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Co-Movement of Steady-State Government Debt and Household Debt

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Keywords: Government debt, household debt, co-movement, uninsurable income risk
JEL Classification: H63, E62, D72

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I. Introduction

Are movements of government debt and aggregate household debt of an economy related? If so, how are they related to each other? As demonstrated in **Figure 1** and **2**, the trends of these two debts are not unrelated. Before 1993, government debt and aggregate household debt of the OECD economies (as % of GDP) increased together. On the other hand, after 1993 until the 2008 Great Recession, while aggregate household debt increased, government debt neither clearly increased nor decreased. In contrast, after the Great Recession, these two debts clearly moved in the opposite directions, because government debt rose, while aggregate household debt declined. This data suggests that the movements of these two debts are correlated in a complicated way. Although the two debts of an economy are quite sizable, relation of their movements has not been well studied. Understanding how movements of steady-state government debt and steady-state aggregate household debt are related also can help us better comprehend mechanisms underlying respective observed trends of these two debts. With a politico-economic model, this paper theoretically analyzes whether and how movements of stationary equilibrium government debt and stationary equilibrium aggregate household debt are correlated.

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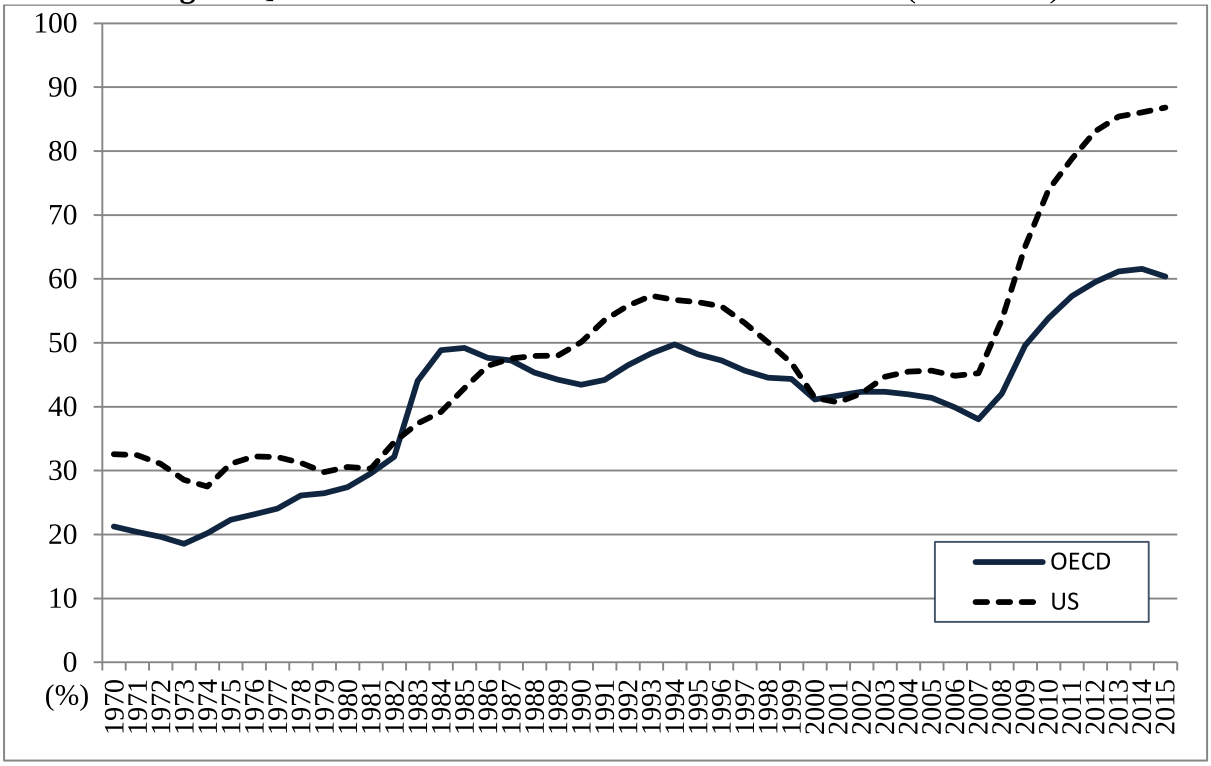
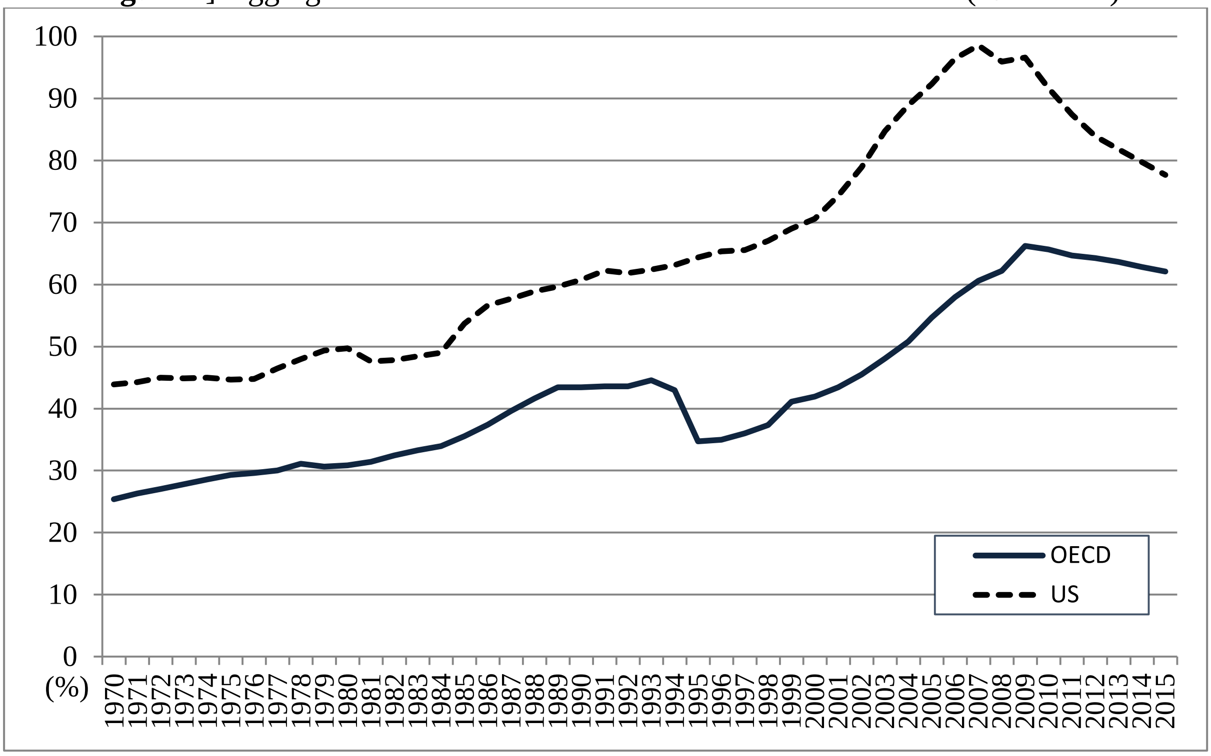
Figure 1] Government Debt of OECD Economies and US (% of GDP)



Note: The data of gross general government debt as % of GDP of the OECD economies is secured from Global Debt Database of IMF. Its average for each year is displayed by the solid line above.

Figure 2] Aggregate Household Debt of OECD Economies and US (% of GDP)



Note: The data of aggregate household debt as % of GDP of the OECD economies is obtained from Global Debt Database of IMF. Its average for each year is displayed by the solid line.

To reflect the reality that fiscal policy is not determined by a benevolent social planner, but by politically-motivated policymakers, this paper assumes that office-seeking policymakers decide government debt, income tax rate and public goods provision to win over voters of overlapping generations. Individual voters decide their own allocation of borrowing/saving, labor supply, and private goods consumption, while the allocation is implemented through competitive markets. Moreover, voters' disposable incomes are exposed to uninsurable idiosyncratic risk. To obtain political supports from voters, policymakers choose fiscal policies that maximize population-weighted sum of utility of their own electorate, while they are subject to term limits. At the same time, individuals' competitive-equilibrium allocation is also affected by fiscal policies. From this model, stationary equilibrium government debt and stationary equilibrium aggregate household debt are derived.

Then, to analyze how movements of stationary equilibrium government debt and stationary equilibrium aggregate household debt are related, a change in the economic parameters of the household-loan collateral value, interest rate, the elderly population share and uninsurable idiosyncratic income risk focuses economic factors is introduced, because these four factors are empirically shown to be important in explaining household debt. By allowing both policymakers and individuals to embrace each other's concurrent response to a change in these economic factors, this paper finds that stationary equilibrium government debt and stationary equilibrium aggregate household debt are related. An increase in the household-loan collateral value or population aging conditionally raises both stationary equilibrium government debt and stationary equilibrium total household debt together. Changes in the value of house or life longevity can lead individual voters to borrow more and to want public goods more. Seeking more public goods, voters can vote for higher government debt, even if higher government debt entails higher income tax rate. On the other hand, an increase in uninsurable income risk unconditionally causes stationary equilibrium government debt and stationary equilibrium total household debt to increase together. As uninsurable income risk increases, risk-averse voters face greater uncertainty on their own private goods consumption so that they value certainly-provided public goods more. Hence, an increase in uninsurable income risk always causes voters to support higher government debt and borrow more for their own lifetime consumption smoothing. In contrast, while an increase in interest rate raises stationary equilibrium government debt, it can lower stationary equilibrium total household debt, leading conditionally these two debts to move in the opposite directions. This negative correlation of these two debt's movements stems from the contrast that policymakers do not need to pay the entire part of the government debt back within their own term, whereas voters should pay the entire part of their own household loan.

This paper unfolds as follows. Section II reviews related literature. Section III describes a theoretical model from which stationary equilibrium household debt and stationary equilibrium government debt are obtained in Section IV and Section V, respectively. Section VI analyzes how movements of stationary equilibrium government debt and stationary equilibrium aggregate household debt are related. The last section concludes the paper.

II. Literature Review

So far, there is no study that theoretically analyzed whether and how movements of government debt and household debts are correlated, although various researches have been conducted on government debt, separated from ones on household debt. At first, studies on government debt were led by the Ricardian invariance theorem (Barro, 1979) which implies that household debt and government debt are unrelated. However, plenty of evidence against the prediction of Barro (1979) that government debt follows a random walk (e.g., Bizer and Durlauf, 1990; Trehan and Walsh, 1991; Bohn, 1998) was presented finding rises in government debt. To explain the observed rises, Azzimonti et al. (2014) showed that an increase in firms' uninsurable income risk (productivity risk) raises government debt. Distinct from Barro (1979) and Azzimonti et al. (2014), this paper adopts a politico-economic model to examine the effect on government and household debts of uninsurable income risk faced by individual workers, instead of firms.

In addition, this paper contributes to the politico-economic theory literature on government debt. Different from the politico-economic theories that attributed government debt rises to political factors such as political polarization (e.g., Alesina and Tabellini, 1990) and pork-barrel spending (Battaglini and Coate, 2008), this paper focuses economic factors. Differentiated from the politico-economic theories that investigated effect of demographics on government debt (Tabellini, 1991; Song et al., 2012), this paper examines effect of population aging on government debt with allowing voters to borrow.

On the other hand, this paper also contributes to the literature on household debt[1]. Unlike studies in the household debt literature which focused on behavioral biases like over-optimism and time consistency (e.g., Stango and Zinman, 2009; Baron and Xiong, 2017), this paper assumes that individuals are rational and focuses economic factors that are shown to be important in explaining household debt. In this regard, numerous empirical studies found that an increase in the value of house as collateral of household loans raises household debt (e.g., Dynan and Kohn, 2007; Mian and Sufi, 2011; Christelis et al., 2015; Coletta et al., 2019). In addition, population aging and interest rate are also considered important factor in the literature (e.g., Pollin, 1988; Barnes and Young, 2003; Campbell, 2006; Dynan and Kohn, 2007; Coletta et al., 2019). Based on simulations with US data, Iacoviello (2008) and Kumhof et al. (2015) showed that an increase in idiosyncratic risk on individual incomes raises income inequality leading to a rise in aggregate household debt. Differentiated from and reflecting the existing literature, this paper conducts a theoretical analysis on how aggregate household debt responds to a change in these four economic factors (the value of

[1] There is a vast literature on boom and bust of household debt and on how they evolve into the Great Recession. However, this paper is not about boom or bust of household debt but about steady-state aggregate household debt.

household-loan collateral, real interest rate, the elderly population share and uninsurable income risk) when government debt also responds to a change in these four economic factors.

III. The Model

Consider an economy populated by overlapping generations of voters. Individuals live up to two periods so that young and old voters coexist in each period. The size of total population stays as one. For each period, new voters are born with no capital and are indexed by *i*. Moreover, young voters survive the next period with the probability of $\frac{\delta}{1-\delta} \in (0,1)$ so that the population share of old voters is $\delta \in (0, 0.5)$. In each period, young voters choose their private goods consumption, labor supply and saving/borrowing, while old voters choose their private consumption. As this paper analyzes steady-state economy, time subscripts are omitted. For any given i, the lifetime utility of young voter *i* is

$$\log(c_{Y,i}) - \frac{l_{Y,i}^{1+\frac{1}{\eta}}}{1+\frac{1}{\eta}} + \chi \log(g) + \beta \frac{\delta}{1-\delta}[\log(c_{O,i}) + \chi \log(g')] \tag{1}$$

where $\beta \in (0,1)$ is time preference; $c_{Y,i} > 0$ and $c_{O,i} > 0$ are private goods consumption of the current period and the next period, respectively, of young voter *i*; $l_{Y,i} \in (0,1)$ is labor supply of young voter *i*; $\eta > 0$ is Frisch elasticity of labor supply; g and g' are public goods provided in the current period and the next period, respectively; $\chi > 0$ is preference for public goods consumption. At the same time, for any given i, the utility of old voter *i* is

$$\log(c_{O,i}) + \chi \log(g). \tag{2}$$

Moreover, young voter *i* faces the following budget constraint:

$$c_{Y,i} - d_i^h + H \leq (1-\tau) w l_{Y,i}, \tag{3}$$

where $\tau \in [0,1)$ is income tax rate; $w > 0$ is wage rate; d_i^h is young voter *i*'s borrowings (i.e., household debt) if it is positive or his savings if it is negative. Because every individual needs a house to live in, let each young voter purchase a house, whose value is equal to H, regardless of whether they borrow or not. Then, before they die, in the second period, they liquidize the house which can be used for their own post-retirement consumption. Because individuals have no bequest motive, the house is cashed out only for their own post-retirement consumption. In this line, old voter *i* meets the following budget constraint:

$$c_{O,i} \leq H - (1+r)d_i^h \tag{4}$$

where $r \in (0,1)$ is interest rate.[2] Putting (3) and (4) together reveals that post-tax labor income is used for lifetime private consumption as well as for house (residence) whose total net present value is $\frac{r}{1+r}H$. Moreover, the value of house in terms of wage is stated as $H = wh$. Reasonably, for strict positive amounts of lifetime private consumption, the value of house is strictly lower than disposable lifetime income.

Reflecting the reality that most of household debts are collateralized by borrowers' house, if a young voter chooses to take out a household loan, he should offer his house as collateral. Therefore, a change in the household-loan collateral value is efficiently represented by a change in the value of *h*. Moreover, if an individual chooses to borrow, he is committed to paying back. Thus, individuals cannot borrow more than the present value of the collateral; thus, $\frac{wh}{(1+r)} > d_i^h$. Consequently, even when voter *i* chooses to take a household loan instead of choosing to save (i.e., $d_i^h > 0$), he still can consume a strictly positive amount of private goods after paying back the loan in the next period (i.e., $c_{O,i} > 0$). Moreover, even when a young voter chooses to take out a household loan and does not survive the next period, his loan still can be paid back from liquidizing his house that is left after his death. Therefore, there is no risk of not paying loans back, which enables young voters to borrow at a risk-free rate of *r*. Noticeably, different from the previous analyses on household debt (e.g., Iacoviello,

[2] Notice that old voters do not save because they do not have bequest motive. For the same reason, they do not borrow either, which is consistent with the empirical finding that old individuals usually do not take out loans (e.g., Yilmazer and DeVaney 2005; Dynan and Kohn, 2007).

