac{c^L}{r}-1\right)}{c^H}}\right)$), we have $\frac{dF(\tau)}{d\tau} < 0$, which implies that Program (A.2) has a unique optimal τ. We also show in the proof of Proposition 8 that $F\left(\tau = \bar{\zeta}\right) < 0$. Overall, there exists a unique optimum $\tau^{SB} \in [\tau_0, \bar{\zeta})$ for Proposition 4.

Similar to the proof of Proposition 8, we find a sufficient condition to guarantee $\tau^{SB} = \tau_0$. Given the sufficient condition for a unique equilibrium, we only need to ensure $F(\tau = \tau_0) \leq 0$, i.e.,

$$\underbrace{\left(c^H - c^L\right) \frac{e^{r(\zeta-\tau)} - 1}{r} \frac{1}{\eta}}_{+} + \left[\underbrace{c^H - v\left(\frac{c^L}{r} - 1\right)}_{-} - r\Pi(\ell) \right] \frac{d\zeta}{d\tau}\omega + \underbrace{\left(\Pi(\ell) - \frac{c^L}{r}\right) \frac{d\omega}{d\tau}}_{-}$$
$$+ \underbrace{(-r) \frac{d\zeta}{d\tau} \left(G(\omega) - \omega\ell\right)}_{-} \leq 0, \tag{A.4}$$

where $\zeta = \zeta_0$, $\ell = 1$, $\Pi(\ell) = \frac{c^L}{r}$, $\omega = 1 - \frac{\zeta_0 - \tau_0}{\eta} = \frac{\kappa}{\beta}\zeta_0$, and $\frac{d\zeta}{d\tau} = \frac{1}{1+\frac{\kappa}{\beta}\eta}$. A sufficient condition for (A.4) to be true is $c^H \frac{e^{r(\zeta_0-\tau_0)}-1}{r}\frac{1}{\eta} + \left[c^H - v\left(\frac{c^L}{r} - 1\right)\right] \frac{1}{1+\frac{\kappa}{\beta}\eta} \frac{\kappa}{\beta}\zeta_0 \leq 0$, that is,

$$\frac{v\left(\frac{c^L}{r} - 1\right)}{c^H} \geq \frac{e^{r(\zeta_0-\tau_0)} - 1}{r\eta} \left(1 + \frac{\kappa}{\beta}\eta\right) \frac{1}{\frac{\kappa}{\beta}\zeta_0} + 1, \tag{A.5}$$

which becomes $\frac{v\left(\frac{c^L}{r}-1\right)}{c^H} \geq \frac{\zeta_0-\tau_0}{\eta}\left(1 + \frac{\kappa}{\beta}\eta\right)\frac{1}{\frac{\kappa}{\beta}\zeta_0} + 1$ as $r \to 0$.

iv) **The decentralized competitive equilibrium** Conditional on the entry strategy of individual banks, the decentralized equilibrium is characterized by the pair (τ^*, ζ). Given ζ, find the optimal strategy τ_i^* for an individual bank t_i. The individual bank t_i's optimization problem in choosing its length of stay, $m + \tau_i$, is given by

$$\tau_i^* = \arg\max_{\tau_i} \left\{ \begin{array}{l} \underbrace{\Pr\left(t_0 + \zeta \in (t_i + m + \tau_i, t_i + m + \zeta]\right)}_{\text{probability of survival}} \underbrace{\left[\int_{t_i}^{t_i+m+\tau_i} e^{-r(t-t_i)} c^H dt + \frac{c^L}{r} e^{-r(m+\tau_i)}\right]}_{\text{payoff in the case of survival}} \\ + \int_{x=0}^{x=\tau_i} \underbrace{f\left(t_0 + \zeta = t_i + m + x\right)}_{\text{density of failure}} \underbrace{\left[\int_{t_i}^{t_i+m+x} e^{-r(t-t_i)} c^H dt + \Pi(\ell) e^{-r(m+x)}\right] dx}_{\text{payoff in the case of failure}} \end{array} \right\}, \tag{A.6}$$

where $\ell = \ell(\zeta)$ given in (A.1) and an individual bank takes ℓ as given, and $\Pi(\ell) = \ell + \ell\left(\frac{c^L}{r} - 1\right)$. The first-order condition of (A.6) implies

$$\frac{\lambda}{1 - \exp\left[-\lambda\left(\zeta - \tau_i^*\right)\right]} = \frac{c^H - c^L}{\frac{c^L}{r} - \Pi(\ell)}. \tag{A.7}$$

In (A.7), ℓ can be $\ell = 1$ (e.g., if $\tau^* = -\eta$ and thus $\zeta = 0$), in which case $\Pi(\ell) = \frac{c^L}{r}$. Also, $\Pi(\ell)$ is a continuous function at $\ell = 1$ (recalling (2)).

Define $\Gamma(\tau) \equiv h(\tau, \zeta) - \frac{c^H - c^L}{\frac{c^L}{r} - \Pi(\ell)}$. The equilibrium solution is given by $\Gamma(\tau^*) = 0$. As in Proposition 2, $\Gamma(\tau)$ is increasing in τ which implies no multiple equilibria. Notice that $\Gamma\left(\tau = \bar{\zeta}\right) > 0$ by considering that $h(\tau, \zeta) = \infty$ when $\tau = \bar{\zeta}$. In addition, $\Gamma(\tau = \tau_0) < 0$ since $\Pi(\ell)|_{\tau=\tau_0} = \frac{c^L}{r}$. Therefore, there exists a unique equilibrium $\tau^* \in \left(\tau_0, \bar{\zeta}\right)$. In equilibrium, $\zeta > \zeta_0$ and $\ell < 1$.

v) **Entry conditions** We have two entry conditions to ensure that banks immediately enter the speculative sector upon receiving their private information about the arrival of t_s. The two entry conditions apply to both the second-best constrained problem and the decentralized competitive equilibrium. First, denote by $EV(t_j)$ the expectation of the sum of the discounted values in the period $t \in [t_j, +\infty)$ for a bank that has not received information by time t_j but decides to enter the speculative sector at time t_j. To ensure that an uninformed bank has no incentive to enter the speculative sector, a sufficient condition is

$$EV(t_j) < \frac{c^L}{r} \text{ for any } t_j, \tag{A.8}$$

where $\frac{c^L}{r}$ is the value a bank can get if it keeps operating in the traditional sector. Second, a bank has incentives to enter the speculative sector immediately after receiving its private information, which requires

$$\int_0^{\zeta-\tau} \frac{\lambda e^{\lambda t}}{e^{\lambda\eta}-1} \left[\begin{array}{c} \frac{c^H}{r}\left(1-e^{-(m+\tau)r}\right) \\ +\frac{c^L}{r}e^{-(m+\tau)r} \end{array} \right] dt + \int_{\zeta-\tau}^{\eta} \frac{\lambda e^{\lambda t}}{e^{\lambda\eta}-1} \left[\begin{array}{c} \frac{c^H}{r}\left(1-e^{-(m+\zeta-t)r}\right) \\ +\Pi(\ell)\, e^{-(m+\zeta-t)r} \end{array} \right] dt > \frac{c^L}{r}. \tag{A.9}$$

In (A.9), the bank expects that it will fall into one of two categories: an early bank receiving information at $t \in [t_s, t_s+\zeta-\tau]$ and a late bank receiving information at $t \in [t_s+\zeta-\tau, t_s+\eta]$.

(1) We examine the first entry condition — the condition where an uninformed bank has no incentive to enter the speculative sector. A sufficient condition is that λ is small enough. Intuitively, when λ is small, with high probability the arrival of the shock time t_s is far away from now, so it is not optimal to enter the speculative sector now as long as there is no private signal yet.

Here we make a simplified assumption that if an uninformed bank entering the speculative sector finds the good fundamentals (α) have not come yet, the bank will stay put and wait until the speculative sector becomes profitable; that is, there are unmodeled frictions such that it is too costly for a bank to exit immediately after entry when the speculative sector is in the infancy stage.

Concretely, we find a sufficient condition to guarantee $EV(t_j) < \frac{c^L}{r}$ for any t_j. We analyze two cases: $t_j > \eta$ and $t_j \le \eta$. For the first case of $t_j > \eta$, the bank knows $t_s \ge t_j - \eta > 0$, that is, $t_s \in [t_j-\eta, t_j) \cup [t_j, +\infty)$. The conditional density of t_s is then given by

$$\phi(t_s|t_j) = \begin{cases} \dfrac{\frac{t_s-(t_j-\eta)}{\eta}\lambda e^{-\lambda[t_s-(t_j-\eta)]}}{\frac{1}{\lambda\eta}(1-e^{-\lambda\eta})} & \text{for } t_s \in [t_j-\eta, t_j) \\ \dfrac{\lambda e^{-\lambda[t_s-(t_j-\eta)]}}{\frac{1}{\lambda\eta}(1-e^{-\lambda\eta})} & \text{for } t_s \in [t_j, +\infty) \end{cases}.$$

Thus,

$$EV(t_j) = \int_{t_j-\eta}^{t_j} \phi(t_s|t_j) V(t_j|t_s)\, dt_s + \int_{t_j}^{+\infty} \phi(t_s|t_j) e^{-r(t_s-t_j)} V(t_j|t_s)\, dt_s, \tag{A.10}$$

where $V(t_j|t_s)$ is the bank value (discounted back to the start point to have positive profits) by entering the speculative sector at time t_j, conditional on the profitable shock occurring at t_s. To evaluate (A.10), we define $V(m+\zeta) = \int_0^{m+\zeta} e^{-rt}c^H dt + \int_{m+\zeta}^{+\infty} e^{-rt}c^L dt$, which is the value for a bank that enters the speculative sector at time t_j, stays there for time length $m+\zeta$, and exits safely. It is easy to show that

$$EV(t_j) < EV(m+\zeta) \equiv \int_{t_j-\eta}^{t_j} \phi(t_s|t_j) V(m+\zeta)\, dt_s + \int_{t_j}^{+\infty} \phi(t_s|t_j) e^{-r(t_s-t_j)} V(m+\zeta)\, dt_s.$$

Clearly, $\lim_{\lambda\to 0} EV(m+\zeta) = 0$. Since $EV(m+\zeta)$ is continuous in λ, it follows that $EV(t_j) < \frac{c^L}{r}$ for any $t_j > \eta$ under a sufficient condition that λ is small enough.

Similarly, we can examine the second case of $t_j \le \eta$, and find that under a sufficient condition that λ is small enough, $EV(t_j) < \frac{c^L}{r}$ for any $t_j \le \eta$.

(2) We examine the second entry condition — the condition where an informed bank has incentives to enter the speculative sector immediately. For a bank that receives information at time t_i and

enters the speculative sector immediately, there are two possibilities: (a) If $t_s \in [t_i - \zeta + \tau, t_i]$, then bank t_i can exit the speculative sector safely, after a period of $m + \tau$. (b) If $t_s \in [t_i - \eta, t_i - \zeta + \tau)$, then bank t_i will get caught by the crisis. The expected value for bank t_i over $t_s \in [t_i - \zeta + \tau, t_i]$ is $\int_0^{\zeta - \tau} \frac{\lambda e^{\lambda t}}{e^{\lambda \eta} - 1} \left[\frac{c^H}{r} \left(1 - e^{-(m+\tau)r}\right) + \frac{c^L}{r} e^{-(m+\tau)r} \right] dt$, and the expected value for bank t_i over $t_s \in [t_i - \eta, t_i - \zeta + \tau)$ is $\int_{\zeta - \tau}^{\eta} \frac{\lambda e^{\lambda t}}{e^{\lambda \eta} - 1} \left[\frac{c^H}{r} \left(1 - e^{-r(m+\zeta-t)}\right) + \Pi(\ell) e^{-r(m+\zeta-t)} \right] dt$. So we have (A.9). Denote by $EU(t_i)$ the LHS of (A.9). It follows that $\lim_{m \to +\infty} EU(t_i) = \frac{c^H}{r} > \frac{c^L}{r}$. Therefore, when m is high enough, an informed bank enters the speculative sector immediately.

vi) Same as Proposition 3, under a sufficient condition of (A.5), it follows that $\tau^{SB} < \tau^*$.

Proof of Lemma 5: As in the model in Section 3, we have the following result. The crisis occurs at $t = t_0 + \zeta$, where ζ as a function of τ is given by (4), which has the property that $\frac{d\zeta}{d\tau} = \frac{1}{1 + \frac{\kappa}{\beta}\eta} \in (0, 1)$. Moreover, ζ is bounded by $\zeta \in [0, \bar{\zeta}]$, where $\zeta = 0$ at $\tau = -\eta$ and $\zeta = \bar{\zeta} = \frac{\beta}{\kappa}$ at $\tau = \bar{\zeta}$. By the crisis condition, $\theta(t_0 + \zeta) + \beta \cdot \omega(t_0 + \zeta) = \alpha$, where $\theta(t_0 + \zeta) = \alpha - \kappa\zeta$. We have $\omega^C = \omega(t_0 + \zeta) = \frac{\kappa}{\beta}\zeta$. Based on the fire-sale price function in (26), it follows that $\ell = \ell(\omega^C) = \begin{cases} 1 & \text{when } \frac{\kappa}{\beta}\zeta \leq \omega_0 \\ \exp\left[-\gamma \cdot \left(\frac{\kappa}{\beta}\zeta - \omega_0\right)\right] & \text{when } \frac{\kappa}{\beta}\zeta > \omega_0 \end{cases}$; that is, $\ell = \begin{cases} 1 & \text{when } \zeta \leq \zeta_0 \\ \exp\left[-v(\zeta - \zeta_0)\right] & \text{when } \zeta > \zeta_0 \end{cases}$, where $v \equiv \gamma\frac{\kappa}{\beta}$ and $\zeta_0 \equiv \omega_0 \frac{\beta}{\kappa}$. It is obvious that $\frac{d\ell}{d\zeta} = -\ell v < 0$ for $\zeta > \zeta_0$.

Proof of Proposition 5: i) Define $\tau_0 \equiv \left[1 + \frac{\kappa}{\beta}\eta\right]\zeta_0 - \eta > -\eta$ given by $\zeta(\tau = \tau_0) = \zeta_0$. As in the proof of Proposition 4, the optimum must be $\tau^{SB} \geq \tau_0$. So we only need to focus on $\tau \geq \tau_0$, which means $\frac{d\ell}{d\zeta} = -\ell v$. The first order condition of Program (29) implies

$$F(\tau) = \left\{ \begin{array}{l} \underbrace{\frac{a-z}{\rho} \frac{e^{(\zeta-\tau)\rho} - 1}{\rho} \frac{1}{\eta}}_{\text{survival banks' gain } (+)} + \underbrace{\frac{d\omega}{d\tau}\left(\frac{\log \ell}{\rho}\right)}_{\text{more failure banks } (-)} + \underbrace{\omega \frac{d\zeta}{d\tau} \left[\underbrace{\left(\frac{a-z}{\rho} - \log \ell\right)}_{\text{higher productivity } (+)} + \underbrace{\frac{1}{\rho}\frac{1}{\ell}\frac{d\ell}{d\zeta}}_{\text{lower price } (-)} \right]}_{\text{failure banks' payoff change}} \\ \underbrace{+ (-1)\frac{d\zeta}{d\tau} \log\left[1 + \frac{\mu(G(q) - q\ell)}{\exp[\log W_0 + (t_0 + \zeta) z]}\right] + \mu \frac{d\left(\frac{(G(q)-q\ell)}{\exp[\log W_0 + (t_0+\zeta)z]}\right)/d\tau}{\left[1 + \frac{\mu(G(q)-q\ell)}{\exp[\log W_0 + (t_0+\zeta)z]}\right]\rho}}_{\text{outside sector's payoff change } (+)} = 0 \end{array} \right\} = 0, \tag{A.11}$$

where $\frac{d\ell}{d\zeta} = -\ell v < 0$. Under $v > a - z$ and if ρ is small enough and μ is small enough, we have $F(\tau = \bar{\zeta}) < 0$ because $\zeta = \tau$ and thus the first term of $F(\tau)$ is 0 and also the third term is negative and the fourth term is close to 0. Consequently, $\tau^{SB} \in [\tau_0, \bar{\zeta})$.

We find the second order condition:

$$F'(\tau) = \frac{a-z}{\rho} \frac{e^{\rho(\zeta-\tau)}}{\eta} \left(\frac{d\zeta}{d\tau} - 1\right) + \frac{d\omega}{d\tau}\frac{d\zeta}{d\tau}\left[\frac{1}{\rho}(a-z) + v\zeta - \frac{1}{\rho}v\right] + \omega\frac{d\zeta}{d\tau}\left[v\frac{d\zeta}{d\tau}\right] + \frac{d\omega}{d\tau}\left[\frac{1}{\rho}(-v)\frac{d\zeta}{d\tau}\right]$$
$$+ (-1)\mu\frac{d\zeta}{d\tau} \frac{d\left(\frac{(G(q)-q\ell)}{\exp[\log W_0 + (t_0+\zeta)z]}\right)/d\tau}{1 + \frac{\mu(G(q)-q\ell)}{\exp[\log W_0 + (t_0+\zeta)z]}} + \mu d\left[\frac{d\left(\frac{(G(q)-q\ell)}{\exp[\log W_0 + (t_0+\zeta)z]}\right)/d\tau}{\left[1 + \frac{\mu(G(q)-q\ell)}{\exp[\log W_0 + (t_0+\zeta)z]}\right]\rho}\right]/d\tau.$$

Consider an extreme case where $\mu = 0$ (so the last line in $F'(\tau)$ is zero). Since $\frac{(a-z)}{\rho} e^{\rho(\zeta-\tau)} \left[\frac{d\zeta}{d\tau} - 1\right] < 0$, a sufficient condition for $F'(\tau) < 0$ can be written as

$$\frac{d\omega}{d\tau}\frac{d\zeta}{d\tau}(v\zeta) + \omega\frac{d\zeta}{d\tau}\left[v\frac{d\zeta}{d\tau}\right] < \frac{d\omega}{d\tau}\frac{d\zeta}{d\tau}\left[\frac{1}{\rho}(2v - (a-z))\right].$$

Under $v > a - z$, the right-hand side goes to $+\infty$ when $\rho \to 0$. Meanwhile, it is obvious that the left-hand side is a bounded function on $\tau \in [-\eta, \bar{\zeta}]$. Therefore, there exists $\hat{\rho}$ such that $F'(\tau) < 0$ holds for $\rho < \hat{\rho}$. Since $F'(\tau)$ is continuous in μ, there exists $\hat{\mu}$ such that $F'(\tau) < 0$ holds for $\mu < \hat{\mu}$. Overall, under a sufficient condition that ρ is small enough and μ is small enough, $F'(\tau) < 0$, which implies that there exists a unique equilibrium for the second best problem.