2008; Stango and Zinman, 2009; Kumhof et al., 2015), this paper assumes that individuals are able to choose to borrow, instead of being born as a borrower or a saver with different values of parameters of time preference or behavioral bias.

Moreover, individual young voters face uninsurable idiosyncratic income shock. Specifically, in each period, after receiving post-tax labor income, disposable income of young voters falls by $100(1-\sigma)\%$ for a given $\sigma \in (0,1)$ with the probability of $\phi \in [0,1)$. That is, for any given i, disposable income of young voter i decreases to $\sigma(1-\tau)wl_{Y,i}$ with the probability of ϕ or remains intact with the probability of $1-\phi$. This negative income shock is independent for each voter. As such, the parameter ϕ represents the degree of uninsurable idiosyncratic risk on individuals' incomes in a tractable way. Moreover, as individuals face higher uninsurable risk on their post-tax incomes (i.e., an increase in ϕ), individuals' post-tax incomes are more dispersed entailing an increase in the level of post-tax income inequality, which is consistent with Iacoviello (2008) and Kumhof et al. (2015). After the uninsurable income shock is realized, private consumption for the remaining life and saving/borrowing are decided. In addition, because an individual's pre- and post- retirement consumption are strictly positive regardless of whether the negative income shock occurs to the individual, the value of house is strictly lower than $\sigma(1-\tau)wl_{Y,i}$ for any given i.

With prices and government policies given, before the uninsurable income shock is realized, individual young voters choose their own labor supply ($l_{Y,i}$) to maximize their own expected lifetime utility subject to their own budget constraints. Thus, reflecting the risk that young voters face, for any given *i*, young voter *i* chooses his labor supply from maximizing the expected lifetime utility of $\phi\{\log(c^{S}_{Y,i}) - (1+\frac{1}{\eta})^{-1} l_{Y,i}^{1+\frac{1}{\eta}} + \chi\log(g) + \beta\frac{\delta}{1-\delta}[\log(c^{S}_{O,i}) + \chi \log(g')]\} + (1-\phi)\{\log(c^{NS}_{Y,i}) - (1+\frac{1}{\eta})^{-1} l_{Y,i}^{1+\frac{1}{\eta}} + \chi\log(g) + \beta\frac{\delta}{1-\delta}[\log(c^{NS}_{O,i}) + \chi\log(g')]\}$ where

$c_{Y,i}^{S}$ and $c_{O,i}^{S}$ are private consumption of young voter *i* in the current and future periods, if negative income shock occurs to his post-tax labor income; and, $c_{Y,i}^{NS}$ and $c_{O,i}^{NS}$ are his private consumption in the current and future periods if no income shock occurs to his post-tax labor income. Private consumption is implemented after realization of the uninsurable income shock, whereas labor is supplied before the shock. Hence, private consumption has the contingency superscript of *S* or *NS*, while labor supply does not. After no shock, the budget constraints that young voter *i* faces in the current and future periods are (3) and (4). After negative income shock, those are $c_{Y,i} - d_i^h + H \le \sigma(1-\tau)wl_{Y,i}$ and (4).

From the constrained maximization, for any given i, optimal labor supply $l_{Y,i}^{*}$ of young voter *i* is defined by the following FOC (first-order condition)

$$(l_{Y,i}^{*})^{\frac{1}{\eta}} = [(1-\phi)\lambda_{Y,i}^{\mathrm{NS}} + \sigma\phi\lambda_{Y,i}^{S}](1-\tau)w \tag{5}$$

where $\lambda_{Y,i}^{NS}$ is Lagrange multiplier for the budget constraint when no income shock occurs to voter *i*'s post-tax labor income; $\lambda_{Y,i}^{S}$ is Lagrange multiplier for the budget constraint when negative income shock occurs to his post-tax labor income. In addition, for any given i, optimal private consumption of young voter *i* for each contingency ($c_{Y,i}^{NS*}$ and $c_{Y,i}^{S*}$) is defined by the following FOCs:

$$\frac{1}{c_{Y,i}^{NS*}} = \lambda_{Y,i}^{NS} \text{ and } \frac{1}{c_{Y,i}^{S*}} = \lambda_{Y,i}^{S}. \tag{6}$$

When young voters make their own contingency plan of private consumption for no-shock and shock, they equalize expected marginal utility of private consumption for each contingency: thus, for any given i,

$$(1-\phi)\frac{1}{c_{Y,i}^{NS*}} = \phi\frac{1}{c_{Y,i}^{S*}}. \tag{7}$$

In addition to these intra-temporal optimality conditions, for any given i, the following Euler equations define inter-temporal optimality condition of voter *i*:

$$\frac{1}{c_{Y,i}^{NS*}} = \beta(1+r)\frac{\delta}{1-\delta}\frac{1}{c_{O,i}^{NS*}} \text{ and } \frac{1}{c_{Y,i}^{S*}} = \beta(1+r)\frac{\delta}{1-\delta}\frac{1}{c_{O,i}^{S*}} . \tag{8}$$

Moreover, (7) and (8) imply that $c_{O,i}^{NS*} = \beta(1+r)\frac{\delta}{1-\delta}c_{Y,i}^{NS*}$ and $c_{O,i}^{S*} = \beta(1+r)\frac{\delta}{1-\delta}c_{Y,i}^{S*} = \beta$ $(1+r)\frac{\delta}{1-\delta}\frac{\phi}{(1-\phi)}c_{Y,i}^{NS*} = \frac{\phi}{(1-\phi)}c_{O,i}^{NS*}$. Thus, $\phi\log(c_{Y,i}^{S*}) + (1-\phi)\log(c_{Y,i}^{NS*}) = \phi\log(\frac{\phi}{(1-\phi)}c_{Y,i}^{NS*}) +$ $(1-\phi)\log(c_{Y,i}^{NS*})$ and $\phi\log(c_{O,i}^{S*}) + (1-\phi)\log(c_{O,i}^{NS*}) = \phi\log(\frac{\phi}{(1-\phi)}c_{O,i}^{NS*}) + (1-\phi)\log(c_{O,i}^{NS*})$. In this light, plugging the optimal allocation defined by (5), (6), (7) and (8) into the utility functions yields the maximized expected utility of voters. For any given i, the maximized expected utility of young voter i before the uninsurable shock on post-tax labor incomes is realized (i.e., when young voter i chooses his labor supply) is stated as

$$\log(c_{Y,i}^{NS*}) + \chi\log(g) - \frac{(l_{Y,i}^{*})^{(1+\frac{1}{\eta})}}{1+\frac{1}{\eta}} + \beta\frac{\delta}{1-\delta}[\log(c_{O,i}^{NS*}) + \chi\log(g')] + \Gamma \tag{9}$$

where $\Gamma = \phi\log(\frac{\phi}{1-\phi}) + \frac{\beta\delta}{1-\delta}\phi\log(\frac{\phi}{1-\phi})$ is non-kernel part of the maximized utility. By the same token, for any given i, the maximized utility of old voter i if he went through negative income shock before retirement is

$$\log(c_{O,i}^{NS*}) + \chi\log(g) + \log(\frac{\phi}{1-\phi}) . \tag{10}$$

The maximized utility of old voter i if he went through no income shock before retirement is

$$\log(c_{O,i}^{NS*}) + \chi\log(g) . \tag{11}$$

In this economy, there is a representative firm whose output is used for private goods and public goods consumption. The firm produces output according to Cobb-Douglas technology.

$$Y = zK^{\alpha}L^{1-\alpha} \tag{12}$$

with $1 > \alpha > 0$, where Y is total output; z is total factor productivity; K is aggregate capital; and, L is aggregate labor. In each period, the firm maximizes its profit of $\max_{\{K,L\}} zK^{\alpha}L^{1-\alpha}$ –

$(r+\upsilon)K - wL$ where $\upsilon \in [0,1]$ is capital depreciation rate. Moreover, this economy is open and small with perfectly mobile capital so that both the government and individuals of this economy take the price to borrow rather than sets the price. While capital is perfectly mobile across different economies, labor is immobile. This assumption of small open economy (given interest rate) is also adopted by previous studies on advanced economies such as Alesina and Tabellini (1990), Battaglini and Coate (2008) and Song et al. (2012) and the like.

The government of this economy finances public goods provision with taxation and issuing debt. In each period, given level of government debt, b, inherited from the previous period, a policymaker who is elected by voters decides government debt issue d^g, income tax rate τ, and public goods provision g for the current period before uninsurable idiosyncratic income shocks are realized to individual voters, with meeting the fiscal budget constraint:

$$d^g = g + (1+r)b - \tau w \int (1-\delta) l^*_{Y,i} dF_i, \tag{13}$$

where F_i is the CDF of the population distribution. The government borrows funds (i.e., issues debt) by selling risk-free one-period bonds. Any government debt issued in the current period should be paid in the next period (no default); hence, government debt serves as state variable linking adjacent periods. In other words, d^g decided in the current period becomes b for the next period, as the policymaker elected in the next period inherits the government debt issued in the current period.

The government faces an upper limit on its borrowings, because it is committed to paying the debt back. Let notate the upper limit of government debt as $\bar{b}$ which is a given parameter. To fulfill government's commitment of paying the debt back, the government cannot borrow more than the maximum tax revenue. Thus, the effective limit of debt that the government can borrow is *strictly* lower than the maximum tax revenue to pay the debt back. That is,

$$\bar{b} < \frac{\bar{\tau} w}{r} \int (1-\delta) l^*_{Y,i}(\bar{\tau}) dF_i, \tag{14}$$

where $\overline{\tau} = \arg\max_{\tau} \tau w \int (1-\delta) l^*_{Y,i}(\tau) dF_i$. Note that the equality between left-hand and right-hand sides of (14) does not hold, because it entails negative infinity values of voters' utility with no provision of public goods. On the other hand, when the government purchases risk-free one-period bonds (i.e., lends funds), it does not buy them more than necessary for providing socially optimal level of public goods g^{sm} only with no taxation, where g^{sm} is defined by the Samuelson condition that equates the sum of individual's marginal benefit of public goods consumption with marginal cost of providing public goods. This defines the lower limit of public debt $\underline{b}$ by $\underline{b} = -\frac{g^{sm}}{r}$ where $\chi g_{sm}^{-1} = 1$. The initial level of government debt of this economy before reaching its steady state is randomly given.

With the government policies (d^g , τ , g) given, a stationary competitive general equilibrium of this economy is defined as a set of all individuals' decision rules of labor supply, private goods consumption and savings/loans as well as factor prices satisfying the following conditions for each period:

(i) With the government policies and prices given, all individuals' decision rules solve the problem of maximizing their own utility subject to their own budget constraints.

(ii) The representative firm maximizes its own profit with labor factor market being cleared by $L = \int (1-\delta) l^*_{Y,i} dF_i$.

(iii) The government budget constraint of (13) is met

Due to Walras' law, once all the condition (i), (ii) and (iii) are met, the aggregate resource constraint (feasibility constraint) is automatically met, clearing goods market as well. Reaching a stationary competitive general equilibrium of this economy, aggregate capital and individuals' savings/borrowings stay the same.

Because this economy is small and open for perfectly mobile capital, a world-wide equilibrium interest rate r is given to this economy, instead of being endogenously decided within this economy. The firm's demand for capital K is always met by capital supply from domestic and/or foreign investors (i.e., savers). Moreover, as young voters do not surely survive in the second period, regardless of their own decision on saving/borrowing, the longevity uncertainty entails some of them to leave accidental bequest. For simplicity, let us assume that the representative firm uses the capital that is accidentally left due to death. At a competitive general equilibrium, marginal capital product of the firm is equated with the given interest rate. At the same time, to meet the above condition (ii), marginal labor product of the firm is equated with wage rate. With the firm's demand for capital and labor being met, it is straightforward to see that at a stationary competitive general equilibrium,

$$w = z^{\frac{1}{1-\alpha}}(1-\alpha)\left(\frac{\alpha}{r+\upsilon}\right)^{\frac{\alpha}{1-\alpha}} . \tag{15}$$

At a stationary competitive general equilibrium, the value of the maximized utility of each voter is uniquely defined for a given set of the government policies. Thus, different sets of the government policies entail different values of the maximized utility of voters. To express that the government policies affect the maximized utility of voters, let $V_{Y,i}(d^g, \tau, g)$ denote for (9) and $V_{O,i}(d^g, \tau, g)$ for (10) or (11) at a stationary competitive general equilibrium.

Above all, the government policies are determined by office-seeking policymakers, instead of a benevolent social planner. In particular, policymakers of this economy are elected and abide by one-period term limit.[3] At the beginning of each period, two candidates, who care only about their own probability of winning the election, run for the government office. Both candidates simultaneously announce their own policy proposals for government debt, income tax rate, and public goods provision. Then, based on both policy proposal and personal appeal of each of the office-seeking candidates, voters decide which candidate to cast their own vote for. Personal appeal of a candidate is not related to policy proposals but revealed over the

[3] The one-period term limit is standard assumption in the political economics literature and adopted by many of the previous studies such as Alesina and Tabellini (1990), Battaglini and Coate (2008) and Song et al. (2012).

course of the election race, reflecting how popular his personality is. When each candidate decides his own policy proposal, he does not know personal appeals of his opponent and himself, which are known to all voters when voters make their own voting decision based on $V_{Y,i}(d^g, \tau, g)$ or $V_{O,i}(d^g, \tau, g)$. Thus, for maximizing their own payoff (winning probability), *each* of the two office-seeking candidates proposes a set of the government policies (d^g, τ, g) that maximizes the population-weighted sum of indirect utility of all voters alive

$$(1-\delta)\int V_{Y,i}(d^g, \tau, g)dF_i + \delta\int V_{O,i}(d^g, \tau, g)dF_i \tag{16}$$

subject to the government budget constraint of (13) and $d^g \in (\underline{b}, \overline{b})$. (For detailed proof, see Appendix A1.)[4] After the election, policy proposal of the elected candidate (winner) is implemented as announced at the beginning of the period.

Then, the timeline is summarized as follows. With the government policies of the elected policymaker known, young voters choose their own labor supply before the uninsurable income shock is realized. After the shock is realized, young voters choose their own savings/borrowings and they buy a house; and, each voter chooses their private goods consumption, reaching a stationary competitive general equilibrium of this economy. In other words, each voter's allocation of labor supply, private consumption and savings/borrowings is determined by voters themselves and implemented through competitive markets.

IV. Equilibrium Household Debt

Having described the politico-economic choice environment, this section derives equilibrium household debt. To this end, at first, each voter's optimal decision on savings/borrowings is delineated. At a stationary competitive general equilibrium, the budget constraint of an individual is binding, regardless of whether uninsurable negative income shock occurs to the individual or not. Thus, using (3) and (4), for any given *i*, the Euler equation (8) of voter *i* is

[4] This is a *variant* (not an original form) of probabilistic voting model of Lindbeck and Weibull (1987). Hence, the details need to be elaborated in Appendix A1.

restated as $\frac{1}{(1-\tau)wl_{Y,i}^{*}+d_{i}^{hNS*}-wh}=\beta(1+r)\frac{\delta}{1-\delta}\frac{1}{[wh-(1+r)d_{i}^{hNS*}]}$ if no uninsurable income shock occurs to voter *i*, where d_{i}^{hNS*} is voter *i*'s optimal capital allocation decision after no shock. This implies that

$$d_{i}^{hNS*}=\frac{1}{(1+r)[1+\frac{\beta\delta}{1-\delta}]}\{wh[1+\frac{\beta\delta}{1-\delta}(1+r)]-(1-\tau)wl_{Y,i}^{*}\frac{\beta\delta}{1-\delta}(1+r)\}. \tag{17}$$

By the same logic, with the budget constraint binding, for any given i, if uninsurable income shock occurs to voter *i*, his Euler equation is restated as $\frac{1}{\sigma(1-\tau)wl_{Y,i}^{*}+d_{i}^{hS*}-wh}=\beta(1+r)$ $\frac{\delta}{1-\delta}\frac{1}{[wh-(1+r)d_{i}^{hS*}]}$, where d_{i}^{hS*} is voter *i*'s optimal capital allocation decision after the income shock, entailing that

$$d_{i}^{hS*}=\frac{1}{(1+r)[1+\frac{\beta\delta}{1-\delta}]}\{wh[1+\frac{\beta\delta}{1-\delta}(1+r)]-\sigma(1-\tau)wl_{Y,i}^{*}\frac{\beta\delta}{1-\delta}(1+r)\}. \tag{18}$$

As individuals are not born as borrower or saver, each voter's optimal savings/borrowings choices of (17) and (18) are endogenously and uniquely driven from a given set of parameters. Thus, different values of parameter yield different household-debt decisions of voters. Aggregating each voter's optimal decision of (17) and (18), the following three kinds of equilibrium can arise: (i) all young voters of this economy choose to take a household loan so that all of domestic capital demands from the firm and borrowers are met only by foreign investors; (ii) no one decides to borrow in this economy; and, (iii) some choose to save while the others chose to borrow. In all the three kinds of equilibrium, old voters only consume without saving or borrowing. The first two kinds of equilibrium are unrealistic extremes and not policy-relevant, although they are theoretically feasible. Hence, among the three possible kinds, this paper focuses on the third kind of equilibrium which is realistic.

Elaborating on the third kind of equilibrium, each voter's optimal savings/borrowings are characterized as below.

Lemma 1] At an equilibrium where only part of young voters borrow, for any given i, voter i chooses to take a household loan when uninsurable income shock occurs to him (i.e., $d_i^{hS*} > 0$). On the other hand, voter i chooses not to borrow when no income shock occurs to him (i.e., $d_i^{hNS*} \leq 0$).

Proof. See Appendix A2.

Although the feature of this model that young voters do not have different earning abilities simplifies the proof and streamlines the theoretical analysis, extending this model by allowing young voters to have different earning abilities does not change the theoretical findings of this paper but adds complexity. Basically, the intuition of **Lemma 1** is that consumption-smoothing rational individuals choose to borrow only if their disposable income for pre-retirement consumption turns out lower than expected.

Importantly, due to **Lemma 1**, (15), (18) and the Law of Large Numbers, aggregate household debt at a stationary competitive general equilibrium, d^{h*}, is obtained as

$$d^{h*} = \frac{(1-\delta)\phi z^{\frac{1}{1-\alpha}}(1-\alpha)(\frac{\alpha}{r+\upsilon})^{\frac{\alpha}{1-\alpha}} \int \{h[1+\frac{\beta\delta}{1-\delta}(1+r)] - \sigma(1-\tau^*) l_{Y,i}^* \frac{\beta\delta}{1-\delta}(1+r)\} dF_i}{(1+r)[1+\frac{\beta\delta}{1-\delta}]} \quad (19)$$

where τ^* refers to stationary politico-economic equilibrium income tax rate that shall be defined later. Because $d_i^{hS*} > 0$ for any given i by **Lemma 1**, $d^{h*} > 0$. Notably, stationary equilibrium aggregate household debt of (19) depends on the economic factors of the household-loan collateral value, interest rate, the elderly population share and uninsurable income risk, which are shown as important factors for explaining aggregate household debt by the existing studies (e.g., Pollin, 1988; Barnes and Young, 2003; Campbell, 2006; Dynan and Kohn, 2007; Iacoviello, 2008; Mian and Sufi, 2011; Christelis et al., 2015; Kumhof et al., 2015; Coletta et al., 2019). In fact, these studies provided no formal theoretical analysis about effects on aggregate household debt of the economic factors which correspond to h, r, δ and ϕ of this paper. With the closed-form formula of (19), this paper can provide a theoretical understanding of how the four economic factors affect aggregate household debt.

First, it is immediate from **Lemma 1** and (19) that if income tax rate is fixed, an increase in the household-loan collateral value raises stationary equilibrium aggregate household debt, because for a given τ^*

$$\left.\frac{\partial d^{h*}}{\partial h}\right|_{\tau^*} = \frac{(1-\delta)\phi z^{\frac{1}{1-\alpha}}(1-\alpha)(\frac{\alpha}{r+\upsilon})^{\frac{\alpha}{1-\alpha}}}{(1+r)[1+\frac{\beta\delta}{1-\delta}]\{1+\frac{1}{\eta}[1+\frac{\beta\delta}{1-\delta}](1+\sigma)(1-\phi)(l_{Y,i}^*)^{-\frac{1}{\eta}-1}\}}\int 1-\sigma\frac{\beta\delta}{1-\delta}r+\frac{\beta\delta}{1-\delta}(1+r)+$$
$$\frac{1}{\eta}[1+\frac{\beta\delta}{1-\delta}(1+r)][1+\frac{\beta\delta}{1-\delta}](1+\sigma)(1-\phi)(l_{Y,i}^*)^{-\frac{1}{\eta}-1}dF_i > 0 \qquad (20)$$

due to $\sigma \in (0,1)$, $\beta \in (0,1)$, $\frac{\delta}{1-\delta} \in (0,1)$, $r \in (0,1)$ and $\frac{\partial l_{Y,i}^*}{\partial h}$ of (A16) in Appendix 5. Notice that an increase in the collateral value (i.e., an increase in the value of house) means less disposable income for pre-retirement consumption and more for post-retirement consumption, if there is no tax rate change. Thus, an increase in the collateral value causes individual young voters to borrow more at a new steady-state competitive general equilibrium reached after the increase with no change in the government policies, because individuals seek to smooth private consumption over their lifetime.

Second, with the formula of (19), it is also straightforward that an increase in interest rate leads to an increase in stationary equilibrium aggregate household debt, if income tax rate is not allowed to respond to the increase. With a given τ^*,

$$\left.\frac{\partial d^{h*}}{\partial r}\right|_{\tau^*}=-\{\frac{1}{1+r}+\frac{\alpha}{1-\alpha}\frac{1}{(r+\upsilon)}\}\frac{(1-\delta)\phi z^{\frac{1}{1-\alpha}}(1-\alpha)(\frac{\alpha}{r+\upsilon})^{\frac{\alpha}{1-\alpha}}}{(1+r)[1+\frac{\beta\delta}{1-\delta}]}\int h[1+\frac{\beta\delta}{1-\delta}(1+r)]-\sigma(1-\tau^*)l^*_{Y,i}$$

$$\frac{\beta\delta}{1-\delta}(1+r)dF_i+\frac{\beta\delta\phi z^{\frac{1}{1-\alpha}}(1-\alpha)(\frac{\alpha}{r+\upsilon})^{\frac{\alpha}{1-\alpha}}}{(1+r)[1+\frac{\beta\delta}{1-\delta}]\{1+\frac{1}{\eta}[1+\frac{\beta\delta}{1-\delta}](1+\sigma)(1-\phi)(l^*_{Y,i})^{-\frac{1}{\eta}-1}\}}\int[\sigma(1-\tau^*)l^*_{Y,i}-h]\{1+\frac{1}{\eta}$$

$$[1+\frac{\beta\delta}{1-\delta}](1+\sigma)(1-\phi)(l^*_{Y,i})^{-\frac{1}{\eta}-1}\}+\frac{\sigma h}{(1+r)}dF_i<0 \tag{21}$$

due to $d^{h*}>0$ and $\frac{\partial l^*_{Y,i}}{\partial r}$ of (A22) in Appendix A6. $[\sigma(1-\tau^*)l^*_{Y,i}-h]w>0$ because the value of house is always strictly lower than disposable lifetime income. Apparently, an increase in interest rate makes borrowing more expensive, while it does not change lifetime post-tax income unless income tax rate changes after the increase. Therefore, an increase in interest rate gives the incentive for individuals to take out a smaller amount of household loans than before the increase.

Third, population aging reduces stationary equilibrium aggregate household debt, if income tax rate is fixed, because, according to (19), for a given τ^*

$$\left.\frac{\partial d^{h*}}{\partial \delta}\right|_{\tau^*}=\frac{\phi z^{\frac{1}{1-\alpha}}(1-\alpha)(\frac{\alpha}{r+\upsilon})^{\frac{\alpha}{1-\alpha}}}{[1+\frac{\beta\delta}{1-\delta}]}\{\frac{(-1)[1+\beta(\frac{1+\delta}{1-\delta})]}{(1+r)[1+\frac{\beta\delta}{1-\delta}]}\int h[1+\frac{\beta\delta}{1-\delta}(1+r)]-\sigma(1-\tau^*)l^*_{Y,i}\frac{\beta\delta}{1-\delta}(1+$$

$$r)dF_i+\frac{(-1)\beta}{(1-\delta)}\int[\sigma(1-\tau^*)l^*_{Y,i}-h]+\frac{\sigma(1-\tau^*)\frac{\beta\delta}{1-\delta}(1+\sigma)(1-\phi)(l^*_{Y,i})^{-\frac{1}{\eta}}}{1+\frac{1}{\eta}[1+\frac{\beta\delta}{1-\delta}](1+\sigma)(1-\phi)(l^*_{Y,i})^{-\frac{1}{\eta}-1}}dF_i\}<0 \tag{22}$$

due to $d^{h*}>0$ and $\frac{\partial l^*_{Y,i}}{\partial \delta}$ of (A28) in Appendix A7. Without a change in income tax rate, post-tax lifetime income is not increased simply by living longer. An increase in life expectancy makes post-retirement consumption more important than before the increase, whereas paying household loans back reduces available resources for post-retirement consumption, as appears in (4). Therefore, with income tax rate being fixed, population aging causes individuals to take out a smaller amount of loans.

Fourth, with the formula of (19), we also find that an increase in uninsurable idiosyncratic income risk always raises stationary equilibrium aggregate household debt, if the increase does not change income tax rate. For a given τ^*,

$$\left.\frac{\partial d^{h*}}{\partial \phi}\right|_{\tau^*} = \frac{(1-\delta)z^{\frac{1}{1-\alpha}}(1-\alpha)(\frac{\alpha}{r+\upsilon})^{\frac{\alpha}{1-\alpha}}}{(1+r)[1+\frac{\beta\delta}{1-\delta}]}\{\int h[1+\frac{\beta\delta}{1-\delta}(1+r)]-\sigma(1-\tau^*)l_{Y,i}^*\frac{\beta\delta}{1-\delta}(1+r)+\phi\sigma(1-\tau^*)$$

$$\frac{\frac{\beta\delta}{1-\delta}(1+r)[1+\frac{\beta\delta}{1-\delta}](1+\sigma)(l_{Y,i}^*)^{-\frac{1}{\eta}}}{1+\frac{1}{\eta}[1+\frac{\beta\delta}{1-\delta}](1+\sigma)(1-\phi)(l_{Y,i}^*)^{-\frac{1}{\eta}-1}}dF_i\}>0 \tag{23}$$

because of $d^{h*}>0$ and $\frac{\partial l_{Y,i}^*}{\partial \phi}$ of (A33) in Appendix A8. Basically, as more individuals are likely to undergo the uninsurable negative income shocks, based on **Lemma 1**, more are likely to borrow, increasing stationary equilibrium total household debt.

Note from (3), (4), (5), (6), (7) and (8) that each individual's optimal allocation decision of borrowing/saving, private consumption and labor supply is affected by the government policies (d^g , τ , g) *only* through income tax rate. This implies that $\left.\frac{\partial d^{h*}}{\partial h}\right|_{\tau^*,d^{g*},g^*} = \left.\frac{\partial d^{h*}}{\partial h}\right|_{\tau^*}$, $\left.\frac{\partial d^{h*}}{\partial r}\right|_{\tau^*,d^{g*},g^*} = \left.\frac{\partial d^{h*}}{\partial r}\right|_{\tau^*}$, $\left.\frac{\partial d^{h*}}{\partial \delta}\right|_{\tau^*,d^{g*},g^*} = \left.\frac{\partial d^{h*}}{\partial \delta}\right|_{\tau^*}$ and $\left.\frac{\partial d^{h*}}{\partial \phi}\right|_{\tau^*,d^{g*},g^*} = \left.\frac{\partial d^{h*}}{\partial \phi}\right|_{\tau^*}$. Importantly, the above analyses of (20), (21), (22) and (23) show only *partial* effects on stationary equilibrium aggregate household debt of the household-loan collateral value, interest rate, population aging and uninsurable income risk, because they assume that government taxation policy is not allowed to change. This assumption is not proper for analyzing long-term behavior of household debt, which is of our focus, although it might be innocuous for analyzing shortterm behavior of household debt. Over the time when individuals adjust their household-loan decisions for responding to changes in the four economic factors, policymakers also do adjust government policies (government debt, income tax rate, and public goods) for responding to the changes.

Thus, this paper also allows policymakers to adjust the government policies for responding to a change in the household-loan collateral value, interest rate, the elderly population share and uninsurable income risk with taking into account of individual voters' responses to the adjustment of the government policies. When policymakers adjust government debt, income tax rate and public goods provision in response to changes in each of the four economic factors (indicated by the parameters of h, r, δ and ϕ), stationary equilibrium aggregate household debt is affected by the policy adjustments as well as by the changes. It is not a priori clear whether the above-noted partial effects of the four economic factors — (20), (21), (22) and (23) — remain the same after allowing concurrent changes in the government policies. To find this out, we first need to characterize stationary politico-economic equilibrium government policies, based on which we can analyze how stationary politico-economic equilibrium government policies respond to a change in the four economic factors. Then, by allowing the mutual influence between policymaker and voters, we examine responses of stationary politico-economic equilibrium government policies and stationary equilibrium aggregate household debt to the four economic factors.

V. Politico-Economic Equilibrium Government Debt

This section characterizes politico-economic equilibrium government debt, income tax rate and public goods provision that maximize policymakers' own probability of getting elected. To maximize the probability, each policymaker chooses a set of government policies that maximizes the population-weighted sum of the utility of his electorate (16), as shown in Appendix A1. As such, politico-economic equilibrium government policies reflect individual voters' responses to a change in the government policies. However, (16) depends on voters' labor supply and private consumption that policymakers cannot directly choose. Rather, labor supply and private consumption are chosen by individual voters. Thus, for characterizing politico-economic equilibrium government policies, it is necessary to find how policymakers' choices of (d^g, τ, g) affect voters' labor supply and private consumption to change voters' utility and thus (16) at a stationary competitive general equilibrium.

To this end, from voters' optimal allocation conditions of (5), (6), (7) and (8), notice that government debt issue d^g does not directly affect any voter's optimal decision of labor supply and private consumption (and borrowing/saving), although it may indirectly do so. Similarly, provision of public goods g does not directly affect any voter's optimal allocation decision, although it directly affects level of voters' utility as it is part of their utility. On the other hand, with voters' budget constraints binding, income tax rate τ affects voters' optimal labor supply and private consumption by changing return on labor supply available for private consumption. In particular, due to (5), for any given i,

$$\frac{\partial l^*_{Y,i}(\tau)}{\partial \tau} = \frac{-\eta l^*_{Y,i}}{(1-\tau)} \tag{24}$$

which is strictly negative as $\eta > 0$, $l^*_{Y,i} > 0$ and $\tau \in [0,1)$. This shows that an increase in income tax rate reduces labor supply of young voters and that the magnitude of this labor-supply distortion increases with labor-supply elasticity. Notably, based on (24), we identify how an increase in income tax rate changes the level of (16) by affecting voters' labor supply.

We also identify how an increase in income tax rate changes the level of (16) by affecting voters' private consumption, based on voters' optimal allocation conditions of (5), (6), (7) and (8) with their own budget constraints binding. For any given i, an increase in income tax rate reduces private consumption utility of young voter *i* because

$$\frac{\partial \log(c_{Y,i}^{NS*}(\tau))}{\partial \tau} = \frac{-(l_{Y,i}^{*})^{\frac{1}{\eta}+1}}{(1+\sigma)(1-\phi)(1-\tau)} \tag{25}$$

On the other hand, for any given i,

$$\frac{\partial \log(c_{O,i}^{NS*}(\tau))}{\partial \tau} = 0 \tag{26}$$

as tax is not imposed on old voters. Based on (25) and (26), we identify how an increase in income tax rate changes the level of (16) by affecting voters' private consumption utility.