Similar to the proof of Proposition 4, we also find a sufficient condition for $\tau^{SB} = \tau_0$. This requires $F(\tau = \tau_0) \leq 0$. Choosing $\mu = 0$, since $\zeta = \zeta_0$, $\ell = 1$, $\frac{d\zeta}{d\tau} = \frac{1}{1+\frac{\kappa}{\beta}\eta}$, and $\omega = 1 - \frac{\zeta_0 - \tau_0}{\eta} = \frac{\kappa}{\beta}\zeta_0$, we have $F(\tau = \tau_0) \leq 0$ which means that $\frac{a-z}{\rho}\frac{e^{(\zeta_0-\tau_0)\rho}-1}{\rho}\frac{1}{\eta} + \frac{\kappa}{\beta}\zeta_0\frac{1}{1+\frac{\kappa}{\beta}\eta}\left[\frac{a-z}{\rho} - \frac{v}{\rho}\right] \leq 0$, which becomes

$$\frac{v}{a-z} \geq \frac{\frac{e^{(\zeta_0-\tau_0)\rho}-1}{\rho}\frac{1}{\eta}}{\frac{\kappa}{\beta}\zeta_0\frac{1}{1+\frac{\kappa}{\beta}\eta}} + 1. \tag{A.12}$$

Since $F(\tau = \tau_0)$ is continuous in μ, there exists $\tilde{\mu}$ such that when $\mu < \tilde{\mu}$, $F(\tau = \tau_0) \leq 0$ under (A.12). In sum, under a sufficient condition that ρ is small enough, μ is small enough, and $\frac{v}{a-z}$ is high enough, the second best optimum is $\tau^{SB} = \tau_0$.

ii) Similar to the model in Section 3, we have two entry conditions. First, denote by $EV(t_j)$ the expectation of the sum of the discounted utility over $t \in [t_j, +\infty)$ for a bank that has not received information by time t_j but decides to enter the speculative sector at time t_j. To ensure that an uninformed bank has no incentive to enter the speculative sector, a sufficient condition is

$$EV(t_j) < \underline{V} \text{ for any } t_j, \tag{A.13}$$

where $\underline{V} \equiv \frac{\log\rho + k(t_j)}{\rho} + \frac{z}{\rho^2}$ is the sum of the discounted utility a bank can get if it keeps operating in the traditional sector. Second, a bank has incentives to enter the speculative sector immediately after receiving its private information, which requires

$$\int_0^{\zeta-\tau} \frac{\lambda e^{\lambda t}}{e^{\lambda\eta} - 1}\left[\begin{array}{c} a\frac{1}{\rho^2} \\ -\frac{(a-z)}{\rho^2}e^{-(m+\tau)\rho} \end{array}\right]dt + \int_{\zeta-\tau}^{\eta} \frac{\lambda e^{\lambda t}}{e^{\lambda\eta} - 1}\left[\begin{array}{c} a\frac{1}{\rho^2} - \frac{(a-z)}{\rho^2}e^{-(m+\zeta-t)\rho} \\ +e^{-(m+\zeta-t)\rho}\frac{\log\ell}{\rho} \end{array}\right]dt > z\frac{1}{\rho^2}. \tag{A.14}$$

(1) We examine the first entry condition — the condition where an uninformed bank has no incentive to enter the speculative sector. A sufficient condition is that λ is small enough. Intuitively, when λ is small, with high probability the arrival of the shock time t_s is far away from now, so it is not optimal to enter the speculative sector now as long as there is no private signal yet. Again, here we make a simplified assumption that if an uninformed bank entering the speculative sector finds the good fundamentals (α) have not come yet, the bank will stay put and wait until the speculative sector becomes profitable; that is, there are unmodeled frictions such that it is too costly for a bank to exit immediately after entry when the speculative sector is in the infancy stage.

The proof is almost the same as the counterpart in the proof of Proposition 4. We only need to redefine $V(m+\zeta)$ as

$$V(m+\zeta) = \frac{\log\rho + k(t_j)}{\rho} + a\frac{1}{\rho^2} - e^{-\rho(m+\zeta)}\frac{a-z}{\rho^2}.$$

(2) We examine the second entry condition — the condition where an informed bank has incentives to enter the speculative sector immediately. For a bank that receives information at time t_i and

enters the speculative sector immediately, there are two possibilities: (a) If $t_s \in [t_i - \zeta + \tau, t_i]$, then bank t_i can exit the speculative sector safely, after a period of $m + \tau$. (b) If $t_s \in [t_i - \eta, t_i - \zeta + \tau)$, then bank t_i will get caught by the crisis. The expected value for bank t_i over $t_s \in [t_i - \zeta + \tau, t_i]$ is

$$\int_0^{\zeta-\tau} \frac{\lambda e^{\lambda t}}{e^{\lambda\eta} - 1} \left[\frac{\log \rho + k(t_i)}{\rho} + a\frac{1}{\rho^2} - \frac{(a-z)}{\rho^2} e^{-(m+\tau)\rho} \right] dt,$$

and the expected value for bank t_i over $t_s \in [t_i - \eta, t_i - \zeta + \tau)$ is

$$\int_{\zeta-\tau}^{\eta} \frac{\lambda e^{\lambda t}}{e^{\lambda\eta} - 1} \left[\frac{\log \rho + k(t_i)}{\rho} + a\frac{1}{\rho^2} - \frac{(a-z)}{\rho^2} e^{-(m+\zeta-t)\rho} + e^{-(m+\zeta-t)\rho} \frac{\log \ell}{\rho} \right] dt.$$

The total expected value needs be higher than $\underline{V} = \frac{\log \rho + k(t_i)}{\rho} + z\frac{1}{\rho^2}$, so we have (A.14). Denote by $EU(t_i)$ the LHS of (A.14), and it follows that $\lim_{m \to +\infty} EU(t_i) = a\frac{1}{\rho^2} > z\frac{1}{\rho^2}$. Therefore, when m is high enough, an informed bank enters the speculative sector immediately.

Proof of Proposition 6: The proof of part 1) on entry conditions is the same as that in the proof of Proposition 5. We proceed to prove part 2). The first order condition of Program (30) implies

$$\frac{f(t_0 + \zeta = t_i + m + x)}{\Pr(t_0 + \zeta \in (t_i + m + \tau_i, t_i + m + \zeta])} = \frac{\frac{\partial U(t_i + m + \tau_i)}{\partial \tau_i}}{U(t_i + m + \tau_i) - \hat{U}(t_i + m + \tau_i)},$$

that is, $\frac{\lambda}{1 - \exp[-\lambda(\zeta - \tau_i^*)]} = \frac{a-z}{-\log \ell}$. Because $\tau_i^* = \tau^*$, we consider the fixed-point problem

$$\frac{\lambda}{1 - \exp[-\lambda(\zeta(\tau) - \tau)]} = \frac{a-z}{-\log \ell}, \tag{A.15}$$

where function $\zeta(\tau)$ is given by (9) and $\ell = \ell(\zeta)$ is given in (27). Clearly, the solution τ to equation (A.15) must satisfy $\tau > \tau_0$, by noting that when $\tau \leq \tau_0$, we have $\zeta \leq \zeta_0$ and $\ell = 1$.

We consider the possible solution τ to equation (A.15) in the range of $\tau \in (\tau_0, \bar{\zeta}]$, under which ℓ in (A.15) is $\ell = \exp[-v(\zeta - \zeta_0)]$. Define $LF(\tau) \equiv \frac{\lambda}{1 - \exp[-\lambda(\zeta - \tau)]}$; since $\zeta - \tau$ is decreasing in τ, $LF(\tau)$ is increasing in τ, and $\lim_{\tau \to \bar{\zeta}} LF(\tau) = +\infty$. Define $RF(\tau) \equiv \frac{a-z}{v[\zeta(\tau) - \zeta_0]}$; it is obvious that $RF(\tau)$ is decreasing in τ, and $\lim_{\tau \to \tau_0} RF(\tau) = \lim_{\zeta \to \zeta_0} \frac{a-z}{v[\zeta - \zeta_0]} \to +\infty$. Combining these results, there exists a unique solution $\tau^* \in (\tau_0, \bar{\zeta})$.

Proof of Proposition 7: Based on the proof of Proposition 5, under a sufficient condition that ρ is small enough, μ is small enough, and $\frac{v}{a-z}$ is high enough, the second best optimum is $\tau^{SB} = \tau_0$. In the proof of Proposition 6, we also show that the competitive equilibrium satisfies $\tau^* \in (\tau_0, \bar{\zeta})$. Therefore, under a sufficient condition that ρ is small enough, μ is small enough, and $\frac{v}{a-z}$ is high enough, we have $\tau^{SB} < \tau^*$. In Appendix B, we also provide a general proof of Proposition 7, similar to the proof of Proposition 3.

Proof of Corollary 2: To simplify the algebra, we assume that the government distributes the tax revenue at time $t = t_0 + \zeta$ in a way that all banks receive a lump-sum capital subsidy Λ. The government breaks even, so

$$\Lambda = \chi \ell \omega. \tag{A.16}$$

The individual bank t_i's optimization problem in (12) is replaced by

$$\tau_i^* = \arg\max_{\tau_i} \left\{ \begin{array}{c} \underbrace{\Pr\left(t_0 + \zeta \in (t_i + \tau_i, t_i + \zeta]\right)}_{\text{probability of survival}} \underbrace{\left[\tau_i c^H + [\Sigma (1 + \Lambda)]\right]}_{\text{payoff in the case of survival}} \\ + \int_{x=0}^{x=\tau_i} \underbrace{f\left(t_0 + \zeta = t_i + x\right)}_{\text{density of failure}} \underbrace{\left[x c^H + \Pi(\ell)\right]}_{\text{payoff in the case of failure}} \quad dx \end{array} \right\},$$

where the term $\Sigma (1 + \Lambda)$ on the first line is the payoff in the case of survival, and the redefined $\Pi(\ell) \equiv \Sigma \cdot [(1 - \chi)\, \ell + \Lambda]$ is the payoff in the case of failure, and an individual bank takes Λ as a given constant. The first-order condition implies

$$\frac{\lambda}{1 - \exp\left[-\lambda\left(\zeta - \tau^*\right)\right]} = \frac{c^H}{\Sigma - \Sigma\ell\,(1 - \chi)}. \tag{A.17}$$

We show that there exists a unique pair (χ, Λ) that makes the solution given by (A.17) and (A.16) satisfy $\tau^* = \tau^{SB}$. On the one hand, substituting $\tau = \tau^{SB}$ and $\zeta = \zeta\left(\tau^{SB}\right)$ in (A.16), ω is fixed and (A.16) gives Λ being an increasing function of χ. On the other hand, substituting $\tau^* = \tau^{SB}$ and $\zeta = \zeta\left(\tau^{SB}\right)$ in (A.17), (A.17) gives Λ being a decreasing function of χ. So there exists a unique pair (χ, Λ) that implements $\tau^* = \tau^{SB}$. Also, under the sufficient condition that Σ is high enough, $\chi < 1$.

The second best efficiency (i.e., the value of the objective function of (7) evaluated at $\tau = \tau^{SB}$) is implemented when τ^{SB} is implemented. For the outside sector, since $\tau = \tau^{SB}$, the payoff is the same as in the second best case. For survival and failure banks, since $\tau = \tau^{SB}$, their profit flow prior to the crisis is the same as in the second best case. After the crisis, since survival banks and failure banks have the same productivity, the capital transfer under tax and subsidy does not affect the aggregate output. Therefore, the second best efficiency is implemented.

Proof of Corollary 3: The total amount of interest income that the government can use as subsidy is $\varrho\,(1 - \ell)\,\omega$. At time $t_0 + \zeta$, all banks receive subsidy Λ. Here Λ denotes the monetary subsidy, while it represents a capital subsidy in the tax policy case. The government breaks even, that is,

$$\Lambda = \varrho\,(1 - \ell)\,\omega. \tag{A.18}$$

The individual bank t_i's optimization problem in (12) is replaced by

$$\tau_i^* = \arg \left\{ \begin{array}{c} \underbrace{\Pr\left(t_0 + \zeta \in (t_i + \tau_i, t_i + \zeta]\right)}_{\text{probability of survival}} \underbrace{\left[\tau_i c^H + [\Sigma + \Lambda]\right]}_{\text{payoff in the case of survival}} \\ + \int_{x=0}^{x=\tau_i} \underbrace{f\left(t_0 + \zeta = t_i + x\right)}_{\text{density of failure}} \underbrace{\left[x c^H + \Pi(\ell)\right]}_{\text{payoff in the case of failure}} \quad dx \end{array} \right\},$$

where the term $\Sigma + \Lambda$ on the first line is the payoff in the case of survival, and the redefined $\Pi(\ell) \equiv \ell + (\Sigma - 1) - \varrho\,(1 - \ell) + \Lambda$ is the payoff in the case of failure, and an individual bank takes Λ as a given constant. The first-order condition implies

$$\frac{\lambda}{1 - \exp\left[-\lambda\left(\zeta - \tau^*\right)\right]} = \frac{c^H}{(1 - \ell)\,(1 + \varrho)}. \tag{A.19}$$

We show that there exists a unique pair (ϱ, Λ) that makes the solution given by (A.19) and (A.18) satisfy $\tau^* = \tau^{SB}$. On the one hand, substituting $\tau = \tau^{SB}$ and $\zeta = \zeta\left(\tau^{SB}\right)$ in (A.18), ω is fixed and (A.18) gives Λ being an increasing function of ϱ. On the other hand, substituting $\tau^* = \tau^{SB}$ and $\zeta = \zeta\left(\tau^{SB}\right)$ in (A.19), (A.19) gives Λ being a decreasing function of ϱ. So there exists a unique pair (ϱ, Λ) that implements $\tau^* = \tau^{SB}$. In addition, a failure bank will choose to refinance when $\ell + (\Sigma - 1) - \varrho(1-\ell) > \ell$ (i.e., the profit under refinancing is higher than ℓ), which is true under the condition that Σ is high enough.

The second best efficiency (i.e., the value of the objective function of (7) evaluated at $\tau = \tau^{SB}$) is implemented when τ^{SB} is implemented. For the outside investors, since $\tau = \tau^{SB}$, the payoff is the same as in the second-best case. For survival and failure banks, since $\tau = \tau^{SB}$, their profit flow prior to the crisis is the same as in the second-best case. After the crisis, the aggregate output does not change and only the wealth transfer between failure banks and survival banks happens. Therefore, the second-best efficiency is implemented.

Internet Appendix

B Additional results

Equilibrium of the baseline model under Assumption 1

In this subsection, we study the equilibrium under the general Assumption 1. Here we give all the differences from the results under Assumption 1′ in Section 2, and we summarize these differences using a series of propositions.

Clearly, Lemmas 1 and 2 and Corollary 1 are not affected by the general Assumption 1.

The social planner's second-best constrained problem

Suppose that the social planner cannot observe the shock time t_0 either. But the social planner can coordinate all banks to choose the same waiting length τ after being informed. Denote the arrival time of the crisis by $t = t_0 + \zeta$. The second-best constrained problem for the social planner is given by

$$\max_{\tau} \left\{ \begin{array}{l} \int_{t_0}^{t_0+\zeta-\tau} \left[\begin{array}{c} \left(\int_{t_0}^{t_i+\tau} e^{-r(s-t_0)} c^H ds\right) \\ +e^{-r(t_i+\tau-t_0)} \cdot \frac{c^L}{r} \end{array} \right] \frac{1}{\eta} dt_i + \int_{t_0+\zeta-\tau}^{t_0+\eta} \left[\begin{array}{c} \left(\int_{t_0}^{t_0+\zeta} e^{-r(s-t_0)} c^H ds\right) \\ +e^{-r(t_0+\zeta-t_0)} \cdot \Pi(\ell) \end{array} \right] \frac{1}{\eta} dt_i \\ +e^{-r\zeta} \left(G(\omega) - \omega \ell\right) \end{array} \right\}$$

$$\begin{aligned} \text{s.t. } \zeta &= \frac{\tau + \eta}{1 + \frac{\kappa}{\beta}\eta} \text{ given by (4)} \\ \omega &= \omega(t_0 + \zeta) = 1 - \frac{\zeta - \tau}{\eta} \\ \ell &= \ell(\zeta) \text{ given by (6), and } \Pi(\ell) = \ell + \ell\left(\frac{c^L}{r} - 1\right). \end{aligned} \tag{B.1}$$

In the objective function, all the payoffs are discounted back to time t_0 without loss of generality.[26] Banks fall into two categories: early banks receiving information at $t_i \in [t_0, t_0 + \zeta - \tau]$ and late banks receiving information at $t_i \in (t_0 + \zeta - \tau, t_0 + \eta]$. Early banks exit before the crisis and survive, while late banks are caught by the crisis and fail. The first term in the objective function is the payoff for the survival banks. A typical survival bank t_i gets the continuous payoff flow c^H in the period $[t_0, t_i + \tau]$ until its exit time $t_i + \tau$, and gets the payoff $\frac{c^L}{r}$ at its exit time by reinvesting its full liquidation value $L = 1$. The second term in the objective function is the payoff for the failure banks. A typical failure bank t_i gets the continuous payoff flow c^H in the period $[t_0, t_0 + \zeta]$ until the crisis arrival time $t = t_0 + \zeta$, and gets the expected payoff $\Pi(\ell)$ at the crisis arrival time by reinvesting its partial liquidation value $L = \ell = \ell(\zeta) < 1$. The third term is the payoff for the asset buyers in the outside investor sector.

[26] If the payoffs are discounted back to time $t = 0$, the objective function of (B.1) is simply altered by multiplying it by a constant e^{-rt_0}.

The first-order condition of (B.1) implies

$$\left\{\begin{array}{l} \underbrace{\underbrace{\left(c^H-c^L\right)\frac{1}{\eta}\frac{e^{r(\zeta-\tau)}-1}{r}}_{=(1-\omega)(c^H-c^L)\text{ when }r\to 0}}_{\text{survival banks' payoff change }(+)}+\underbrace{\left[c^H-r\Pi\left(\ell\right)+\frac{d\Pi\left(\ell\right)}{d\ell}\frac{d\ell}{d\zeta}\right]\frac{d\zeta}{d\tau}\omega}_{\text{failure banks'payoff change }(-)}+\underbrace{\left(\Pi\left(\ell\right)-\frac{c^L}{r}\right)\frac{d\omega}{d\tau}}_{\text{more banks caught }(-)} \\ \qquad\qquad\qquad+\underbrace{\left[(-r)\frac{d\zeta}{d\tau}\left(G\left(\omega\right)-\omega\ell\right)-\frac{d\ell}{d\zeta}\frac{d\zeta}{d\tau}\omega\right]}_{\text{outside sector's payoff change }(+)} \end{array}\right\}=0, \quad \text{(B.2)}$$

where $\frac{d\zeta}{d\tau}=\frac{1}{1+\frac{\kappa}{\beta}\eta}$, $\frac{d\ell}{d\zeta}=-v$, $\frac{d\Pi(\ell)}{d\ell}=\frac{c^L}{r}$, and $\frac{d\omega}{d\tau}=\frac{d\left(1-\frac{\zeta-\tau}{\eta}\right)}{d\tau}=-\frac{1}{\eta}\left(\frac{d\zeta}{d\tau}-1\right)$.

The first-order condition (B.2) highlights the benefit-cost tradeoff for the social planner in choosing the optimal waiting length τ. An increase in τ has four effects on the payoffs in the objective function of (B.1). First, a survival bank obtains the interest flow c^H for a longer period because the crisis is delayed. These banks' exit time spreads over $[t_0+\tau, t_0+\zeta]$, so the total discounted incremental payoffs are given by the first term on the LHS of (B.2). Second, a failure bank also obtains the interest flow c^H for a longer period; however, its expected reinvestment payoff $\Pi\left(\ell\right)$ is decreased due to a more delayed crisis. The net incremental payoff for failure banks is given by the second term, which is negative by considering $v\left(\frac{c^L}{r}-1\right)>c^H$. Third, a delayed crisis results in some banks switching from survival banks to failure banks and each of such banks loses $\frac{c^L}{r}-\Pi\left(\ell\right)$, which is the third term. The fourth term represents the payoff change for the outside investor sector.

Denote the LHS of (B.2) by $F\left(\tau\right)$. First, observe that $F\left(\tau=\bar{\zeta}\right)<0$ under the parameter condition $v\left(\frac{c^L}{r}-1\right)>c^H$. This is because $\zeta=\tau$ when $\tau=\bar{\zeta}$, meaning the first term of $F\left(\tau\right)$ is equal to zero, while the sum of the second term and the fourth term is negative. So the optimum of (B.1) cannot be $\tau=\bar{\zeta}$. Second, under the sufficient condition that r relative to $\frac{v\left(\frac{c^L}{r}-1\right)}{c^H}$ is small enough (e.g., $r<\frac{\kappa}{\beta}\left(2-\frac{\frac{v}{c^H}+1}{\frac{v}{c^H}+\frac{v\left(\frac{c^L}{r}-1\right)}{c^H}}\right)$), we have $\frac{dF(\tau)}{d\tau}<0$, which implies that Program (B.1) has a unique optimal τ.