Based on (24), (25) and (26), each policymaker can know how a change in his choice of the government policies (d^g, τ, g) changes his winning probability of (16), even though he cannot choose any voter' labor supply or private consumption. As such, by deciding a set of (d^g, τ, g), office-seeking policymakers can induce allocation of voters' labor supply and private consumption that maximizes (16). In this maximization, each policymaker faces the fiscal budget constraints of (13) and $d^g \in (\underline{b}, \overline{b})$. Moreover, due to the one-period term limit, each policymaker cannot credibly choose the government policies for the future periods after his term. Each policymaker only can credibly choose the government policies of his own term with treating government debt (b) inherited from the previous period as state variable given. For tractability, let us characterize politico-economic equilibrium government policies that depend only on the current state variable b and are Markov perfect. When each policymaker decides government debt issue, income tax rate, and public goods provision for his own term, he takes what other policymakers (of the past and the future periods) would do as given. Restating the winning probability of (16) in terms of competitive-equilibrium labor supply and private consumption as well as competitive-equilibrium wage rate of (15), the maximization problem[5] that each policymaker solves is summarized as follows: for any given

[5] The statement of (27) should not be misunderstood that voters' optimal private consumption and labor supply depend only on the current income tax rate. It is just to indicate that policymaker's decision affects voters' private consumption and labor supply through income tax rate by changing voters' post-tax labor incomes disposable for private consumption. In addition, for efficient use of space, constant terms that do not affect policymaker's maximization are abstracted from (27).

$b \in (\underline{b}, \overline{b})$,

$$\max_{(d^g,\tau,g)} \int [(1-\delta)\log(c_{Y,i}^{NS*}(\tau)) + \delta \log(c_{O,i}^{NS*}(\tau)) + \chi \log(g) - (1-\delta)\frac{l_{Y,i}^{*}(\tau)^{1+\frac{1}{\eta}}}{1+\frac{1}{\eta}} + \beta\delta\{\log(c_{O,i}^{NS*}(\tau'(d^g)))$$

$+\chi \log(g'(d^g))\}]dF_i$ subject to $d^g = g + (1+r)b - \tau z^{\frac{1}{1-\alpha}}(1-\alpha)(\frac{\alpha}{r+\upsilon})^{\frac{\alpha}{1-\alpha}}\int (1-\delta) l_{Y,i}^{*} dF_i$ and $d^g \in (\underline{b}, \overline{b})$. (27)

Notice that all the three conditions for the ensuing allocation of voters to be implemented as a competitive general equilibrium of this economy are satisfied by (27). The condition (iii) is embedded as the fiscal budget constraint of (27). By plugging competitive-equilibrium wage rate of (15) into the fiscal budget constraint of (27), the condition (ii) is also met. Moreover, the condition (i) is also satisfied, because for any given i, $c_{Y,i}^{NS*}$, $c_{O,i}^{NS*}$ and $l_{Y,i}^{*}$ in (27) are defined by optimality conditions of (5), (6), (7) and (8).

By finding optimal policy functions, denoted by $\{d^g(b),\ \tau(b),\ g(b)\}$, which solve (27), politico-economic equilibrium government policies are characterized. Based on (24), (25) and (26), combining the FOCs for optimal public goods provision and income tax rate obtains the intra-temporal optimality condition for the government policies: for $\forall b \in (\underline{b}, \overline{b})$,

$$\frac{\chi}{g(b)} = \{\frac{1}{(1+\sigma)(1-\phi)} - \eta\}\frac{1}{1-\tau(b)(1+\eta)}\zeta, \tag{28}$$

where $\zeta = \dfrac{\int l_{Y,i}^{*\ \frac{1}{\eta}+1} dF_i}{z^{\frac{1}{1-\alpha}}(1-\alpha)(\frac{\alpha}{r+\upsilon})^{\frac{\alpha}{1-\alpha}}\int l_{Y,i}^{*} dF_i}$. While each policymaker wants to win over his electorate by providing more public goods, he also needs to consider the cost for financing the current public expenditures borne by his electorate. In this regard, each policymaker equates marginal benefit of public goods provision, which is the left-hand side of (28), with marginal cost of financing the provision, which is the right-hand side of (28). In addition, from the FOCs for optimal government debt and income tax rate, inter-temporal optimality condition for the government policies is derived as follows: for $\forall b \in (\underline{b}, \bar{b})$,

$$\{\frac{1}{(1+\sigma)(1-\phi)} - \eta\}\frac{1}{1-\tau(b)(1+\eta)}\zeta = -\beta\delta\frac{\chi}{g'(d^g(b))}\frac{\partial g'(d^g(b))}{\partial d^g(b)}. \tag{29}$$

To finance the current public expenditures, each policymaker utilizes *both* taxation and government debt issue (i.e., taxing now and later), because voters prefer him to do so for their own consumption smoothing. For allocating public finance cost over time, as shown by the left-hand and right-hand sides of (29) respectively, each policymaker equates marginal cost of taxation with the present value of marginal disutility of his electorate from an increase in government debt which reduces the available resources for public goods provision in the next period. As demonstrated by the right-hand side of (29), the marginal cost of issuing government debt is not borne by old voters, because old voters do not survive in the next period. Furthermore, note from (29) that the cost on unborn future voters is not internalized, although they are going to pay part of the current government debt in the future, because they are not part of the incumbent policymaker's electorate. Above all, based on (28) and (29), optimal policy functions of government debt, income tax rate and public goods provision are identified as below.

Lemma 2] Politico-economic equilibrium policy functions $\{d^g(b), \tau(b), g(b)\}$ are defined as follows. For $\forall b \in (\underline{b}, \bar{b})$,

$$d^g(b) = \bar{b} - \rho(\bar{b} - b), \tag{30}$$

$$\tau(b) = \frac{1}{(1+\eta)} - \{\frac{1}{(1+\sigma)(1-\phi)} - \eta\}\frac{\zeta}{\delta\beta\chi(1+\eta)}\rho(\bar{b} - b), \tag{31}$$

$$g(b) = \frac{1}{\delta\beta}\rho(\bar{b} - b), \tag{32}$$

where ρ is defined by

$$\rho(b) = \frac{\bar{b} - (1+r)b + z^{\frac{1}{1-\alpha}}(1-\alpha)(\frac{\alpha}{r+\upsilon})^{\frac{\alpha}{1-\alpha}}\frac{(1-\delta)}{(1+\eta)}\int l^*_{Y,i}(\tau^*)dF_i}{(\bar{b} - b)[1 + \frac{1}{\delta\beta} + \{\frac{1}{(1+\sigma)(1-\phi)} - \eta\}\frac{(1-\delta)}{\delta\beta\chi(1+\eta)}\int l^{*\,\frac{1}{\eta}+1}_{Y,i} dF_i]} > 0. \tag{33}$$

Proof. See Appendix A3.

According to (30), (31) and (32) of **Lemma 2**, which are nonlinear functions of a given level of inherited debt *b*, we identify politico-economic equilibrium government debt, income tax rate and public goods provision. As a result, politico-economic equilibrium government policies vary depending on different levels of *b* (government debt in the previous period). As this paper focuses on steady-state economy, we characterize stationary politico-economic equilibrium government policies. Specifically, politico-economic equilibrium government policies are stationary if politico-economic equilibrium government debt stays the same (i.e., $d^{g*} = b^*$). Once politico-economic equilibrium government debt reaches its steady state, stationary politico-economic equilibrium income tax rate and public goods provision are identified by the optimal policy functions of (31) and (32) respectively.

Lemma 3] Stationary politico-economic equilibrium government debt, income tax rate and public goods provision are

$$d^{g*} = \frac{\dfrac{\{\frac{1}{(1+\sigma)(1-\phi)} - \eta\}\bar{b}(1-\delta)}{\delta\beta\chi(1+\eta)} \int l_{Y,i}^{*\,\frac{1}{\eta}+1} dF_i + \dfrac{\bar{b}}{\delta\beta} - \dfrac{z^{\frac{1}{1-\alpha}}(1-\alpha)(\frac{\alpha}{r+\upsilon})^{\frac{\alpha}{1-\alpha}}(1-\delta)}{(1+\eta)} \int l_{Y,i}^{*} dF_i}{[\{\frac{1}{(1+\sigma)(1-\phi)} - \eta\} \dfrac{(1-\delta)}{\delta\beta\chi(1+\eta)} \int l_{Y,i}^{*\,\frac{1}{\eta}+1} dF_i + \dfrac{1}{\delta\beta} - r]}, \quad (34)$$

$$\tau^* = \frac{1}{(1+\eta)} - \{\frac{1}{(1+\sigma)(1-\phi)} - \eta\} \frac{\zeta}{\delta\beta\chi(1+\eta)} (\bar{b} - d^{g*}), \quad (35)$$

$$g^* = \frac{z^{\frac{1}{1-\alpha}}(1-\alpha)(\frac{\alpha}{r+\upsilon})^{\frac{\alpha}{1-\alpha}} \dfrac{(1-\delta)}{(1+\eta)} \int l_{Y,i}^{*} dF_i - r\bar{b}}{\delta\beta[\{\frac{1}{(1+\sigma)(1-\phi)} - \eta\} \dfrac{(1-\delta)}{\delta\beta\chi(1+\eta)} \int l_{Y,i}^{*\,\frac{1}{\eta}+1} dF_i + \dfrac{1}{\delta\beta} - r]}. \quad (36)$$

Proof. See Appendix A4.

Above all, from (34) of **Lemma 3**, it is immediate that stationary politico-economic equilibrium government debt is affected by interest rate, the elderly population share and the uninsurable idiosyncratic income risk. In light of (A16) of Appendix A4, $\frac{\partial l_{Y,i}^*}{\partial h} < 0$ for any given i; hence, (34) implies that stationary politico-economic equilibrium government debt is also affected by the household-loan collateral value. Furthermore, as optimal policy functions of (30), (31) and (32) show, through d^{g^*}, the economic factors of the household-loan collateral value, interest rate, the elderly population share and the uninsurable income risk also affect stationary politico-economic equilibrium income tax rate and public goods provision, as appears in (35) and (36). Recall that stationary equilibrium aggregate household debt of (19) is affected not only by a change in these four economic factors but also by a concurrent response of stationary politico-economic equilibrium income tax rate to the change. Thus, an increase in each of the economic parameters of h, r, δ and ϕ concurrently changes stationary politico-economic equilibrium government debt and stationary equilibrium aggregate household debt (d^{g^*} and d^{h^*}). Because in this analysis equilibrium government policies unequivocally refer to politico-economic equilibrium government policies, we use the former expression henceforth.

VI. Movements of Steady-State Government Debt and Aggregate Household Debt

Thus far, stationary equilibrium government debt and stationary equilibrium aggregate household debt have been obtained with all the parameters being fixed. Now, we introduce an increase in each of the economic parameters of the household-loan collateral value, interest rate, the elderly population share and uninsurable idiosyncratic income risk, to examine how movements of stationary equilibrium government debt and stationary equilibrium aggregate household debt are related. Because stationary equilibrium government debt, income tax rate and public goods provision incorporate individuals' optimal allocation, optimal policy adjustments to an increase in each of the four economic parameters of h, r, δ and ϕ inherently reflect the concurrent responses of individuals' optimal allocation to the increase. At the same time, individuals' optimal borrowings/savings are affected by government's optimal policy adjustments to a change in h, r, δ or ϕ as well as by the change itself. With allowing the mutual influence between responses of policymakers and their voters to an increase in each of the four economic parameters of h, r, δ and ϕ, we analyze movements of stationary equilibrium government debt and stationary equilibrium aggregate household debt responding to each increase.

First, as mentioned above, the government policies directly affect each voter's allocation decision only through income tax rate; hence, how an increase in the household-loan collateral value (h) affects stationary equilibrium total household debt with reflecting optimal adjustments, if any, of the government policies to the increase is identified by

$$\frac{\partial d^{h*}}{\partial h} = \left.\frac{\partial d^{h*}}{\partial h}\right|_{\tau^*} + \frac{\partial d^{h*}}{\partial \tau^*}\frac{\partial \tau^*}{\partial h}. \tag{37}$$

The closed-form formulae of stationary equilibrium aggregate household debt d^{h*} and stationary equilibrium income tax rate τ^*, (19) and (35), enable us to obtain the second term as well. In particular,

$$\frac{\partial \tau^*}{\partial h} = \frac{\{\frac{1}{(1+\sigma)(1-\phi)} - \eta\}\zeta}{\delta\beta\chi(1+\eta)}\frac{\partial d^{g*}}{\partial h} - \frac{\{\frac{1}{(1+\sigma)(1-\phi)} - \eta\}}{\delta\beta\chi(1+\eta)}(\bar{b} - d^{g*})\frac{\partial \zeta}{\partial h}. \tag{38}$$

Combining (37) and (38) illustrates that income tax rate is the channel through which movements of stationary equilibrium government debt and stationary equilibrium aggregate household debt are related. Above all, capitalizing on (19), (20) and **Lemma 3**, we find that a change in the household-loan collateral value (h) *conditionally* causes stationary equilibrium government debt and total household debt to move in the same direction.

Proposition 1] An increase in the value of household-loan collateral causes an increase or a decrease in stationary equilibrium government debt. If an increase in the household-loan collateral value raises stationary equilibrium government debt by a margin large enough to raise stationary equilibrium income tax rate (i.e., $\frac{\partial d^{g*}}{\partial h} > \frac{(\bar{b} - d^{g*})}{\zeta}\frac{\partial \zeta}{\partial h}$), then it always raises stationary equilibrium aggregate household debt. If not (i.e., $\frac{\partial d^{g*}}{\partial h} \leq 0$ or $0 < \frac{\partial d^{g*}}{\partial h} \leq \frac{(\bar{b} - d^{g*})}{\zeta} \frac{\partial \zeta}{\partial h}$), then it can lower or raise stationary equilibrium aggregate household debt.

Proof. See Appendix A5.

As mentioned above, an increase in the household-loan collateral value (i.e., an increase in the value of house) with no change in tax rate means less disposable income for pre-retirement consumption, which makes young voters borrow more and work more, as shown by (20) and $\frac{\partial l^{*}_{Y,i}}{\partial h} > 0$ in (A16) of Appendix 5. However, borrowing more will reduce young voters' post-retirement consumption in the future and working more will reduce their utility now, if there is no change in the government policies. Moreover, young voters' borrowing more and working more does not necessarily entail increases in their pre-retirement consumption enough to regain pre-increase level of their utility. Thus, young voters consider voting for higher level of government debt to get more utility from more public goods provision. Even if young voters face higher rate of income tax by voting for higher level of government debt, their working more may end up with giving them higher level of post-tax income. Alternatively, young voters also consider voting for lower level of government debt to lower income tax rate, which essentially increase private goods consumption at the cost of public goods consumption for the first and second periods. Facing this trade-off in the two alternatives, it is not a priori clear whether higher level of government debt gives higher level of utility to young voters than lower level of government debt. On the other hand, as old voters do not pay tax, they always vote for higher level of government debt to get more public goods, although the majority of the electorate is young voters. Hence, an increase in the household-loan collateral value causes office-seeking policymakers to either increase or decrease government debt, which largely depends on young voters' political support. Either case is fully feasible. As such, the household-loan collateral value does affect stationary equilibrium government debt, either positively or negatively.

If an increase in the household-loan collateral value causes office-seeking policymakers to raise government debt (i.e., $\frac{\partial d^{g*}}{\partial h} > 0$), then it does not necessarily raise income tax rate. Only if an increase in the collateral value causes voters to support for policymakers to raise government debt large enough to increase income tax rate (i.e., only when $\frac{\partial \tau^{*}}{\partial h} > 0$), an increase in h raises both stationary equilibrium government debt and aggregate household debt together. In this regard, (38) implies that $\frac{\partial \tau^{*}}{\partial h} > 0$ if $\frac{\partial d^{g*}}{\partial h} > \frac{(\bar{b} - d^{g*})}{\zeta}\frac{\partial \zeta}{\partial h}$. As (A18) shows, $\frac{\partial d^{h*}}{\partial \tau^{*}} > 0$, meaning that an increase in income tax rate leads young voters to take a larger amount of household loans, while it provide more public goods.

On the other hand, if $\frac{\partial d^{g*}}{\partial h} \le \frac{(\bar{b} - d^{g*})}{\zeta}\frac{\partial \zeta}{\partial h}$, then an increase in the household-loan collateral value can lead to an increase or a decrease in stationary equilibrium government debt to entail $\frac{\partial \tau^{*}}{\partial h} \le 0$. Due to (38) and (A18), $\frac{\partial d^{h*}}{\partial h}$ can be negative or positive, if $\frac{\partial \tau^{*}}{\partial h} \le 0$ that counteracts the partial effect of the collateral value shown by (20). Furthermore, if an increase in the household-loan collateral value reduces government debt (and thus income tax rate) large enough to dominate the positive partial effect of (20) (i.e., if $\left.\frac{\partial d^{h*}}{\partial h}\right|_{\tau^{*}} + \{\frac{1}{(1+\sigma)(1-\phi)} - \eta\}\frac{1}{\delta\beta\chi(1+\eta)}\frac{\partial d^{h*}}{\partial \tau^{*}}[\zeta\frac{\partial d^{g*}}{\partial h} - (\bar{b} - d^{g*})\frac{\partial \zeta}{\partial h}] < 0$), then an increase in the value of collateral lowers both stationary equilibrium government debt and aggregate household debt together. Thus, co-movement of both debts also can occur even when the household-loan collateral value negatively affects stationary equilibrium government debt.

Taking government's optimal policy response to an increase in the household-loan collateral value into account of household-debt response to the increase, **Proposition 1** shows that the effect of the household-loan collateral value on aggregate household debt is no longer always positive but depends on government's policy changes. This theoretical finding is not inconsistent with the existing empirical studies on aggregate household debt and house price (e.g., Barnes and Young, 2003; Dynan and Kohn, 2007; Mian and Sufi, 2011; Christelis et al., 2015; Coletta et al., 2019). Notably, in the existing theoretical and empirical studies on government debt, the value of household-loan collateral (or the value of house) has long been overlooked as an irrelevant factor. By allowing mutual influence of government's debt decision and voters' household-loan decision, **Proposition 1** newly discovers that an increase in the household-loan collateral value does affect steady-state government debt as well.

Second, by the same token, how an increase in interest rate (r) affects stationary equilibrium aggregate household debt with reflecting optimal adjustments of the government policies to the increase is identified by

$$\frac{\partial d^{h*}}{\partial r} = \left.\frac{\partial d^{h*}}{\partial r}\right|_{\tau^*} + \frac{\partial d^{h*}}{\partial \tau^*}\frac{\partial \tau^*}{\partial r}. \tag{39}$$

With the closed-form formulae of d^{h*} and τ^*, we obtain $\frac{\partial \tau^*}{\partial r}$ in the second term of (39) as

$$\frac{\partial \tau^*}{\partial r} = \frac{\{\frac{1}{(1+\sigma)(1-\phi)} - \eta\}\zeta}{\delta\beta\chi(1+\eta)}\frac{\partial d^{g*}}{\partial r} - \frac{\{\frac{1}{(1+\sigma)(1-\phi)} - \eta\}}{\delta\beta\chi(1+\eta)}(\bar{b} - d^{g*})\frac{\partial \zeta}{\partial r}. \tag{40}$$

Capitalizing on (19), (21) and **Lemma 3**, we find that an increase in interest rate *conditionally* causes stationary equilibrium government debt and stationary equilibrium total household debt to move together in the opposite directions.

Proposition 2] An increase in interest rate raises stationary equilibrium government debt. If an increase in interest rate raises stationary equilibrium government debt by a margin small enough to lower equilibrium income tax rate (i.e., $\frac{\partial d^{g*}}{\partial r} < \frac{(\bar{b}-d^{g*})}{\zeta}\frac{\partial \zeta}{\partial r}$), then it always lower stationary equilibrium aggregate household debt. If not (i.e., $\frac{\partial d^{g*}}{\partial r} \geq \frac{(\bar{b}-d^{g*})}{\zeta}\frac{\partial \zeta}{\partial r}$), then it can lower or raise stationary equilibrium aggregate household debt.

Proof. See Appendix A6.

While office-seeking policymakers with the term limit do not have incentive to reduce the level of government debt to zero for winning over voters with more public goods provision, they should service the inherited debt issued by the previous policymakers which costs higher as interest rate increases. Hence, an increase in interest rate certainly causes policymakers to raise government debt. In contrast, because borrower voters should reduce the level of household debt to zero after they are retired, higher interest rate leads young voter to borrow less by making borrowing more expensive, unless tax rate decreases to raise their disposable income. Due to this contrast, an increase in interest rate causes stationary equilibrium government debt and aggregate household debt to move in the *opposite* directions, as long as it does not increase stationary equilibrium income tax rate. Notice that higher level of government debt does not necessarily entail higher rate of income tax, because an increase in interest rate makes young voters work more by reducing disposable income for private goods consumption other than residence (as shown by $\frac{\partial l^*_{Y,i}}{\partial r} > 0$ in (A22) of Appendix 6) to yield more tax revenue without a change in tax rate.

On the other hand, if an increase in interest rate causes policymakers to raise both government debt and income tax rate, then this policy adjustment counteracts the partial effect of interest rate on aggregate household debt shown by (21). As more public goods provision gives higher consumption utility to voters at the cost of paying more tax, it is feasible for voters to vote for this policy adjustment to an increase in interest rate. In this case, an increase in interest rate can lower or raise stationary equilibrium total household debt, while it increases stationary equilibrium government debt. Thus, when concurrent policy adjustment is allowed, household-debt response to an increase in interest rate is no longer certainly negative, which is different from (21).

Third, how population aging affects stationary equilibrium total household debt with reflecting optimal policy response to an increase in δ is identified by

$$\frac{\partial d^{h*}}{\partial \delta} = \left.\frac{\partial d^{h*}}{\partial \delta}\right|_{\tau^*} + \frac{\partial d^{h*}}{\partial \tau^*}\frac{\partial \tau^*}{\partial \delta}. \tag{41}$$

Using the closed-form formulae of d^{h*} and τ^*, we obtain $\frac{\partial \tau^*}{\partial \delta}$ in the second term of (41) as

$$\frac{\partial \tau^*}{\partial \delta} = \frac{\{\frac{1}{(1+\sigma)(1-\phi)} - \eta\}\zeta}{\delta\beta\chi(1+\eta)}\frac{\partial d^{g*}}{\partial \delta} - \frac{\{\frac{1}{(1+\sigma)(1-\phi)} - \eta\}}{\delta\beta\chi(1+\eta)}(\bar{b} - d^{g*})\frac{\partial \zeta}{\partial \delta}. \tag{42}$$

Based on (19), (22) and **Lemma 3**, how population aging affects stationary equilibrium government debt and stationary equilibrium aggregate household debt.

Proposition 3] Population aging causes an increase or a decrease in stationary equilibrium government debt. If population aging raises stationary equilibrium government debt, then it can lower or raise stationary equilibrium aggregate household debt. If population aging lowers stationary equilibrium government debt, then it always lowers stationary equilibrium aggregate household debt.

Proof. See Appendix A7.

Facing population aging, young workers work more to prepare private consumption for longer retirement, as shown by $\frac{\partial l^*_{Y,i}}{\partial \delta} > 0$ in (A28) of Appendix 7. Due to disutility of labor supply, young voters can consider voting for lower level of government to lower income tax rate or for higher level of government debt to get more public goods to increase consumption utility. On the other hand, old voters always vote for higher level of government debt, as they do not pay tax. Because policymakers care about the *entire* electorate, when facing inter-generational disagreement over the government debt, population aging does not necessarily raise stationary equilibrium government debt. This ambiguity of effect on government debt of population aging resonates with the findings of Tabellini (1991) and Song et al. (2012).

If population aging makes policymakers raise government debt by a sufficiently large margin (i.e., $\frac{\partial d^{g*}}{\partial \delta} > \left\| \left. \frac{\partial d^{h*}}{\partial \delta} \right|_{\tau^*} \right\| [\{\frac{1}{(1+\sigma)(1-\phi)} - \eta\} \frac{\zeta}{\delta\beta\chi(1+\eta)} \frac{\partial d^{h*}}{\partial \tau^*}]^{-1} + \frac{(\bar{b} - d^{g*})}{\zeta} \frac{\partial \zeta}{\partial \delta}$), then it always raises stationary equilibrium aggregate household debt. If not, population aging can lower or raise stationary equilibrium total household debt. In particular, if population aging makes policymakers lower government debt, it always lowers income tax rate, as implied from (42) and $\frac{\partial \zeta}{\partial \delta} > 0$ in (A30) of Appendix 7, which increases post-tax income so that voters take out lower amount of household loans strengthening the partial effect of (22). As such, population aging also can bring about co-movement of both stationary equilibrium government debt and aggregate household debt by increasing or decreasing both together.

With optimal policy response to population aging taken into account, it is no longer clear whether population aging decreases stationary equilibrium aggregate household debt, unlike (22). In fact, there is no clear consensus among the empirical studies regarding effect of population aging on aggregate household debt. By conducting regression analyses, Pollin (1988) found no statistically significant effect of demographics, whereas Coletta et al. (2019) found positive effect. In contrast, with time-series data of baby-boomers, Barnes and Young (2003) and Dynan and Kohn (2007) suggested negative correlation between population aging and aggregate household debt.

Fourth, we examine how uninsurable idiosyncratic income risk affects stationary equilibrium total household debt with reflecting optimal policy response to an increase in the risk by identifying the sign of

$$\frac{\partial d^{h*}}{\partial \phi} = \left.\frac{\partial d^{h*}}{\partial \phi}\right|_{\tau^*} + \frac{\partial d^{h*}}{\partial \tau^*}\frac{\partial \tau^*}{\partial \phi}. \tag{43}$$

Using the closed-form formulae of d^{h*} and τ^*, we obtain $\frac{\partial \tau^*}{\partial \phi}$ in the second term of (43) as

$$\frac{\partial \tau^*}{\partial \phi} = \frac{\{\frac{1}{(1+\sigma)(1-\phi)} - \eta\}\zeta}{\delta\beta\chi(1+\eta)}\frac{\partial d^{g*}}{\partial \phi} - \frac{\{\frac{1}{(1+\sigma)(1-\phi)} - \eta\}}{\delta\beta\chi(1+\eta)}(\bar{b} - d^{g*})\frac{\partial \zeta}{\partial \phi}. \tag{44}$$

Based on (19), (23) and **Lemma 3**, we find that uninsurable idiosyncratic income risk unconditionally entails co-movement of stationary equilibrium government debt and aggregate household debt.

Proposition 4] An increase in uninsurable idiosyncratic risk on individuals' incomes always causes stationary equilibrium government debt and stationary equilibrium aggregate household debt to increase together.

Proof. See Appendix A8.

As the uninsurable idiosyncratic risk on post-tax labor income increases, the return of supplying labor is exposed to higher risk, which gives for risk-averse young voters to reduce labor supply, as shown by $\frac{\partial l^*_{Y,i}}{\partial \phi} < 0$ of (A33) in Appendix A8. In addition, because an increase in uninsurable income risk makes individual young voters face greater uncertainty on their own private goods consumption, public goods become more valuable to them because public goods are *certainly* provided. Therefore, responding to an increase in the risk, young voters vote for higher level of government debt for more public goods provision, although they have to face higher rate of income tax. Because an increase in the risk negatively affects labor supply, higher level of government debt necessarily entails higher rate of income tax. Young voters are willing to pay more tax than before an increase in uninsurable income risk, as the increase makes them value public goods higher than before the increase. At the same time, old voters also vote for higher level of government debt, because they prefer more public goods although they do not face the income risk. Thus, an increase in uninsurable income risk causes higher level of government debt to gain voters' political supports for more public goods provision. Hence, an increase in uninsurable income risk raises stationary equilibrium government debt, increasing stationary equilibrium income tax rate.

As an increase in uninsurable income risk raises stationary equilibrium government debt and income tax rate, it reduces post-tax incomes of young voters, causing young voters to borrow more as well as making them more likely to borrow. As such, the policy adjustment to an increase in uninsurable income risk strengthens the partial positive effect on aggregate household debt of uninsurable income risk shown by (23). Thus, an increase in uninsurable idiosyncratic risk on individuals' incomes raises both stationary equilibrium government debt and total household debt together. By the same logic, a decrease in uninsurable income risk lowers both debts together. Therefore, unlike the other three economic factors of the household-loan collateral value, interest rate and population aging, a change in uninsurable risk on individuals' incomes *unconditionally* causes stationary equilibrium government debt and aggregate household debt to move in the *same* direction.

Markedly, this paper shows that movements of government debt and aggregate household debt are related and that the relation depends on which economic factor brings out the movements. To apply the theoretical findings **Proposition 1**, **2**, **3** and **4** for measuring how much of each of the four economic factors (h, r, δ and ϕ) contributed to the observed movements of government debt and aggregate household debt in advanced economies (**Figure 1** and **2**), we need data of the ratio of house (household-loan collateral) value to wage and data of uninsurable idiosyncratic risk on individuals' disposable incomes over 1970-

2015. However, currently, these data or their estimates are not available. Thus, quantitative analysis of the theoretical findings is delegated to future studies.

Comparing **Proposition 1**, **2** and **3** with (20), (21) and (22), respectively, reveals the importance of reflecting concurrent change in government policies for properly analyzing aggregate household debt behavior. In particular, explicitly illustrated by the first term in (38), (40), (42) and (44), respectively, the movements of steady-state government debt and steady-state aggregate household debt, brought by a change in the household-loan collateral value, interest rate, the elderly population or uninsurable idiosyncratic income risk, are correlated. While voters' own debt decisions are affected by government debt decision through income tax rate, voters' political supports affect policymakers' decision on government debt and income tax rate. Nonetheless, the decisions of policymakers and their voters or the mutual influences between their decisions never internalize the public finance cost borne by unborn future generations, who shall pay part of the current government debt in the future.

VII. Concluding Remarks

In sum, this paper theoretically analyzes movements of steady-state government debt and aggregate household debt in a politico-economic model where office-seeking policymakers with a term limit decide government debt to win over voters who can choose to take household loans with the collateral of house. From this model, stationary equilibrium government debt and household debt are obtained, based on which their movements brought by a change in the four highly-plausible economic factors are analyzed. This paper finds that movements of stationary equilibrium government debt and stationary equilibrium aggregate household debt of an economy are correlated, in contrast to what Ricardian invariance theorem implies. An increase in uninsurable idiosyncratic risk on individual voters' income unconditionally raises stationary equilibrium government debt and aggregate household debt together. On the other hand, an increase in the household-loan collateral value or the elderly population share conditionally causes these two debts to increase together. In contrast to these three economic factors that bring about positive correlations between these two debts' movements, an increase in interest rate conditionally causes these two debts to move in the opposite directions.

References

Alesina, Alberto and Guido Tabellini. (1990) "A Positive Theory of Fiscal Deficits and Government Debt," *Review of Economic Studies*, 57: 403-414.

Azzimonti, Marina, Eva De Francisco and Vincenzo Quadrini. (2014) "Financial Globalization, Inequality, and the Rising Public Debt," *American Economic Review*, 104: 2267-2302.

Baron, Matthew and Wei Xiong. (2017) "Credit Expansion and Neglected Crash Risk," *Quar terly Journal of Economics*, 132: 713–764.

Barnes, Sebastian and Garry Young. (2003) "The Rise in US Household Debt: Assessing Its Causes and Sustainability," *Bank of England Working Papers 206*, Bank of England.

Barro, Robert. (1979) "On the Determination of the Public Debt," *Journal of Political Economy*, 87: 940-971.

Battaglini, Marco and Stephen Coate. (2008) "A Dynamic Theory of Public Spending, Taxation, and Debt," *American Economic Review*, 98: 201-236.

Bizer, David and Steven Durlauf. (1990) "Testing the Positive Theory of Government Finance," *Journal of Monetary Economics*, 26: 123-141.

Bohn, Henning. (1998) "The Behavior of U.S. Public Debt and Deficits," *Quarterly Journal of Economics*, 113: 949-963.

Campbell, John. (2006) "Household Finance," *Journal of Finance*, 61: 1553-1604.

Christelis, Dimitris, Michael Ehrmann and Dimitris Georgarakos. (2015) "Exploring Differences in Household Debt across Euro Area Countries and the United States," No. 2015-16. *Bank of Canada Working Paper*.

Coletta, Massimo, Riccardo De Bonis and Stefano Piermattei. (2019) "Household Debt in OECD Countries: The Role of Supply-Side and Demand-Side Factors," *Social Indicators Research*, 143(3): 1185-1217.

Dynan, Karen and Donald Kohn. (2007) "The Rise in U.S. Household Indebtedness: Causes and Consequences," *Federal Reserve Working Paper No. 2007-37*.

Iacoviello, Matteo. (2008) "Household Debt and Income Inequality, 1963–2003," *Journal of*

Money, Credit and Banking, 40(5): 929-965.

Kumhof, Michael, Romain Rancière and Pablo Winant. (2015) “Inequality, Leverage, and Crises,” *American Economic Review*, 105(3): 1217-1245.