Proposition 8. *Under a sufficient condition that r relative to $\frac{v\left(\frac{c^L}{r}-1\right)}{c^H}$ is small enough, the social planner has a unique optimal τ, denoted by τ^{SB}, which lies in $\tau^{SB}\in\left[0,\bar{\zeta}\right)$. Moreover, under a sufficient condition that $\frac{v\left(\frac{c^L}{r}-1\right)}{c^H}$ is high enough,[27] it follows that $\tau^{SB}=0$.*

[27] A sufficient condition is $\frac{v\left(\frac{c^L}{r}-1\right)}{c^H}\geq\left[\exp\left(\frac{r\eta}{1+\frac{\kappa}{\beta}\eta}\right)-1\right]\left(\frac{1}{r\eta}\right)\left(1+\frac{\kappa}{\beta}\eta\right)^2\left(\frac{1}{\frac{\kappa}{\beta}\eta}\right)+1$ (which implies $\frac{v\left(\frac{c^L}{r}-1\right)}{c^H}\geq\frac{\beta}{\kappa\eta}+2$ when $r\to 0$).

The decentralized competitive equilibrium

The individual bank t_i's optimization problem is given by

$$\tau_i^* = \arg\max_{\tau_i} \left\{ \begin{array}{l} \underbrace{\Pr\left(t_0+\zeta \in (t_i+\tau_i, t_i+\zeta]\right)}_{\text{probability of survival}} \underbrace{\left[\int_{t_i}^{t_i+\tau_i} e^{-r(t-t_i)} c^H dt + \frac{c^L}{r} e^{-r\tau_i}\right]}_{\text{payoff in the case of survival}} \\ + \int_{x=0}^{x=\tau_i} \underbrace{f\left(t_0+\zeta = t_i + x\right)}_{\text{density of failure}} \underbrace{\left[\int_{t_i}^{t_i+x} e^{-r(t-t_i)} c^H dt + \Pi\left(\ell\right) e^{-rx}\right]}_{\text{payoff in the case of failure}} dx \end{array} \right\}, \tag{B.3}$$

where $\ell = \ell(\zeta)$ given in (6) and an individual bank takes ℓ as given, and $\Pi(\ell) = \ell + \ell\left(\frac{c^L}{r} - 1\right)$ by (2). The two probability terms in (12) are the same as those in Section 2.

The first-order condition of (B.3) implies

$$h\left(t_0+\zeta = t_i+\tau_i^* \middle| t_i, \tau_i^*\right) \equiv \frac{f\left(t_0+\zeta = t_i+\tau_i^*\right)}{\Pr\left(t_0+\zeta \in (t_i+\tau_i^*, t_i+\zeta]\right)} = \frac{c^H - c^L}{\frac{c^L}{r} - \Pi(\ell)}, \tag{B.4}$$

that is,

$$\frac{\lambda}{1-\exp\left[-\lambda\left(\zeta-\tau_i^*\right)\right]} = \frac{c^H - c^L}{\frac{c^L}{r} - \Pi(\ell)}. \tag{B.5}$$

As in Section 2, we also define

$$\Gamma(\tau) \equiv h(\tau, \zeta) - \frac{c^H - c^L}{\frac{c^L}{r} - \Pi(\ell)}.$$

We have the following proposition.

Proposition 9. *The decentralized competitive equilibrium, characterized by the pair (τ^*, ζ), is given by (9), (B.5) and (16). There exists a unique equilibrium. Moreover, if parameter ζ_0 is close to $\underline{\zeta}$ enough such that $\Gamma(\tau = 0) < 0$, the unique equilibrium satisfies $\tau^* > 0$ (non-corner solution).*

Comparison of the second best and the competitive equilibrium

Proposition 10. *Under a sufficient condition that $\frac{v\left(\frac{c^L}{r}-1\right)}{c^H}$ is high enough, it follows that $\tau^{SB} \leq \tau^*$ with strict inequality holding whenever $\tau^* > 0$ (non-corner solution).[28] That is, the banks exit too late in comparison with the second-best optimum.*

When the first-order condition for the social planner is evaluated at the competitive equilibrium solution pair $(\tau^*, \zeta(\tau^*))$, it follows that

$$\begin{aligned} &\digamma\left(\tau^*, \zeta(\tau^*)\right) = \digamma\left(\tau^*, \zeta(\tau^*)\right) - \Gamma\left(\tau_i^* = \tau^*, \zeta(\tau^*)\right) \\ &= \left.\left\{ \begin{array}{l} \left(c^H - c^L\right) \left[\underbrace{\frac{e^{r(\zeta-\tau)}-1}{r\eta} - \frac{\frac{d\omega}{d\tau}}{h}}_{>0}\right] + \left[c^H + \underbrace{(-v)\left(\frac{c^L}{r} - 1\right)}_{\text{not internalized}} - r\Pi(\ell)\right] \frac{d\zeta}{d\tau}\omega \\ -r\frac{d\zeta}{d\tau}\left(G(\omega) - \omega\ell\right) \end{array} \right\}\right|_{(\tau,\zeta)=(\tau^*,\zeta(\tau^*))}. \end{aligned} \tag{B.6}$$

[28] Clearly, a sufficient condition to have the strict inequality $\tau^{SB} < \tau^*$ is the condition in Proposition 1 to guarantee $\tau^{SB} = 0$ jointly with the condition in Proposition 2 to guarantee $\tau^* > 0$.

To fully characterize the externality, we conduct the following exercise. Suppose that a tiny proportion ϖ of banks (labelled as type A) increase their τ^* to $\tau^* + \Delta$ while other banks (type B) keep their τ^*. (Type-A banks are randomly drawn from the entire population; that is, type-A banks and type-B banks have the identical information distribution over $[t_0, t_0 + \eta]$ and differ only in waiting strategy.) We examine how the payoff of type-B banks is affected. Denote the objective function of the second best in (B.1) by $\Psi(\tau, \zeta)$. When $\Delta \to 0$, the externality to type-B banks (together with outside investors as a whole) is characterized by

$$\begin{aligned}\frac{\partial \Psi(\tau,\zeta)}{\partial \zeta} &= \underbrace{e^{-r\zeta}\frac{1}{\eta}\left(\frac{c^L}{r} - \Pi(\ell)\right)}_{\text{part 1 externality }(+)} + \omega\left[(-r)\Pi(\ell) + \underbrace{c^H}_{\text{part 2 }(+)} + \underbrace{(-v)\left(\frac{c^L}{r} - 1\right)}_{\text{part 3 }(-)}\right]e^{-r\zeta} \\ &\quad + (-r)e^{-r\zeta}\left(G(\omega) - \omega\ell\right). \end{aligned} \tag{B.7}$$

In (B.7), when type-A banks increase τ^*, the crisis is delayed for longer (i.e., ζ is increased). An increase in ζ has three effects (externality) on type-B banks which keep τ^*.[29] First, there is positive externality, including two parts. A more delayed crisis causes some among type-B banks which would otherwise fail to be able to successfully escape from being caught by the crisis and each of such banks gains (part 1), by noting $\frac{\partial(1-\omega)}{\partial \zeta} = \frac{1}{\eta}$. A more delayed crisis also causes those eventually failed banks among type-B banks to obtain the higher interest flow c^H for a longer period (part 2). Second, there is also negative externality on type-B banks (together with outside investors as a whole). A more delayed crisis results in a lower liquidation price $\ell(\zeta)$ for those eventually failed banks among type-B banks and hence a lower social surplus $\Pi(\ell) - \ell$ (part 3), by noting that $\frac{d\ell}{d\zeta}\frac{d(\Pi(\ell)-\ell)}{d\ell} = (-v)\left(\frac{c^L}{r} - 1\right)$. The part 3 corresponds to the "price impact" of a more delayed crisis that individual banks do not internalize. Under a sufficient condition that $\frac{v\left(\frac{c^L}{r}-1\right)}{c^H}$ is high enough, the negative externality outweighs the positive externality (so the net externality is negative), which is the root cause of the result $\tau^{SB} < \tau^*$.

Proof of Propositions 1 and 8: We prove Proposition 8. Proposition 1 is just a special case of Proposition 8. By Lemma 2, $\ell(\zeta) < 1$ for any $\tau \geq 0$. Hence, $\Pi(L) = \Pi(\ell) = \ell + \ell\left(\frac{c^L}{r} - 1\right)$ with $\ell < 1$ in (B.1). We can simplify the objective function in (B.1) as follows:

$$\max_{\tau}\left\{\int_{\tau}^{\zeta}\left[e^{-rt}\frac{c^L}{r} + c^H\frac{1-e^{-rt}}{r}\right]\frac{1}{\eta}dt + \omega\left[e^{-r\zeta}\Pi(\ell) + c^H\frac{1-e^{-r\zeta}}{r}\right] + e^{-r\zeta}\left(G(\omega) - \omega\ell\right)\right\}. \tag{B.8}$$

The first-order condition of (B.8) implies

$$\left\{\begin{array}{r}\left(c^H - c^L\right)\frac{1}{\eta}\frac{e^{r(\zeta-\tau)}-1}{r} + \left[c^H - r\Pi(\ell) + \frac{d\Pi(\ell)}{d\ell}\frac{d\ell}{d\zeta}\right]\frac{d\zeta}{d\tau}\omega + \left(\Pi(\ell) - \frac{c^L}{r}\right)\frac{d\omega}{d\tau} \\ + \left[(-r)\frac{d\zeta}{d\tau}\left(G(\omega) - \omega\ell\right) - \frac{d\ell}{d\zeta}\frac{d\zeta}{d\tau}\omega\right]\end{array}\right\} = 0, \tag{B.9}$$

where we use $\frac{d\omega}{d\tau} = \frac{d\left(1-\frac{\zeta-\tau}{\eta}\right)}{d\tau} = -\frac{1}{\eta}\left(\frac{d\zeta}{d\tau} - 1\right)$. Under the parameter condition $v\left(\frac{c^L}{r} - 1\right) > c^H$, the second term in (B.9) is negative because $c^H + \frac{d\Pi(\ell)}{d\ell}\frac{d\ell}{d\zeta} < 0$ by using $\frac{d\Pi(\ell)}{d\ell} = 1 + \left(\frac{c^L}{r} - 1\right)$.

[29] The externality also includes the interest rate r-related terms in (B.7). But these terms are negative, so the net externality must be negative if the sum of the other terms is negative.