Lindbeck, Assar and Jörgen Weibull. (1987) “Balanced-budget Redistribution as Political Equilibrium,” *Public Choice*, 52: 273-297.

Mian, Atif and Amir Sufi. (2011) “House Prices, Home Equity-Based Borrowing, and the US Household Leverage Crisis,” *American Economic Review*, 101(5): 2132-2156.

Pollin, Robert. (1988) “The Growth of US Household Debt: Demand-side Influences,” *Journal of Macroeconomics*, 10(2): 231-248.

Stango, Victor and Jonathan Zinman. (2009) “Exponential Growth Bias and Household Finance,” *Journal of Finance*, 64(6): 2807-2849.

Song, Zheng, Kjetil Storesletten and Fabrizio Zilibotti. (2012) “Rotten Parents and Disciplined Children: A Politico-Economic Theory of Public Expenditure and Debt,” *Econometrica*, 80: 2785-2803.

Tabellini, Guido. (1991) “The Politics of Intergenerational Redistribution,” *Journal of Political Economy*, 99: 335-357.

Trehan, Bharat and Carl Walsh. (1991) “Testing Intertemporal Budget Constraints: Theory and Applications to US Federal Budget and Current Account Deficits,” *Journal of Money, Credit and Banking*, 23: 206-223.

Yilmazer, Tansel and Sharon DeVaney. (2005) “Household Debt over the Life Cycle,” *Financial Services Review*, 14: 285-304.

Appendix

A1. *Election of a Policymaker and Choice of Policy Proposal*

Let us label two office-seeking candidates as A and B and denote their policy proposals by (d_A^g, τ_A, g_A) and (d_B^g, τ_B, g_B) respectively. Moreover, how much policy-irrelevant popularity of candidate B’s personality relative to candidate A’s is denoted by γ, which is unknown to the two candidates when they simultaneously announce their own policy proposals. Each of the two candidates has to estimate γ when he chooses his policy proposal. In particular, γ follows a zero-median uniform distribution of $Uni\,[-\frac{1}{2\psi}, \frac{1}{2\psi}]$ with $\psi > 0$. After both candidates announce their own policy proposals and the relative personal appeal (the value of γ) is revealed, each voter casts his (her) own vote for one of the two candidates who gives him (her) higher level of the maximized lifetime utility. According to (9), for any given i, young voter i chooses to vote for candidate A if and only if

$$V_{Y,i}(d_A^g, \tau_A, g_A) - V_{Y,i}(d_B^g, \tau_B, g_B) - \gamma > 0 . \tag{A1}$$

At the same time, according to (10) and (11), for any given i, old voter i chooses to vote for candidate A if and only if

$$V_{O,i}(d_A^g, \tau_A, g_A) - V_{O,i}(d_B^g, \tau_B, g_B) - \gamma > 0 . \tag{A2}$$

Based on the above voting decision rules of all individual voters, the winning probability of candidate A is

$$\Pr(\pi_A \geq \frac{1}{2}) = \frac{1}{2} + \psi\{(1-\delta)\int [V_{Y,i}(d_A^g, \tau_A, g_A) - V_{Y,i}(d_B^g, \tau_B, g_B)]dF_i + \delta \int_{\Theta} [V_{O,i}(d_A^g, \tau_A, g_A) - V_{O,i}(d_B^g, \tau_B, g_B)]dF_i\} , \tag{A3}$$

where π_A is the share of votes for candidate A. Since $\pi_A = 1 - \pi_B$, the winning probability of candidate B is defined symmetrically. Therefore, in each period, for choosing policy proposal that maximizes their own winning probability, each of the two candidates maximizes the population-weighted sum of maximized utility of all voters, (16), subject to the government budget constraint of (13) and $d^g \in (\underline{b}, \overline{b})$. As both candidates alike only want to win the election (i.e., their payoff is just their own winning probability), both basically solve the same maximization problem for choosing their own policy proposal. As a result, both candidates end up with announcing the same policy proposal. Then, one of the two candidates will be selected randomly with the same chance for each (i.e., at equilibrium, $\pi_A^* = \pi_B^* = \frac{1}{2}$). ■

A2. *Proof for Lemma 1*

At the outset, notice that young voters are identical except for whether they undergo uninsurable negative income shock or not. Thus, for any two different young voters whose post-tax labor income is not hit by the negative shock, their optimal capital allocation decisions of (17) take the same sign. Likewise, for any two different young voters whose post-tax labor income is hit by the negative shock, their optimal capital allocation decisions of (18) take the same sign. As a result, there are only four possible cases that can occur at an equilibrium: (i) both (17) and (18) are positive; (ii) both (17) and (18) are negative; (iii) while (17) is negative, (18) is positive; and, (iv) while (17) is positive, (18) is negative. The case (i) and (ii) are not relevant to an equilibrium where only part of young voters borrow, whereas the case (iii) and (iv) are relevant. Furthermore, because of $\sigma \in (0,1)$, for any given i,

$$d_i^{hS*} > d_i^{hNS*} . \tag{A4}$$

Hence, the case (iv) is not feasible. This implies that $d_i^{hS*} > 0$ and $d_i^{hNS*} \leq 0$ for any given i, which means that an individual voter chooses to borrow if uninsurable income shock occurs to him and does not otherwise. ■

A3. *Proof for Lemma 2*

[step 1] To begin, let us guess that (30), (31) and (32) are the politico-economic equilibrium policy functions $\{d^g(b), \tau(b), g(b)\}$ with the coefficient ρ being unknown. First, plug (30), (31) and (32) into the government budget constraint of $d^g(b) = g(b) + (1+r)b - \tau(b)z^{\frac{1}{1-\alpha}}$ $(1-\alpha)(\frac{\alpha}{r+\upsilon})^{\frac{\alpha}{1-\alpha}}\int(1-\delta)l_{Y,i}^* dF_i$ in (27) to get

$$\bar{b} - \rho(\bar{b} - b) = \frac{\rho(\bar{b}-b)}{\delta\beta} + (1+r)b - z^{\frac{1}{1-\alpha}}(1-\alpha)(\frac{\alpha}{r+\upsilon})^{\frac{\alpha}{1-\alpha}}\int(1-\delta)l_{Y,i}^* dF_i[\frac{1}{(1+\eta)} - \{\frac{1}{(1+\sigma)(1-\phi)}$$

$$-\eta\}\frac{1}{\delta\beta\chi(1+\eta)}\rho(\bar{b}-b)\frac{\int l_{Y,i}^{*\,\frac{1}{\eta}+1} dF_i}{z^{\frac{1}{1-\alpha}}(1-\alpha)(\frac{\alpha}{r+\upsilon})^{\frac{\alpha}{1-\alpha}}\int l_{Y,i}^* dF_i}. \qquad \text{(A5)}$$

From the definition of ζ, (A5) entails the coefficient ρ to be equal to (33).

[step 2] For verifying the guess of the above step 1, firstly, notice that the optimality condition of (28) is satisfied by (30), (31), (32) and (33) because

$$\{\frac{1}{(1+\sigma)(1-\phi)} - \eta\}\frac{1}{1-\tau(b)(1+\eta)}\zeta = \frac{\{\frac{1}{(1+\sigma)(1-\phi)} - \eta\}}{\{\frac{1}{(1+\sigma)(1-\phi)} - \eta\}\frac{\zeta}{\delta\beta\chi}\rho(\bar{b}-b)}\zeta = \frac{\chi}{\frac{1}{\delta\beta}\rho(\bar{b}-b)} = \frac{\chi}{g(b)}$$

for any given $b \in (\underline{b}, \bar{b})$. Secondly, the remaining optimality condition of (29) is also met by (30), (31), (32) and (33) because

$$-\beta\delta\frac{\chi}{\mathrm{g}'(d^g(b))}\frac{\partial \mathrm{g}'(d^g(b))}{\partial d^g(b)} = -\beta\delta\frac{\chi}{\mathrm{g}'(d^g)}\frac{-1}{\delta\beta}\rho(d^g) = \frac{\chi}{\frac{1}{\delta\beta}\rho(d^g)(\bar{b}-d^g)}\rho(d^g) = \frac{\chi}{\frac{1}{\delta\beta}(\bar{b}-d^g)} =$$

$$\frac{\chi}{\frac{1}{\delta\beta}\rho(\bar{b}-b)} = \frac{\chi}{g(b)} = \{\frac{1}{(1+\sigma)(1-\phi)} - \eta\}\frac{1}{1-\tau(b)(1+\eta)}\zeta$$

for any given $b \in (\underline{b}, \bar{b})$. Therefore, the policy functions of (30), (31) and (32) with (33) satisfy all the optimality conditions for politico-economic equilibrium government debt, income tax rate and public goods provision, verifying the guess.

[step 3] From the above step 2, $g(b) = \frac{1}{\delta\beta}\rho(\bar{b}-b)$ for $\forall b \in (\underline{b}, \bar{b})$ due to (32). Notice that

politico-economic equilibrium public goods provision is strictly positive to prevent the utility of voters from plunging into $-\infty$; that is, $g(b)=\frac{1}{\delta\beta}\rho(\bar{b}-b)>0$. Because $\delta\beta>0$, this means that $\rho(b)>0$ for $\forall b\in(\underline{b},\bar{b})$. ■

A4. *Proof for Lemma 3*

A set of politico-economic equilibrium government debt, income tax rate and public goods provision is stationary if the politico-economic equilibrium government debt stays the same for the current and next periods. That is, d^{g^*}, τ^* and g^* are defined by $d^{g^*}=b^*$. According to (30) of **Lemma 2**, $d^{g^*}=b^*$ if and only if

$$\rho(b^*)=\rho(d^{g^*})=1. \tag{A6}$$

Thus, (A6) is the defining condition of stationary politico-economic equilibrium government policies. By solving (A6) for d^{g^*} using (30) and (33) of **Lemma 2**, stationary politico-economic equilibrium government debt of (34) is obtained. Then, according to (34) and the politico-economic equilibrium policy functions (31) and (32) of **Lemma 2**, stationary politico-economic equilibrium income tax rate and public goods provision are derived as (35) and (36), respectively. ■

A5. *Proof for Proposition 1*

[step 1] Although decisions of household debt and government debt are made in the same period, as clearly noted in the text, individuals' borrowing decisions are made after the government policies are decided. Thus, it is necessary to first know how an increase in the household-loan collateral value affects stationary equilibrium government debt for showing how the increase affects stationary equilibrium aggregate household debt. As the value of household-loan collateral is represented by the parameter h, whether an increase in the household-loan collateral value raises stationary equilibrium government debt is identified by the sign of $\frac{\partial d^{g^*}}{\partial h}$. As d^{g^*} is defined by (A6), using the Implicit Function Theorem, we get

$$\frac{\partial d^{g^*}}{\partial h}=-\frac{\partial\rho}{\partial h}\{\frac{\partial\rho}{\partial d^{g^*}}\}^{-1}. \tag{A7}$$

Thus, we identify the sign of $\frac{\partial d^{g^*}}{\partial h}$ by finding the respective signs of $\frac{\partial\rho}{\partial d^{g^*}}$ and $\frac{\partial\rho}{\partial h}$.

[step 2] At first, we find the sign of $\frac{\partial\rho}{\partial d^{g^*}}$. To this end, according to (33) of **Lemma 2**,

$$\frac{\partial\rho}{\partial d^{g*}}=\frac{-r\bar{b}+z^{\frac{1}{1-\alpha}}(1-\alpha)(\frac{\alpha}{r+\upsilon})^{\frac{\alpha}{1-\alpha}}\frac{(1-\delta)}{(1+\eta)}\int l_{Y,i}^{*}(\tau^{*})dF_i}{(\bar{b}-d^{g*})^2[1+\frac{1}{\delta\beta}+\{\frac{1}{(1+\sigma)(1-\phi)}-\eta\}\frac{(1-\delta)}{\delta\beta\chi(1+\eta)}\int l_{Y,i}^{*\frac{1}{\eta}+1}dF_i]}. \qquad (A8)$$

Because the denominator of (A8) is greater than zero, the sign of $\frac{\partial\rho}{\partial d^{g*}}$ is equal to the sign of the numerator $-r\bar{b}+w\frac{(1-\delta)}{(1+\eta)}\int l_{Y,i}^{*}(\tau^{*})dF_i$ due to (15). In this regard, due to (14)

$$w\frac{(1-\delta)}{(1+\eta)}\int l_{Y,i}^{*}(\tau^{*})dF_i-r\bar{b}>w(1-\delta)\{\frac{1}{(1+\eta)}\int l_{Y,i}^{*}(\tau^{*})dF_i-\bar{\tau}\int l_{Y,i}^{*}(\bar{\tau})dF_i\}. \qquad (A9)$$

Moreover, as $\bar{\tau}=\arg\max_{\tau}\tau w\int(1-\delta)l_{Y,i}^{*}(\tau)dF_i$, $\bar{\tau}$ is defined by the following FOC

$$w(1-\delta)\int l_{Y,i}^{*}(\bar{\tau})dF_i+\bar{\tau}w(1-\delta)\int\frac{\partial l_{Y,i}^{*}(\bar{\tau})}{\partial\tau}dF_i=0. \qquad (A10)$$

Using (24), (A10) implies that $\bar{\tau}=\frac{1}{(1+\eta)}$. Thus,

$$\{\frac{1}{(1+\eta)}\int l_{Y,i}^{*}(\tau^{*})dF_i-\bar{\tau}\int l_{Y,i}^{*}(\bar{\tau})dF_i\}=\frac{1}{(1+\eta)}\{\int l_{Y,i}^{*}(\tau^{*})dF_i-\int l_{Y,i}^{*}(\bar{\tau})dF\}. \qquad (A11)$$

Moreover, due to $\tau^{*}\in[0,1)$ and (35) of **Lemma 3**, $\bar{\tau}>\tau^{*}$. Because (24) shows that higher tax rate entails lower labor supply, $\bar{\tau}>\tau^{*}$ implies that $l_{Y,i}^{*}(\tau^{*})>l_{Y,i}^{*}(\bar{\tau})$ for any given i. Hence, (A11) is strictly positive, which implies that

$$w(1-\delta)\{\frac{1}{(1+\eta)}\int l_{Y,i}^{*}(\tau^{*})dF_i-\bar{\tau}\int l_{Y,i}^{*}(\bar{\tau})dF_i\}>0 \qquad (A12)$$

as $\delta\in(0,0.5)$ and $w>0$ .With (A9) and (A8), this means

$$\frac{\partial\rho}{\partial d^{g*}}>0. \qquad (A13)$$

[step 3] Now, we examine the sign of $\frac{\partial\rho}{\partial h}$. To this end, according to (33) of **Lemma 2**,

$$\frac{\partial\rho}{\partial h}=\{z^{\frac{1}{1-\alpha}}(1-\alpha)(\frac{\alpha}{r+\upsilon})^{\frac{\alpha}{1-\alpha}}\frac{(1-\delta)}{(1+\eta)}\int\frac{\partial l_{Y,i}^{*}}{\partial h}dF_i[1+\frac{1}{\delta\beta}+\{\frac{1}{(1+\sigma)(1-\phi)}-\eta\}\frac{(1-\delta)}{\delta\beta\chi(1+\eta)}\int l_{Y,i}^{*\frac{1}{\eta}+1}dF_i]-$$

$$-\{\frac{1}{(1+\sigma)(1-\phi)}-\eta\}\frac{(1-\delta)}{\delta\beta\chi(1+\eta)}\int\frac{\partial l_{Y,i}^{*\frac{1}{\eta}+1}}{\partial h}dF_i[\bar{b}-(1+r)d^{g*}+z^{\frac{1}{1-\alpha}}(1-\alpha)(\frac{\alpha}{r+\upsilon})^{\frac{\alpha}{1-\alpha}}\frac{(1-\delta)}{(1+\eta)}\int l_{Y,i}^{*}dF_i]$$

$$\}\{(\bar{b}-d^{g*})^{-1}[1+\frac{1}{\delta\beta}+\{\frac{1}{(1+\sigma)(1-\phi)}-\eta\}\frac{(1-\delta)}{\delta\beta\chi(1+\eta)}\int l_{Y,i}^{*\frac{1}{\eta}+1}dF_i]^{-2}\} \qquad (A14)$$

Clearly, the denominator of $\frac{\partial \rho}{\partial h}$ is strictly positive as $(\bar{b} - d^{g*})[1 + \frac{1}{\delta\beta} + \{\frac{1}{(1+\sigma)(1-\phi)} - \eta\}$ $\frac{(1-\delta)}{\delta\beta\chi(1+\eta)}\int l_{Y,i}^{*\,\frac{1}{\eta}+1} dF_i]^2 > 0$. Thus, the sign of $\frac{\partial \rho}{\partial h}$ is determined by the sign of its numerator, which also depends on the value of $\frac{\partial l_{Y,i}^*}{\partial h}$. It is necessary to identify $\frac{\partial l_{Y,i}^*}{\partial h}$.