Denote the LHS of (B.9) by $F(\tau)$. First, observe that $F\left(\tau=\bar{\zeta}\right)<0$, because $\zeta=\tau$ when $\tau=\bar{\zeta}$ and thus the first term of $F(\tau)$ is equal to zero and also the sum of the second term and the fourth term is negative by $c^H+\frac{d\Pi(\ell)}{d\ell}\frac{d\ell}{d\zeta}-\frac{d\ell}{d\zeta}=c^H-\left(\frac{c^L}{r}-1\right)v<0$. Second, we have

$$\frac{dF(\tau)}{d\tau}=\underbrace{\left(c^H-c^L\right)\frac{e^{r(\zeta-\tau)}}{\eta}\left(\frac{d\zeta}{d\tau}-1\right)}_{-}+\underbrace{\frac{d\Pi(\ell)}{d\ell}(-v)\frac{d\zeta}{d\tau}\frac{d\omega}{d\tau}}_{-}+\underbrace{(-r)\frac{d\zeta}{d\tau}\left(\omega v\frac{d\zeta}{d\tau}\right)}_{-}$$
$$+\left[\underbrace{r\frac{d\Pi(\ell)}{d\ell}v\left(\frac{d\zeta}{d\tau}\right)^2\omega}_{+}+\left(\underbrace{c^H+\left(\frac{d\Pi(\ell)}{d\ell}-1\right)\frac{d\ell}{d\zeta}}_{+/-}-r\Pi(\ell)\right)\frac{d\zeta}{d\tau}\frac{d\omega}{d\tau}\right], \tag{B.10}$$

by considering $c^H>c^L$, $0<\frac{d\zeta}{d\tau}<1$, and $\frac{d\omega}{d\tau}=-\frac{1}{\eta}\left(\frac{d\zeta}{d\tau}-1\right)$. Collecting all positive terms and r-independent negative terms in (B.10) and considering $\omega\leq 1$, we find a sufficient condition to guarantee $\frac{dF(\tau)}{d\tau}<0$, which is $r\frac{d\Pi(\ell)}{d\ell}v\left(\frac{d\zeta}{d\tau}\right)^2+\left[c^H+\left(\frac{d\Pi(\ell)}{d\ell}-1\right)(-v)\right]\frac{d\zeta}{d\tau}\frac{d\omega}{d\tau}+\frac{d\Pi(\ell)}{d\ell}(-v)\frac{d\zeta}{d\tau}\frac{d\omega}{d\tau}<0$, rewritten as

$$r<\frac{\kappa}{\beta}\left[2-\frac{v+c^H}{v+v\left(\frac{c^L}{r}-1\right)}\right], \tag{B.11}$$

by using $\frac{d\zeta}{d\tau}=\frac{1}{1+\frac{\kappa}{\beta}\eta}$. Note that the RHS of (B.11) is decreasing in r, so the inequality (B.11) gives a unique threshold of r. To summarize, under a sufficient condition of (B.11) (which is guaranteed under Assumpiton 2 and $r<\frac{\kappa}{\beta}$), Program (B.1) has a unique optimum $\tau^{SB}\in\left[0,\bar{\zeta}\right)$.

Next, we show a sufficient condition to guarantee $\tau^{SB}=0$. Given the sufficient condition for a unique equilibrium in (B.11), we only need to ensure $F(\tau=0)\leq 0$, that is,

$$\underbrace{\left(c^H-c^L\right)\frac{e^{r(\zeta-\tau)}-1}{r}\frac{1}{\eta}}_{+}+\left[\underbrace{c^H-v\left(\frac{c^L}{r}-1\right)}_{-}-r\Pi(\ell)\right]\frac{d\zeta}{d\tau}\omega+\underbrace{\left(\Pi(\ell)-\frac{c^L}{r}\right)\frac{d\omega}{d\tau}}_{-}$$
$$+\underbrace{(-r)\frac{d\zeta}{d\tau}\left(G(\omega)-\omega\ell\right)}_{-}\leq 0, \tag{B.12}$$

where $\ell=\ell(\zeta)<1$, $\zeta=\frac{\tau+\eta}{1+\frac{\kappa}{\beta}\eta}$ and $(\zeta-\tau)|_{\tau=0}=\frac{\eta}{1+\frac{\kappa}{\beta}\eta}$, and $\omega=1-\frac{\zeta-\tau}{\eta}=1-\frac{1}{1+\frac{\kappa}{\beta}\eta}=\frac{\frac{\kappa}{\beta}\eta}{1+\frac{\kappa}{\beta}\eta}$. A sufficient condition for (B.12) to be true is $c^H\frac{e^{r(\zeta-\tau)}-1}{r}\frac{1}{\eta}+\left[c^H-v\left(\frac{c^L}{r}-1\right)\right]\frac{d\zeta}{d\tau}\omega\Big|_{\tau=0}\leq 0$, that is,

$$\frac{v\left(\frac{c^L}{r}-1\right)}{c^H}\geq\left[\exp\left(\frac{r\eta}{1+\frac{\kappa}{\beta}\eta}\right)-1\right]\left(\frac{1}{r\eta}\right)\left(1+\frac{\kappa}{\beta}\eta\right)^2\left(\frac{1}{\frac{\kappa}{\beta}\eta}\right)+1, \tag{B.13}$$

which becomes $\frac{v\left(\frac{c^L}{r}-1\right)}{c^H}\geq\frac{\beta}{\kappa\eta}+2$ when $r\to 0$.

Proof of Propositions 2 and 9: We prove Proposition 9. Proposition 2 is just a special case of Proposition 9. We study scenario $t_0\geq\eta+m$. For clarity of the proof, we distinguish between two cases of $\zeta<\eta$ and $\zeta\geq\eta$, which will nevertheless give the same first-order condition. We first consider the case of $\zeta < \eta$, which is guaranteed under a sufficient parameter condition $\bar{\zeta} = \frac{\beta}{\kappa} < \eta$ (recalling Lemma 1). Under this case, an individual bank's optimal waiting time must satisfy $\tau_i \in [0, \zeta]$. For the individual bank that receives information at t_i, it expects that the crisis will occur at $t_0+\zeta \in (t_i, t_i + \zeta]$. The first order condition of (B.3) implies

$$\left\{ \begin{array}{l} -f\left(t_0+\zeta = t_i+\tau_i\right)\left[\int_{t_i}^{t_i+\tau_i} e^{-r(t-t_i)}c^H dt + \frac{c^L}{r}e^{-r\tau_i}\right] \\ +\Pr\left(t_0+\zeta \in (t_i+\tau_i, t_i+\zeta]\right)\left(e^{-r\tau_i}c^H + (-r)\frac{c^L}{r}e^{-r\tau_i}\right) \\ +f\left(t_0+\zeta = t_i+\tau_i\right)\left[\int_{t_i}^{t_i+\tau_i} e^{-r(t-t_i)}c^H dt + \Pi\left(\ell\right)e^{-r\tau_i}\right] \end{array} \right\} = 0, \tag{B.14}$$

rewritten as $\Pr\left(t_0+\zeta \in (t_i+\tau_i, t_i+\zeta]\right)\left(c^H - c^L\right) = f\left(t_0+\zeta = t_i+\tau_i\right)\left(\frac{c^L}{r} - \Pi\left(\ell\right)\right)$. As shown in the main text, the two terms are calculated as $f\left(t_0+\zeta = t_i+\tau_i\right) = f\left(t_0 = t_i+x-\zeta\right) = \phi\left(t_0 = t_i+x-\zeta|t_i\right) = \frac{\lambda e^{\lambda(\zeta-\tau_i)}}{e^{\lambda\eta}-1}$ and $\Pr\left(t_0+\zeta \in (t_i+\tau_i, t_i+\zeta]\right) = \Pr\left(t_0 \in (t_i+\tau_i-\zeta, t_i]\right) = \Phi\left(t_0 = t_i|t_i\right) - \Phi\left(t_0 = t_i+\tau_i-\zeta|t_i\right) = \frac{e^{\lambda(\zeta-\tau_i)}-1}{e^{\lambda\eta}-1}$. In addition, $h\left(t_0+\zeta = t_i+\tau_i^* \middle| t_i, \tau_i^*\right) \equiv \frac{f\left(t_0+\zeta = t_i+\tau_i^*\right)}{\Pr\left(t_0+\zeta\in\left(t_i+\tau_i^*, t_i+\zeta\right]\right)} = \frac{\lambda}{1-\exp\left[-\lambda\left(\zeta-\tau_i^*\right)\right]}$. Hence, we have (B.4).

We then consider the case of $\zeta \geq \eta$. Under this case, an individual bank's optimal waiting time must satisfy $\tau_i \in [\zeta-\eta, \zeta]$; that is, even for the last bank in the queue which receives information at time $t_0+\eta$, it still takes time length $\zeta-\eta$ for the crisis to come after the bank receives information, so an individual bank must choose $\tau_i \geq \zeta - \eta$. The individual bank t_i's optimization problem is

$$\tau_i^* = \arg\max_{\tau_i\in[\zeta-\eta,\zeta]} \left\{ \begin{array}{l} \Pr\left(t_0+\zeta \in (t_i+\tau_i, t_i+\zeta]\right)\left[\int_{t_i}^{t_i+\tau_i} e^{-r(t-t_i)}c^H dt + \frac{c^L}{r}e^{-r\tau_i}\right] \\ +\int_{x=\zeta-\eta}^{x=\tau_i} f\left(t_0+\zeta = t_i+x\right)\left[\int_{t_i}^{t_i+x} e^{-r(t-t_i)}c^H dt + \Pi\left(\ell\right)e^{-rx}\right]dx \end{array} \right\}. \tag{B.15}$$

In (B.15), for the individual bank t_i, it knows that the crisis will occur at the earliest at $t = t_i + (\zeta-\eta)$ and at the latest at $t = t_i+\zeta$. Thus, when the individual bank chooses its exiting time as $t_i+\tau_i$, it knows that there are two possibilities: $t_0+\zeta \in [t_i+\zeta-\eta, t_i+\tau_i] \cup (t_i+\tau_i, t_i+\zeta]$. If $t_0+\zeta \in (t_i+\tau_i, t_i+\zeta]$ is realized, bank t_i survives. If $t_0+\zeta \in [t_i+\zeta-\eta, t_i+\tau_i])$ is realized, the bank fails at the crisis arrival time t_i+x, where $x \in [\zeta-\eta, \tau_i]$. The first-order condition of (B.15) also yields (B.14).

Next, we analyze the equilibrium solution given by $\Gamma(\tau^*) = 0$, where $\Gamma(\tau) \equiv h(\tau,\zeta) - \frac{c^H-c^L}{\frac{c^L}{r}-\Pi(\ell)}$. From Lemma 1, we have $0 < \frac{d\zeta}{d\tau} < 1$, and thus $h(\tau, \zeta(\tau))$ is increasing in τ. Also, $\Pi(\ell(\zeta))$ is decreasing in τ. Overall, $\Gamma(\tau)$ is decreasing in τ. Moreover, when $\tau = \bar{\zeta}$, it follows $h(\tau,\zeta) = \infty$ and $\Gamma\left(\tau = \bar{\zeta}\right) > 0$. Therefore, if $\Gamma(\tau = 0) < 0$, there is a unique equilibrium with non-corner solution $\tau^* > 0$; otherwise, there is a unique equilibrium with corner solution $\tau^* = 0$. Also, considering that $\frac{c^L}{r} - \Pi(\ell) = \frac{c^L}{r} - \left[\ell + \ell\left(\frac{c^L}{r} - 1\right)\right]$ with $\ell = 1 - v\cdot(\zeta-\zeta_0)$ (so $\frac{c^L}{r} - \Pi(\ell)\Big|_{\tau=0} \to 0$ when $\zeta_0 \to \underline{\zeta}$), we have $\Gamma(\tau=0) < 0$ if parameter ζ_0 is close to $\underline{\zeta}$ enough.

We also consider scenario $t_0 < \eta+m$. For clarity of presentation of the proof, we simply set $m = 0$ here. The proof for this scenario closely follows Abreu and Brunnermeier (2002) (see the proof of Proposition 1 on pages 358-359 in their paper). In the first step, given that the crisis occurs at $t_0+\zeta$, find an individual bank's optimal τ_i. 1) For bank $t_i > \zeta$, it does not have additional information. It chooses $\tau_i^* = \tau^*$, where τ^* solves the first order condition

$$\frac{\lambda}{1-\exp\left[-\lambda\left(\zeta-\tau^*\right)\right]} = \frac{c^H-c^L}{\frac{c^L}{r}-\Pi\left(\ell\right)} \tag{B.16}$$

shown in (B.5). 2) For bank $t_i < \zeta$, the bank knows that the crisis occurs earliest at time ζ (when $t_0 = 0$) and it still takes at least time length $\zeta - t_i$ for the crisis to come after receiving information before the crisis will hit, so it must choose $\tau_i \geq \zeta - t_i$. Hence, $\tau_i^* = \max\left(\tau^*, \zeta-t_i\right)$ for bank $t_i < \zeta$, where τ^* solves the first order condition (B.16). In the second step, given banks' updated strategy ($\tau_i^* = \max(\tau^*, \zeta - t_i)$ for bank $t_i < \zeta$, and $\tau_i^* = \tau^*$ for other banks), we confirm that the crisis occurs at time $t_0+\zeta$, where ζ is given by $\zeta = \frac{\tau^*+\eta}{1+\frac{\kappa}{\beta}\eta}$. By the condition $\tau_i^* = \max(\tau^*, \zeta - t_i)$, only those banks with $t_i \leq \zeta - \tau^*$choose $\tau_i^* = \zeta - t_i$ and hence they exit the speculative sector no later than time ζ, implying that those banks exit before the crisis arrival time by considering $\zeta \leq t_0 + \zeta$. Therefore, up to the crisis arrival time $t_0 + \zeta$, the accumulated pressure of exiting is still $x(t_0+\zeta) = \frac{(t_0+\zeta)-(t_0+\tau^*)}{\eta} = \frac{\zeta-\tau^*}{\eta}$. By the crisis condition, we have $\zeta = \frac{\tau^*+\eta}{1+\frac{\kappa}{\beta}\eta}$. Intuitively, the updated strategy of some banks only changes the density distribution of exiting before the crisis arrival time, but does not change the accumulated amount of exiting up to the crisis arrival time.

Proof of Propositions 3 and 10: We prove Proposition 10. Propositions 3 is just a special case of Proposition 10. Since $\hat{F}(\tau_i^* = \tau^*, \zeta(\tau^*)) \propto \Gamma(\tau_i^* = \tau^*, \zeta(\tau^*)) = 0$, we have

$$
\begin{aligned}
&F(\tau^*, \zeta(\tau^*)) = F(\tau^*, \zeta(\tau^*)) - \Gamma(\tau_i^* = \tau^*, \zeta(\tau^*)) = F(\tau^*, \zeta(\tau^*)) - \Gamma(\tau_i^* = \tau^*, \zeta(\tau^*))\,\frac{\frac{d\omega}{d\tau}}{h} \\
&= \left\{ \begin{array}{l} \left(c^H - c^L\right)\Bigg[\underbrace{\frac{e^{r(\zeta-\tau)}-1}{r\eta} - \frac{\frac{d\omega}{d\tau}}{h}}_{>0}\Bigg] + \Bigg[c^H - \underbrace{v\left(\frac{c^L}{r}-1\right)}_{\text{not internalized}} - r\Pi(\ell)\Bigg]\frac{d\zeta}{d\tau}\omega \\ -r\frac{d\zeta}{d\tau}\left(G(\omega) - \omega\ell\right) \end{array} \right\}\Bigg|_{(\tau,\zeta)=(\tau^*,\zeta(\tau^*))}, \qquad \text{(B.17)}
\end{aligned}
$$

where $h = \frac{\lambda}{1-\exp[-\lambda(\zeta-\tau)]} \geq \lim_{\lambda\to 0}\frac{\lambda}{1-\exp[-\lambda(\zeta-\tau)]} = \frac{1}{\zeta-\tau}$ and $\frac{d\omega}{d\tau} = \frac{1}{\eta}\left(1 - \frac{d\zeta}{d\tau}\right)$, and hence $\frac{\frac{d\omega}{d\tau}}{h} \leq \left(1-\frac{d\zeta}{d\tau}\right)\frac{\zeta-\tau}{\eta} < 1-\omega$. Also because $\frac{e^{r(\zeta-\tau)}-1}{r\eta} \geq \lim_{r\to 0}\frac{e^{r(\zeta-\tau)}-1}{r\eta} = 1-\omega$, we have $\frac{e^{r(\zeta-\tau)}-1}{r\eta} - \frac{\frac{d\omega}{d\tau}}{h} > 0$.

A sufficient condition to ensure $F(\tau^*, \zeta(\tau^*)) < 0$ in (B.17) is

$$
c^H\frac{e^{r(\zeta-\tau)}-1}{r\eta} + \left[c^H - v\left(\frac{c^L}{r}-1\right)\right]\frac{d\zeta}{d\tau}\omega\Bigg|_{(\tau,\zeta)=(\tau^*,\zeta(\tau^*))} \leq 0. \qquad \text{(B.18)}
$$

As the LHS of (B.18) is decreasing in τ, a sufficient condition for (B.18) to be true is

$$
c^H\frac{e^{r(\zeta-\tau)}-1}{r}\frac{1}{\eta} + \left[c^H - v\left(\frac{c^L}{r}-1\right)\right]\frac{d\zeta}{d\tau}\omega\Bigg|_{(\tau,\zeta)=(0,\underline{\zeta})} \leq 0,
$$

which gives the same condition as (B.13).

Considering that parameter ω_0 and thereby ζ_0 help to pin down the condition for the non-corner solution of τ^*, there are three cases for the comparison between τ^{SB} and τ^*: $\tau^{SB} = \tau^* = 0$ if $\zeta_0 \leq \zeta_0^*$, and $\tau^{SB} = 0 < \tau^*$ or $0 < \tau^{SB} < \tau^*$ if $\zeta_0 > \zeta_0^*$, where ζ_0^* is a threshold lying in $\zeta_0^* \in [0, \underline{\zeta})$.

Proof of Proposition 7: We give a general proof of Proposition 7, similar to the proof of Proposition 3. Recalling function F in (A.11), we find $F(\tau^*, \zeta(\tau^*))$, the first-order condition for the social planner evaluated at the competitive equilibrium solution pair $(\tau^*, \zeta(\tau^*))$. Based on (A.15), define $\Gamma(\tau_i^*, \zeta) =$

$h-\frac{a-z}{-\log\ell}$, where $h=\frac{\lambda}{1-\exp\left[-\lambda\left(\zeta-\tau_i^*\right)\right]}$. Since $\Gamma\left(\tau_i^*=\tau^*,\zeta\left(\tau^*\right)\right)=0$, it follows that

$$\digamma\left(\tau^*,\zeta\left(\tau^*\right)\right)=\digamma\left(\tau^*,\zeta\left(\tau^*\right)\right)-\Gamma\left(\tau_i^*=\tau^*,\zeta\left(\tau^*\right)\right)=\digamma\left(\tau^*,\zeta\left(\tau^*\right)\right)-\Gamma\left(\tau_i^*=\tau^*,\zeta\left(\tau^*\right)\right)\frac{\frac{d\omega}{d\tau}}{h\rho}$$

$$=\left.\left\{\begin{array}{l}\frac{a-z}{\rho}\frac{e^{(\zeta-\tau)\rho}-1}{\rho}\frac{1}{\eta}+\left(\frac{a-z}{\rho}-\frac{v}{\rho}-\log\ell\right)\omega\frac{d\zeta}{d\tau}+\left(\frac{\log\ell}{\rho}\right)\frac{d\omega}{d\tau}\\+(-1)\frac{d\zeta}{d\tau}\log\left[1+\frac{\mu(G(q)-q\ell)}{\exp[\log W_0+(t_0+\zeta)z]}\right]+\mu\frac{d\left(\frac{(G(q)-q\ell)}{\exp[\log W_0+(t_0+\zeta)z]}\right)/d\tau}{\left[1+\frac{\mu(G(q)-q\ell)}{\exp[\log W_0+(t_0+\zeta)z]}\right]\rho}\\-\left[(a-z)-(-\log\ell)h\right]\frac{\frac{d\omega}{d\tau}}{h\rho}\end{array}\right\}\right|_{(\tau,\zeta)=(\tau^*,\zeta(\tau^*))}$$