At a stationary competitive general equilibrium, the budget constraint binds for each young voter. Capitalizing upon the conditions for individuals' optimal allocation of (5), (6), (7) and (8) as well as (17), the budget constraint of (3) is restated as follows: for any given i,

$$(1-\tau)l_{Y,i}^* - [1 + \frac{\beta\delta}{1-\delta}](1+\sigma)(1-\phi)(l_{Y,i}^*)^{-\frac{1}{\eta}}(1-\tau) - \frac{rh}{(1+r)} = 0. \tag{A15}$$

Applying the Implicit Function Theorem to (A15), for any given i,

$$\frac{\partial l_{Y,i}^*}{\partial h} = \frac{\frac{r}{(1+r)}}{(1-\tau^*) + \frac{1}{\eta}[1 + \frac{\beta\delta}{1-\delta}](1+\sigma)(1-\phi)(l_{Y,i}^*)^{-\frac{1}{\eta}-1}(1-\tau^*)} > 0 \tag{A16}$$

because $r > 0$, $\beta > 0$, $\frac{\delta}{1-\delta} \in (0,1)$, $\sigma > 0$, $\eta > 0$, $\tau^* \in [0,1)$, and $l_{Y,i}^* > 0$. Utilizing **Lemma 3**, (A6), (A14) and (A16), the numerator of $\frac{\partial \rho}{\partial h}$ is restated as

$$[\frac{(1+\eta)^2\tau^* - 1}{\eta}][\bar{b} - (1+r)d^{g*} + z^{\frac{1}{1-\alpha}}(1-\alpha)(\frac{\alpha}{r+\upsilon})^{\frac{\alpha}{1-\alpha}}\frac{(1-\delta)}{(1+\eta)}\int l_{Y,i}^* dF_i]\frac{z^{\frac{1}{1-\alpha}}(1-\alpha)(\frac{\alpha}{r+\upsilon})^{\frac{\alpha}{1-\alpha}}}{(\bar{b} - d^{g*})}\frac{(1-\delta)}{(1+\eta)}$$

$$\int \frac{\frac{r}{(1+r)}}{(1-\tau^*) + \frac{1}{\eta}[1 + \beta\frac{\delta}{1-\delta}](1+\sigma)(1-\phi)(l_{Y,i}^*)^{-\frac{1}{\eta}-1}(1-\tau^*)} dF_i. \tag{A17}$$

Because the first bracket of (A17) can be positive or negative, (A17) can be either positive or negative. Consequently, the sign of $\frac{\partial \rho}{\partial h}$ can be either positive or negative. (A7) and (A13) imply that $\frac{\partial d^{g*}}{\partial h} > 0$ if $\frac{\partial \rho}{\partial h} < 0$ and $\frac{\partial d^{g*}}{\partial h} \le 0$ if $\frac{\partial \rho}{\partial h} \ge 0$. Therefore, an increase in the household-loan collateral value causes an increase or a decrease in stationary equilibrium government debt.

[step 4] Now, we identify whether an increase in the household-loan collateral value raises stationary equilibrium aggregate household debt or not which is indicated by the sign of (37). While the first term of (37) is shown to be always positive due to (20), we need to find the

sign of $\frac{\partial d^{h*}}{\partial \tau^*}\frac{\partial \tau^*}{\partial h}$ the second term of (37). To this end, first, based on (19),

$$\frac{\partial d^{h*}}{\partial \tau^*} = \frac{\sigma\beta\delta\phi}{[1+\frac{\beta\delta}{1-\delta}]} z^{\frac{1}{1-\alpha}}(1-\alpha)(\frac{\alpha}{r+\upsilon})^{\frac{\alpha}{1-\alpha}}\int(1+\eta)l^*_{Y,i}dF_i > 0 \tag{A18}$$

as $z^{\frac{1}{1-\alpha}}(1-\alpha)(\frac{\alpha}{r+\upsilon})^{\frac{\alpha}{1-\alpha}} > 0$, $\beta > 0$, $\frac{\delta}{1-\delta} \in (0,1)$, $\phi \in (0,1)$, $\eta > 0$, $\sigma \in (0,1)$ and $l^*_{Y,i} > 0$.

Thus, (A18) implies that the sign of $\frac{\partial d^{h*}}{\partial \tau^*}\frac{\partial \tau^*}{\partial h}$ is determined by the sign of $\frac{\partial \tau^*}{\partial h}$ which is (38). Although the step 3 above shows that the first term of (38) can be positive or negative, we need to examine the sign of the second term of (38). Due to (A16) and the definition of ζ, we obtain that

$$\frac{\partial \zeta}{\partial h} = \frac{\int \frac{\frac{r}{(1+r)} l^{*\,\frac{1}{\eta}+1}_{Y,i}}{\eta(1-\tau^*)+[1+\beta\frac{\delta}{1-\delta}](1+\sigma)(1-\phi)(l^*_{Y,i})^{-\frac{1}{\eta}-1}(1-\tau^*)} dF_i}{z^{\frac{1}{1-\alpha}}(1-\alpha)(\frac{\alpha}{r+\upsilon})^{\frac{\alpha}{1-\alpha}}[\int l^*_{Y,i}dF]^2} > 0 \tag{A19}$$

because $z^{\frac{1}{1-\alpha}}(1-\alpha)(\frac{\alpha}{r+\upsilon})^{\frac{\alpha}{1-\alpha}} > 0$ and $\frac{\partial l^*_{Y,i}}{\partial h} > 0$ of (A16).

Putting (38), (A18), (A19) and the step 3 above together shows that if $\frac{\partial d^{g*}}{\partial h} \le 0$, then $\frac{\partial \tau^*}{\partial h} < 0$ and $\frac{\partial d^{h*}}{\partial \tau^*}\frac{\partial \tau^*}{\partial h} < 0$, whereas if $\frac{\partial d^{g*}}{\partial h} > 0$, then $\frac{\partial \tau^*}{\partial h}$ and $\frac{\partial d^{h*}}{\partial \tau^*}\frac{\partial \tau^*}{\partial h}$ can be positive or negative. Furthermore, (38) implies that if $\frac{\partial d^{g*}}{\partial h} > \frac{(\bar{b}-d^{g*})}{\zeta}\frac{\partial \zeta}{\partial h}$ (the left-hand side of which is positive due to (A19), $\bar{b} > d^{g*}$ and $\zeta > 0$), then $\frac{\partial \tau^*}{\partial h} > 0$ and $\frac{\partial d^{h*}}{\partial \tau^*}\frac{\partial \tau^*}{\partial h} > 0$; if not, $\frac{\partial \tau^*}{\partial h} \le 0$ and $\frac{\partial d^{h*}}{\partial \tau^*}\frac{\partial \tau^*}{\partial h} \le 0$.

[step 5] Finally, (37) and the step 4 above imply that if $\frac{\partial d^{g*}}{\partial h} > \frac{(\bar{b}-d^{g*})}{\zeta}\frac{\partial \zeta}{\partial h}$, then $\frac{\partial d^{h*}}{\partial h} > 0$, whereas if $\frac{\partial d^{g*}}{\partial h} \le 0$ or $0 < \frac{\partial d^{g*}}{\partial h} \le \frac{(\bar{b}-d^{g*})}{\zeta}\frac{\partial \zeta}{\partial h}$, $\frac{\partial d^{h*}}{\partial h}$ can be either positive or negative (i.e., ambiguous sign of $\frac{\partial d^{h*}}{\partial h}$). ■

A6. *Proof for Proposition 2*

[step 1] By the same logic of proof of **Proposition 1**, whether an increase in interest rate

raises stationary equilibrium government debt or not is identified by the sign of $\frac{\partial d^{g*}}{\partial r}$. As d^{g*} is defined by (A6), using the Implicit Function Theorem

$$\frac{\partial d^{g*}}{\partial r} = -\frac{\partial \rho}{\partial r}\{\frac{\partial \rho}{\partial d^{g*}}\}^{-1}. \tag{A20}$$

Thus, we can identify the sign of $\frac{\partial d^{g*}}{\partial r}$ by finding the sign of $\frac{\partial \rho}{\partial r}$ as $\frac{\partial \rho}{\partial d^{g*}} > 0$ from (A13). According to (33) of **Lemma 2**,

$$\frac{\partial \rho}{\partial r} = \{[z^{\frac{1}{1-\alpha}}(1-\alpha)(\frac{\alpha}{r+\upsilon})^{\frac{\alpha}{1-\alpha}}\frac{(1-\delta)}{(1+\eta)}\int \frac{\partial l^*_{Y,i}}{\partial r}dF - d^{g*} - (\frac{\alpha}{1-\alpha})(\frac{1}{r+\upsilon})z^{\frac{1}{1-\alpha}}(1-\alpha)(\frac{\alpha}{r+\upsilon})^{\frac{\alpha}{1-\alpha}}\frac{(1-\delta)}{(1+\eta)}$$
$$\int l^*_{Y,i}dF_i][1+\frac{1}{\delta\beta}+\frac{\{\frac{1}{(1+\sigma)(1-\phi)}-\eta\}(1-\delta)}{\delta\beta\chi(1+\eta)}\int l^{*\,\frac{1}{\eta}+1}_{Y,i}dF_i] - \frac{\{\frac{1}{(1+\sigma)(1-\phi)}-\eta\}(1-\delta)}{\delta\beta\chi(1+\eta)}\int \frac{\partial l^{*\,\frac{1}{\eta}+1}_{Y,i}}{\partial r}dF_i$$
$$[\bar{b}-(1+r)d^{g*}+z^{\frac{1}{1-\alpha}}(1-\alpha)(\frac{\alpha}{r+\upsilon})^{\frac{\alpha}{1-\alpha}}\frac{(1-\delta)}{(1+\eta)}\int l^*_{Y,i}dF_i]\}(\bar{b}-d^{g*})^{-1}[1+\frac{1}{\delta\beta}+\{\frac{1}{(1+\sigma)(1-\phi)}-\eta\}$$
$$\frac{(1-\delta)}{\delta\beta\chi(1+\eta)}\int l^{*\,\frac{1}{\eta}+1}_{Y,i}dF_i]^{-2} \tag{A21}$$

Clearly, the denominator of $\frac{\partial \rho}{\partial r}$ is strictly positive as $(\bar{b}-d^{g*})[1+\frac{1}{\delta\beta}+\{\frac{1}{(1+\sigma)(1-\phi)}-\eta\}$ $\frac{(1-\delta)}{\delta\beta\chi(1+\eta)}\int l^{*\,\frac{1}{\eta}+1}_{Y,i}dF_i]^2 > 0$. Thus, the sign of $\frac{\partial \rho}{\partial r}$ is determined by the sign of its numerator. Moreover, applying the Implicit Function Theorem to (A15), at a stationary equilibrium, for any given i,

$$\frac{\partial l^*_{Y,i}}{\partial r} = \frac{\frac{h}{(1+r)^2}}{(1-\tau^*)+\frac{1}{\eta}[1+\frac{\beta\delta}{1-\delta}](1+\sigma)(1-\phi)(l^*_{Y,i})^{-\frac{1}{\eta}-1}(1-\tau^*)} > 0 \tag{A22}$$

because $r>0$, $\beta>0$, $\frac{\delta}{1-\delta}\in(0,1)$, $\phi\in(0,1)$, $\eta>0$, $\tau^*\in[0,1)$, $\sigma>0$ and $l^*_{Y,i}>0$. Using **Lemma 3**, (A6), (A21) and (A22), the numerator of $\frac{\partial \rho}{\partial r}$ is stated as

$$(-1)[d^{g*}+(\frac{\alpha}{1-\alpha})(\frac{1}{r+\upsilon})\int l^*_{Y,i}dF_i + \int \frac{\{1-(1+\eta)\tau^*\}\frac{h}{(1+r)^2}}{(1-\tau^*)\eta+[1+\frac{\beta\delta}{1-\delta}](1+\sigma)(1-\phi)(l^*_{Y,i})^{-\frac{1}{\eta}-1}(1-\tau^*)}dF_i]z^{\frac{1}{1-\alpha}}(1-$$
$$\alpha)(\frac{\alpha}{r+\upsilon})^{\frac{\alpha}{1-\alpha}}\frac{(1-\delta)}{(1+\eta)}[\bar{b}-(1+r)d^{g*}+z^{\frac{1}{1-\alpha}}(1-\alpha)(\frac{\alpha}{r+\upsilon})^{\frac{\alpha}{1-\alpha}}\frac{(1-\delta)}{(1+\eta)}\int l^*_{Y,i}dF_i](\bar{b}-d^{g*})^{-1} < 0 \tag{A23}$$

because $\dfrac{1}{1+\eta} > \tau^*$, $z^{\frac{1}{1-\alpha}}(1-\alpha)(\dfrac{\alpha}{r+\upsilon})^{\frac{\alpha}{1-\alpha}} > 0$ and $\alpha \in (0,1)$. Thus, $\dfrac{\partial \rho}{\partial r} < 0$. Then, (A13), (A20) and (A23) imply that

$$\frac{\partial d^{g*}}{\partial r} > 0\,. \tag{A24}$$

Therefore, an increase in interest rate increases stationary equilibrium government debt.

[step 2] Now, we identify the sign of (39). The signs of the first term and $\dfrac{\partial d^{h*}}{\partial \tau^*}$ of (39) are identified by (21) and (A18), respectively. We need to find the sign of (40) which determines the sign of the second term of (39). To this end, using (A22), we get

$$\frac{\partial \zeta}{\partial r} = \frac{\displaystyle\int \frac{\dfrac{h}{(1+r)^2} l_{Y,i}^{*\,\frac{1}{\eta}+1}}{\eta(1-\tau^*) + [1+\dfrac{\beta\delta}{1-\delta}](1+\sigma)(1-\phi)(l_{Y,i}^*)^{-\frac{1}{\eta}-1}(1-\tau^*)} dF_i}{z^{\frac{1}{1-\alpha}}(1-\alpha)(\dfrac{\alpha}{r+\upsilon})^{\frac{\alpha}{1-\alpha}}[\int l_{Y,i}^* dF]^2} > 0\,. \tag{A25}$$

Thus, (A18), (A24), (A25) and (40) together imply that $\dfrac{\partial \tau^*}{\partial r}$ and $\dfrac{\partial d^{h*}}{\partial \tau^*}\dfrac{\partial \tau^*}{\partial r}$ can be positive or negative. In particular, if $\dfrac{\partial d^{g*}}{\partial r} < \dfrac{(\bar{b}-d^{g*})}{\zeta}\dfrac{\partial \zeta}{\partial r}$, then $\dfrac{\partial d^{h*}}{\partial \tau^*}\dfrac{\partial \tau^*}{\partial r} < 0$ entailing that $\dfrac{\partial d^{h*}}{\partial r} < 0$ due to the above step 1, (21) and (39). If not (i.e., $\dfrac{\partial d^{g*}}{\partial r} \geq \dfrac{(\bar{b}-d^{g*})}{\zeta}\dfrac{\partial \zeta}{\partial r}$ or $\dfrac{\partial d^{h*}}{\partial r} < 0$), then $\dfrac{\partial d^{h*}}{\partial \tau^*}\dfrac{\partial \tau^*}{\partial r} \geq 0$ so that $\dfrac{\partial d^{h*}}{\partial r}$ can be positive or negative, because the first term of (39) takes the opposite sign of the second term of (39). ■

A7. *Proof for Proposition 3*

[step 1] By the same logic, whether population aging raises stationary equilibrium government debt is identified by the sign of $\dfrac{\partial d^{g*}}{\partial \delta}$. As d^{g*} is defined by (A6), using the Implicit Function Theorem

$$\frac{\partial d^{g*}}{\partial \delta} = -\frac{\partial \rho}{\partial \delta}\{\frac{\partial \rho}{\partial d^{g*}}\}^{-1}. \tag{A26}$$

Thus, we identify the sign of $\dfrac{\partial d^{g*}}{\partial \delta}$ by finding the sign of $\dfrac{\partial \rho}{\partial \delta}$ as $\dfrac{\partial \rho}{\partial d^{g*}} > 0$ from (A13). According to (33) of **Lemma 2**,

$$\frac{\partial \rho}{\partial \delta} = \{[\bar{b} - (1+r)d^{g*} + z^{\frac{1}{1-\alpha}}(1-\alpha)(\frac{\alpha}{r+\upsilon})^{\frac{\alpha}{1-\alpha}}\frac{(1-\delta)}{(1+\eta)}\int l_{Y,i}^* dF_i][\{\frac{1}{(1+\sigma)(1-\phi)} - \eta\}\frac{1}{\delta^2\beta\chi(1+\eta)}\int$$

$$l_{Y,i}^{*\frac{1}{\eta}+1}dF_i + \frac{1}{\delta^2\beta} - \{\frac{1}{(1+\sigma)(1-\phi)} - \eta\}\frac{(1-\delta)}{\delta\beta\chi(1+\eta)}\int \frac{\partial l_{Y,i}^{*\frac{1}{\eta}+1}}{\partial\delta}dF_i] + z^{\frac{1}{1-\alpha}}(1-\alpha)(\frac{\alpha}{r+\upsilon})^{\frac{\alpha}{1-\alpha}}\frac{1}{(1+\eta)}[(1-$$

$$\delta)\int\frac{\partial l_{Y,i}^*}{\partial\delta}dF_i - \int l_{Y,i}^* dF_i][1+\frac{1}{\delta\beta}+\frac{\{\frac{1}{(1+\sigma)(1-\phi)}-\eta\}(1-\delta)}{\delta\beta\chi(1+\eta)}\int l_{Y,i}^{*\frac{1}{\eta}+1}dF_i]\}(\bar{b}-d^{g*})^{-1}[1+\frac{1}{\delta\beta}+\frac{(1-\delta)}{(1+\eta)}$$

$$\frac{1}{\delta\beta\chi}\{\frac{1}{(1+\sigma)(1-\phi)}-\eta\}\int l_{Y,i}^{*\frac{1}{\eta}+1}dF_i]^{-2} \tag{A27}$$

Clearly, the denominator of $\frac{\partial\rho}{\partial\delta}$ is strictly positive as $(\bar{b}-d^{g*})[1+\frac{1}{\delta\beta}+\{\frac{1}{(1+\sigma)(1-\phi)}-\eta\}$ $\frac{(1-\delta)}{\delta\beta\chi(1+\eta)}\int l_{Y,i}^{*\frac{1}{\eta}+1}dF_i]^2 > 0$. Thus, the sign of $\frac{\partial\rho}{\partial\delta}$ is determined by the sign of its numerator. Moreover, applying the Implicit Function Theorem to (A15), for any given i,

$$\frac{\partial l_{Y,i}^*}{\partial\delta} = \frac{\frac{\beta}{(1-\delta)^2}(1+\sigma)(1-\phi)(l_{Y,i}^*)^{-\frac{1}{\eta}}}{1+\frac{1}{\eta}[1+\frac{\beta\delta}{1-\delta}](1+\sigma)(1-\phi)(l_{Y,i}^*)^{-\frac{1}{\eta}-1}} > 0 \tag{A28}$$

because $\beta > 0$, $\frac{\delta}{1-\delta}\in(0,1)$, $\sigma > 0$, $\eta > 0$, $\phi\in(0,1)$ and $l_{Y,i}^* > 0$. Using **Lemma 3**, (A6), (A27) and (A28), the numerator of $\frac{\partial\rho}{\partial\delta}$ is restated as

$$[\frac{\{\frac{1}{(1+\sigma)(1-\phi)}-\eta\}}{\delta^2\beta\chi(1+\eta)}\int l_{Y,i}^{*\frac{1}{\eta}+1}dF_i + \frac{1}{\delta^2\beta} - \frac{z^{\frac{1}{1-\alpha}}(1-\alpha)(\frac{\alpha}{r+\upsilon})^{\frac{\alpha}{1-\alpha}}}{(\bar{b}-d^{g*})(1+\eta)}\{\int l_{Y,i}^* dF_i + \frac{(1-(1+\eta)^2\tau^*)(1-\delta)}{\eta}\int$$

$$\frac{\partial l_{Y,i}^*}{\partial\delta}dF_i\}][\bar{b}-(1+r)d^{g*}+z^{\frac{1}{1-\alpha}}(1-\alpha)(\frac{\alpha}{r+\upsilon})^{\frac{\alpha}{1-\alpha}}\frac{(1-\delta)}{(1+\eta)}\int l_{Y,i}^* dF_i] \tag{A29}$$

According to (33) of **Lemma 2**, the second bracket of (A29) is clearly positive, while the terms in the first bracket of (A29) take opposite signs. Thus, the sign of $\frac{\partial\rho}{\partial\delta}$ can be either positive or negative. (A8) and (A26) imply that $\frac{\partial d^{g*}}{\partial\delta}\geq 0$ if $\frac{\partial\rho}{\partial\delta}\leq 0$ and $\frac{\partial d^{g*}}{\partial\delta} < 0$ if $\frac{\partial\rho}{\partial\delta} > 0$. Hence, population aging causes an increase or a decrease in stationary equilibrium government debt.

[step 2] Now, we identify whether population aging raises stationary equilibrium aggregate household debt by finding the sign of (41). The signs of the first term and $\frac{\partial d^{h*}}{\partial\tau^*}$ of (41) are identified by (22) and (A18) respectively. To find the sign of (42) that is the part of the

second term of (41), based on the definition of ζ and (A28), we obtain

$$\frac{\partial\zeta}{\partial\delta}=\frac{1}{z^{\frac{1}{1-\alpha}}(1-\alpha)(\frac{\alpha}{r+\upsilon})^{\frac{\alpha}{1-\alpha}}}\int\frac{\frac{\beta}{(1-\delta)^2}(1+\sigma)(1-\phi)}{\eta l_{Y,i}^*+[1+\frac{\beta\delta}{1-\delta}](1+\sigma)(1-\phi)(l_{Y,i}^*)^{-\frac{1}{\eta}}}dF_i>0 \tag{A30}$$

because $z^{\frac{1}{1-\alpha}}(1-\alpha)(\frac{\alpha}{r+\upsilon})^{\frac{\alpha}{1-\alpha}}>0$, $\beta>0$, $\frac{\delta}{1-\delta}\in(0,1)$, $\sigma>0$, $\eta>0$, $\phi\in(0,1)$ and $l_{Y,i}^*>0$.

Putting (22), (41), the above step 1, (42), (A18) and (A30) together shows that if $\frac{\partial d^{g*}}{\partial\delta}<0$, then $\frac{\partial\tau^*}{\partial\delta}<0$ and $\frac{\partial d^{h*}}{\partial\tau^*}\frac{\partial\tau^*}{\partial\delta}<0$ to entail $\frac{\partial d^{h*}}{\partial\delta}<0$, whereas if $\frac{\partial d^{g*}}{\partial\delta}\geq 0$, then $\frac{\partial\tau^*}{\partial\delta}$ and $\frac{\partial d^{h*}}{\partial\tau^*}\frac{\partial\tau^*}{\partial\delta}$ can be positive or negative so that $\frac{\partial d^{h*}}{\partial\delta}$ can be positive or negative. Furthermore, if $\frac{\partial d^{g*}}{\partial\delta}\geq 0$ is large enough to entail $\frac{\partial d^{h*}}{\partial\tau^*}\frac{\partial\tau^*}{\partial\delta}>-\left.\frac{\partial d^{h*}}{\partial\delta}\right|_{\tau^*}$ (i.e., $\frac{\partial d^{g*}}{\partial\delta}>\left\|\left.\frac{\partial d^{h*}}{\partial\delta}\right|_{\tau^*}\right\|[\frac{\zeta}{\delta\beta\chi(1+\eta)}\{\frac{1}{(1+\sigma)(1-\phi)}-\eta\}\frac{\partial d^{h*}}{\partial\tau^*}]^{-1}+\frac{(\bar{b}-d^{g*})}{\zeta}\frac{\partial\zeta}{\partial\delta}$), then $\frac{\partial d^{h*}}{\partial\delta}$ is positive; otherwise, $\frac{\partial d^{h*}}{\partial\delta}$ can be positive or negative. ■

A8. *Proof for Proposition 4*

[step 1] As uninsurable idiosyncratic income risk is represented by ϕ, whether an increase in the uninsurable income risk raises stationary equilibrium government debt is identified by the sign of $\frac{\partial d^{g*}}{\partial\phi}$. As d^{g*} is defined by (A6), using the Implicit Function Theorem

$$\frac{\partial d^{g*}}{\partial\phi}=-\frac{\partial\rho}{\partial\phi}\{\frac{\partial\rho}{\partial d^{g*}}\}^{-1}. \tag{A31}$$

Thus, we identify the sign of $\frac{\partial d^{g*}}{\partial\phi}$ by finding the sign of $\frac{\partial\rho}{\partial\phi}$ as $\frac{\partial\rho}{\partial d^{g*}}>0$ from (A13). According to (33) of **Lemma 2**,

$$\frac{\partial\rho}{\partial\phi}=\{z^{\frac{1}{1-\alpha}}(1-\alpha)(\frac{\alpha}{r+\upsilon})^{\frac{\alpha}{1-\alpha}}\frac{(1-\delta)}{(1+\eta)}\int\frac{\partial l_{Y,i}^*}{\partial\phi}dF_i[1+\frac{1}{\delta\beta}+\{\frac{1}{(1+\sigma)(1-\phi)}-\eta\}\frac{(1-\delta)}{\delta\beta\chi(1+\eta)}\int l_{Y,i}^{*\frac{1}{\eta}+1}dF_i]-[\bar{b}-(1+r)d^{g*}+z^{\frac{1}{1-\alpha}}(1-\alpha)(\frac{\alpha}{r+\upsilon})^{\frac{\alpha}{1-\alpha}}\frac{(1-\delta)}{(1+\eta)}\int l_{Y,i}^*dF_i][\frac{1}{(1+\sigma)(1-\phi)^2}\frac{(1-\delta)}{\delta\beta\chi(1+\eta)}\int l_{Y,i}^{*\frac{1}{\eta}+1}dF_i+\frac{1}{\delta\beta\chi}\frac{(1-\delta)}{(1+\eta)}\{\frac{1}{(1+\sigma)(1-\phi)}-\eta\}\int\frac{\partial l_{Y,i}^{*\frac{1}{\eta}+1}}{\partial\phi}dF_i]\}(\bar{b}-d^{g*})^{-1}[1+\frac{1}{\delta\beta}+\{\frac{1}{(1+\sigma)(1-\phi)}-\eta\}\frac{(1-\delta)}{\delta\beta\chi(1+\eta)}\int l_{Y,i}^{*\frac{1}{\eta}+1}dF_i]^{-2} \tag{A32}$$

Because the denominator of $\frac{\partial\rho}{\partial\phi}$ is strictly positive as $(\bar{b}-d^{g*})[1+\frac{1}{\delta\beta}+\{\frac{1}{(1+\sigma)(1-\phi)}-\eta\}$

$\frac{(1-\delta)}{\delta\beta\chi(1+\eta)}\int l_{Y,i}^{*\,\frac{1}{\eta}+1}dF_i]^2>0$, the sign of $\frac{\partial\rho}{\partial\phi}$ is decided by the sign of its numerator. Applying the Implicit Function Theorem to (A15), for any given i,

$$\frac{\partial l_{Y,i}^{*}}{\partial\phi}=-\frac{[1+\frac{\beta\delta}{1-\delta}](1+\sigma)(l_{Y,i}^{*})^{-\frac{1}{\eta}}}{1+\frac{1}{\eta}[1+\frac{\beta\delta}{1-\delta}](1+\sigma)(1-\phi)(l_{Y,i}^{*})^{-\frac{1}{\eta}-1}}<0 \tag{A33}$$

because $\beta>0$, $\frac{\delta}{1-\delta}\in(0,1)$, $\phi\in(0,1)$, $\eta>0$, $\sigma>0$ and $l_{Y,i}^{*}>0$. Using **Lemma 3**, (A6), (A32) and (A33), the numerator of $\frac{\partial\rho}{\partial\phi}$ is restated as

$$[-\frac{\tau^{*}(1-\delta)}{(\bar{b}-d^{g*})}z^{\frac{1}{1-\alpha}}(1-\alpha)(\frac{\alpha}{r+\upsilon})^{\frac{\alpha}{1-\alpha}}\int\frac{[1+\frac{\beta\delta}{1-\delta}](1+\sigma)(l_{Y,i}^{*})^{-\frac{1}{\eta}}}{1+\frac{1}{\eta}[1+\frac{\beta\delta}{1-\delta}](1+\sigma)(1-\phi)(l_{Y,i}^{*})^{-\frac{1}{\eta}-1}}dF_i-\frac{(1-\delta)}{\delta\beta\chi(1+\eta)}$$

$$\int\frac{\eta(l_{Y,i}^{*})^{\frac{1}{\eta}+1}+[1+\frac{\beta\delta}{1-\delta}](1+\sigma)(1-\phi)[\eta(1+\sigma)(1-\phi)]}{(1+\sigma)(1-\phi)^2\{\eta+[1+\frac{\beta\delta}{1-\delta}](1+\sigma)(1-\phi)(l_{Y,i}^{*})^{-\frac{1}{\eta}-1}\}}dF_i][\bar{b}-(1+r)d^{g*}+z^{\frac{1}{1-\alpha}}(1-\alpha)(\frac{\alpha}{r+\upsilon})^{\frac{\alpha}{1-\alpha}}$$

$$\frac{(1-\delta)}{(1+\eta)}\int l_{Y,i}^{*}dF_i]<0 \tag{A34}$$

because $z^{\frac{1}{1-\alpha}}(1-\alpha)(\frac{\alpha}{r+\upsilon})^{\frac{\alpha}{1-\alpha}}>0$, $\beta>0$, $\frac{\delta}{1-\delta}\in(0,1)$, $\tau^{*}\in[0,1)$, $\eta>0$, $\sigma>0$ and $l_{Y,i}^{*}>0$.

Thus, (A8), (A31) and (A34) imply that

$$\frac{\partial d^{g*}}{\partial\phi}>0. \tag{A35}$$

Hence, an increase in uninsurable idiosyncratic risk on individuals' incomes raises stationary equilibrium government debt.

[step 2] Now, we identify whether uninsurable idiosyncratic income risk also positively affects stationary equilibrium aggregate household debt, which is indicated by the sign of (43). The signs of the first term and $\frac{\partial d^{h*}}{\partial\tau^{*}}$ of (43) are identified by (23) and (A18), respectively. Thus, we need to find the sign of (44) that determines the sing of the second term of (43). To this end, based on the definition of ζ and (A33), we obtain

$$\frac{\partial \zeta}{\partial \phi}=\frac{\int \frac{-[1+\frac{\beta\delta}{1-\delta}](1+\sigma)(l_{Y,i}^{*})}{\eta+[1+\frac{\beta\delta}{1-\delta}](1+\sigma)(1-\phi)(l_{Y,i}^{*})^{-\frac{1}{\eta}-1}}dF_i}{z^{\frac{1}{1-\alpha}}(1-\alpha)(\frac{\alpha}{r+\upsilon})^{\frac{\alpha}{1-\alpha}}[\int l_{Y,i}^{*}dF_i]^2}<0 \tag{A36}$$

because $z^{\frac{1}{1-\alpha}}(1-\alpha)(\frac{\alpha}{r+\upsilon})^{\frac{\alpha}{1-\alpha}}>0$, $\beta>0$, $\frac{\delta}{1-\delta}\in(0,1)$, $\phi\in(0,1)$, $\eta>0$, $\sigma>0$ and $l_{Y,i}^{*}>0$.

Putting the above step 1, (44), (A18), (A35) and (A36) together implies that

$$\frac{\partial d^{h*}}{\partial \phi}>0. \tag{A37}$$

Putting (A35) and (A37) together shows that an increase in uninsurable idiosyncratic risk on individuals' incomes unconditionally raises both stationary equilibrium government debt and stationary equilibrium aggregate household debt together. For the same reason, a decrease in uninsurable idiosyncratic risk on individuals' incomes lowers both debts together. ■