$$=\left.\left\{\begin{array}{l}\frac{a-z}{\rho}\left[\underbrace{\frac{e^{(\zeta-\tau)\rho}-1}{\rho\eta}-\frac{\frac{d\omega}{d\tau}}{h}}_{>0}\right]+\left(\frac{a-z}{\rho}-\underbrace{\frac{v}{\rho}}_{\text{not internalized}}-\log\ell\right)\frac{d\zeta}{d\tau}\omega\\+(-1)\frac{d\zeta}{d\tau}\log\left[1+\frac{\mu(G(q)-q\ell)}{\exp[\log W_0+(t_0+\zeta)z]}\right]+\mu\frac{d\left(\frac{(G(q)-q\ell)}{\exp[\log W_0+(t_0+\zeta)z]}\right)/d\tau}{\left[1+\frac{\mu(G(q)-q\ell)}{\exp[\log W_0+(t_0+\zeta)z]}\right]\rho}\end{array}\right\}\right|_{(\tau,\zeta)=(\tau^*,\zeta(\tau^*))},$$

where the result of $\frac{e^{\rho(\zeta-\tau)}-1}{\rho\eta}-\frac{\frac{d\omega}{d\tau}}{h}>0$ is proved in the proof of Proposition 10. Since $\frac{\frac{d\omega}{d\tau}}{h}>0$, a sufficient condition to ensure $\digamma\left(\tau^*,\zeta\left(\tau^*\right)\right)<0$ is

$$\left.\left\{\begin{array}{l}\frac{a-z}{\rho}\frac{e^{(\zeta-\tau)\rho}-1}{\rho\eta}+\frac{a-z}{\rho}\frac{d\zeta}{d\tau}\omega\\+(-1)\frac{d\zeta}{d\tau}\log\left[1+\frac{\mu(G(q)-q\ell)}{\exp[\log W_0+(t_0+\zeta)z]}\right]+\mu\frac{d\left(\frac{(G(q)-q\ell)}{\exp[\log W_0+(t_0+\zeta)z]}\right)/d\tau}{\left[1+\frac{\mu(G(q)-q\ell)}{\exp[\log W_0+(t_0+\zeta)z]}\right]\rho}\end{array}\right\}\right|_{(\tau,\zeta)=(\tau^*,\zeta(\tau^*))}$$

$$<\left.\left(\frac{v}{\rho}+\log\ell\right)\frac{d\zeta}{d\tau}\omega\right|_{(\tau,\zeta)=(\tau^*,\zeta(\tau^*))}=\left.v\left(\frac{1}{\rho}-(\zeta-\zeta_0)\right)\frac{d\zeta}{d\tau}\omega\right|_{(\tau,\zeta)=(\tau^*,\zeta(\tau^*))},$$

by noticing that $\tau^*>\tau_0$, $\zeta>\zeta_0$, and thus $\log\ell=-v\left(\zeta-\zeta_0\right)$.

If ρ is small enough such that $\frac{1}{\rho}>\zeta\left(\tau^*\right)-\zeta_0$, then the above sufficient condition can be rewritten as

$$v>\frac{\left.\left\{\frac{a-z}{\rho}\frac{e^{(\zeta-\tau)\rho}-1}{\rho\eta}+\frac{a-z}{\rho}\frac{d\zeta}{d\tau}\omega-\frac{d\zeta}{d\tau}\log\left[1+\frac{\mu(G(q)-q\ell)}{\exp[\log W_0+(t_0+\zeta)z]}\right]+\mu\frac{d\left(\frac{(G(q)-q\ell)}{\exp[\log W_0+(t_0+\zeta)z]}\right)/d\tau}{\left[1+\frac{\mu(G(q)-q\ell)}{\exp[\log W_0+(t_0+\zeta)z]}\right]\rho}\right\}\right|_{(\tau,\zeta)=(\tau^*,\zeta(\tau^*))}}{\left.\left[\frac{1}{\rho}-(\zeta-\zeta_0)\right]\frac{d\zeta}{d\tau}\omega\right|_{(\tau,\zeta)=(\tau^*,\zeta(\tau^*))}}.\tag{B.19}$$

Since $\tau^*\in\left(\tau_0,\bar{\zeta}\right)$, $\zeta\left(\tau^*\right)\in\left(\zeta_0,\bar{\zeta}\right)$ and $\omega\in\left(\frac{\kappa}{\beta}\zeta_0,1\right)$ are all bounded, the RHS of (B.19) is bounded.

Combining these results, we have $\digamma\left(\tau^*,\zeta\left(\tau^*\right)\right)<0$ and thus $\tau^{SB}<\tau^*$ under a sufficient condition that ρ is small enough (such that $\frac{1}{\rho}>\zeta\left(\tau^*\right)-\zeta_0$) and v is high enough (such that (B.19) is true), ceteris paribus.

Results in Section 4.3: We give the expression of $Y(t)$. We divide time into five stages: $t\in[0,t_s)$, $[t_s,t_s+\eta)$, $[t_s+\eta,t_s+m+\tau=t_0+\tau)$, $[t_0+\tau,t_0+\zeta)$, and $[t_0+\zeta,+\infty)$. In the first stage $t\in[0,t_s)$, all banks are operating in the traditional sector, so the aggregate output is given by

$$Y(t)=\exp\left(k_0+zt\right)\cdot Z.$$

In the second stage $t \in [t_s, t_s + \eta)$, some banks have already entered the speculative sector by using technology A while others are staying in the traditional sector and using technology Z. Hence,

$$\begin{aligned} Y(t) &= \exp(k_0 + zt_s)\left[A\int_{t_s}^{t}\exp[(s-t_s)z + (t-s)a]\frac{1}{\eta}ds + Z\frac{t_s+\eta-t}{\eta}\exp[(t-t_s)z]\right] \\ &= \exp(k_0 + zt_s)\left[\frac{1}{\eta}\frac{1}{a-z}A\left[e^{a(t-t_s)} - e^{z(t-t_s)}\right] + \frac{t_s+\eta-t}{\eta}Ze^{(t-t_s)z}\right]. \end{aligned}$$

In the third stage $t \in [t_s + \eta, t_0 + \tau)$, all banks have already entered the speculative sector by using technology A. Thus,

$$\begin{aligned} Y(t) &= \exp(k_0 + zt_s)A\int_{t_s}^{t_s+\eta}\exp[(s-t_s)z + (t-s)a]\frac{1}{\eta}ds \\ &= \exp(k_0 + zt_s)A\frac{1}{\eta}\frac{1}{a-z}e^{a(t-t_s)}\left[1 - e^{-(a-z)\eta}\right]. \end{aligned}$$

In the fourth stage $[t_0 + \tau, t_0 + \zeta)$, some banks have already safely exited the speculative sector by switching to technology Z while others are staying in the speculative sector and using technology A. It follows that

$$\begin{aligned} Y(t) &= \exp(k_0 + zt_s)\left[Z\frac{t-(t_0+\tau)}{\eta}e^{a(m+\tau)+z[t-(t_0+\tau)]} + A\int_{t-m-\tau}^{t_s+\eta}\exp[z(s-t_s) + a(t-s)]\frac{1}{\eta}ds\right] \\ &= \exp(k_0 + zt_s)\left[Z\frac{t-(t_0+\tau)}{\eta}e^{a(m+\tau)+z[t-(t_0+\tau)]} + \frac{A\frac{1}{\eta}}{a-z}\left[e^{(a-z)(m+\tau)+z(t-t_s)} - e^{-(a-z)\eta+a(t-t_s)}\right]\right]. \end{aligned}$$

The fifth stage $[t_0 + \tau, t_0 + \zeta)$ is the post-crisis period, in which all banks are operating in the traditional sector. The banks that are caught by the crisis at $t = t_0 + \zeta$ lose a portion of their capital at the crisis arrival time $t = t_0 + \zeta$. Hence,

$$Y(t) = K(t_0 + \zeta)Ze^{z[t-(t_0+\zeta)]},$$

where $K(t_0 + \zeta) = \exp(k_0 + zt_s)\left[\frac{\zeta-\tau}{\eta}e^{a(m+\tau)+z(\zeta-\tau)} + \frac{1}{\eta}\frac{\ell(\zeta)}{a-z}\left[e^{(a-z)(m+\tau)+z(m+\zeta)} - e^{-(a-z)\eta+a(m+\zeta)}\right]\right]$.

We give the expression of $Y_s(t)$. In the first stage $t \in [0, t_s)$, all banks are operating in the traditional sector, so the aggregate output is given by $Y_s(t) = 0$. In the second stage $t \in [t_s, t_s + \eta)$, some banks have already entered the speculative sector by using technology A while others are staying in the traditional sector and using technology Z. Hence,

$$Y_s(t) = \exp(k_0 + zt_s)A\int_{t_s}^{t}\exp[(s-t_s)z + (t-s)a]\frac{1}{\eta}ds = \exp(k_0 + zt_s)\frac{1}{\eta}\frac{1}{a-z}A\left[e^{a(t-t_s)} - e^{z(t-t_s)}\right].$$

In the third stage $t \in [t_s + \eta, t_0 + \tau)$, all banks have already entered the speculative sector by using technology A. Thus,

$$Y_s(t) = \exp(k_0 + zt_s)A\int_{t_s}^{t_s+\eta}\exp[(s-t_s)z + (t-s)a]\frac{1}{\eta}ds = \exp(k_0 + zt_s)\frac{A}{a-z}\frac{1}{\eta}e^{a(t-t_s)}\left[1 - e^{-(a-z)\eta}\right].$$

In the fourth stage $[t_0 + \tau, t_0 + \zeta)$, some banks have already safely exited the speculative sector by switching to technology Z while others are staying in the speculative sector and using technology A. It follows that

$$\begin{aligned} &Y_s(t) = \exp(k_0 + zt_s)\left[A\int_{t-m-\tau}^{t_s+\eta}\exp[z(s-t_s) + a(t-s)]\frac{1}{\eta}ds\right] \\ &= \exp(k_0 + zt_s)\frac{A}{a-z}\frac{1}{\eta}\left[e^{(a-z)(m+\tau)+z(t-t_s)} - e^{-(a-z)\eta+a(t-t_s)}\right]. \end{aligned}$$

In the post-crisis period $[t_0 + \zeta, +\infty)$, all banks operate in the traditional sector. Hence, $Y_s(t) = 0$.